Winners from Winners: A Tale of Risk Factors*

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Abstract

Starting from the twelve distinct risk factors in four well-established asset pricing models, a pool we refer to as the winners, we construct and compare 4,095 asset pricing models and find that the model with the risk factors, Mkt, SMB, MOM, ROE, MGMT, and PEAD, performs the best in terms of Bayesian posterior probability, out-of-sample predictability, and Sharpe ratio. A more extensive model comparison of 8,388,607 models, constructed from the twelve winners plus eleven principal components of anomalies unexplained by the winners, shows the benefit of incorporating information in genuine anomalies in explaining the cross-section of expected equity returns.

JEL Classification: G12, C11, C12, C52, C58

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1 Introduction

The question of which risk factors best explain the cross-section of expected equity returns continues to draw attention on account of the large importance of this topic for theoretical and empirical finance (Cochrane, 2011). Along with the market factor, hundreds of additional risk factors have emerged (Harvey, Liu, and Zhu, 2016; Hou, Xue, and Zhang, 2017), and the set of possible such factors continues to grow. Rather than add to this list, we ask a straightforward question: could we start with the risk factor collections that have generated support in the recent literature, take the union of the factors in those collections as the pool of winners, and then find a new set of risk factors (winners from winners) that gather even more support from the data, on both statistical and financial grounds?

To answer this question, in what we call the tale of risk factors, we consider the four risk factor collections that we believe have spawned consensus support within the profession, namely those in the papers by Fama and French (1993, 2015, 2018), Hou, Xue, and Zhang (2015), Stambaugh and Yuan (2017), and Daniel, Hirshleifer, and Sun (2020). These collections, which cover the market, fundamental and behavioral factors, listed by author initials and risk factor abbreviations, are as follows[1]:

- FF6 collection: {Mkt, SMB, HML, RMW, CMA, MOM};
- HXZ collection: {Mkt, SMB, IA, ROE};
- SY collection: {Mkt, SMB, MGMT, PERF};
- DHS collection: {Mkt, PEAD, FIN}

In the first part of the analysis, the *winners model scan*, we focus on the set of these winners, and ask what collection of winners from winners emerge when each factor is allowed to play

[1] There are slight differences in these collections in relation to the size factor, which we ignore for simplicity.

the role of a risk factor, or a non-risk factor, to produce different groupings of risk factors in a combinatorial fashion. Each grouping consists of a collection of factors that are risk factors in that grouping, and a complementary collection (the remaining factors) that are not risk factors in that grouping. We compare the resulting set of 4,095 asset pricing models, each of which is internally consistent with its assumptions of the risk factors, from a Bayesian model comparison perspective (Avramov and Chao, 2006; Barillas and Shanken, 2018; Chib and Zeng, 2019; Chib, Zeng, and Zhao, 2020).

Our first main result, from the winners model scan, is that a six-factor model, consisting of Mkt, SMB, MOM, ROE, MGMT, and PEAD from the twelve factors, gets the most support from the data. This model is closely followed by a second model, a seven-factor model that has PERF as an additional risk factor, and a third model, a five-factor model, that does not have MOM as a risk factor. In terms of probabilities, these models have posterior model probabilities of around 0.13, 0.11, 0.10, respectively. Though one would have liked to witness even more decisive posterior support for the best model, this is unrealistic on a large model space composed of models constructed from factors that are a prior winners. Nevertheless, the data evidence is clear in isolating the winners from winners as the posterior probability distribution beyond the top three slumps sharply. For example, the posterior probabilities of the fourth and fifth-best models are around 0.03, and the sixth is about 0.025. The posterior probabilities of the remaining models in the model space barely register, being roughly of the same size as the prior probability of 1/4,095 = 0.00025, and even below.

Interestingly, models with the same risk factor set as FF6, HXZ, SY, and DHS do not appear in the top model set. As we demonstrate later, this relative under performance stems from a failure to clear an *internal consistency condition*. We say the risk factor set of a particular model satisfies an internal consistency condition if its risk factors can price the set of non risk factors in that model without incurring a penalty. In other words, a penalty is incurred, and the marginal likelihood suffers if the constraint that the intercepts in the conditional model of the non risk factors, which by the assumption must be all zero, is binding. If the constraints are not binding, then its marginal likelihood is significantly higher. Empirically, we find that the FF6, HXZ, SY, and DHS risk factor sets fail this internal consistency condition, while the risk factor sets of our three top factor models pass it.

We also document the performance of top models on other important statistical and economic dimensions. For one, we examine the performance of the top models in forecasting out-of-sample. Also, we compare the Sharpe ratios of the mean-variance portfolio constructed from the risk factors of the top models, and the Sharpe ratios of the portfolios from the risk factors in the FF6, HXZ, SY, and DHS collections. We find that the top models perform well in all comparisons.

In the second part of our analysis, in what we call the *winners plus genuine anomalies model scan*, we consider a more extensive model comparison of 8,388,607 asset pricing models, constructed from the twelve winners plus eleven principal components of anomalies unexplained by the winners, the *genuine anomalies*. The general question is to see if one can get an improved set of risk factors by considering models that involve the set of winners and additional factors based on the genuine anomalies, i.e., anomalies that cannot be explained by the winners. We show that from the set of 125 anomalies in Green, Hand, and Zhang (2017) and Hou, Xue, and Zhang (2017), only twenty anomalies cannot be explained by the winners. These constitute the set of genuine anomalies. For our winners plus anomalies model scan, we construct our different asset pricing models from the twelve winners and the first eleven principal components (PCs) of the genuine anomalies. In other words, each of the twelve winners, and each of the eleven PC's, is allowed to play the role of a risk-factor or a non-risk factor, leading to a model space of $2^{23} - 1$ possible asset pricing models that we compare by using our Bayesian model comparison technique.

In this analysis, our tactic of reducing the 125 anomalies to the set of twenty genuine anomalies can be viewed as a *dimension-reduction strategy*. Furthermore, our idea of converting these anomalies to the space of principal components, is another element of the same strategy. Little is lost (and much is gained) by converting the genuine anomalies to PCs since, in either case, the genuine anomalies, or PCs, are portfolios of assets. What is important, however, is that we retain the identity of our winners, thus allowing us to understand whether the PC factors provide incremental information for pricing assets. And if so, whether the winners from the winners model scan continue to be risk factors in this broader space of models.

