Sharing of Control versus Monitoring as Corporate Governance Mechanisms*

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Abstract

Do large shareholders monitor firms on behalf of minority shareholders, or share control with other insiders to maximize their own gains? We show how firm characteristics and governance laws determine the role of large shareholders. If investment opportunities are hard for insiders to evaluate, letting a large shareholder monitor the firm is efficient because shared control creates disagreement costs that are more likely to destroy profitable opportunities than to prevent bad investments. In contrast, sharing control is efficient if investment opportunities are hard for outsiders to evaluate, when financing requirements are large, and in countries that poorly protect minority shareholders.

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Abstract

Do large shareholders monitor firms on behalf of minority shareholders, or share control with other insiders to maximize their own gains? We show how firm characteristics and governance laws determine the role of large shareholders. If investment opportunities are hard for insiders to evaluate, letting a large shareholder monitor the firm is efficient because shared control creates disagreement costs that are more likely to destroy profitable opportunities than to prevent bad investments. In contrast, sharing control is efficient if investment opportunities are hard for outsiders to evaluate, when financing requirements are large, and in countries that poorly protect minority shareholders.

I Introduction

Since Berle and Means (1932), an extensive literature has investigated the consequences for firm value of a separation between ownership and control. In most of this literature, ownership structure matters because part of the firm's value consists of benefits of control that are not enjoyed by outside investors: the appropriation of corporate assets for personal use, empire building motives in the selection of projects, etc. To preserve private benefits, insiders may, for instance, forgo profitable projects (e.g., down sizing) at a cost to minority shareholders. Private benefits of control, therefore, imply conflicts of interest that led academics and practitioners to search for mechanisms that protect the rights of minority shareholders.

One such mechanism that the corporate governance literature has emphasized is the monitoring of investment decisions by outside shareholders (e.g., Shleifer and Vishny (1986), Kahn and Winton (1998), Bloch and Hege (2001), and Noe (2002)). Yet, it is unlikely that monitoring assures value-maximizing policies. For one, outside investors may lack incentives to monitor because the benefits of monitoring are spread over all minority shareholders, while the monitor bears all of its costs (see Grossman and Hart (1980)). Moreover, outside monitors are self-interested agents that do not necessarily aim to maximize firm value. Indeed, as Pagano and Roell (1998), Bolton and Von Thadden (1998), and Burkart, Gromb, and Panunzi (1997) argue, outside investors have incentives to block business decisions that reduce cash flows even if this loss is more than offset by an increase of private benefits of control. Undermonitoring, therefore, is not the only reason for looking for alternative mechanisms to protect the rights of minority shareholders; excessive monitoring may also be a problem.

In this paper, we argue that entrepreneurs may find it optimal to share control with other large shareholders, rather than relying on them to monitor business decisions on behalf of the minority shareholders. Under shared control, a governance structure arises in which multiple controlling shareholders enjoy private benefits and minority shareholders can no longer count on the presence of a peer that monitors corporate decisions on their behalf. Still, sharing control may increase firm value, for two reasons. First, it increases the equity stake of the decision makers, making them internalize firm value to a greater extent. This equity effect reduces incentives for business decisions that increase private benefits at a high efficiency cost. Second, ex-post bargaining problems among controlling shareholders may prevent business decisions

that are in the collective interest of the controlling group but harm minority shareholders.

Sharing control is not always efficient, though. Bargaining problems may result in corporate paralysis, reducing the firm's overall efficiency and possibly hurting the minority shareholders as well. A trade-off thus exists between the inefficient monitoring of outside investors and the net bargaining costs associated with a governance structure with multiple controlling shareholders.

By solving this trade-off, we show that sharing control dominates monitoring in firms with investment opportunities that are hard for outsiders to evaluate. In these firms, controlling shareholders should have an informational advantage in evaluating projects, while monitoring is more likely to harm firm value than overturning inefficient business decisions. In contrast, monitoring by outside investors is efficient if the investment opportunities yield private benefits that are hard for insiders to evaluate. Here, outside monitors ignore noisy signals of the value of private benefits that could make firms under shared control lose profitable investment opportunities. As we shall argue later, these results imply that, conditioned on a governance structure with shared control, controlling shareholders should have similar business backgrounds, as is likely to be the case in family firms and in joint ventures.

Yet, firm characteristics do not fully explain the role of large shareholders in corporate governance. Governance law is also an important determinant of the costs and benefits of monitoring and sharing control. In particular, our model implies that shared control is more common in countries with legal systems that offer weak protection to minority shareholders. The intuition for this implication is as follows. Weakening of governance laws can be thought of as a shift in investment opportunities that enhances the importance of the private benefits vis-à-vis the public cash flows. This shift increases the controlling shareholders' incentives for taking inefficient projects that increase private benefits, and makes it more difficult for monitors to block corporate decisions that harm minority shareholders. As a result, firm value under monitoring decreases when governance laws weaken. In contrast, we shall show that a control group can mitigate the inefficiency of weak governance laws by improving their own incentives through a larger equity stake. Hence, while strong governance laws enhance the value of monitoring, they are much less important for firms under shared control.

But is there evidence of ownership structures with shared control? As it turns out, shared

control is pervasive in close corporations and public corporations. In close corporations, numerous studies in the legal literature (e.g., O'Neal and Thompson (1992) and Clark (1986)) discuss mechanisms that minimize the costs of disagreements among controlling shareholders; and a growing empirical literature documents governance structures in private firms with multiple large shareholders (e.g., Ball and Shivkumar (2004) and Nagar, Petroni, and Wolfenzon (2004)).

Likewise, in a sample of 865 public firms of 13 Western European countries, Laeven and Levine (2004) show that about one-third – and over forty percent of the public firms with one large shareholder – have two or more owners holding more than 10 percent of the voting rights each.[1] Moreover, they show that firm value increases with the equity stake of a second large shareholder only if the gap in voting rights between the first and the second largest shareholder is small, as one would expect to occur in governance structures under shared control. These findings confirm in a broader sample previous research by Volpin (2002) that shows that, from 1987 to 1996, 15 percent of the firms listed in the Milan Stock Exchange were controlled by large shareholders that entered in explicit agreements to vote as a block (voting syndicates). These agreements are meaningful only in firms under shared control. More importantly, Volpin shows that managerial turnover is more sensitive to performance in firms with voting syndicates, consistent with our argument that shared control plays an important role in corporate governance.[2]

The papers closest to ours are Aghion and Bolton (1992), Bennedsen and Wolfenzon (2000), and Boot, Gopalan, and Thakor (2005). Aghion and Bolton look for the optimal ownership structure of a firm whose initial shareholder is credit constrained. In their paper, a single shareholder – the initial entrepreneur – enjoys private benefits of control, which lead to conflicting objectives between him and the remaining shareholders. Our paper departs from theirs

[1] Faccio and Lang (2002) also document the presence of a second large shareholder in 46 percent of a sample of 3,300 Western European corporations with at least one large shareholder (20 percent or more of the shares).

[2] In the US, Black and Gilson (1997) report that shareholders' agreements similar to voting syndicates are often present in the venture capital industry. These agreements are not limited to partnerships and start-up companies; they also seem to be pervasive in public firms that experienced privately negotiated block transactions. The Securities and Exchange Commission (SEC) requires that firms with publicly traded securities disclose shareholders' agreements in form 13-D. We searched for shareholders' agreements among a random sample of 115 public firms announcing privately negotiated block transactions of five percent or more from 1996 to 2000. These privately negotiated transactions triggered board rights agreements, veto power provisions, or voting agreements in 53 of the 115 firms (46 percent).

by letting multiple controlling shareholders enjoy private benefits. In this setting, we obtain new trade-offs that link firm characteristics and governance laws to the choice of the governance structure, and we show that, in contrast to Aghion and Bolton, shared control may be an efficient mechanism to protect minority shareholders.

In Bennedsen and Wolfenzon (2000), the presence of a large outside shareholder forces the controlling group to amass a greater equity stake or else control may be lost. The larger control stake increases efficiency because it makes the controlling group internalize more of the firm's value. Bennedsen and Wolfenzon ignore bargaining problems and a monitor's ability to discipline the controlling shareholders, focusing instead on the coalition games that determine the size of the controlling stake. In contrast, we ignore the coalition games and focus on a trade-off between inefficient monitoring and bargaining costs.

Boot, Gopalan, and Thakor (2005) focus on the trade-offs involved in the choice between private and public ownership. In their setting, public capital markets provide better liquidity than private markets, but private markets are better than public capital markets in allowing managers and investors resolve their disagreements, which, unlike in our model, are driven by differences in beliefs about the value-maximizing actions.

The remainder of the paper is organized as follows. Section II describes the model. Section III characterizes firm value under shared control and monitoring. Section IV endogenizes the governance structure and shows how firm characteristics and governance laws determine the role of large shareholders in corporate governance. Section V discusses the stability of ownership structures with shared control and the robustness of our results. Section VI concludes the paper. Proofs of the propositions that are not present in the text can be found in the appendix.

II The Model

Figure 1 summarizes the timing and the main events of the model. At time $t = 0$, a firm with a single shareholder (the entrepreneur) seeks outside investors to finance the cost I of a project, in an economy with risk-neutral agents and a zero risk-free interest rate. To create a link between the investment opportunity and the firm's ownership structure, we assume that the value of the existing assets is V_0 but both the entrepreneur and the firm have exhausted

Figure 1: Timing of events

their debt capacities.[3] Hence, to finance the project, the entrepreneur must sell equity shares. After raising the investment requirement, additional information about the project's payoffs is released at $t = 1$ and then the firm decides whether to undertake it or not at time $t = 2$. Cash flows realize at $t = 3$, when the firm is liquidated.

The entrepreneur's problem is to choose an ownership structure that maximizes the firm's value conditioned on raising the investment requirement I. Of course, ownership structure is relevant to the firm's value only if shareholders may have different incentives to undertake the project. As in the modern literature on the theory of the firm (e.g., Grossman and Hart (1986) and Hart and Moore (1990)), we obtain conflicting incentives over the investment decision by introducing nonverifiable cash flows, that is, the private benefits of control.

The project's cash flows thus consist of two parts: a public cash flow that is paid to all shareholders in proportion to their shareholdings – the sum of the investment requirement I and its return y – and a private benefit component, b, that is captured by the controlling shareholders only. The impact of the project on the firm's public and private cash flows can be either positive or negative. For instance, down sizing the firm may increase profits, $y > 0$, and yet reduce the utility of controlling shareholders who are empire builders, in which case $b < 0$. And expanding the firm may increase the private benefits of empire builders, $b > 0$, at a loss of profits, $y < 0$.

[3] Given that our main goal is to explore the role that large shareholders play in corporate governance, adding debt financing unnecessarily complicates the model. Financing the project thus requires attracting new shareholders through an equity issue which, for simplicity, makes the existing debt safe. (Assuming that the firm's debt is safe after the equity issue let us ignore the existing debt in the analysis.)

The private benefits of control distort the investment incentives of the entrepreneur. Private benefits induce the entrepreneur to overinvest if they are high enough to offset his share of the negative public return of an inefficient project. In turn, incentives for underinvestment arise if the project is efficient, but the entrepreneur's share of the public payoffs does not offset a loss of private benefits. More formally, if α_{uc} is the equity share of the entrepreneur under unilateral control (i.e., an ownership structure with a single controlling shareholder), then he will seek to undertake the project at time $t = 2$ if $b + \alpha_{uc} y > 0$, which leads to overinvestment if $b + y < 0$. In contrast, incentives for underinvestment arise if $b + \alpha_{uc} y < 0$ and $b + y > 0$.

Anticipating the incentives for an inefficient investment policy, investors will buy equity at reduced prices only. The entrepreneur, therefore, bears the cost of his sub-optimal incentives. As such, it may not be in his best interest to finance the project by selling equity to minority investors who will passively accept the firm's investment decisions. Instead, the entrepreneur may find it optimal to sell shares to a large investor who will challenge investment decisions that do not maximize public cash flows.

Accordingly, we assume that the large outside investor – the monitor from now on – observes the firm's investment decision at $t = 2$ and, by incurring a private cost $c(m) = \frac{\rho m^2}{2}$, can overturn the decision with probability $m \in [0, 1]$.[4] The monitor may thus influence the firm's actions, but we do not allow him to receive side payments from the entrepreneur or extract private benefit. This modelling of the monitor is standard, and typical examples in the corporate finance of monitors that seem to match this description are pension fund managers and institutional money managers.

One way to interpret the monitor's action is that, with probability m, he can convince an uninformed court to overturn the investment decision, on the basis that it harms the minority shareholders. In this interpretation, convincing a court to overturn managements' decisions is likely to be more difficult if the project has a high volatility of returns (e.g., an R&D project); a reluctance that we model as a large ρ. It then follows that, in the presence of a monitor, an

[4] Large outside investors will observe the investment decision if, for example, the corporate charter forces the control group to disclose major investments ahead of time. To rule out uninteresting solutions to the conflicts between the controlling group and the other shareholders, we do not allow for the controlling group to commit to never undertake the project or to always undertake it. This assumption is justified on the basis that we analyze one of several investment opportunities (the one with conflicts of interests), which make the never-invest and always-invest strategies too costly for shareholders.

entrepreneur's decision to overinvest (or underinvest) will carry on with probability $1-m<1$.

Monitoring the firm on behalf of minority shareholders is not the only governance role that a large investor can play, though. The entrepreneur may let the large investor join the control group by giving him board seats or veto power over major investment decisions.[5] In this case, the large investor becomes an insider, with access to the private benefits of control. Indeed, access to private benefits is what distinguishes in our model an outside monitor from a controlling shareholder. Unlike the controlling shareholders, we assume that monitors cannot use their power to overturn investment decisions to capture (possibly through side payments) part of the private benefits of control. Otherwise, they become insiders who fight not only for public cash flows but also for private benefits, and minority shareholders can no longer count on them to defend their interests.

Note, however, that, under shared control, the private benefits from the project are not necessarily equally split between the controlling shareholders. It may be the case that the project's payoff is positive for one of the controlling shareholders, $b_1+\alpha_1 y>0$, and negative for the other, $b_2+\alpha_2 y<0$, where $b_i\in[\underline{b}_i,\bar{b}_i]$ is the private benefit and α_i is the equity share of controlling shareholder $i\in\{1,2\}$. As in Aghion and Bolton (1992), we model this joint investment decision as a bargaining game with veto power. The control group will undertake the project at $t=1$ if and only if both controlling shareholders agree with the investment, possibly after transfer payments are made.[6] These transfers change the payoffs from the project. More importantly, ex-post bargaining problems within the controlling group may block projects that are in their collective interest but harm minority shareholders. In this case, the firm's private benefits do not change and the firm's value remains at V_0+I. The presence of a second large shareholder, therefore, may protect minority shareholders even if he does not play the role of a monitor.