Our second main result, from the winners plus genuine anomalies scan, shows that there is much to gain by incorporating information in the genuine anomalies. For example, the Sharp ratios increase by more than 30%, which shows the benefit of incorporating information in genuine anomalies in explaining the cross-section of expected equity returns. Nonetheless, the risk factors from the winners set are the key risk factors, even though some prove to be redundant.

Our paper is part of a new wave of Bayesian approaches to risk factor selection. For instance, Kozak, Nagel, and Santosh (2020) focus on PC factors that are constructed from well-known factors and anomaly factors, and utilize interesting economic priors to isolate the relevant PCs in a classical penalized regression estimation. In contrast to their study, we retain the identity of the winners and construct PCs only from the genuine anomalies and approach the estimation from a fully Bayesian perspective. Bryzgalova, Huang, and Julliard (2019) is also part of this new wave of Bayesian work which delivers, for each factor, the marginal posterior probability that that factor is a risk factor, while our approach is concerned with the question of which collection of factors are jointly risk factors.

This paper adds broadly to the recent Bayesian literature in finance, for example, Pástor (2000), Pástor and Stambaugh (2000), Pástor and Stambaugh (2001), Avramov (2002), Ang and Timmermann (2012) and Goyal, He, and Huh (2018).

The rest of the paper is organized as follows. In Section 2, we briefly outline the methodology that we use to conduct our model comparisons. In Section 3, we consider the winners model scan, and in Section 4 the winners plus genuine anomalies model scan. Section 5 and Section 6 contain results and Section 7 concludes.

2 Methodology

Suppose that the potential risk factor set is denoted by $\boldsymbol{f}_t : K \times 1$, where t denotes the t-th month. We now allow each factor to play the role of a risk factor (i.e., an element of the stochastic discount factor), or a non-risk factor, to produce different groupings of risk factors in a combinatorial fashion. Starting with K initial possible risk factors, there are, therefore, $J = 2^K - 1$ possible risk factor combinations (assuming that the risk-factor set cannot be empty). Each of these risk factor combinations defines a particular asset pricing model $\mathbb{M}_j$, $j = 1, \ldots, J$.

Consider now a specific model $\mathbb{M}_j$, $j = 1, \ldots, J$ consisting of the risk factors $\boldsymbol{x}_{j,t} : k_{x,j} \times 1$, and the complementary set of factors (the non risk factors) $\boldsymbol{w}_{j,t} : k_{x,j} \times 1$, where $K = k_{x,j} + k_{w,j}$. By definition, factors are risk factors if they are in the stochastic discount factor $M_{j,t}$. Following Hansen and Jagannathan (1991), we specify the SDF as

$$M_{j,t} = 1 - \boldsymbol{\lambda}'_{x,j} \Omega_{x,j}^{-1} \left(\boldsymbol{x}_{j,t} - \mathbb{E}[\boldsymbol{x}_{j,t}] \right), \tag{1}$$

where $\boldsymbol{b}_{x,j} \triangleq \Omega_{x,j}^{-1} \boldsymbol{\lambda}_{x,j} : k_{x,j} \times 1$ are the unknown risk factor loadings, and $\Omega_{x,j} : k_{x,j} \times k_{x,j}$ is the covariance matrix of $\boldsymbol{x}_j$. Enforcing the pricing restrictions implied by the no-arbitrage condition

$$\mathbb{E}[M_{j,t} \boldsymbol{x}'_{j,t}] = \mathbf{0} \quad \text{and} \quad \mathbb{E}[M_{j,t} \boldsymbol{w}'_{j,t}] = \mathbf{0}$$

for all t, we get that $\mathbb{E}[\boldsymbol{x}_{j,t}] = \boldsymbol{\lambda}_{x,j}$, and $\mathbb{E}[\boldsymbol{w}_{j,t}] = \Gamma_j \boldsymbol{\lambda}_{x,j}$, for some matrix $\Gamma_j : k_{w,j} \times k_{x,j}$. If we assume that the joint distribution of $(\boldsymbol{x}_j, \boldsymbol{w}_j)$ is Gaussian, then the latter pricing restrictions imply that under a marginal-conditional decomposition of the factors, $\mathbb{M}_j$ has the restricted reduced form given by

$$\boldsymbol{x}_{j,t} = \boldsymbol{\lambda}_{x,j} + \varepsilon_{x,j,t}, \tag{2}$$

$$\boldsymbol{w}_{j,t} = \Gamma_j \boldsymbol{x}_{j,t} + \varepsilon_{w \cdot x,j,t}, \tag{3}$$

where the errors are block independent Gaussian

$$\begin{pmatrix} \varepsilon_{x,j,t} \\ \varepsilon_{w \cdot x,j,t} \end{pmatrix} \sim \mathcal{N}_K \left(\mathbf{0}, \begin{pmatrix} \Omega_{x,j} & \mathbf{0} \\ \mathbf{0} & \Omega_{w \cdot x,j} \end{pmatrix} \right), \tag{4}$$

and $\Omega_{w \cdot x,j} : k_{w,j} \times k_{w,j}$ is the covariance matrix of the conditional residuals $\varepsilon_{w \cdot x,j,t}$.

The goal of the analysis is to calculate the support for these models, $\mathbb{M}_j$, $j = 1, ..., J$, given the sample data on the factors. To explain how this is done, we start with the prior distributions of the parameters across models.

The parameters of model $\mathbb{M}_j$ are given by

$$\boldsymbol{\theta}_j \triangleq (\boldsymbol{\lambda}_{x,j}, \boldsymbol{\eta}_j),$$

where $\boldsymbol{\lambda}_{x,j}$ are the risk premia parameters, and $\boldsymbol{\eta}_j = (\Gamma_j, \Omega_{x,j}, \Omega_{w \cdot x,j})$ are nuisance parameters.