It then follows that the entrepreneur may constrain his incentive to expropriate the value of outside investors by raising the investment requirement I in one of two ways: Selling shares

[5] Although all the arguments of this paper apply to ownership structures with more than two large shareholders, the analysis is simpler if we restrict attention to two large shareholders. (Zwiebel (1995) reports that, in the US, ownership structures with more than two large shareholders are rare.)

[6] In Aghion and Bolton (1992), future actions must be chosen by unanimous consent when more than one investor is in control. Indeed, governance structures with veto power do exist. In a sample of 200 joint ventures, Bai, Tao and Wu (2004) show that 190 firms require unanimity for approval of mergers, change in corporate charter, and capital increases.

to a large investor who will act as an outside monitor (unilateral control) or selling shares to a large investor who will join the control group (shared control). To better compare these two alternatives, we assume that the private benefits do not depend on the number of controlling shareholders. That is, if b is the private benefit of the project with a single controlling shareholder then $\sum_{i=1}^{2} b_i = b$.[7] Moreover, whether control is shared or not, we allow for – but do not require – the presence of minority shareholders. We do require, though, that controlling shareholders hold at least a fraction $\underline{\alpha} > 0$ of the equity shares.[8] And we rule out investors who can afford the whole firm; or else the first best would be trivially obtained by selling the firm to a single investor, who would internalize all the costs and benefits of business decisions.

To complete the model, we describe the information structure. At time $t = 0$, when the ownership structure is chosen, the entrepreneur and the potential investors are symmetrically informed, knowing the value of the public cash flow y and the distribution of the project's private benefits.[9] At time $t = 1$ (before making their investment decision) the control group learns new information about the private benefits of the project. In case of unilateral control, the single controlling shareholder privately learns the realization of the project's private benefits. Outside investors (including the monitor) observe the investment decision of the entrepreneur at $t = 2$ but do not learn the realization of the private benefits.[10] Under shared control, controlling shareholder $i \in \{1, 2\}$ observes his own private benefit b_i and a noisy signal s_j of the private benefit of controlling shareholder $j \neq i$. The noisy signals s_1 and s_2 are observed by both controlling shareholders at $t = 1$, and they satisfy $b_j = s_j + \epsilon_j$ with $\epsilon_j \in [-\frac{\epsilon}{2}, \frac{\epsilon}{2}]$ and $\epsilon > 0$ for $j \in \{1, 2\}$. Outside investors observe the investment decision of the controlling group but do not learn either the realizations of the private benefits or the signals (s_1, s_2).

Conditioned on s_j, the density function of private benefit b_j is $f_j(b_j|s_j)$, which we assume

[7] The direct effect of the number of controlling shareholders on private benefits is uncertain. A large number of controlling shareholders may increase the efforts of unlocking private benefits. If so, the private benefits should stochastically increase with the number of controlling shareholders. But, a large number of controlling shareholders may also lead to a destructive fight for private benefits. Hence, private benefits may stochastically decrease with the number of controlling shareholders. Note also that this assumption does not make the private benefit an exogenous variable. Its equilibrium amount is endogenously determined by the investment decision.

[8] The exogenous lower bound on the equity stake of the controlling shareholders, $\underline{\alpha}$, is not essential to the analysis. It simply implies that, to directly participate in the management, investors must have some equity stake in the firm.

[9] Assuming that public cash flows are certain and private benefits are random allows us to focus the attention on the key problems created by the private benefits of control.

[10] Since we assume that the investors observe the project's public cash flow and the firm's investment decision, their actions would not change had we assumed that they learn the value of the private benefits.

to be uniformly distributed in the interval $[s_j - \frac{\epsilon}{2}, s_j + \frac{\epsilon}{2}]$. Hence, the realization of s_j implies that controlling shareholder i's posterior about b_j is independent of b_i. Note also that we do not impose restrictions on the joint distribution of the signals (s_1, s_2). Accordingly, our results do not rely on the mechanism that splits the private benefits between the controlling shareholders. In particular, the model is consistent with a sharing rule that, with a large probability, gives no private benefits to the controlling shareholder with the smallest equity stake.

III Firm Value under Monitoring and Shared Control

Left unchecked, the entrepreneur has incentives to overinvest if private benefits are high enough to offset his share of the negative public return of an inefficient project. In turn, incentives for underinvestment arise if the project is efficient, but the entrepreneur's share of the public payoffs does not offset a loss of private benefits. Anticipating an inefficient investment policy, investors will buy equity at reduced prices only. The entrepreneur, therefore, bears the cost of his sub-optimal incentives, making it in his best interest to look for mechanisms that constrain sub-optimal investment decisions. This section analyzes two such mechanisms: monitoring by an outside investor and shared control.

A Firm value with monitoring

Suppose that the entrepreneur wants to stay as the single controlling shareholder but, to avoid being left unchecked, sells a fraction β of the shares to an outside investor who will play the role of a monitor. As an outsider, the monitor cannot enjoy private benefits of control. As such, he has incentives to overturn firm decisions that do not maximize public cash flows. Yet, overturning the entrepreneur's decisions is costly. The monitor bears a cost $c(m) = \frac{\rho m^2}{2}$ to assure a probability $m \in [0, 1]$ of overturning business decisions.[11] In this setting, the investor's

[11] In this paper, we analyze situations in which there is only one monitor. In contrast, Noe (2002) allows for multiple monitors, demonstrating that the relation between shareholdings and activism may not be monotonic: among the investors who monitor with positive probability, those with smaller holdings are the most active.

monitoring effort is $m(y,\beta) = min\{\frac{\beta}{\rho}|y|, 1\}$, which is the solution of

$$\max_{\{m\in[0,1]\}} m\beta|y| - \frac{\rho m^2}{2}. \tag{1}$$

To understand program (1), recall that, in our model, the entrepreneur may act against the interests of the monitor in two ways. He may invest in a project with a negative y that imposes on the monitor a cost of $-\beta y$. Or, he may pass up a project with a positive y, at a cost of βy to the monitor. We can thus write the monitor's benefit from overturning an undesirable investment decision with probability m as $m\beta|y|$, where $|y|$ is the absolute value of the public return. Subtracting the cost of monitoring from $m\beta|y|$ yields the objective function that the monitor maximizes.

Note that a larger equity stake increases the investor's incentive to monitor, reducing the undermonitoring problems that Grossman and Hart (1980) point out. However, an increase of the monitor's equity stake may lead to overmonitoring problems if the entrepreneur's decision implies a loss of public cash flows that is more than offset by gains in private benefits. A value-maximizing ownership structure, therefore, must take into account not only the entrepreneur's incentives to distort the investment policy but also the monitor's own incentive problems. This trade-off determines the monitor's and the dispersed shareholders' equity stakes.

To characterize the optimal ownership structure under unilateral control, consider first a project with negative public returns ($y < 0$). In this case, the monitor will try to block the project, being successful with probability $m(y,\beta) = min\{\frac{\beta}{\rho}|y|, 1\}$. Of course, the monitor's strategy is vacuous if the entrepreneur has no incentive to undertake the project either. Hence, monitoring is relevant only if, despite the negative public return, private benefits make it worthwhile for an entrepreneur to undertake the project, that is, $b+\alpha_{uc}y > 0$. If this condition is satisfied, the value of the investment opportunity is $(y+b)(1-m(y,\beta))$ – i.e., the value of the project times the probability that the monitor does not block it – minus the cost of monitoring, $\frac{\rho m(y,\beta)^2}{2}$. Conditioned on the entrepreneur's equity stake α_{uc}, the monitor's stake that maximizes firm value solves:

$$\max_{\beta\in[0,1-\alpha_{uc}]} E\left[(y+b)(1-m(y,\beta)) - \frac{\rho m(y,\beta)^2}{2}\middle| b > -\alpha_{uc}y\right] P(b > -\alpha_{uc}y). \tag{2}$$

The program that solves for the monitor's stake with positive public returns ($y > 0$) is analogous. If, despite the positive y, the investment implies a sufficiently large loss of private benefits ($b + \alpha_{uc} y < 0$), then the entrepreneur may pass up the project. In this case, the monitor will try to force the undertaking of the project, succeeding with probability $m(y, \beta)$. The monitor's stake that maximizes firm value solves:

$$\max_{\beta \in [0, 1-\alpha_{uc}]} E[y + b | \, b > -\alpha_{uc} y] P(b > -\alpha_{uc} y) + E[(y+b) m(y, \beta) - \frac{\rho m(y, \beta)^2}{2} | b < -\alpha_{uc} y] P(b < -\alpha_{uc} y), \tag{3}$$

where the first term in program (3) is the value of the investment opportunity conditioned on being in the entrepreneur's interest to undertake it, and the second term is the value conditioned on the monitor's forcing it upon the firm despite its being against the entrepreneur's interest. Proposition 1 below characterizes the solutions of programs (2) and (3).

Proposition 1 *Define* $sign(y) = 1$ *if* $y \geq 0$, $sign(y) = -1$ *if* $y < 0$, *and let* $\hat{b}(\alpha_{uc}) = E\left[b | \, (b + \alpha_{uc} y) \, sign(y) < 0\right]$. *Thus, under unilateral control, a firm whose entrepreneur has an equity stake* α_{uc} *rules out monitoring,* $\beta^*(\alpha_{uc}) = 0$, *if and only if* $\left(\hat{b}(\alpha_{uc}) + y\right) sign(y) \leq 0$. *For* $\left(\hat{b}(\alpha_{uc}) + y\right) sign(y) > 0$, *the optimal monitoring stake is* $\beta^*(\alpha_{uc}) = \min\{1 - \alpha_{uc}, \widehat{\beta}(\alpha_{uc})\} > 0$, *where*

$$\widehat{\beta}(\alpha_{uc}) = \begin{cases} \frac{\rho}{|y|} & \text{if } \left(\hat{b}(\alpha_{uc}) + y\right) sign(y) \geq \rho \\ \frac{1}{y}\left(\hat{b}(\alpha_{uc}) + y\right) & \text{if } \rho > \left(\hat{b}(\alpha_{uc}) + y\right) sign(y) > 0. \end{cases}$$

The dispersed shareholders' total stake is $1 - \alpha_{uc} - \beta^*(\alpha_{uc})$.

The intuition for Proposition 1 is the following. For projects that destroy public cash flows, $y < 0$, overinvestment is the problem that the monitor mitigates. In this case, monitoring is efficient ($\beta^*(\alpha_{uc}) > 0$) if the project yields a negative expected payoff conditioned on being in the entrepreneur's interest to undertake it, that is, $E\left[b | \, (b + \alpha_{uc} y) > 0\right] + y < 0$. And the optimal ownership structure should induce as much monitoring as possible ($m(y, \beta) = 1$ at $\beta = \frac{\rho}{|y|}$) if the loss from undertaking the project is sufficiently large: $E\left[b | \, (b + \alpha_{uc} y) > 0\right] + y \leq -\rho$. Conversely, $E\left[b | \, (b + \alpha_{uc} y) > 0\right] + y > 0$ implies that monitoring is not efficient, $\beta^*(\alpha_{uc}) = 0$, because it is more likely that the project is valuable when the entrepreneur wants to undertake it. For projects that increase public cash flows, $y > 0$, underinvestment is the problem that the monitor will try to avoid. Here, monitoring is valuable if it is more likely that the project is efficient when the entrepreneur does not want to undertake it ($E\left[b | \, (b + \alpha_{uc} y) < 0\right] + y > 0$).

And monitoring will be as intensive as possible when the loss from forgoing the project is sufficiently large, $E\left[b\right|\left(b+\alpha_{uc}y\right)<0]\geq\rho$. As one can easily check, the definitions of $\hat{b}(\alpha_{uc})$ and $sign(y)$ let us write all of the above conditions as those in Proposition 1.

To complete the characterization of the ownership structure under unilateral control, it suffices to determine the optimal equity stake of the entrepreneur, α_{uc}^{*}. Given α_{uc}^{*}, the ownership structure that maximizes firm value under unilateral control is $(\alpha_{uc}^{*},\beta^{*}(\alpha_{uc}^{*}),1-\alpha_{uc}^{*}-\beta^{*}(\alpha_{uc}^{*}))$, where the triple denotes, respectively, the stakes of the entrepreneur, the monitor, and the dispersed shareholders. As Proposition 2 shows, the optimal equity stake of the monitor is the largest one that allows for the financing of the investment requirement I.

Proposition 2 *Under unilateral control, the optimal equity stake of the entrepreneur, α_{uc}^{*}, is the largest one that allows for the entrepreneur to raise the investment requirement I. As such, the ownership structure that maximizes firm value under unilateral control is $(\alpha_{uc}^{*},\beta^{*}(\alpha_{uc}^{*}),1-\alpha_{uc}^{*}-\beta^{*}(\alpha_{uc}^{*}))$, where $\beta^{*}(\alpha_{uc})$ is characterized in Proposition 1 as the monitor's equity stake as a function of the entrepreneur's stake.*

The intuition for Proposition 2 is quite simple. By selling the minimum amount of shares that raises I, the entrepreneur internalizes his actions as much as possible, bringing the investment policy closer to efficiency without harming the disciplinary role of the monitor.

Our next task is to analyze our alternative for monitoring, that is, sharing control.

B Firm value with shared control

A vast literature on corporate law (e.g., O'Neal and Thompson (1992) and Clark (1986)) discusses conflicts of interest within a controlling group that may lead to deadlock problems. The Wall Street Journal of May 13, 1998 (page B10) describes an example of such type of conflict. In 1998, Ted Turner, then Vice Chairman and the largest individual shareholder (11 percent) of Time Warner, vetoed the sale of the group's legal channel, Court TV, to Discovery Communications Inc. Allegedly, Ted Turner was concerned with a new owner transforming the legal channel into a competitor to CNN, the flagship of Turner Broadcasting's own cable channel and also a member of the Time-Warner group. According to the Wall Street Journal, Mr. Turner prevailed over Gerald Levin, Time Warner's Chairman, who did not internalize

the consequences for CNN of the sale of Court TV as much as Mr. Turner. Sharing control, therefore, opens the door for bargaining problems.

In our paper, conflicts of interest between the controlling shareholders arise if $b_i + \alpha_i^{sc} y > 0$ and $b_j + \alpha_j^{sc} y < 0$, where α_i^{sc} is the equity stake of controlling shareholder i and $b_i + \alpha_i^{sc} y$ is his valuation for the project. In this case, the firm undertakes the project if controlling shareholder i convinces the opposing shareholder j not to use his veto power. The investment decision, therefore, amounts to a bargaining game.

By their very nature, the benefits of control of each controlling shareholder are likely to be privately known. We thus model the investment decision as a bargaining game under asymmetric information, which, for ease of exposition, we solve using a simple mechanism first analyzed by Chatterjee and Samuelson (1983).[12]

In the mechanism of Chatterjee and Samuelson, which is a natural generalization of the Nash bargaining solution to a setting with imperfect information, the controlling shareholders simultaneously announce their valuations of the project – call them V_i for $i \in \{1, 2\}$. The project is undertaken if and only if $V_1 + V_2 \geq 0$, in which case the two controlling shareholders split their announced benefits. To implement this split, there is a side payment $t = \frac{V_1 - V_2}{2}$ (possibly negative) from the first controlling shareholder (the entrepreneur) to the second one. If the project is not undertaken, no side payment is required.

We show in Lemma 1 that, if necessary, transfers of shares can implement the side payment t, provided that the equity value that arises from the firm's existing assets is sufficiently large. As such, despite the entrepreneur's wealth constraint, we shall proceed as if both controlling shareholders can afford the transfer payments. (Section V discusses the case that transfers of shares cannot support the side payments.)

Lemma 1 *Assume that the equity value from the assets in place, $V_0 + I$, satisfies*

$$V_0 + I \geq -y + max_{i \in \{1,2\}} \frac{\bar{b}_i + \alpha_i^{sc} y}{\alpha_i^{sc} - \underline{\alpha}}, \tag{4}$$

where y is the public cash flow, $\bar{b}_i$ is the maximum private benefit of controlling shareholder i, α_i^{sc} is the equity stake of controlling shareholder i, and $\underline{\alpha}$ is the minimum equity stake that

[12] We also analyzed the bargaining problem using the direct mechanism approach of Myerson and Satterthwaite (1983), and the results are analogous to the one using the Chatterjee and Samuelson (1983) mechanism.

allows a shareholder to gain control. Then transfers of shares can implement any incentive-compatible payment of the bargaining game.

Now, assume for a while that the control stakes α_1^{sc} and α_2^{sc} are given. The transfer payment $t = \frac{V_1 - V_2}{2}$ implies that, conditioned on the investment being made, the two controlling shareholders gain by shading their valuations of the project. Of course, reducing the announced valuation will also increase the chances that the project will not be undertaken (remember that the investment happens if and only if $V_1 + V_2 \geq 0$). Hence, when shading their valuations, each controlling shareholder will weigh a higher gain in the event that the project is undertaken against a higher probability that they forgo a valuable project.

To solve this trade-off, we look for a Bayesian equilibrium in which the announcements of the controlling shareholders depend on their own valuations for the project, $b_i + \alpha_i^{sc} y$, and their guesses of the announcement of the other controlling shareholder. The Bayesian equilibrium is described by a pair of functions, $(V_1(b_1 + \alpha_1^{sc} y, s_1, s_2), V_2(b_2 + \alpha_2^{sc} y, s_1, s_2))$, such that the announcement of the first controlling shareholder, $V_1(b_1 + \alpha_1^{sc} y, s_1, s_2)$, solves

$$\max_{V_1} \int_{V_2^{-1}(-V_1, s_1, s_2) - \alpha_2^{sc} y}^{s_2 + \frac{\epsilon}{2}} (b_1 + \alpha_1^{sc} y - \frac{V_1 - V_2(b_2 + \alpha_2^{sc} y, s_1, s_2)}{2}) f_2(b_2 | s_2) db_2. \tag{5}$$

The objective function in program (5) is the expected payoff of the entrepreneur given his announcement of V_1; his true valuation of the project, $b_1 + \alpha_1^{sc} y$; and the signals s_1 and s_2. This payoff is uncertain, for two reasons. First, the transfer payment $t = \frac{V_1 - V_2}{2}$ depends on the second controlling shareholder's announcement, V_2, which is a function of his valuation for the project. Second, announcing V_1 will block the project if $V_1 + V_2 < 0$, or equivalently, $V_2 < -V_1$. So, the lower the announced V_1, the higher the chances that the project will not be undertaken. In fact, given V_1, the lowest V_2 that leads to the acceptance of the project solves $V_1 + V_2(b_2 + \alpha_2^{sc} y, s_1, s_2) = 0$, which implies that $V_1 + V_2(b_2 + \alpha_2^{sc} y, s_1, s_2) > 0$ if and only if $b_2 + \alpha_2^{sc} y > V_2^{-1}(-V_1, s_1, s_2)$, where $V_2^{-1}(.)$ is the inverse function of $V_2(b_2 + \alpha_2^{sc} y, .)$.[13] Therefore, the expectation of the entrepreneur's payoff is taken with respect to b_2 (using the density that the signal s_2 induces, $f_2(b_2|s_2)$) for values higher than the cut-off $V_2^{-1}(-V_1, s_1, s_2) - \alpha_2^{sc} y$.

[13] It can be shown that, in any Bayesian Equilibrium, the functions $V_1(.)$ and $V_2(.)$ increase with the valuation of the project. If these functions are not strictly increasing, $V_i^{-1}(x)$ should be understood as the minimum valuation of the project that makes the controlling shareholder i announce x.

Analogously to program (5), the optimal announcement of the second controlling shareholder solves

$$\max_{V_2} \int_{V_1^{-1}(-V_2,s_1,s_2)-\alpha_1^{sc}y}^{s_1+\frac{\epsilon}{2}} (b_2 + \alpha_2^{sc}y + \frac{V_1(b_1+\alpha_1^{sc}y, s_1, s_2) - V_2}{2}) f_1(b_1|s_1)db_1. \tag{6}$$

Proposition 3 characterizes the solution of the bargaining game for best responses $V_1(.)$ and $V_2(.)$ that are linear functions of the controlling shareholders' own valuations.

Proposition 3 *Suppose that the value from the existing assets, V_0+I, satisfies condition (4). Then, there is a Bayesian Equilibrium in which the investment is undertaken if and only if*

$$b_1 + b_2 + (\alpha_1^{sc} + \alpha_2^{sc})y \geq \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2). \tag{7}$$

The left-hand side of the investment rule (equation (7)) is the combined valuation of the controlling shareholders. Under shared control, all that matters for investment decisions is the total equity stake of the controlling group. And transfer payments should square off their difference, provided that their total gains from the project overcome the hurdle $\frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$.

Since ϵ_i is uniformly distributed in the interval $[-\frac{\epsilon}{2}, \frac{\epsilon}{2}]$, the hurdle $\frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$ is strictly positive with probability one. As a result, a project that reduces the combined payoff of the control group ($b_1 + b_2 + (\alpha_1^{sc} + \alpha_2^{sc})y < 0$) does not satisfy the decision rule (7). But the controlling group will pass up projects that are in their collective interest if the payoffs satisfy $0 < b_1 + b_2 + (\alpha_1^{sc} + \alpha_2^{sc})y < \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$. We have thus established

Proposition 4 *With shared control, the controlling shareholders will not undertake projects that are against their collective interest. However, they may pass up projects that would have increased the sum of their expected payoffs.*

From Proposition 4, bargaining under asymmetric information biases the investment decision against the undertaking of the project. These ex-post bargaining problems are costly if an efficient project ($b + y > 0$) fails to pass the hurdle for investment under shared control (i.e., $b_1 + b_2 + (\alpha_1^{sc} + \alpha_2^{sc})y < \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$). This inefficiency result is robust to different information structures; all that it requires is that the controlling shareholders are asymmetrically informed about the project's payoffs. If, for example, the controlling shareholders were also

asymmetrically informed about the public cash flows, the ability to veto the project would still give a stronger bargaining power to the most pessimistic shareholder. And the controlling shareholders would have incentive to shade their valuations – at the risk of passing up an efficient project – to reduce the expected transfer payment. The veto power is thus the driving force of the bias toward underinvestment.

Nonetheless, rejecting projects that are in the collective interest of the controlling shareholders may well be efficient. Disagreements among the controlling shareholders may prevent them from undertaking a project that, although in their collective interest, inefficiently harm minority shareholders. In other words, bargaining problems associated with shared control mitigate overinvestment problems. A value-maximizing ownership structure under shared control, therefore, will not necessarily minimize the bargaining problems of the controlling group.

Now, since the only aspect of the ownership structure that matters for the investment decision is the total stake of the controlling group, we shall proceed by characterizing the control stake – call it α_{sc}^* – that maximizes the value of the investment opportunity. As we show later, moving from α_{sc}^* to the optimal ownership structure is fairly simple.

Let the control stake be $\alpha_{sc} = \alpha_1^{sc} + \alpha_2^{sc}$ and write the hurdle for undertaking a project with private benefits $b = b_1 + b_2$ as $b + \alpha_{sc} y < \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$. Since we assume that ϵ_1 and ϵ_2 are independently and uniformly distributed in the interval $[-\epsilon/2, \epsilon/2]$, straightforward calculations yield that the probability that the two controlling shareholders agree to undertake the project is $\frac{1}{\epsilon^2}\left[\int_{-\epsilon/2}^{\epsilon/2}\int_{-\epsilon/2}^{\epsilon/2} I(b + \alpha_{sc} y > \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2))d\epsilon_1 d\epsilon_2\right]$, where $I(b + \alpha_{sc} y > \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2))$ is an indicator function that takes value one if the hurdle for undertaking the project is satisfied. It then follows that the value of the investment opportunity under a control stake α_{sc} and public cash flow y is

$$V_{sc}(\alpha_{sc}, y) = \int (b + y)\left[\frac{1}{\epsilon^2}\int_{-\epsilon/2}^{\epsilon/2}\int_{-\epsilon/2}^{\epsilon/2} I(b + \alpha_{sc} y > \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2))d\epsilon_1 d\epsilon_2\right] f(b)db, \qquad (8)$$

where $f(b)$ is the density function of the sum of the private benefits of control. Lemma 2 obtains a closed form solution for the value of the investment opportunity under shared control.

Lemma 2 *The value of the investment opportunity for a given control stake α_{sc} is $V_{sc}(\alpha_{sc}, y) =$*

$\int (b+y)\,\Gamma(\frac{b+\alpha_{sc}y}{\epsilon})f(b)db$, *where*

$$\Gamma(x) = \begin{cases} 1 & \text{if } x \geq \frac{2}{3} \\ 1 - \frac{1}{2}(3x-2)^2 & \text{if } x \in \left[\frac{1}{3}, \frac{2}{3}\right] \\ \frac{1}{2}(3x)^2 & x \in \left[0, \frac{1}{3}\right] \\ 0 & \text{if } x \leq 0. \end{cases}$$

Using Lemma 2, Proposition 5 characterizes the control stake that maximizes the value of the investment opportunity under shared control.

Proposition 5 *Shared control with no minority shareholders achieves the first best if* $P(b+y \in \left(0, \frac{2\epsilon}{3}\right)) = 0$. *Otherwise, shared control may distort the investment policy and the optimal controlling stake,* α^*_{sc}, *depends on the public cash flows of the project as follows:*
(i) If $y < 0$ *(i.e., the project decreases public cash flows):* $1 - \frac{2\epsilon}{3|y|} < \alpha^*_{sc} < 1$. *In this case, the presence of minority shareholders (with stake* $1 - \alpha^*_{sc} > 0$*) increases firm value under shared control.*
(ii) If $y > 0$ *(i.e., the project increases public cash flows):* $\alpha^*_{sc} = 1$. *In this case, it is not optimal to sell equity to minority shareholders.*

Curiously, Proposition 5 shows that the presence of minority shareholders maximizes the value of the investment opportunity precisely when it is likely to harm them ($y < 0$). To get some intuition for this result, lets first understand the condition for shared control to achieve the first best. From Proposition 4, shared control implies ex-post bargaining costs that make it more difficult for the firm to undertake the project. As a result, underinvestment is likely to be the most relevant problem in firms under shared control. In our model, underinvestment arises if the project is valuable, $b + y > 0$, and the hurdle for undertaking the project is not met: $b + \alpha_{sc}y < \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$. In the absence of minority shareholders, $\alpha_{sc} = 1$, these two conditions are equivalent to $b+y \in \left(0, \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)\right)$. Hence, a sufficient condition for ruling out underinvestment under shared control – and for $\alpha_{sc} = 1$ to be optimal – is the event $b + y \in \left(0, \frac{2\epsilon}{3}\right)$ to have zero probability. In this case, shared control achieves the first best.

Suppose now that shared control implies bargaining problems that allow for underinvestment (i.e., $P(b+y \in \left(0, \frac{2\epsilon}{3}\right)) > 0$). Clearly, bargaining problems are less severe if the controlling

group can shift some of the costs of undertaking the project to minority shareholders. But this is what happens if the project destroys public cash flows ($y < 0$); the controlling group captures all of the benefits of control while the minority shareholders bear the cost $-(1-\alpha_{sc})y$. It is thus not surprising that the presence of minority shareholders enhances the value of the investment opportunity when $y < 0$. In contrast, the controlling group cannot shift costs to minority shareholders if the project increases public cash flows. The optimal control structure thus gives full ownership to the controlling shareholders so that they internalize as much as possible the cost of passing up profitable projects.

Now, the total control stake α^*_{sc} that maximizes the value of the investment opportunity is consistent with any profile of control stakes that satisfies $\alpha^{sc}_1 + \alpha^{sc}_2 = \alpha^*_{sc}$. To be sure, there is an infinite number of such profiles that fail to raise the required investment I. It is easy to see, nonetheless, that, fixed α^*_{sc}, the ownership structure that maximizes the entrepreneur's ability to raise funds is $(\alpha^{sc}_1, \alpha^{sc}_2, 1-\alpha^*_{sc}) = (\underline{\alpha}, \alpha^*_{sc} - \underline{\alpha}, 1-\alpha^*_{sc})$. In this ownership structure, the entrepreneur holds the minimum number of shares that let's him keep control, $\underline{\alpha}$, sells to the other controlling shareholder what is needed to assure the value-maximizing control stake α^*_{sc}, and leaves the remaining shares, $1-\alpha^*_{sc}$, to the dispersed shareholders. Hence, $(\underline{\alpha}, \alpha^*_{sc} - \underline{\alpha}, 1-\alpha^*_{sc})$ is the best case scenario for shared control to raise I. In the next section, we shall compare monitoring and shared control under α^*_{sc} while assuming that they can both raise the investment requirement.

IV Monitoring versus Shared Control

In this section, we endogenize the firm's governance structure. The chosen governance structure, a single controlling shareholder monitored by an outside investor (unilateral control) or two large shareholders sharing control, is the one that maximizes the firm value. We shall demonstrate that the relative efficiency of unilateral and shared control depends not only on firm characteristics but also on governance laws. We discuss the stability of the ex-ante governance choice in the next section.