Note the key point that the dimension of these nuisance parameters equals

$$\begin{aligned} & \{k_{w,j}k_{x,j} + k_{x,j}(k_{x,j}+1)/2 + k_{w,j}(k_{w,j}+1)/2\} \\ = & \{k_{x,j}^2 + k_{w,j}^2 + 2k_{x,j}k_{w,j} + (k_{x,j}+k_{w,j})\}/2 \\ = & (K^2+K)/2, \end{aligned}$$

which is the same across models. Chib and Zeng (2019) exploit this fact to develop proper priors, and Chib, Zeng, and Zhao (2020) to develop improper priors, of $\boldsymbol{\eta}_j,\ j = 1,2,...,J$, from a *single prior distribution*.

Let $\mathbb{M}_1$ denote the model in which all K potential risk factors are risk factors. Then, $\boldsymbol{\eta}_1$ just consists of $\Omega_{x,1}$. Now let this parameter have the Jeffreys' improper prior

$$\pi(\Omega_{x,1}|\mathbb{M}_1) = c|\Omega_{x,1}|^{-\frac{K+1}{2}}. \tag{5}$$

By derivations given in Chib, Zeng, and Zhao (2020), we get that the priors of $\boldsymbol{\eta}_j,\ j > 1$ are

$$\psi(\boldsymbol{\eta}_j|\mathbb{M}_j) = c|\Omega_{x,j}|^{-\frac{2k_{x,j}-K+1}{2}}|\Omega_{w\cdot x,j}|^{-\frac{K+1}{2}},\ j > 1, \tag{6}$$

where c is an arbitrary constant that by construction is the same across these priors, and, hence, irrelevant in the comparison of models.

Finally, complete the prior distributions by supposing that, conditional on $\boldsymbol{\eta}_j$, the priors of $\boldsymbol{\lambda}_{x,j}$ are the multivariate normal distributions

$$\boldsymbol{\lambda}_{x,j}|\mathbb{M}_j,\boldsymbol{\eta}_j \sim \mathcal{N}_{k_{x,j}}(\boldsymbol{\lambda}_{x,j,0},\kappa_j\Omega_{x,j}),$$

where $\boldsymbol{\lambda}_{x,j,0}$ and κ_j are model-specific hyperparameters that are determined from a training sample.

2.1 Marginal Likelihoods

Assume that each model $\mathbb{M}_j \in \mathscr{M}$ has a prior model probability of $\Pr(\mathbb{M}_j)$ of being the correct model. The objective is to calculate the posterior model probability $\Pr(\mathbb{M}_j|\boldsymbol{y}_{1:T})$, where $\boldsymbol{y}_{1:T} = (\boldsymbol{f}_1,...,\boldsymbol{f}_T)$ is the estimation sample of the potential risk factors.

The key quantities for performing this prior-posterior update for the models in $\mathscr{M}$ are the marginal likelihoods, defined as

$$m_j(\boldsymbol{y}_{1:T}|\mathbb{M}_j) = \int_{\Theta_j} p(\boldsymbol{y}_{1:T}|\mathbb{M}_j,\boldsymbol{\theta}_j)\pi(\boldsymbol{\lambda}_{x,j}|\mathbb{M}_j,\boldsymbol{\eta}_j)\psi(\boldsymbol{\eta}_j|\mathbb{M}_j)\mathrm{d}\boldsymbol{\theta}_j,$$

where Θ_j is the domain of $\boldsymbol{\theta}_j$,

$$p(\boldsymbol{y}_{1:T}|\mathbb{M}_j,\boldsymbol{\theta}_j) = \prod_{t=1}^{T} \mathscr{N}_{k_{x,j}}(\boldsymbol{x}_{j,t}|\boldsymbol{\lambda}_{x,j},\Omega_{x,j})\mathscr{N}_{k_{w,j}}(\boldsymbol{w}_{j,t}|\Gamma_j\boldsymbol{x}_{j,t},\Omega_{w\cdot x,j})$$

is the density of the data and $\mathscr{N}_d(\cdot|\mu,\Omega)$ is the d-dimensional multivariate normal density function with mean μ and covariance matrix Ω.

Notice that the phrase marginal likelihood encapsulates two concepts: one that it is a function that is marginalized over the parameters of model j, hence the word marginal; and second that it is the likelihood of the model parameter $\mathbb{M}_j$, hence the word likelihood.

Under our assumptions, the *log* marginal likelihoods are in closed form. Specifically,

$$\log m_1(\boldsymbol{y}_{1:T}|\mathbb{M}_1) = -\frac{TK}{2}\log\pi - \frac{K}{2}\log(T\kappa_1+1) - \frac{T}{2}\log|\Psi_1| + \log\Gamma_K\left(\frac{T}{2}\right),$$

and

$$\log m_j(\boldsymbol{y}_{1:T}|\mathbb{M}_j) = -\frac{Tk_{x,j}}{2}\log\pi - \frac{k_{x,j}}{2}\log(T\kappa_j+1) - \frac{\left(T+k_{x,j}-K\right)}{2}\log|\Psi_j| + \log\Gamma_{k_{x,j}}\left(\frac{T+k_{x,j}-K}{2}\right)$$
$$-\frac{(K-k_{x,j})(T-k_{x,j})}{2}\log\pi - \frac{\left(K-k_{x,j}\right)}{2}\log|W_j^*| - \frac{T}{2}\log|\Psi_j^*| + \log\Gamma_{K-L_j}\left(\frac{T}{2}\right),\ j>1\ ,$$

where we have set $c=1$ (as this choice is irrelevant, by construction), and

$$\Psi_j = \sum_{t=1}^{T}(\boldsymbol{x}_{j,t}-\hat{\boldsymbol{\lambda}}_{x,j})(\boldsymbol{x}_{j,t}-\hat{\boldsymbol{\lambda}}_{x,j})' + \frac{T}{T\kappa_j+1}\left(\hat{\boldsymbol{\lambda}}_{x,j}-\boldsymbol{\lambda}_{xj0}\right)\left(\hat{\boldsymbol{\lambda}}_{x,j}-\boldsymbol{\lambda}_{xj0}\right)'$$
$$W_j^* = \sum_{t=1}^{T}\boldsymbol{x}_{j,t}\boldsymbol{x}_{j,t}'\ ,\ \ \Psi_j^* = \sum_{t=1}^{T}(\boldsymbol{w}_{j,t}-\hat{\Gamma}_j\boldsymbol{x}_{j,t})(\boldsymbol{w}_{j,t}-\hat{\Gamma}_j\boldsymbol{x}_{j,t})'.$$

Note that the variables in hat in the above expressions are the least squares estimates calculated using the estimation sample, and $\Gamma_d(\cdot)$ denotes the d dimensional multivariate gamma function.