A Firm characteristics

Whether it is optimal for large shareholders to monitor the firm (unilateral control) or join the controlling group (shared control) depends on firm characteristics. To see why, consider first a firm under shared control. Here, underinvestment is the main source of inefficiency: an uneven distribution of private benefits may induce one of the controlling shareholders to veto a profitable project. An uneven distribution of private benefits does not suffice for underinvestment to obtain, though. Some friction on the controlling shareholders' ability to square off their differences is also required.

One such a friction is asymmetry of information on the private benefits of control. In our model, asymmetric information between the controlling shareholder increases with the uncertainty ϵ on the noisy signal each controlling shareholder receives about the private benefits of the other controlling shareholder (i.e., the true private benefits are $b_j = s_j + \epsilon_j$ where s_j is the signal and $\epsilon_j \sim [-\frac{\epsilon}{2}, \frac{\epsilon}{2}]$ is a uniformly distributed noise with uncertainty ϵ). To determine how the relative efficiency of shared control and unilateral control change with the asymmetry of information, we keep the overall distribution of private benefits unaltered and only change the precision of the signal of the private benefits.[14]

Accordingly, Proposition 6 below demonstrates that the value of a firm under shared control decreases with ϵ. In contrast, bargaining problems within a controlling group cannot exist if there is a single controlling shareholder (unilateral control), and thus the value under unilateral control is unaffected by changes in ϵ. As such, the relative efficiency of unilateral control increases with ϵ, which we interpret as a measure of how difficult it is for insiders to evaluate the private benefits associated with the investment opportunities.

Proposition 6 *Firm value with shared control is non-increasing in the measure ϵ of the degree of information asymmetry between the controlling shareholders. Hence, large shareholders are more likely to monitor the firm on behalf of outside investors in firms with investment opportunities that are hard for insiders to evaluate (large ϵ).*

[14] This is accomplished by changing the joint distribution of the signal (s_1, s_2) with ϵ, as follows. Letting the density of the distribution of the private benefits be $f(b_1, b_2)$, then the density, $g(s_1, s_2)$, of the distribution of the signals $s_j = b_j - \epsilon_j$ is the convolution of the densities of b_j and ϵ_j. More precisely, $g(s_1, s_2) = \frac{1}{\epsilon^2} \int_{-\frac{\epsilon}{2}}^{\frac{\epsilon}{2}} \int_{-\frac{\epsilon}{2}}^{\frac{\epsilon}{2}} f(b_1 - \epsilon_1, b_2 - \epsilon_2) d\epsilon_1 d\epsilon_2$, which is a function of the uncertainty ϵ in the signal.

From Proposition 6, an entrepreneur should not simply look for a deep-pocket investor to share control with. It is in the entrepreneur's interest to find an investor with similar background. Conceivably, a similar background should decrease as much as possible the level of asymmetry of information (small ϵ), increasing the value of sharing the control. Assuming that family members are likely to have common backgrounds, our model thus predicts that shared control should be pervasive in family businesses. Likewise, our model predicts that joint ventures, which are typical examples of shared control, are likely to be formed by firms in related industries.

Of course, outsiders are also likely to have a hard time agreeing with the value of a project. While these disagreements are harmless when their source is the value of the private benefits, they can be costly in a governance structure with unilateral control if their source is the value of the public cash flows. In this case, it is more difficult for the monitor to pressure the entrepreneur to overturn an investment decision (i.e., large ρ). Therefore, while large shareholders are likely to act as outside monitors in firms with investment opportunities that are hard for insiders to evaluate, large shareholders are likely to join control when the investment opportunities are hard for outsiders to evaluate.

Finally, Proposition 7 below tells us that shared control is more likely to be efficient in firms with large financing requirements.

Proposition 7 *An increase in the financing requirement that can be financed by a sale of control shares does not change the value of a firm under shared control. In contrast, an increase in the investment requirement reduces firm value under unilateral control. Therefore, the likelihood of shared control versus monitoring increases with the investment requirement.*

The intuition for Proposition 7 is fairly simple. Under unilateral control, a larger financing requirement forces the entrepreneur to sell more minority shares, making him internalize a smaller fraction of the firm's value. The entrepreneur's increased incentives to distort the investment policy makes it harder for a monitor to overturn inefficient decisions, reducing firm value under unilateral control. More interestingly, a larger financing requirement does not necessarily imply a reduction of the control stake under shared control. Indeed, rather than selling more minority shares to finance the larger investment, the entrepreneur may be able to raise extra funds, while keeping the control stake constant, by selling more shares to the

other controlling shareholder. Since the investment rule under shared control depends only on the total control stake, this financing strategy insulates firm value under shared control from the size of the financing requirement. Hence, unless the increase in the financing requirement makes it impossible for the project to be financed by a sale of control shares, a larger investment increases the chances that a large investor joins the control group rather than acting as an outside investor.

While this section compares the relative efficiency of shared control and monitoring across firms, our model also yields implications on the relative efficiency of these governance structures in countries with different judicial systems. As we show next, shared control is more likely to be efficient in countries that offer weak protection to minority shareholders.

B Governance law

Is there a relation between governance laws and the role that large shareholders play in corporate governance? To answer this question, we consider a change in governance laws (e.g., stricter disclosure requirements) that makes it harder for controlling shareholders to expropriate the value of minority shareholders, but keeps the total payoff of the project constant. For instance, with a project that increases private benefits, the improvement of governance laws reduces the project's impact on the private benefits to $b-g>0$, while increasing its impact on the public cash flows to $y+g$. This modelling of the improvement in the law allows us to keep the focus of the analysis on the link between firm value and the relative efficiency between shared control and monitoring.

It is not surprising that this improvement of governance law increases firm value under unilateral control. For one, the improvement increases the value of minority shares for any level of monitoring, letting the entrepreneur raise the financing requirement with fewer minority shares. The entrepreneur's equity stake thus rises, making him internalize firm value to a greater extent and moving the investment decision closer to the first best.

In contrast, laws that make it harder for firms to harm minority shareholders are much less important to the value of a firm under shared control. Intuitively, an improvement of the law is not crucial under shared control because the entrepreneur can more efficiently change the controlling group's preferences vis-à-vis public and private cash flows by properly choosing

the size of the minority stake. For example, the entrepreneur can replicate the impact of an improvement of governance laws by selling more shares to the new controlling shareholder and fewer shares to minority investors. By doing so, he substitutes control shares for minority shares, aligning the control group's incentives more closely to the minority investors, similarly to what an improvement of governance laws accomplishes under unilateral control.

It then follows that, under shared control, a weak legal protection of minority shareholders should not be as harmful to firm value as under unilateral control. Accordingly, large shareholders should directly participate in the management and the control stakes should be large in countries that offer weak protection to minority shareholders.

Proposition 8, below, formalizes the link between the governance role of large shareholders and the legal protection of minority shareholders.

Proposition 8 *Consider an improvement of governance laws that maintains the total payoff of the project constant while reducing the importance of private benefits relative to public cash flows by* $|g| < Min\{|b|, |y| - \frac{2\epsilon}{3(1-\underline{\alpha})}\}$. *Thus, under shared control, the improvement of the governance laws decreases the optimal controlling stake but it does not affect the value of the firm. In contrast, the improvement of governance laws increases firm value under unilateral control. Hence, shared control and large control stakes are more likely to prevail in countries with legal systems that offer weak protection to minority shareholders.*

The assumption $|g| < Min\{|b|, |y| - \frac{2\epsilon}{3(1-\underline{\alpha})}\}$ in Proposition 8 is a technical condition that limits the extent of the improvement of governance laws. As it turns out, this is a sufficient condition for ruling out improvements in the law that perfectly align the interests of all shareholders under unilateral and shared control. For example, if, contrary to the assumption, $|g|$ is smaller than $|b|$ and bigger than $|y|$, then the improvement in the law would make all shareholders agree with a project that previously would have increased private benefits ($b > 0$) at the cost of the minority shareholders ($y < 0$). Of course, improvements of laws that eliminate agency conflicts increase firm value, leaving no governance role for large shareholders. As La-Porta, Lopez-Silanes, Shleifer and Vishny (1998, 2000) show, however, governance laws are far from perfect, with a significant variation on the quality of laws protecting minority shareholders around the world.

Finally, Proposition 8 is consistent with recent research (e.g., La Porta, Lopez-De-Silanes

and Shleifer (1999)) that associates concentrated ownership and underdeveloped capital markets to judicial systems that offer weak protection to minority shareholders. Weak governance laws increase the relative efficiency of sharing control over unilateral control. Moreover, from Proposition 5, selling minority shares to finance investment requirements is suboptimal if the project increases public cash flows that cannot be diverted from minority shareholders. As such, our model predicts that, in countries that offer weak protection to minority shareholders, firms that go public are likely to have investment opportunities that facilitate the diversion of value from minority shareholders. The low capitalization of stocks in emerging markets, therefore, arises in our model as a self-selection phenomenon.

V Discussion and Extensions

A Shared control without side payments

In the bargaining game under shared control, we have assumed that the controlling shareholders can use cash or shares to make side payments. Yet, there are reasons for ruling out these side payments. First, the court may perceive them as evidence of bribes. Second, liquidity constrained controlling shareholders may be unwilling to dispose of their shares if the lowering of the equity stake makes them vulnerable to a control fight.[15] Accordingly, we explore below how our main results change in the polar case that no side payments are allowed.

In the absence of side payments, the firm will undertake a project only if it increases the expected payoff of both controlling shareholders; projects that are highly profitable for one (and only one) controlling shareholder will not be implemented if side-payments to convince the opposing controlling shareholder are not allowed. It is thus more difficult for the firm to undertake the project, further increasing the underinvestment problems of shared control.

The lack of side payments does not necessarily rule out shared control in firms with valuable investment opportunities, though. In these firms, controlling shareholders are unlikely to disagree if their payoffs from the project ($b_1 + \alpha_1^{sc} y$ and $b_2 + \alpha_2^{sc} y$) are positively correlated. After all, in this case, both controlling shareholders lose if the firm does not undertake the project. As it turns out, a positive correlation of the controlling shareholders' payoffs is more

[15] More formally, transfers of shares will not support the side payments if condition (4) of Lemma 1 does not hold.

likely if they have similar equity stakes and/or their private benefits are positively correlated.[16] Hence, our model predicts that controlling shareholders of firms with valuable investment opportunities hold similar equity stakes and have positively correlated private benefits.

B The stability of governance structures

In our model, we assume that the initial entrepreneur chooses the ownership structure to maximize firm value. But what guarantees that a given ownership structure is going to be stable to ex-post deviations? We show that the stability of a desired governance structure can be guaranteed by adding commonly used contractual clauses such as lock-up provisions, resale restrictions, anti-greenmail provisions, or supermajority rules. Moreover, we also show that the stability of the shared control structure is robust against ex-post dissolutions of the controlling group.

The stability of the monitor can be upset by the large shareholder deviating from the optimal monitoring stake. For example, the large outside shareholder may want to increase (decrease) its monitoring intensity by buying (selling) shares in the open market. Alternatively, the entrepreneur may attempt to repurchase the monitor's shares at a premium to avoid monitoring. Contractual provisions commonly used in corporate law can curtail these deviations. Specifically, the repurchase of target shares at premium can be controlled by the adoption of anti-greenmail provisions; the accumulation of a stake beyond a certain threshold can be controlled by standstill provisions or poison pills; and finally, the sale of shares in the open market can be prevented by lock-up provisions or resale restrictions.[17]

The stability of shared control can also be upset by ex-post changes in the control stake. A lower equity stake makes public cash flows less important to the control group. Thus, lowering the control stake decreases the stock price in anticipation of the control group's enhanced willingness to substitute private benefits for public cash flows. Despite the fall of the stock price, the control group may be better off with the lower equity stake because, in contrast to the private benefits, the fall of the stock price is shared with the minority shareholders. Hence,

[16] Conceivably, an even split of private benefits is more likely if the controlling shareholders hold similar equity stakes in the firm.

[17] Moreover, remember that we ruled out the possibility that the entrepreneur successfully bribes the monitor by offering side payments or other private benefits in exchange for a lower monitoring intensity.

the bias of the control group is toward reducing the ex-ante optimal control stake.

As it turns out, supermajority rules can prevent ex post incentives to reduce an ex-ante optimal control stake. To see how, suppose that an ex ante optimal ownership structure requires that a fraction α_{sc} of the shares be held by the controlling group. For a given voting structure, the stake α_{sc} is associated with a number of votes, say v. The initial shareholder can avoid ex-post incentives to reduce the controlling stake below α_{sc} by giving control to a group of investors who holds a fraction v of the votes. With this mechanism, which can be interpreted as a supermajority rule, the controlling shareholders cannot divest below v without bearing the risk of losing control.

Finally, the stability of shared control can be destroyed by dissolving the controlling group. In the absence of credit constraints, a buyout is a natural mechanism to eliminate bargaining problems that, although ex-ante optimal, are ex-post inefficient from the perspective of the controlling shareholders. In our model, the second controlling shareholder may have some debt capacity left after his purchasing of the control stake. If so, he may try to acquire full control by making an offer for the remaining control shares.

Consider then that a controlling shareholder can make a buyout offer between the time he learns his valuation of the project and the time the investment decision has to be made. We ask whether there is an incentive compatible direct mechanism that lets the controlling shareholders dissolve their partnership with probability one. If so, ex-post bargaining problems can be solved by a buyout, and the temporary presence of multiple controlling shareholders would not increase value.

Proposition 9 shows that the same asymmetry of information that prevents the controlling shareholders from agreeing with the investment decision may prevent them from dissolving the control group.

Proposition 9 *Assume that controlling shareholder j receives a signal s_i of the private benefits of controlling shareholder $i \neq j$, with $\underline{b}_i = s_i - \frac{\epsilon}{2}$, $\bar{b}_i = s_i + \frac{\epsilon}{2}$, and $\sum_{i=1}^{2}(\underline{b}_i + \alpha_i^{sc} y) < 0 < \sum_{i=1}^{2}(\bar{b}_i + \alpha_i^{sc} y)$. Then there is no ex-post efficient mechanism that dissolves the controlling group after the controlling shareholders have privately learned their valuations.*

Proposition 9 departs from Cramton, Gibbons, and Klemperer (1987), who argue that a partnership can always be efficiently dissolved if the equity holdings are evenly spread across

several partners. The way we model the private benefits of control is the key to explaining the difference in the results. In Cramton, Gibbons, and Klemperer, the value of the firm to each controlling shareholder is proportional to the fraction of shares that they own. As a result, in an evenly distributed ownership structure, the cost of extracting a truthful announcement of the firm's value decreases with the number of controlling shareholders. In our model, a controlling shareholder may have large private benefits of control in spite of an evenly distributed ownership structure.

A buyout is just one example of a coalition of shareholders that can overturn an existing control structure. In principle, any member of a controlling group may be co-opted to participate in a new coalition that aims to defeat the incumbent controlling group. Bennedsen and Wolfenzon (2000) argue that the size of the controlling stake is determined by these coalition games, which our model ignores. As it turns out, shareholders' agreements can prevent exclusions and defections that unravel the controlling group. For instance, voting agreements assure that each member of the agreement – the voting syndicate – will vote together. Voting syndicates, therefore, can prevent coalition games from unravelling governance structures based on shared control.

VI Conclusion

In the corporate control literature, large shareholders are usually assumed to monitor managers on behalf of all shareholders. Recent empirical evidence, however, suggests that large shareholders often share control in closely held firms, joint ventures, and public corporations in Western Europe. Such evidence is consistent with a vast literature on corporate law that does not view large shareholders as monitoring each other on behalf of minority shareholders. Instead, large shareholders are perceived as decision makers who seek to influence corporate decisions in a way that favors their personal agendas. Large shareholders, therefore, may play very different roles in corporate governance.

This paper argues that firm characteristics and governance laws determine the role that large shareholders play in corporate governance. In firms whose investment opportunities are hard for insiders to evaluate, sharing control creates bargaining problems that exacerbate the risk of corporate paralysis. Control should not be divided and, as in the corporate control

literature, monitoring by a large investor arises as the most efficient way to protect minority shareholders. In contrast, sharing control increases efficiency in countries with legal systems that offer weak protection to minority shareholders, in firms with projects that are hard for outsiders to evaluate, and when external financing requirements are large. In these cases, multiple large shareholders should participate in the firm's management, as assumed in the corporate law literature.

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Appendix

Proof of Proposition 1: We divide the proof in two cases: when the project destroys public cash flows ($y < 0$) and when it doesn't ($y \geq 0$).

Case 1: $y < 0$. The monitor will always try to block a project that destroys public cash flows, succeeding with a probability $m(y, \beta) = \min\{\frac{\beta}{\rho}|y|, 1\}$ that is non-decreasing on the amount of public cash flows that the project destroys, $|y|$, and on the equity stake of the monitor, β. Hence, the firm will undertake the project if it is in the interest of the entrepreneur ($b + \alpha_{uc} y > 0$) and the monitor fails to block it. Defining $\hat{b}(\alpha_{uc}) = E\left[b | \left(b + \alpha_{uc} y\right) sign(y) < 0\right]$ and using that $sign(y) = -1$ for $y < 0$, we can write the value of the investment opportunity as function of the monitor's, β, as

$$V(\beta) = \left[\left(y + \hat{b}(\alpha_{uc})\right)\left(1 - m(y, \beta)\right) - \frac{\rho m(y, \beta)^2}{2}\right] P(b > -\alpha_{uc} y). \tag{9}$$

For $\beta \in [0, \frac{\rho}{|y|}]$, the probability of blocking the project is $m(y, \beta) = \frac{\beta}{\rho}|y|$ and $V(\beta) = \left[\left(y + \hat{b}(\alpha_{uc})\right)\left(1 - \frac{\beta}{\rho}|y|\right) - \frac{\rho\left(\frac{\beta}{\rho}|y|\right)^2}{2}\right] P(b > -\alpha_{uc} y)$ is a quadratic and strictly concave function of β. The derivative of $V(\beta)$ with respect to the monitor's stake is

$$\frac{dV(\beta)}{d\beta} = \left[-\frac{1}{\rho}|y|\left(\hat{b}(\alpha_{uc}) + y\right) - \frac{1}{\rho}\beta|y|^2\right] P(b > -\alpha_{uc} y). \tag{10}$$

It then follows from equation (10) that the value of the investment opportunity strictly decreases with β in the interval $[0, \frac{\rho}{|y|})$, if $\hat{b}(\alpha_{uc}) + y \geq 0$. Since $V(\beta)$ is continuous in β in the interval $[0, \frac{\rho}{|y|}]$ and $V(\beta) = -\frac{\rho}{2} P(b > -\alpha_{uc} y)$ for $\beta \geq \frac{\rho}{|y|}$, we conclude that the optimal equity stake of the monitor is $\beta^*(\alpha_{uc}) = 0$ if $\hat{b} + y \geq 0$. In contrast, $\hat{b}(\alpha_{uc}) + y < 0$ implies that $\frac{dV(0)}{d\beta} > 0$ and monitoring increases firm value valuable for any β sufficiently small. Hence, $\beta^*(\alpha_{uc}) = 0$ if and only if $\hat{b}(\alpha_{uc}) + y \geq 0$, or equivalently, $(\hat{b}(\alpha_{uc}) + y) sign(y) \leq 0$.

To obtain $\beta^*(\alpha_{uc})$ when $(\hat{b}(\alpha_{uc}) + y) sign(y) > 0$, let us ignore for a while the constraint $\beta^*(\alpha_{uc}) \leq 1 - \alpha_{uc}$, focusing instead on the program $\max_{\beta \in [0, \frac{\rho}{|y|}]} V(\beta)$, which can be viewed as a relaxed program because $V(\beta)$ does not vary with β in the interval $[\frac{\rho}{|y|}, \infty]$. From the argument in the previous paragraph, this relaxed program implies a strictly positive equity stake, call it $\bar{\beta}(\alpha_{uc})$, if and only if $(\hat{b}(\alpha_{uc}) + y) sign(y) > 0$, which is the case that we analyze

now.

Concavity of $V(\beta)$ in $[0, \frac{\rho}{|y|}]$ implies an optimum at the corner, $\bar{\beta}(\alpha_{uc}) = \frac{\rho}{|y|}$, if and only if $\frac{dV(\frac{\rho}{|y|},\alpha_{uc})}{d\beta} \geq 0$. This latter inequality implies $\hat{b}(\alpha_{uc}) + y \leq -\rho$, or equivalently, $(\hat{b}(\alpha_{uc}) + y)sign(y) \geq \rho$. Taking into account the constraint $\beta \leq 1 - \alpha_{uc}$, concavity of $V(\beta, \alpha_{uc})$ in $[0, \frac{\rho}{|y|}]$ implies that the optimum of the original problem is $\beta^*(\alpha_{uc}) = \min\{1 - \alpha_{uc}, \frac{\rho}{|y|}\}$ if $(\hat{b}(\alpha_{uc}) + y)sign(y) \leq \rho$. For $\rho > (\hat{b}(\alpha_{uc}) + y)sign(y) > 0$, the necessary first order condition is not satisfied at zero and $\frac{\rho}{|y|}$, and an interior solution obtains. This interior optimum solves $\frac{dV(\beta)}{d\beta} = 0$ in equation (10), yielding $\bar{\beta}(\alpha_{uc}) = -\frac{1}{|y|}\left(\hat{b}(\alpha_{uc}) + y\right)$, which is lower than $\frac{\rho}{|y|}$ if $\hat{b}(\alpha_{uc}) + y > -\rho$, or equivalently, $\rho > (\hat{b}(\alpha_{uc}) + y)sign(y)$. Once more, concavity of $V(\beta)$ implies that the optimum of the original problem is $\beta^* = \min\{1 - \alpha_{uc}, -\frac{1}{|y|}(\underline{b} + y)\}$ if and only if $\rho > (\hat{b} + y)sign(y) > 0$

<u>Case 2: $y \geq 0$</u>. If the project increases public cash flows, the firm will undertake it if either it is in the interest of the entrepreneur $(b + \alpha_{uc}y > 0)$ or it isn't but the monitor succeeds to overturn an entrepreneur's decision to pass the project. Using that $sign(y) = 1$ for $y \geq 0$, we can write the value of the investment opportunity, $V(\beta)$, as

$$(E\left[b|b > -\alpha_{uc}y\right] + y)P(b > -\alpha_{uc}y) + \left[(\hat{b}(\alpha_{uc}) + y)sign(y)m(y,\beta) - \frac{\rho m(y,\beta)^2}{2}\right]P(b < -\alpha_{uc}y),$$

which is continuous in β.

For $\beta \in [0, \frac{\rho}{|y|}]$, $m(y,\beta) = \frac{\beta}{\rho}|y|$ which implies that $V(\beta)$ is a quadratic and strictly concave function of β. Since $V(\beta)$ does not change with β for $\beta > \frac{\rho}{|y|}$, a necessary and sufficient condition for $\beta^* = 0$ is $\frac{dV_{uc}(0)}{d\beta} \leq 0$, or equivalently, $\hat{b}(\alpha_{uc}) + y \leq 0$.

For $\hat{b}(\alpha_{uc}) + y > 0$, ignore for a while the constraint $\beta \leq 1 - \alpha_{uc}$, focusing on the relaxed program $\max_{\beta \in [0, \frac{\rho}{|y|}]} V(\beta)$, which, from the argument in the previous paragraph, implies a strictly positive solution, $\widehat{\beta}(\alpha_{uc}) > 0$. Concavity of $V(\beta)$ in $[0, \frac{\rho}{|y|}]$ implies an optimum at the upper corner if and only if $\frac{dV(\frac{\rho}{|y|})}{d\beta} \geq 0$, or equivalently, $\hat{b}(\alpha_{uc}) + y > \rho$. For $0 < \hat{b}(\alpha_{uc}) + y < \rho$, we have an interior optimum at $\widehat{\beta}(\alpha_{uc}) = \frac{1}{|y|}\left(\hat{b}(\alpha_{uc}) + y\right)$, which is the monitor's stake that solves $\frac{dV(\beta)}{d\beta} = 0$. Finally, as in case 1, concavity of $V(\beta)$ implies that the solution of the original program, which takes into account the constraint $\beta \leq 1 - \alpha_{uc}$, is $\beta^*(\alpha_{uc}) = \min\{1 - \alpha_{uc}, \widehat{\beta}(\alpha_{uc})\}$.

Q.E.D.

Proof of Proposition 2: We prove the proposition in two steps. First, we show that an increase in the equity stake of the entrepreneur moves his incentives closer to the first best, increasing the value of the investment opportunity, $V(\alpha_{uc})$, accordingly. In the second step we characterize the optimal equity stake of the entrepreneur.

Step 1: To show that $V(\alpha_{uc})$ increases with α_{uc}, assume first that the project destroys public cash flows ($y < 0$) and that it is inefficient to undertake the project ($b + y < 0$). Then an increase in the equity stake of the entrepreneur reduces his willingness to invest because his payoff from the inefficient project, $b + \alpha_{uc}y$, decreases with α_{uc} when $y < 0$. In turn, if the project is efficient, then $b+y \geq 0$ and $y < 0$ imply that $b+\alpha_{uc}y \geq 0$ for any $\alpha_{uc} \in [0, 1]$. Hence, the entrepreneur has the correct incentives to undertake the project, regardless of his equity stake. Assume now that the project increases public cash flows ($y \geq 0$) and that it is efficient to undertake the project ($b+y \geq 0$). Thus, a larger equity stake increases $b+\alpha_{uc}y$, increasing the entrepreneur's incentive to undertake the project. In turn, if the project is inefficient, then $b + y < 0$ and $y \geq 0$ imply that $b + \alpha_{uc}y = b + y - (1 - \alpha_{uc})y < 0$. So, the entrepreneur has no incentive to undertake the project, regardless of his equity stake. We thus conclude that a larger equity stake can only improve the incentives of the entrepreneur. Hence, for a larger equity stake to reduce firm value, it must be the case that it makes monitoring less efficient. This cannot happen, though, because the optimal level of monitoring, $m(y, \beta) = \min\{\frac{\beta}{\rho}|y|, 1\}$, does not depend on the entrepreneur's equity stake.

Step 2: In our model, the value-maximizing ownership structure must raise enough funds to finance the investment requirement I. To formalize this constraint, recall that the value of the investment opportunity, $V(\alpha_{uc})$, consists of two parts: the expected public cash flows and the expected private benefits. If we call $b(\alpha_{uc})$ the expected private benefits induced by α_{uc}, then we can write the expected public cash flows as $V(\alpha_{uc}) - b(\alpha_{uc})$, and the part of the firm's value that goes to the outside shareholders is $(1 - \alpha_{uc})(V_0 + I + V(\alpha_{uc}) - b(\alpha_{uc}))$. As a result, we can write the entrepreneur's problem as

$$\begin{aligned} &\max_{\alpha_{uc} \in [\underline{\alpha}, 1]} V(\alpha_{uc}) \\ \text{subject to } (1 - \alpha_{uc})(V_0 + I + V(\alpha_{uc}) - b(\alpha_{uc})) &\geq I, \end{aligned}$$

where $\underline{\alpha}$ is the smallest equity stake that gives control to the entrepreneur.

Since $P(b < -\alpha_{uc}y)$ is a continuous function of α_{uc}, so is the expected public cash flow. Hence, $V(\alpha_{uc}) - b(\alpha_{uc})$ is continuous, implying that at the optimum, the constraint must be binding, or else a small increase of α_{uc} would raise the objective function without violating the constrain. Assuming that selling up to the minimum number of shares that retain control, $\alpha_{uc} = \underline{\alpha}$, allows the entrepreneur to finance the investment requirement I, then continuity of $V(\alpha_{uc}) - b(\alpha_{uc})$ implies that there is an equity stake that satisfies the constraint with equality because $(1-\alpha_{uc})(V_0 + I + V(\alpha_{uc}) - b(\alpha_{uc})) < I$ for $\alpha_{uc} = 1$. It then follows that the optimal ownership structure is $(\alpha^*_{uc}, \beta^*(\alpha^*_{uc}), 1-\alpha^*_{uc}-\beta^*(\alpha^*_{uc}))$, where α^*_{uc} is the largest equity in $[\underline{\alpha}, 1]$ that solves $(1-\alpha_{uc})(V_0 + I + V(\alpha_{uc}) - b(\alpha_{uc})) = I$.

Q.E.D.

Proof of Lemma 1: Let $V_0 + I$ be the value of equity that arises from the assets in place (i.e., excluding the project). In case the project is undertaken, the public cash flow changes by y. Since both controlling shareholders know y, they agree that controlling shareholder j's wealth changes by $\alpha(y+V_0+I) \geq 0$ if he receives a fraction α of the equity stake of controlling shareholder i. Given that $\underline{\alpha}$ suffices for any controlling shareholder to retain control, controlling shareholder i can offer up to $(\alpha_i^{sc} - \underline{\alpha})(y + V_0 + I)$ as a side payment in any bargaining game.