2.2 Model Scan Approach

We conduct a prior-posterior analysis on the model space denoted by $\mathscr{M} = \{\mathbb{M}_1, \mathbb{M}_2, \ldots, \mathbb{M}_J\}$. Assume that each model in the model space is given an uninformative and equalized prior model probability, that is, for any j

$$\Pr(\mathbb{M}_j) = 1/J. \tag{7}$$

Applying Bayes theorem to the unknown model parameter $\mathbb{M}_j$, the posterior model probability is given by

$$\Pr(\mathbb{M}_j|\boldsymbol{y}_{1:T}) = \frac{m_j(\boldsymbol{y}_{1:T}|\mathbb{M}_j)}{\sum\limits_{l=1}^{J} m_l(\boldsymbol{y}_{1:T}|\mathbb{M}_l)}\ , \tag{8}$$

as the model prior probabilities in the numerator and the denominator cancel out.

Both the prior and posterior probability distributions on model space acknowledge the notion of *model uncertainty*. The prior distribution on model space represents our beliefs about the models before we see the data. A discrete uniform prior is our default, but, of course, it is possible to consider other prior distributions. The posterior distribution retains model uncertainty unless the sample size is large in relation to the dimension of the model space. By this we mean that the posterior distribution on model space will not concentrate on a single model. As T becomes large, however, the asymptotic theory of the marginal likelihood (see, e.g., Chib, Shin, and Simoni (2018)), implies that the posterior model probabilities will concentrate on the true model (if it is in the set of models), or on the model that is closest to the true model in the Kullback-Leibler distance.

Regardless of the sample size, however, the end-product of our analysis is a ranking of models by posterior model probabilities (equivalently, by marginal likelihoods given that the denominator in the posterior probability calculation is just a normalization constant). We indicate these ranked models by

$$\{\mathbb{M}_{1*}, \mathbb{M}_{2*}, \ldots, \mathbb{M}_{J*}\}$$

such that

$$m_{1*}(\boldsymbol{y}_{1:T}|\mathbb{M}_{1*}) > m_{2*}(\boldsymbol{y}_{1:T}|\mathbb{M}_{2*}) > \cdots > m_{J*}(\boldsymbol{y}_{1:T}|\mathbb{M}_{J*}).$$

This ranking provides the basis for our empirical Bayesian model comparison.

3 Winners Model Scan

As mentioned in the introduction, our first analysis is based on twelve factors from the studies of Fama and French (1993, 2015, 2018), Stambaugh and Yuan (2017), and Daniel, Hirshleifer, and Sun (2020). Details of these factors, {Mkt, SMB, HML, RMW, CMA, MOM, IA, ROE, MGMT, PERF, PEAD, FIN}, are given in Table 1. While Mkt captures the overall market risk, SMB, HML, RMW, CMA, MOM, IA, and ROE are well-known characteristic-based factors and are constructed by sorting stocks in a relatively simple way. For those remaining novel four mispricing factors, MGMT and PERF are constructed based on average rankings of eleven anomalies of stocks, and PEAD and FIN are related to investors' psychology. Although the construction and motivation of those factors are different, those twelve factors are named "winners" as they are believed and proved to have strong power in explaining the cross-section of expected equity returns.

The data on these winners are monthly, spanning the period from January 1974 to December 2018, for a total of 540 starting observations. Of these the last 12 months of data are held-out for out-of-sample prediction validation purpose. For the other 528 in-sample monthly observations, the first 10 percent is used as a training sample to construct the prior distributions of the risk premia parameters, leaving a sample size of $T = 475$ as estimation sample.

3.1 Two special cases

To understand how the framework is applied, consider first the model in which all twelve winners are risk factors. In this case, model $\mathbb{M}_1$ (say), the general model reduces to

$$\boldsymbol{x}_{1,t} = \underbrace{\boldsymbol{\lambda}_{x,1}}_{12\times 1} + \boldsymbol{\varepsilon}_{x,1,t}, \;\; \boldsymbol{\varepsilon}_{x,1,t} \sim \mathscr{N}_{12}(\mathbf{0}, \boldsymbol{\Omega}_{x,1}), \tag{9}$$

where $\boldsymbol{x}_{1,t} = (\mathrm{Mkt}, \mathrm{SMB}, \mathrm{SML}, \mathrm{RMW}, \mathrm{CMA}, \mathrm{MOM}, \mathrm{IA}, \mathrm{ROE}, \mathrm{MGMT}, \mathrm{PERF}, \mathrm{PEAD}, \mathrm{FIN})'_t$ and, since the non-risk factor collection $\boldsymbol{w}_{1,t}$ is empty, $k_{x,1} = 12$ and $\boldsymbol{\Omega}_{x,1} : 12 \times 12$.

Now consider $\mathbb{M}_2$ (say) with the FF6 risk factors $\boldsymbol{x}_{2,t} = (\mathrm{Mkt}, \mathrm{SMB}, \mathrm{SML}, \mathrm{RMW}, \mathrm{CMA}, \mathrm{MOM})'_t$. Then, we have

$$\boldsymbol{x}_{2,t} = \underbrace{\boldsymbol{\lambda}_{x,2}}_{6\times 1} + \boldsymbol{\varepsilon}_{x,2,t}, \;\; \boldsymbol{\varepsilon}_{x,2,t} \sim \mathscr{N}_{6}(\mathbf{0}, \boldsymbol{\Omega}_{x,2}),$$

while for $\boldsymbol{w}_{2,t} = (\text{IA}, \text{ROE}, \text{MGMT}, \text{PERF}, \text{PEAD}, \text{FIN})'_t$ we have

$$\boldsymbol{w}_{2,t} = \underbrace{\Gamma_2}_{6\times 6} \boldsymbol{x}_{2,t} + \varepsilon_{w\cdot x,2,t}, \ \ \varepsilon_{w\cdot x,2,t} \sim \mathcal{N}_6(\mathbf{0}, \Omega_{w\cdot x,2}),$$

where $k_{x,2} = 6$, $k_{w,2} = 6$, $\Omega_{x,2} : 6 \times 6$, and $\Omega_{w\cdot x,2} : 6 \times 6$. The latter model embodies the pricing restrictions that the assumed risk factors of this model price the non-risk factors $\boldsymbol{w}_{2,t}$.