Now, let $\bar{V}_i \equiv \bar{b}_i + \alpha_i^{sc}y$ be the controlling shareholder i's maximum valuation for the project, with $\bar{b}_i$ an upper bound on the private benefits of controlling shareholder i. Clearly, controlling shareholder i will not announce a valuation that implies a transfer larger than $\bar{V}_i$. In equilibrium, therefore, the transfer payment is bounded by $\bar{b}_i + \alpha_i^{sc}y$. Hence, a sufficient condition for transfers of shares to implement any incentive-compatible bargaining mechanism is that, with probability one, any controlling shareholder can afford the upper bound on the transfer payment by transferring an amount of shares that does not lead to a loss of control. Formally,

$$(\alpha_i^{sc} - \underline{\alpha})(y + V_0 + I) \geq \bar{b}_i + \alpha_i^{sc}y, \text{ for } i \in \{1, 2\},$$

which implies condition (4) when $\alpha_i^{sc} > \underline{\alpha}$ for $i \in \{1, 2\}$. Q.E.D.

Proof of Proposition 3: From Lemma 1, condition 4 implies that the controlling shareholders can honor any incentive compatible payment that may arise from the Chatterjee-Samuelson mechanism by transferring their shares. Thus, let $V_i(x_i, s_1, s_2)$ be the best announcement of controlling shareholder $i \in \{1, 2\}$, where $x_i = b_i + \alpha_i^{sc}y$ is controlling shareholder i's valuation

of the project. To simplify the notation, we will henceforth omit the arguments s_1 and s_2 in $V_i(x_i, s_1, s_2)$. In addition, let $u_i \equiv s_i + \frac{\epsilon}{2} + \alpha_i^{sc} y$ and $l_i \equiv s_i - \frac{\epsilon}{2} + \alpha_i^{sc} y$ be, respectively, the upper and lower bounds of controlling shareholder i's valuation of the project given the signal s_i and the public return y.

Standard arguments in the mechanism design literature show that, in any Bayesian equilibrium, $V_i(x_i)$ increases with the valuation of the project x_i. Moreover, Lemma 3, below, shows that the equilibrium announcements must satisfy a system of differential equations.

Lemma 3 *In any Bayesian equilibrium in which the best policies $V_1(x_1)$ and $V_2(x_2)$ are differentiable, the following linked differential equations hold:*

$$V_1^{-1}\left(-V_2\left(x_2\right)\right) + V_2\left(x_2\right) = \frac{1}{2}\frac{\left(1 - F_2(x_2)\right)}{f_2(x_2)} V_2'\left(x_2\right) \tag{11}$$

$$V_2^{-1}\left(-V_1\left(x_1\right)\right) + V_1\left(x_1\right) = \frac{1}{2}\frac{\left(1 - F_1(x_1)\right)}{f_1(x_1)} V_1'\left(x_1\right), \tag{12}$$

where $F_i(x_i)$ and $f_i(x_i)$ are, respectively, the distribution and the density of $x_i \equiv b_i + \alpha_i^{sc} y$ conditioned on the signals s_1 and s_2.

Proof. Given V_1, the minimal announcement $V_2^\star$ that implies the undertaking of the project must satisfy $V_1 + V_2^\star = 0$. Since $V_2(x_2)$ increases with x_2, $V_2^\star$ induces a cutoff for the valuation x_2 of the second controlling shareholder: $V_1 + V_2(x_2^\star) = 0 \Rightarrow x_2^\star = V_2^{-1}(-V_1)$. The expected payoff of the initial shareholder given an announcement V_1 and a valuation x_1 is then equal to

$$\Pi_1(V_1, x_1) = \int_{V_2^{-1}(-V_1)}^{u_2} [x_1 - \frac{1}{2}(V_1 - V_2(x_2))] f_2(x_2) dx_2.$$

Assume first that any small perturbation from $V_1(x_1)$ affects the probability that the project will be undertaken. Then V_1 maximizes the expected payoff off the initial shareholder if and only if

$$\begin{aligned}
\frac{\partial \Pi_1(V_1, x_1)}{\partial V_1} &= -\left(x_1 - V_1\right) f_2(V_2^{-1}\left(-V_1\right)) \frac{dV_2^{-1}\left(-V_1\right)}{dV_1} - \frac{1}{2}\int_{V_2^{-1}(-V_1)}^{u_2} f_2(x_2) dx_2 = \\
&= \frac{\left(x_1 - V_1\right) f_2(V_2^{-1}\left(-V_1\right))}{V_2'\left(V_2^{-1}\left(-V_1\right)\right)} - \frac{1}{2}\left(1 - F_2(V_2^{-1}\left(-V_1\right))\right) = 0
\end{aligned}$$

Since $x_2^\star = V_2^{-1}(-V_1)$, the above equation can be rewritten as

$$(x_1 + V_2(x_2^\star)) - \frac{1}{2}\frac{(1 - F_2(x_2^\star))}{f_2(x_2^\star)} V_2'(x_2^\star) = 0 \tag{13}$$

If V_1 is an optimal response, $V_1 + V_2(x_2^\star) = 0$ implies that the initial shareholder's valuation, $x_1^\star$, that led to V_1 solves $V_1(x_1^\star) + V_2(x_2^\star) = 0$, which implies $x_1^\star = V_1^{-1}(-V_2(x_2^\star))$. Plugging $x_1^\star$ into equation (13) yields equation (11):

$$V_1^{-1}(-V_2(x_2^\star)) + V_2(x_2^\star) - \frac{1}{2}\frac{(1 - F_2(x_2^\star))}{f_2(x_2)} V_2'(x_2^\star) = 0.$$

The proof that equation (12) holds when any small perturbation of V_2 affects the chances that the project will be undertaken is analogous.

Suppose now that a perturbation of $V_1(x_1)$ does not change the probability that the project will be undertaken. In particular, x_1 may be so large that, given $V_1(.)$ and $V_2(.)$, the project will be undertaken regardless of the announcement of the second controlling shareholder. In this case, there is a $x_2^\star \in [l_2, u_2)$ such that $V_1(l_1) + V_2(x_2^\star) = 0$. Still, the differential equation associated with the initial shareholder's announcement remains unchanged, as we show below.

$$\Pi_1^\star(V_1, x_1) = \int_{V_2^{-1}(-V_1)}^{x_2^\star} [x_1 - \frac{1}{2}(V_1 - V_2(x_2))] f_2(x_2) dx_2 + \int_{x_2^\star}^{u_2} [x_1 - \frac{1}{2}(V_1 - V_2(x_2^\star))] f_2(x_2) dx_2.$$

$$\begin{aligned}
\frac{\partial \Pi_1^\star(V_1, x_1)}{\partial V_1} &= -(x_1 - V_1) f_2(V_2^{-1}(-V_1)) \frac{dV_2^{-1}(-V_1)}{dV_1} - \frac{1}{2}\int_{V_2^{-1}(-V_1)}^{x_2^\star} f_2(x_2) dx_2 - \frac{1}{2}\int_{x_2^\star}^{u_2} f_2(x_2) dx_2 \\
&= \frac{(x_1 - V_1) f_2(V_2^{-1}(-V_1))}{V_2'\left(V_2^{-1}(-V_1)\right)} - \frac{1}{2}\left(1 - F_2(V_2^{-1}(-V_1))\right) = 0,
\end{aligned}$$

which is the first order condition that yields equation (11).

A second boundary case happens when the valuation of a controlling shareholder is so low that it blocks the project regardless of the announcement of the other controlling shareholder. To characterize this situation, let $x_1^{\star\star}$ be the minimum valuation of the initial shareholder when the second controlling shareholder's announcement is as large as possible, that is, $V_2(u_2)$. Then, $V_1(x_1^{\star\star}) + V_2(u_2) = 0$, and the project will not be undertaken for any $x_1 < x_1^{\star\star}$. Since the

project will not be undertaken, the announcement of the initial shareholder is irrelevant. It is then optimal to set $V_1(x_1)$ satisfying equation (11) for $x_1 < x_1^{\star\star}$ with the understanding that the project will not be undertaken. Similarly, $V_1(u_1) + V_2(x_2^{\star\star}) = 0$ implies that the project will not be undertaken for $x_2 < x_2^{\star\star}$, and we can assign $V_2(x_2)$ satisfying equation (12).

Q.E.D.

The proof of the Proposition follows from equations (11) and (12). Conditioned on s_i, b_i is uniformly distributed in the interval $[l_i - \alpha_i y, u_i - \alpha_i y]$. Standard computations then show that, conditioned on s_i, the hazard rate of the random variable $x_i \equiv b_i + \alpha_i y$ is $\frac{(1-F_i(x_i))}{f_i(x_i)} = u_i - x_i$. Plugging this hazard ratio into equations (11) and (12) yields

$$\begin{aligned} V_1^{-1}\left(-V_2\left(x_2\right)\right) &= \frac{1}{2}\left(u_2 - x_2\right) V_2'\left(x_2\right) - V_2\left(x_2\right) \\ V_2^{-1}\left(-V_1\left(x_1\right)\right) &= \frac{1}{2}\left(u_1 - x_1\right) V_1'\left(x_1\right) - V_1\left(x_1\right). \end{aligned}$$

Assume now that there is a solution for the above system of differential equations that is linear in the valuation x_i, that is, $V_1(x_1) = Ax_1 + B$ and $V_2(x_2) = Cx_2 + D$. Thus

$$\begin{aligned} V_1^{-1}\left(-\left(Cx_2 + D\right)\right) &= \frac{1}{2}\left(u_2 - x_2\right) C - \left(Cx_2 + D\right) \\ V_2^{-1}\left(-\left(Ax_1 + B\right)\right) &= \frac{1}{2}\left(u_1 - x_1\right) A - \left(Ax_1 + B\right). \end{aligned}$$

Plugging $V_1^{-1}(-(Cx_2 + D)) = \frac{-(Cx_2+D)-B}{A}$, $V_2^{-1}(-(Ax_1 + B)) = \frac{-(Ax_1+B)-D}{C}$, and collecting terms gives us

$$\begin{aligned} (\frac{-C}{A} + \frac{3}{2}C)x_2 - \frac{B+D}{A} &= \frac{1}{2}Cu_2 - D \qquad (14) \\ (\frac{-A}{C} + \frac{3}{2}A)x_1 - \frac{B+D}{C} &= \frac{1}{2}Au_1 - B. \end{aligned}$$

This system of equations must hold for all values of x_1 and x_2, which requires that $-\frac{C}{A} + \frac{3}{2}C = 0 \Rightarrow A = \frac{2}{3}$ and $-\frac{A}{C} + \frac{3}{2}A = 0 \Rightarrow C = \frac{2}{3}$. Plugging $A = \frac{2}{3}$ and $C = \frac{2}{3}$ into the system of equations (14) obtains

$$\begin{aligned} -\left(B + D\right) &= \frac{2}{9}u_2 - \frac{2}{3}D \\ -\left(B + D\right) &= \frac{2}{9}u_1 - \frac{2}{3}B \end{aligned}$$

Solving this system of equation gives us $D = -\frac{1}{4}u_1 + \frac{1}{12}u_2$, $B = \frac{1}{12}u_1 - \frac{1}{4}u_2$. The optimal announcements of the controlling shareholders as a function of their valuations are then

$$\begin{aligned} V_1(x_1) &= \frac{2}{3}x_1 + \frac{1}{12}u_1 - \frac{1}{4}u_2, \\ V_2(x_2) &= \frac{2}{3}x_2 + \frac{1}{12}u_2 - \frac{1}{4}u_1. \end{aligned}$$

From above, $V_1(x_1) + V_2(x_2) \geq 0$ is equivalent to $\frac{2}{3}x_1 + \frac{1}{12}u_1 - \frac{1}{4}u_2 + \frac{2}{3}x_2 + \frac{1}{12}u_2 - \frac{1}{4}u_1 \geq 0$, which implies $x_1 + x_2 \geq \frac{1}{4}(u_1 + u_2)$. Plugging $x_i = b_i + \alpha_i y$ and $u_i = s_i + \frac{\epsilon}{2} + \alpha_i y$ into this last inequality yields $b_1 + b_2 + (\alpha_1 + \alpha_2)y \geq \frac{1}{4}\{s_1 + s_2 + (\alpha_1 + \alpha_2)y + \epsilon\}$. And using $s_i = b_i + \epsilon_i$, we can rewrite this decision rule as $b_1 + b_2 + (\alpha_1^{sc} + \alpha_2^{sc})y \geq \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$.

We now show that the investment rule holds in the boundaries as well. If, for instance, x_2 is large enough to imply the investment regardless of the announcement of the initial shareholder, Lemma 3 shows that $V_2 = V_2(x_2^\star)$ for $x_2 \geq x_2^\star$, where $x_2^\star$ solves $V_1(l_1) + V_2(x_2^\star) = 0$. Thus, $V_1(x_1) + V_2(x_2) \geq V_1(x_1) + V(x_2^\star) \geq 0 \Rightarrow V_1(x_1) + V_2(x_2) \geq 0 \Rightarrow b_1 + b_2 + (\alpha_1 + \alpha_2)y \geq \frac{1}{4}\{s_1 + s_2 + (\alpha_1 + \alpha_2)y + \epsilon\}$. Conversely, $V_1(x_1) + V_2(x_2) < 0 \Rightarrow V_1(x_1) + V(x_2^\star) < 0$, which is not consistent with the assumption that the investment will happen for $x_2 \geq x_2^\star$ with probability one. Therefore, investing if and only if $b_1+b_2+(\alpha_1+\alpha_2)y \geq \frac{1}{4}\{s_1+s_2+(\alpha_1+\alpha_2)y+\epsilon\}$ is optimal when some realization of x_2 implies the undertaking of the project regardless of the realization of x_1. The same argument can be used to show that $b_1 + b_2 + (\alpha_1 + \alpha_2)y \geq \frac{1}{4}\{s_1 + s_2 + (\alpha_1 + \alpha_2)y + \epsilon\}$ characterizes the optimal investment rule when a large x_1 implies the undertaking of the project regardless of x_2. Finally, Lemma 3 shows that $V_1(x_1)$ and $V_2(x_2)$ are optimal announcements when x_1 and x_2 are such that the probability that the project will be undertaken is zero. Moreover, $V_1(x_1) + V_2(x_2) < 0$ in these cases. Hence, the investment rule $b_1 + b_2 + (\alpha_1 + \alpha_2)y \geq \frac{1}{4}\{s_1 + s_2 + \alpha y + \epsilon\}$ still applies, and the proof follows by plugging $s_i = b_i + \epsilon_i$ into this inequality. Q.E.D.

Proof of Lemma 2: Define $K_\epsilon(x) = \frac{1}{\epsilon^2}\int_{-\epsilon/2}^{\epsilon/2}\int_{-\epsilon/2}^{\epsilon/2} I(\epsilon_1 + \epsilon_2 > x))d\epsilon_1 d\epsilon_2$. By construction, $K_\epsilon(x) = 0$ for all $x \geq \epsilon$, and for all $x \leq -\epsilon$, $K_\epsilon(x) = 1$. Moreover, in the interval $[0, \epsilon]$, $K_\epsilon(x) = \frac{1}{\epsilon^2}\int_{x-\epsilon/2}^{\epsilon/2}\left(\frac{\epsilon}{2} - (x - \epsilon_2)\right)d\epsilon_2 = \frac{(\epsilon-x)^2}{2\epsilon^2}$, and in the interval $[-\epsilon, 0]$, $K_\epsilon(x) = \frac{1}{\epsilon^2}\int_{x+\epsilon/2}^{\epsilon/2}\epsilon d\epsilon_2 + \frac{1}{\epsilon^2}\int_{-\epsilon/2}^{x+\epsilon/2}\left(\frac{\epsilon}{2} - (x - \epsilon_2)\right)d\epsilon_2 = 1 - \frac{(\epsilon+x)^2}{2\epsilon^2}$.

One can then easily check that $\Gamma(\frac{x}{\epsilon}) = K_\epsilon(\epsilon - 3x)$, where $\Gamma(x)$ is the function defined in the

statement of the Lemma. Therefore, since $b+\alpha_{sc}y > \frac{1}{3}(\epsilon-\epsilon_1-\epsilon_2)$ is equivalent to $\epsilon_1+\epsilon_2 > \epsilon - 3(b+\alpha_{sc}y) = x$, we have that $\Gamma(\frac{b+\alpha_{sc}y}{\epsilon}) = \frac{1}{\epsilon^2}\int_{-\epsilon/2}^{\epsilon/2}\int_{-\epsilon/2}^{\epsilon/2} I(b+\alpha_{sc}y > \frac{1}{3}(\epsilon-\epsilon_1-\epsilon_2))d\epsilon_1 d\epsilon_2$, which concludes the proof. Q.E.D.

Proof of Proposition 5: The paragraph following the statement of the proposition shows that if $P(b+y \in \left(0, \frac{2\epsilon}{3}\right)) = 0$, then shared control with $\alpha^*_{sc} = 1$ achieves the first-best. Assume now that $P(b+y \in \left(0, \frac{2\epsilon}{3}\right)) > 0$. From Lemma 2, $V_{sc}(\alpha_{sc}, y) = \int (b+y)\,\Gamma(\frac{b+\alpha_{sc}y}{\epsilon})f(b)db$ and since $\Gamma(x)$ is continuous and differentiable, so is $V_{sc}(\alpha_{sc}, y)$. An interior optimum for the control stake must thus satisfy $\frac{dV_{sc}(\alpha_{sc},y)}{d\alpha_{sc}} = \frac{y}{\epsilon}\int (b+y)\,\Gamma'(\frac{b+\alpha_{sc}y}{\epsilon})f(b)db = 0$ with $\frac{dV_{sc}(1,y)}{d\alpha_{sc}} \geq 0$ for a corner solution at $\alpha_{sc} = 1$. To evaluate $\frac{dV_{sc}}{d\alpha_{sc}}$, we consider two cases ($y < 0$ and $y > 0$) after noting that $\Gamma'(x)$ is a continuous and *positive* triangular function that is equal to

$$\Gamma'(x) = \begin{cases} 0 & \text{if } x \geq \frac{2}{3} \\ 6-9x & \text{if } x \in \left[\frac{1}{3}, \frac{2}{3}\right] \\ 9x & x \in \left[0, \frac{1}{3}\right] \\ 0 & \text{if } x \leq 0. \end{cases}$$

<u>Case 1: $y < 0$</u>. Continuity of $V_{sc}(\alpha_{sc}, y)$ and items (1.a) and (1.b) imply that the optimum α^*_{sc} lies in open interval $(1-\frac{2\epsilon}{3|y|}, 1)$

(1.a) At $\alpha_{sc} = 1$, we show that $\frac{dV_{sc}}{d\alpha_{sc}} < 0$: Indeed, $\left.\frac{dV_{sc}}{d\alpha_{sc}}\right|_{\alpha_{sc}=1} = \frac{y}{\epsilon}\int (b+y)\,\Gamma'(\frac{b+y}{\epsilon})f(b)db < 0$, because $P(b+y \in \left(0, \frac{2\epsilon}{3}\right)) > 0$ while $\Gamma'(\frac{b+y}{\epsilon})$ is strictly positive in the region $b+y \in \left(0, \frac{2\epsilon}{3}\right)$ and zero elsewhere.

(1.b) At any $\alpha_{sc} \leq 1-\frac{2\epsilon}{3|y|}$, we show that $\frac{dV_{sc}}{d\alpha_{sc}} \geq 0$: Since $\Gamma'(\frac{b+\alpha_{sc}y}{\epsilon})$ is strictly positive in the region $b+\alpha_{sc}y \in \left(0, \frac{2\epsilon}{3}\right)$ and zero elsewhere, it suffices to show that $\frac{dV_{sc}(\alpha_{sc},y)}{d\alpha_{sc}} = \frac{y}{\epsilon}\int (b+y)\,\Gamma'(\frac{b+\alpha_{sc}y}{\epsilon})f(b)db \geq 0$ for $b+\alpha_{sc}y \in \left(0, \frac{2\epsilon}{3}\right)$. But this is true because, for all $b+\alpha_{sc}y \in (0, \frac{2\epsilon}{3})$, we have that $b+y \in ((1-\alpha_{sc})\,y, \frac{2\epsilon}{3}+(1-\alpha_{sc})\,y)$, with $\frac{2\epsilon}{3}+(1-\alpha_{sc})\,y \leq 0$ for any $\alpha_{sc} \leq 1-\frac{2\epsilon}{3|y|}$.

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<u>Case 2: $y \geq 0$</u>. We show that $V_{sc}(\alpha_{sc}, y) = \int (b+y)\,\Gamma(\frac{b+\alpha_{sc}y}{\epsilon})f(b)db$ is non-decreasing in α_{sc}. Because $\Gamma(x)$ is a non-decreasing function of x, $\Gamma(\frac{b+\alpha_{sc}y}{\epsilon})$ is non-decreasing in α_{sc} when $y > 0$. Since $b+y < 0$ implies $b+\alpha_{sc}y < 0$ for any $y \geq 0$ and $\alpha_{sc} \in [0,1]$, by construction of $\Gamma(.)$, $\Gamma(\frac{b+\alpha_{sc}y}{\epsilon}) = 0$ and so $V_{sc}(\alpha_{sc}, y) = \int (b+y)\,\Gamma(\frac{b+\alpha_{sc}y}{\epsilon})f(b)db = 0$. We can thus assume that

$(b+y)\,f(b) \geq 0$, implying that $(b+y)\,\Gamma(\frac{b+\alpha_{sc}y}{\epsilon})f(b)$ is non-decreasing in α_{sc}. Q.E.D.

Proof of Proposition 6: Let the value of the investment opportunity under shared control be $V_{sc}(\epsilon, \alpha^*_{sc}(\epsilon)) = \int (b+y)\,\Gamma(\frac{b+\alpha^*_{sc}(\epsilon)y}{\epsilon})f(b)db$, where $\alpha^*_{sc}(\epsilon)$ is the optimal control stake given $\epsilon > 0$. We analyze two cases.

<u>Case 1: $y \geq 0$</u>. From Proposition 5, $\alpha^*_{sc}(\epsilon) = 1$ when $y \geq 0$, implying that $V_{sc}(\epsilon, \alpha^*_{sc}(\epsilon)) = \int (b+y)\,\Gamma(\frac{b+y}{\epsilon})f(b)db$. By construction (see the statement of Lemma 2), $\Gamma(z)$ is a nondecreasing function of z with $\Gamma(z) = 0$ for $z < 0$. Hence, $\Gamma(\frac{b+y}{\epsilon})$ is a nonincreasing function of ϵ for all $b+y \geq 0$, implying that $\int (b+y)\,\Gamma(\frac{b+y}{\epsilon})f(b)db$ is nonincreasing on ϵ when $b+y \geq 0$. For $b+y < 0$, the definition of $\Gamma(.)$ implies $\Gamma(\frac{b+y}{\epsilon}) = 0$. Hence, $\int (b+y)\,\Gamma(\frac{b+y}{\epsilon})f(b)db = 0$ for any $b+y < 0$ and $V_{sc}(\epsilon, \alpha^*_{sc}(\epsilon))$ is once again nonincreasing in the disagreement measure ϵ.

<u>Case 2: $y < 0$</u>. Our goal is to show that, at any point $\epsilon = \epsilon_0$, we have $\frac{dV_{sc}(\epsilon,\alpha^*_{sc}(\epsilon))}{d\epsilon}_{|\epsilon=\epsilon_0} \leq 0$. Let $\alpha^*_{sc}(\epsilon)$ be the optimal solution and let the function $\alpha(\epsilon)$ be implicitly defined by $\Gamma(\frac{(-y)+\alpha(\epsilon)y}{\epsilon}) = \Gamma(\frac{(-y)+\alpha^*_{sc}(\epsilon_0)y}{\epsilon_0})$. Note that $\alpha(\epsilon_0) = \alpha^*_{sc}(\epsilon_0)$, and that $\alpha(\epsilon)$ exists and is uniquely determined by the monotonicity of $\Gamma(\cdot)$. From Proposition 5, $y < 0$ implies that $\alpha^*_{sc}(\epsilon)$ is an interior solution. Hence, by the the envelope theorem, $\frac{\partial V_{sc}(\epsilon,\alpha^*(\epsilon))}{\partial\alpha} = 0$ and:

$$\begin{aligned}
\frac{dV_{sc}(\epsilon, \alpha^*_{sc}(\epsilon))}{d\epsilon}_{|\epsilon=\epsilon_0} &= \frac{\partial V_{sc}(\epsilon_0, \alpha^*_{sc}(\epsilon_0))}{\partial\epsilon} + \frac{\partial V_{sc}(\epsilon_0, \alpha^*_{sc}(\epsilon_0))}{\partial\alpha}\frac{d\alpha^*_{sc}(\epsilon)}{d\epsilon}_{|\epsilon=\epsilon_0} = \frac{\partial V_{sc}(\epsilon_0, \alpha^*_{sc}(\epsilon_0))}{\partial\epsilon} = \\
&= \frac{\partial V_{sc}(\epsilon_0, \alpha(\epsilon_0))}{\partial\epsilon} + \frac{\partial V_{sc}(\epsilon_0, \alpha(\epsilon_0))}{\partial\alpha}\frac{d\alpha(\epsilon)}{d\epsilon}_{|\epsilon=\epsilon_0} = \frac{dV_{sc}(\epsilon, \alpha(\epsilon))}{d\epsilon}_{|\epsilon=\epsilon_0}.
\end{aligned}$$

Now, defining $g_\epsilon(b) = \Gamma(\frac{b+\alpha(\epsilon)y}{\epsilon})$, we can write $\frac{dV_{sc}(\epsilon,\alpha(\epsilon))}{d\epsilon}_{|\epsilon=\epsilon_0} = \int (b+y)\,\frac{dg_\epsilon(b)}{d\epsilon}_{|\epsilon=\epsilon_0} f(b)db$. We now show that $\frac{dg_\epsilon(b)}{d\epsilon}_{|\epsilon=\epsilon_0} \leq 0$ for all $b \geq -y$, and $\frac{dg_\epsilon(b)}{d\epsilon}_{|\epsilon=\epsilon_0} \geq 0$ for $b \leq -y$. The derivative $\frac{dg_\epsilon(b)}{d\epsilon}_{|\epsilon=\epsilon_0}$ is equal to

$$\frac{dg_\epsilon(b)}{d\epsilon}_{|\epsilon=\epsilon_0} = \frac{d\Gamma(\frac{b+\alpha(\epsilon_0)y}{\epsilon_0})}{dz}\left(\frac{\alpha'(\epsilon_0)\,\epsilon_0 y - (b+\alpha(\epsilon_0)y)}{\epsilon_0^2}\right).$$

But $\frac{d\Gamma(\frac{b+\alpha(\epsilon_0)y}{\epsilon_0})}{dz} \geq 0$, so the sign of the expression $\frac{dg_\epsilon(b)}{d\epsilon}_{|\epsilon=\epsilon_0}$ is the same as the sign of the monotonically decreasing linear expression $\alpha'(\epsilon_0)\,\epsilon_0 y - (b+\alpha(\epsilon_0)y)$. Note that at $b = -y$, $\frac{dg_\epsilon(-y)}{d\epsilon}_{|\epsilon=\epsilon_0} = 0$, because by the definition of $\alpha(\epsilon)$, $g_\epsilon(-y) = g_{\epsilon_0}(-y)$ for all ϵ. But $\frac{d\Gamma(\frac{-y+\alpha^*_{sc}(\epsilon_0)y}{\epsilon_0})}{dz} > 0$, by the optimality condition of Proposition **5,** $1 - \frac{2\epsilon_0}{3|y|} < \alpha^*_{sc} < 1$, and

because $\Gamma'(z) > 0$ for $z \in (0, \frac{2}{3})$. Therefore, $\alpha'(\epsilon_0)\,\epsilon_0 y - (-y + \alpha(\epsilon_0)y) = 0$. It then follows that $\frac{dg_\epsilon(b)}{d\epsilon}_{|\epsilon=\epsilon_0} \leq 0$ for all $b \geq -y$, and $\frac{dg_\epsilon(b)}{d\epsilon}_{|\epsilon=\epsilon_0} \geq 0$ for $b \leq -y$. Finally this yields that $\frac{dV_{sc}(\epsilon,\alpha(\epsilon))}{d\epsilon}_{|\epsilon=\epsilon_0} = \int (b+y)\,\frac{dg_\epsilon(b)}{d\epsilon}_{|\epsilon=\epsilon_0} f(b)db \leq 0$, which concludes the proof. Q.E.D.

Proof of Proposition 7: As the proof of Proposition 2 shows, the firm's value under unilateral control is monotonically increasing on the controlling shareholder's stake. Therefore, a large financing requirement reduces firm value under unilateral control because it forces the entrepreneur to sell more shares, decreasing the control stake accordingly.

In contrast, firm value under shared control does not depend on the size of the investment requirement. To see this, let $\alpha^*_{sc}(I)$ be an optimal controlling stake given the investment requirement I. Provided that a sale of control shares can finance the increase in the investment requirement, the entrepreneur does not need to reduce the control stake. Instead, he can simply sell more of his own shares to the second controlling shareholder. Inspection of the investment rule under shared control (equation (7)) reveals that the decision of undertaking the project depends only on the total control stake. Hence, if a controlling stake $\alpha^*_{sc}(I) = \alpha^{sc}_1 + \alpha^{sc}_2$ is optimal for investment requirement I, then $\alpha^*_{sc}(I) = (\alpha^{sc}_1 - x) + (\alpha^{sc}_2 + x)$ must remain optimal for an investment requirement $I' > I$, if x is the additional amount of control shares that the entrepreneur must sell to finance I'. It then follows that the increase in the financing requirement does not affect the firm's value. Q.E.D.