There are $J = 4,095$ such models in the model space $\mathscr{M}$. Our goal is to compare these J models using the model scan approach described in Section 2.

3.2 Winners Model Scan Results

3.2.1 Top Model Set

To get a clear picture of the prior-posterior update on the model space $\mathscr{M}$, we view each model as a point in that space. The prior distribution of models on that space is uniform. The posterior probabilities of the models are proportional to the product of the uniform prior and the marginal likelihoods. We can use these posterior probabilities to plot these points (or models) in that space in decreasing order. From Figure 1, which plots the posterior model probability of the top 220 models. We can see from the figure that the posterior model probabilities drop sharply beyond the top three models. For example, the posterior probabilities of the fourth and fifth-best models are around 0.03, and the sixth is about 0.025. The posterior probabilities of the remaining models in the model space barely register, being roughly of the same size as the prior probability of 1/4,095 = 0.00025, and even below.

In Figure 2 we plot these posterior model probabilities but, this time, only for the top 5 models. We see that the top three models have a joint posterior probability of 0.3407. In a sense, we can think of these models as being indistinguishable, or equivalent. To make this more precise, in the notation introduction above, let $\mathbb{M}_{1*}$ denote the highest posterior probability model. If we now let the Bayes factor of the best model against any other model be denoted by

$$B_{1j} = \frac{m_{1*}(\boldsymbol{y}_{1:T}|\mathbb{M}_{1*})}{m_{j*}(\boldsymbol{y}_{1:T}|\mathbb{M}_{j*})},$$

then, according to Jeffreys' scale, if $B_{1j} \leq 10^{\frac{1}{2}}$, the evidence supporting $\mathbb{M}_{1*}$ over $\mathbb{M}_{j*}$ is barely worth mentioning. Equivalently, in terms of the log Bayes factor, the indistinguishably condition above can be expressed as

$$\log B_{1j} = \log m_{1*}(\boldsymbol{y}_{1:T}|\mathbb{M}_{1*}) - \log m_{j*}(\boldsymbol{y}_{1:T}|\mathbb{M}_{j*}) \leq 1.15.$$

We can, therefore, refer to $\mathbb{M}_{j*}$ that is in the radius of the best model in this way as being indistinguishable from the best model.

Applying this criterion, we conclude that $\mathbb{M}_{1*}$, $\mathbb{M}_{2*}$, and $\mathbb{M}_{3*}$ constitute the top model set $\mathscr{M}_*$ in the winners scan, while $\mathbb{M}_{4*}$ and $\mathbb{M}_{5*}$ also given in Figure 2 are not in the top model set.

Table 2 shows that the six-factor model $\mathbb{M}_{1*}$ consisting of Mkt, SMB, MOM, ROE, MGMT, and PEAD as risk factors gets the most support from the data. This model is closely followed by the seven-factor model $\mathbb{M}_{2*}$ that has PERF as an additional risk factor, and the five-factor model $\mathbb{M}_{3*}$ that does not have MOM as a risk factor.

Interestingly, Mkt, SMB, ROE, MGMT, and PEAD, are present in each of the three top groupings. It appears that both fundamental and behavioral factors play an important role in pricing the cross-section of expected equity returns. It should also be noted that the top groupings feature between five and seven-factors, similar to the number of factors in most of the literature.

Besides, the ratio of the posterior model probability and the prior model probability of any given model $\mathbb{M}_j$, denoted by $\frac{\Pr(\boldsymbol{y}_{1:T}|\mathbb{M}_{j*})}{\Pr(\mathbb{M}_{j*})}$, is provided in Table 2. That ratio reflects the information improvement of posterior over the same prior for $\mathbb{M}_j$ when data are observed. Therefore it is a good measure for evaluating the joint superiority of candidate models. For comparison, Panel B of Table 2 reports the marginal likelihoods and that ratios for $\mathbb{M}_1$ and models with the same risk factor sets as CAPM, Fama and French family, HXZ, SY, and DHS. It can seen that the information improvement of each of those models is substantially smaller than that of top models. Because $\mathbb{M}_1$ is also not supported by the data, we can conclude that the twelve factors together contain information redundancies.

3.2.2 Parameter Updating for the Best Model of the Winners Model Scan

We now provide more details about the best model in the winners model scan $\mathbb{M}_{1*}$, which takes the form

$$\mathbb{M}_{1*}: \begin{pmatrix} \mathrm{Mkt}_t \\ \mathrm{SMB}_t \\ \mathrm{MOM}_t \\ \mathrm{ROE}_t \\ \mathrm{MGMT}_t \\ \mathrm{PEAD}_t \end{pmatrix} = \underbrace{\boldsymbol{\lambda}_{x,1*}}_{6\times 1} + \varepsilon_{x,1*,t}, \;\; \varepsilon_{x,1*,t} \sim \mathcal{N}_6(\mathbf{0}, \Omega_{x,1*}),$$

$$\begin{pmatrix} \mathrm{HML}_t \\ \mathrm{RMW}_t \\ \mathrm{CMA}_t \\ \mathrm{IA}_t \\ \mathrm{PERF}_t \\ \mathrm{FIN}_t \end{pmatrix} = \underbrace{\Gamma_{1*}}_{6\times 6} \begin{pmatrix} \mathrm{Mkt}_t \\ \mathrm{SMB}_t \\ \mathrm{MOM}_t \\ \mathrm{ROE}_t \\ \mathrm{MGMT}_t \\ \mathrm{PEAD}_t \end{pmatrix} + \varepsilon_{w\cdot x,1*,t}, \;\; \varepsilon_{w\cdot x,1*,t} \sim \mathcal{N}_6(\mathbf{0}, \Omega_{w\cdot x,1*}).$$

In calculating the marginal likelihood of this model, which equals 14186.43, as reported earlier in Table 2, we used the prior on $\boldsymbol{\eta}_{1*} = (\Gamma_{1*}, \Omega_{x,1*}, \Omega_{w\cdot x,1*})$ from (6) equal to $|\Omega_{x,1*}|^{-\frac{1}{2}}|\Omega_{w\cdot x,1*}|^{-\frac{13}{2}}$, and the proper prior of the risk premia parameters $\boldsymbol{\lambda}_{x,1*}$ from the training sample equal to

$$\pi(\boldsymbol{\lambda}_{x,1*}|\mathbb{M}_{1*}, \boldsymbol{\eta}_{1*}) = \mathscr{N}_6(\boldsymbol{\lambda}_{x,1*}|\boldsymbol{\lambda}_{x,1*,0}, 0.1915 \times \Omega_{x,1*}),$$

where $\boldsymbol{\lambda}_{x,1*,0} = (0.0017, 0.0130, 0.0044, 0.0041, 0.0084, 0.0085)'$.