Proof of Proposition 8: Our first task is to model an improvement of governance law that keeps the total payoffs of the projects constant. For projects that increase private benefits, $b > 0$, there is a quite natural way to model the improvement of the law: It reduces the private benefits to $b - g > 0$, while increasing the public cash flows to $y + g$, with $g > 0$. The characterization of the improvement of the law is subtle, though, for projects that destroy private benefits, $b < 0$. In this case, the improvement should leave fewer private benefits for the projects to destroy. We thus have that if $b < 0$ before the improvement of the law, then, after the improvement, the loss of private benefits is reduced to $|b - g| < |b|$, or equivalently, $0 > b - g > b$ with $g < 0$. Since, by assumption, we require that the improvement of the law does not change the total payoffs, the smaller reduction of the private benefits must be matched by a decrease of public cash flows to $y + g < y$.

In summary, our modelling of the improvement of governance laws can be interpreted as

a shift of firms' private benefits toward zero, which is akin to assuming that the impact of the project on the existing private benefits is reduced by $|g|$. We assume that $|g|$ is publicly known. Whether g is positive or negative, however, depends on the realization of the private benefits of control b: g and b will always have the same sign.

We are now ready to characterize the impact of the improvement of governance laws on firm value. We do this by first analyzing its impact on firms under shared control, and then we look at its impact on firms under unilateral control.

i) *Shared control:* For a given control stake, changes in the financing requirement can be accommodated by changing the sale of shares to the large investor. Hence, we can treat the control stake as a control variable in the entrepreneur's maximization problem under shared control. We have two cases to analyze: projects that increase public cash flows and projects that destroy public cash flows.

i.a) Case $y \geq 0$: From Proposition 5**,** the optimum controlling stake before the governance improvement is $\alpha_{sc} = 1$ if $y \geq 0$. Moreover, for $y \geq 0$, the assumption $|g| < Min\{|b|, |y| - \frac{2\epsilon}{3(1-\underline{\alpha})}\}$ implies that $y + g \geq 0$. Hence, from Proposition 5**,** the optimal controlling stake after the improvement of the law is still $\alpha_{sc}(g) = 1$. And so, the value of the investment opportunity after the improvement is $\int ((b-g)+(y+g))\,\Gamma\left(\frac{(b-g)+(y+g)}{\epsilon}\right) f(b)db = \int (b+y)\,\Gamma(\frac{b+y}{\epsilon}) f(b)db$. But the latter integral is the value of the investment opportunity before the improvement of the law, proving that it does not change the firm's value when $y \geq 0$.

i.b) Case $y < 0$: Consider first that the project destroys private benefits ($b < 0$), then the project is inefficient before the improvement of the governance law ($b + y < 0$) and so is after the improvement because $|g| < Min\{|b|, |y| - \frac{2\epsilon}{3(1-\underline{\alpha})}\}$ implies that $y + g < 0$ and $b - g < 0$. Hence, regardless of the law and the control stake, the control group will reject the project because, from Proposition 4, there cannot be overinvestment under shared control. To determine the optimal control stake and the firm's value under shared control we can thus restrict attention to projects that increase private benefits.

Assuming $y < 0$ and $b > 0$, the value of the project under shared control after the improvement of the law is

$$V_{sc}(g) = \max_{\alpha \in [\underline{\alpha}, 1]} \int ((b-g)+(y+g))\,\Gamma\left(\frac{(b-g)+\alpha\,(y+g)}{\epsilon}\right) f(b|b>0)dbP(b>0). \qquad (15)$$

Taking into account that the value of the project is zero for $y < 0$ and $b \leq 0$, and defining $\gamma = \alpha + \frac{(1-\alpha)g}{|y|}$ with $g > 0$, we can rewrite the value of the project (equation (15)) as

$$V_{sc}(g) = \max_{\gamma \in [\underline{\gamma},1]} \int (b+y)\,\Gamma\left(\frac{b+\gamma y}{\epsilon}\right) f(b)db, \tag{16}$$

where $\underline{\gamma} = \underline{\alpha} + \frac{(1-\underline{\alpha})g}{|y|}$, and $\underline{\gamma} > \underline{\alpha}$. By the assumption of the proposition, we have $|g| < |y| - \frac{2\epsilon}{3(1-\underline{\alpha})}$, which is equivalent to $\underline{\alpha} + \frac{(1-\underline{\alpha})g}{|y|} < 1 - \frac{2\epsilon}{3|y|}$. Thus, from Proposition 5, the solution of $\max_{\gamma \in [\underline{\alpha},1]} \int (b+y)\,\Gamma\left(\frac{b+\gamma y}{\epsilon}\right) f(b)db$ satisfies $\alpha^*_{sc} > 1 - \frac{2\epsilon}{3|y|}$. And we can write $\max_{\gamma \in [\underline{\gamma},1]} \int (b+y)\,\Gamma\left(\frac{b+\gamma y}{\epsilon}\right) f(b)db = \max_{\alpha \in [\underline{\alpha},1]} \int (b+y)\,\Gamma\left(\frac{b+\alpha y}{\epsilon}\right) f(b)db$, proving that the values of the investment opportunity before and after the change in the law are equal.

Moreover, the solution of (16) is $\gamma = \alpha^*_{sc}$ where α^*_{sc} is the optimal controlling stake before the improvement of the governance laws. Undoing the transformation of variables that defines γ, we have that the optimal controlling stake after the change in the law is $\alpha^*_{sc}(g) = \alpha^*_{sc} - \frac{(1-\alpha^*_{sc})g}{|y+g|} < \alpha^*_{sc}$ because $b > 0$ implies $g > 0$. Therefore, the improvement of the law reduces the optimal controlling stake under shared control.

ii) *Unilateral control:* Monitoring is irrelevant whenever the project increases or destroys simultaneously the public and the private payoffs. Moreover, by an assumption of the proposition, $|g| < Min\{|b|, |y| - \frac{2\epsilon}{3(1-\underline{\alpha})}\}$, the improvement of the governance laws does not alter the events under which the public and private payoffs of the project have the same sign. To determine the impact of governance laws on the value of a firm under unilateral control we can thus restrict attention to projects with $b > 0$ and $y < 0$, and projects with $b < 0$ and $y > 0$.

Consider then a project with $b > 0$ and $y < 0$, and an arbitrary control stake α_{uc}. For this type of project, the improvement of the governance law is modelled as a transfer $g > 0$ from the value of the private benefits to the public cash flows. We shall evaluate the derivative of the firm value at $g = 0$, showing that it is positive. (For a project with $b < 0$ and $y > 0$, the improvement of the law is modelled by a $g < 0$. The proof that the derivative at $g = 0$ is negative if $b < 0$ and $y > 0$ is analogous).

Given the improvement of the law, the entrepreneur has incentive to undertake the project if $b - g + \alpha_{uc}(y+g) \geq 0$, or equivalently, $b \geq -\alpha_{uc} y + g(1-\alpha_{uc})$. Conditioned on the project increasing private benefits, the value of the investment opportunity with governance g and no monitoring is $V_{uc}(g) = \int_{-\alpha_{uc}y+g(1-\alpha_{uc})} (b+y) f(b|b>0)dbP(b>0)$. As in the case of shared

control, the entrepreneur will not undertake the project if $y < 0$ and $b \leq 0$. Hence, without loss of generality, we can drop $P(b > 0)$ in the valuation of the investment opportunity and ignore the conditioning in the density. Accordingly, the derivative at $g = 0$ of the firm value is $\frac{dV_{uc}(g)}{dg}\Big|_{g=0} = -(1-\alpha_{uc})(-\alpha_{uc}y + y)f(-\alpha_{uc}y) = -(1-\alpha_{uc})^2 y f(-\alpha_{uc}y) \geq 0$ for $y < 0$.

Likewise, if $\beta^*(g, \alpha_{uc})$ is the monitor's optimal stake as characterized in the Proposition 1, we can write the firm's value under unilateral control and an optimal level of positive monitoring as

$$V_m(g) = \int_{-\alpha_{uc}y+g(1-\alpha_{uc})} \left[(y+b)\left(1 - m(y+g, \beta^*(g))\right) - \frac{\rho m(y+g, \beta^*(g, \alpha_{uc}))^2}{2} \right] f(b)db.$$

To show that $\frac{dV_m(g)}{dg}\Big|_{g=0} \geq 0$, we shall use throughout that $F(g) = \int_{L(g)}^{H(g)} f(b,g)db$ implies that

$$\frac{dF(g)}{dg} = \int_{L(g)}^{H(g)} \frac{\partial f(b,g)}{\partial g} db + H'(g)f(H(g), g) - L'(g)f(L(g), g).$$

Define $F(y, b, \beta, g) = \left[(y+b)\left(1 - m(y+g, \beta)\right) - \frac{\rho m(y+g,\beta)^2}{2}\right] = (y+b)\left(1 - \frac{\beta}{\rho}|y+g|\right) - \frac{\rho\left(\frac{\beta}{\rho}|y+g|\right)^2}{2}$. We can thus write the derivative of $V_m(g)$ as $\frac{dV_m(g)}{dg}\Big|_{g=0} = \int_{-\alpha_{uc}y} \frac{\partial F(y,b,\beta^*,0)}{\partial g} f(b)db - (1-\alpha_{uc})F(y, -\alpha_{uc}y, \beta^*, 0)f(-\alpha_{uc}y)$, where $\beta^* = \beta^*(0, \alpha_{uc})$ is the optimal monitoring stake (by the envelope theorem).

Note now that, at the optimal β^*, $\frac{dF(y,b,\beta,g)}{dg}|_{g=0} = \frac{\beta}{\rho}\left((b+y) - y\beta\right)$, which implies that $\int_{-\alpha_{uc}y} \frac{\partial F(y,b,\beta^*,0)}{\partial g} f(b)db$ is equal to

$$\int_{-\alpha_{uc}y} \frac{\beta^*}{\rho}\left((b+y) - y\beta^*\right) f(b)db = \frac{\beta^*}{\rho}\left(\int_{-\alpha_{uc}y} bf(b)db + P(b > -\alpha_{uc}y)\left(y - y\beta^*\right)\right) = 0,$$

because $\beta^* = \frac{1}{y}\left(\underline{b} + y\right)$, where $\underline{b} = E[b|b > -\alpha_{uc}y] = \int_{-\alpha_{uc}y} bf(b)db/P(b > -\alpha_{uc}y)$.

It then follows that $\frac{dV_m(g)}{dg}\Big|_{g=0} = -(1-\alpha_{uc})F(y, \underline{b}, \beta^*, 0)f(\underline{b}) \geq 0$ because $F(y, -\alpha_{uc}y, \beta^*, 0) = (y - \alpha_{uc}y)\left(1 - \frac{\beta^*}{\rho}|y|\right) - \frac{\rho\left(\frac{\beta^*}{\rho}|y|\right)^2}{2} = (1-\alpha_{uc})y\left(1 - \frac{\beta^*}{\rho}|y|\right) - \frac{\rho\left(\frac{\beta^*}{\rho}|y|\right)^2}{2} \leq 0$ because $F(y, -\alpha_{uc}y, \frac{\rho}{|y|}, 0) = -\frac{\rho}{2} < 0$, $\beta^* \leq \frac{\rho}{|y|}$ and $\frac{dF(y,-\alpha_{uc}y,\beta,0)}{d\beta} > 0$ for $\beta \leq 1 - \alpha_{uc}$.

Q.E.D.

Proof of Proposition 9: Assume by absurd that, for any (b, y), there is a controlling shareholder $i(b, y)$ who can successfully acquire full control. From the Revelation Principle, we can ignore the signals of the private benefits, restricting attention to direct mechanisms, in

which the controlling shareholder $i(b,y)$ pays $t_j(b,y)$ to the controlling shareholder $j \neq i(b,y)$.

The new single controlling shareholder internalizes all of the private benefits, investing if and only if $b + \alpha_{sc} y > 0$, where $\alpha_{sc} \equiv \sum_{i=1}^{2} \alpha_i^{sc}$. Conditioned on the existence of the buy out mechanism, this investment rule can be replicated in the ownership structure with multiple controlling shareholders. If x is the probability that the controlling group undertakes the project, set $x(b,y) = 1$ if and only if $b + \alpha_{sc} y > 0$, with transfers $t_j(b,y)$ for $j \neq i(b,y)$, and $t_i(b,y) = -t_j(b,y)$. Thus, we have obtained a direct mechanism for the investment decision that, from the perspective of the two controlling shareholders, is ex-post efficient. Now, standard arguments in the mechanism design literature show that an ex-post efficient mechanism $(x(.), t(.))$ must satisfy the following inequality

$$I = \int \int \sum_{i=1}^{2} \left(b_i + \alpha_i^{sc} y - \frac{1 - F_i(b_i)}{f_i(b_i)} \right) \prod_{k=1}^{2} f(b_k) db_k \geq 0. \tag{17}$$

Consider the following change of variables: $x_1 = -\left(b_1 + \alpha_1^{sc} y\right), x_2 = b_2 + \alpha_2^{sc} y$. Let the density and cumulative distribution of x_i be, respectively, f_i and F_i with support in the interval $[\underline{x}_i, \overline{x}_i]$ where $\underline{x}_1 = -\left(\overline{b}_1 + \alpha_1^{sc} y\right)$, $\overline{x}_1 = -\left(\underline{b}_1 + \alpha_1^{sc} y\right)$, $\underline{x}_2 = \underline{b}_2 + \alpha_2^{sc} y$, and $\overline{x}_2 = \overline{b}_2 + \alpha_2^{sc} y$. One can easily check that the assumption of the proposition implies $\underline{x}_2 < \overline{x}_1$ and $\overline{x}_2 < \underline{x}_1$. Using the formula for the integral with a transformation of variables we have that,

$$I = \int_{\underline{x}_2}^{\overline{x}_2} \int_{\underline{x}_1}^{\min\{x_2, \overline{x}_1\}} ([x_2 - \frac{1 - F_2(x_2)}{f_2(x_2)}] - [x_1 + \frac{F_1(x_1)}{f_1(x_1)}]) f_1(x_1) f_2(x_2) dx_1 dx_2 \tag{18}$$

Myerson and Satterthwaite (1983) show that the above integral is negative under the assumptions of the Proposition. It then follows that the ex-post efficient mechanism to dissolve the partnership is not feasible. Q.E.D.