Under our prior distributions, it is easy to confirm that the posterior distributions $\pi(\boldsymbol{\theta}_j|\mathbb{M}_j, \boldsymbol{y}_{1:T})$ of parameters $\boldsymbol{\theta}_j$ of any given model $\mathbb{M}_j$ have the marginal-conditional forms given by

$$\pi(\Omega_{x,j}|\mathbb{M}_j, \boldsymbol{y}_{1:T}) = \mathscr{W}_{k_{x,j}}^{-1}(\Omega_{x,j}|\Psi_j, T + k_{x,j} - K), \tag{10}$$

$$\pi(\boldsymbol{\lambda}_{x,j}|\mathbb{M}_j, \boldsymbol{y}_{1:T}, \Omega_{x,j}) = \mathscr{N}_{k_{x,j}}(\boldsymbol{\lambda}_{x,j}|\boldsymbol{\lambda}_{xj1}, \left(1/\kappa_j + T\right)^{-1}\Omega_{x,j}), \tag{11}$$

and

$$\pi(\Omega_{w\cdot x,j}|\mathbb{M}_j, \boldsymbol{y}_{1:T}) = \mathscr{W}_{k_{w,j}}^{-1}(\Omega_{w\cdot x,j}|\Psi_j^*, T), \tag{12}$$

$$\pi(\mathrm{vec}(\Gamma_j)|\mathbb{M}_j, \boldsymbol{y}_{1:T}, \Omega_{w\cdot x,j}) = \mathscr{N}_{k_{w,j}\times k_{x,j}}(\mathrm{vec}(\Gamma_j)|\mathrm{vec}(\hat{\Gamma}_j), W_j^{*-1} \otimes \Omega_{w\cdot x,j}), \tag{13}$$

where $\boldsymbol{\lambda}_{x,j,1} = \frac{1}{T\kappa_j+1}\boldsymbol{\lambda}_{x,j,0} + \frac{T\kappa_j}{T\kappa_j+1}\hat{\boldsymbol{\lambda}}_{x,j}$ and $\mathscr{W}^{-1}(\Psi, \nu)$ denotes the inverse Wishart distribution with ν degrees of freedom and scale matrix Ψ. Thus, the posterior distribution $\pi(\boldsymbol{\theta}_j|\mathbb{M}_j, \boldsymbol{y}_{1:T})$ is given by the product of equations (10), (11), (12), and (13). We can apply this result to generate a large number of simulated draws, first by sampling the marginal distribution, and then by the conditional distribution given the draws from the marginal distributions. These sampled draws can be used to make marginal posterior distributions of relevant parameters, and other summaries.

Applying the above sampling procedure to $\mathbb{M}_{1*}$, we obtain the marginal posterior distributions of the risk premia parameters $\boldsymbol{\lambda}_{x,1^*}$, given in Figure 3, and the posterior means, standard deviations and quantiles, given in Table 3. It is interesting that the posterior means of the risk premia parameters are similar, except for that of SMB, while the posterior standard deviations of the risk premia of Mkt and MOM are almost twice as large as the rest.

3.3 Why do FF6, HXZ, SY, and DHS not win?

It is crucial and meaningful to understand why models with the same risk factor set as FF6, HXZ, SY, and DHS do not appear in the top model set. The reason for this is interesting. Essentially, those models do not satisfy an internal consistency condition. We say that a particular model satisfies the internal consistency condition if its risk factors can price the set of non risk factors in that model without incurring a penalty. In other words, a penalty is incurred, and the marginal likelihood suffers if the constraint that the intercepts in the conditional model of the non risk factors, which by assumption must be all zero, is binding. If the constraints are not binding, then its marginal likelihood is significantly higher.

Consider model $\mathbb{M}_j$

$$\boldsymbol{x}_{j,t} = \boldsymbol{\lambda}_{x,j} + \varepsilon_{x,j,t},$$

$$\boldsymbol{w}_{j,t} = \Gamma_j \boldsymbol{x}_{j,t} + \varepsilon_{w\cdot x,j,t},$$

with risk factors $\boldsymbol{x}_{j,t}$ and non risk factors $w_{j,t} = (w_{j,1,t}, \ldots, w_{j,k_{w,j},t})'$ with dimension $k_{w,j} \times 1$. Now for each non risk factor $w_{j,i,t}$, $i = 1, 2, \ldots, k_{w,j}$, we compare the two models,

$$\mathbb{M}^i_{j,0}: \; w_{j,i,t} = \gamma'_{j,i} \boldsymbol{x}_{j,t} + \varepsilon_{j,i,t} \tag{14}$$

and

$$\mathbb{M}^i_{j,1}: \; w_{j,i,t} = \alpha_{j,i} + \gamma'_{j,i}\boldsymbol{x}_{j,t} + \varepsilon_{j,i,t} \tag{15}$$

using marginal likelihoods. If the log marginal likelihood of the second model does not exceed that of the first model by more than 1.15, then, by an application of Jeffreys' rule, we can conclude that imposing the zero $\alpha_{j,i}$ pricing restriction does not result in a marginal likelihood penalty. Stated yet another way, this means that the non risk factor $w_{j,i}$ can be priced by the risk factor set $\boldsymbol{x}_j$ of that model $\mathbb{M}_j$. If this condition holds for each of the factors in w_j, we conclude that the risk factor set of that model satisfies the ICC condition.

Our analysis shows that models with the same risk factor sets as CAPM, Fama and French family, HXZ, SY, and DHS do not satisfy ICC. Specifically, the single Mkt factor cannot explain the remaining 11 non risk factors. The risk factor sets of the Fama and French family models can explain at most one non risk factor (IA). The risk factor set of HXZ can explain all of the Fama and French factors, but cannot explain MGMT and PEAD. The risk factor sets of SY and DHS models cannot explain one non risk factor, PEAD and MGMT, respectively. In contrast, the top models in $\mathscr{M}_*$ satisfy the ICC condition fully, which helps to explain why those models rank high in the winners model scan.

3.4 Prediction

It is worthwhile to consider how well the top models perform out-of-sample. From the Bayesian perspective, an elegant way to examine this question is by calculating the *predictive likelihood* for a set of future observations. This predictive likelihood, which like the marginal likelihood, is a number when evaluated at a particular sample of future observations, can be used to rank the predictive performance of each model in the model space.

To define the predictive likelihood, let $\pi(\boldsymbol{\theta}_j|\mathbb{M}_j, \boldsymbol{y}_{1:T})$ denote the posterior distributions of the parameters $\boldsymbol{\theta}_j$ of $\mathbb{M}_j$, and let $\boldsymbol{y}_{(T+1):(T+12)} = (\boldsymbol{f}_{T+1}, ..., \boldsymbol{f}_{T+12})$ denote 12 months of held-out out-of-sample data on those winners. Then, for any given model $\mathbb{M}_j$, the predictive likelihood is defined as

$$m_j\left(\boldsymbol{y}_{(T+1):(T+12)}|\mathbb{M}_j, \boldsymbol{y}_{1:T}\right) = \int_{\Theta_j} p(\boldsymbol{y}_{(T+1):(T+12)}|\mathbb{M}_j, \boldsymbol{\theta}_j)\pi(\boldsymbol{\theta}_j|\mathbb{M}_j, \boldsymbol{y}_{1:T})\mathrm{d}\boldsymbol{\theta}_j,$$

where

$$p(\boldsymbol{y}_{(T+1):(T+12)}|\mathbb{M}_j, \boldsymbol{\theta}_j) = \prod_{s=1}^{12} \mathscr{N}_{k_{x,j}}(\boldsymbol{x}_{j,T+s}|\boldsymbol{\lambda}_{x,j}, \Omega_{x,j})\mathscr{N}_{k_{w,j}}(\boldsymbol{w}_{j,T+s}|\Gamma_j\boldsymbol{x}_{j,T+s}, \Omega_{w\cdot x,j})$$

is the out-of-sample density of the factors given the parameters. We can compute this integral by Monte Carlo. Taking draws $\{\boldsymbol{\theta}_j^{(1)}, ..., \boldsymbol{\theta}_j^{(G)}\}$ from $\pi(\boldsymbol{\theta}_j|\mathbb{M}_j, \boldsymbol{y}_{1:T})$, with G being a large integer, we calculate the predictive likelihood as the Monte Carlo average

$$m_j\left(\boldsymbol{y}_{(T+1):(T+12)}|\mathbb{M}_j, \boldsymbol{y}_{1:T}\right) = \frac{1}{G}\sum_{g=1}^{G} p(\boldsymbol{y}_{(T+1):(T+12)}|\mathbb{M}_j, \boldsymbol{\theta}_j^{(g)}).$$

Table 4 reports the log predictive likelihoods of the top three models as well as those competing models. We can see that the top three models also have larger predictive likelihoods, which means that they outperform the competing models on the predictive dimension.

4 Winners Plus Genuine Anomalies Model Scan

We now show that there are some benefits to including additional potential risk factors along with the winners. There are many additional risk factors to draw upon and the approach we describe can be used with any set of additional potential risk factors. For our analysis here we focus on the

125 anomalies in Green et al. (2017) and Hou, Xue, and Zhang (2017). These anomalies, which are re-balanced at an annual or quarterly frequency, exclude anomalies that are re-balanced at a monthly frequency, because the latter cease to be anomalies once transaction costs are taken into account (Novy-Marx and Velikov, 2016; Patton and Weller, 2020). All these portfolios are value-weighted and held for one month. Just as in the case of the winners, we have monthly observations on these anomalies spanning the period from January 1974 to December 2018, for a total of 540 observations. We partition these observations into out-of-sample and in-sample, which consists of the training sample and the estimation sample, just as in Section 3.

What we aim to show is that whether there is information in these anomalies that can be captured to produce better statistical performance (in terms of marginal likelihoods) and higher Sharpe-ratios of portfolios built from the best fitting risk-factors. In order to show this, we recognize that the winners are already carefully vetted factors and, therefore, the anomalies that are allowed to enter the pool of augmented potential risk factors must be those that cannot be priced by these winners. This point helps to limit the dimension of the model space and allows us to design a full model scan approach, as we now detail.

4.1 Genuine Anomalies

The model space with all 125 anomalies is 2^{137}, which is astronomically large. However, it is unnecessary to consider such a large model space because many of the anomalies can actually be priced by the winners. In fact, Harvey, Liu, and Zhu (2016) and Hou, Xue, and Zhang (2017) have cast doubt on the credibility of these anomalies and Cochrane (2011) has raised similar concerns.

The first step, therefore, is to eliminate anomalies that are not *genuine anomalies*. An anomaly is a genuine anomaly if it cannot be priced by the winners. Here is how we sort this issue out. Let z_i, $i = 1, 2, \dots, 125$, denote the anomalies. Let $\boldsymbol{x} =$ (Mkt, SMB, HML, RMW, CMA, MOM, IA, ROE, MGMT, PERF, PEAD, FIN) denote the twelve winners. Now for each anomaly z_i as the response, and x as the covariates, we estimate two models, one without an intercept and one with:

$$\mathbb{M}_0^i : \; z_{i,t} = \gamma_i' \boldsymbol{x}_t + \varepsilon_{i,t}, \; \varepsilon_{i,t} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_i^2) \tag{16}$$

and

$$\mathbb{M}_1^i : \; z_{i,t} = \alpha_i + \gamma_i' \boldsymbol{x}_t + \varepsilon_{i,t} \; \varepsilon_{i,t} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_i^2). \tag{17}$$

In estimating these models, the first 10 percent of the data are used as a training sample to pin down the hyperparameters of the proper prior distributions. We then use standard Bayesian Markov chain Monte Carlo methods to estimate model on the remaining 90 percent of the data, $\boldsymbol{y}_{1:T}$. The log marginal likelihood of each model is computed by the Chib (1995) method. Denote these marginal likelihoods by $\log m_0^i(\boldsymbol{y}_{1:T} | \mathbb{M}_0^i)$ and $\log m_1^i(\boldsymbol{y}_{1:T} | \mathbb{M}_0^i)$. Then, based on the Jeffreys (1961) scale, if the following condition holds

$$\log m_1^i(\boldsymbol{y}_{1:T} | \mathbb{M}_1^i) - \log m_0^i(\boldsymbol{y}_{1:T} | \mathbb{M}_0^i) > 1.15, \tag{18}$$

then x is not able to price z_i. In this case, we assert that z_i is a genuine anomaly; otherwise z_i is not a genuine anomaly.

Applying the procedure described above, twenty genuine anomalies emerge, namely, acc, age, currat, hire, lev, quick, salecash, sgr, Em, Lbp, dFin, Cop, Lfe, SA, sue, cash, OLAQ, CLAQ, TBIQ, and BLQ. Details about these anomalies are given in Table 5.

4.2 The Potential Risk Factor Set

Now instead of conducting our model scan on the original twelve winners and these twenty genuine anomalies, which leads to a model space of around four billion models ($2^{32}-1=4,294,967,295$), we apply a second dimension reduction step by converting the genuine anomalies to principal components (with the rotated mean added back in), of which we then consider the first eleven that explain in total approximating 91% of the variation in the genuine anomalies, as can be seen from Table 6. This set, of the twelve winners and the first eleven PCs of the genuine anomalies, constitutes our potential risk factor set which we use to launch our extended risk factor analysis.

We note that this strategy of blending of winners and the PCs in this way appears to be new to the literature. By this strategy, we are able to limit the model space to a reasonable dimension (of around eight million models), while simultaneously avoiding the problem of working with twenty correlated PCs that are also quite correlated with the winners. For instance, some anomalies are related to leverage (currat, lev, quick, Lbp, and BLQ) and some are linked to sales status (salecash and sgr). Considering two groups of risk factors in this way, where some are in their original form and some are PCs, appears to be novel. It allows us to show the value of including anomalies as potential risk factors.

We also note that the idea of transforming our genuine anomalies into their corresponding principal components is similar to Kozak, Nagel, and Santosh (2020) who argued that "a relatively small set of principal component from the universe of potential characteristics-based factors can approximate the SDF quite well." Our idea is related, but distinct, as we keep the key factors (the winners) as is, but only covert the less important (the genuine anomalies) into principal components.

We emphasize again that our approach of reducing the 125 anomalies to the set of twenty genuine anomalies is a *dimension-reduction strategy*. Furthermore, our idea of converting these anomalies to the space of principal components, is another element of that same strategy. Of course, whether as anomalies, or as PC's, these factors are portfolios of assets. We believe that it is meaningful and useful to retain the identity of the winners, thus allowing us to understand whether the PC factors provide incremental information for pricing assets.

5 Winners Plus Genuine Anomalies Model Scan Results

5.1 Top Model Set

Starting with the potential risk factor set of dimension $\tilde{K} = 23$, twelve winners plus eleven PCs, and applying the methodology given in Section 2, we calculate the marginal likelihood of each of the $\tilde{J} = 8,388,607$ models in $\tilde{\mathscr{M}}$. Converting these marginal likelihoods into posterior model probabilities, the ratios of these posterior model probabilities and the prior model probability (assumed equal to $1/\tilde{J}$) can be calculated. The ratio, $\frac{\Pr(\tilde{\boldsymbol{y}}_{1:T}|\tilde{\mathbb{M}}_{j*})}{\Pr(\tilde{\mathbb{M}}_{j*})}$, defined earlier in Section 3.2.1, makes it easier to see which models receive more support from the data. We report these ratios for the top 220 models in Figure 4, in which the dashed blue vertical line represents the cutoff of the top model set.

The top model set, denoted by $\tilde{\mathscr{M}}_*$ as in Section 3.2.1, can be defined in relation to the best model $\tilde{\mathbb{M}}_{1*}$. A model is in the top model set if its distance to the best model on the log marginal likelihood scale is less than 1.15. These 29 models, along with the including associated risk factor sets, log marginal likelihoods, and the ratios of posterior model probability and prior model probability are provided in Panel A of Table 7. The risk factors common to all these top models are Mkt, MOM, ROE, PEAD, MGMT, PC1, PC4, and PC5. We also note that the risk factors that are common to the top 3 models in the winners model scan, {Mkt, SMB, ROE, PEAD, MGMT}, are also the risk factors that are common to the top 29 models in the extended model scan except that SMB is replaced by MOM, which is also risk factors of the top 2 models of the winners scan.

5.1.1 Parameter Updating for the Best Model of the Winners Plus Genuine Model Scan

We now provide more details about the best model in the winners plus genuine model scan $\tilde{\mathbb{M}}_{1*}$, in which the risk factor set is given by $\tilde{\boldsymbol{x}}_{1*,t} = (\text{Mkt}, \text{RMW}, \text{MOM}, \text{IA}, \text{ROE}, \text{MGMT}, \text{PEAD}, \text{FIN}, \text{PC1}, \text{PC3}, \text{PC4}, \text{PC5}, \text{PC7})_t'$ and the non risk factor set is given by $\tilde{\boldsymbol{w}}_{1*,t} = (\text{SMB}, \text{HML}, \text{CMA}, \text{PERF}, \text{PC2}, \text{PC6}, \text{PC8}, \text{PC9}, \text{PC10}, \text{PC11})_t'$:

$$\begin{aligned}
\tilde{\mathbb{M}}_{1*}:\ \tilde{\boldsymbol{x}}_{1*,t} &= \underbrace{\tilde{\boldsymbol{\lambda}}_{x,1*}}_{13\times 1} + \tilde{\boldsymbol{\varepsilon}}_{x,1*,t},\ \ \tilde{\boldsymbol{\varepsilon}}_{x,1*,t} \sim \mathcal{N}_{13}(\mathbf{0}, \tilde{\Omega}_{x,1*}),\\
\tilde{\boldsymbol{w}}_{1*,t} &= \underbrace{\tilde{\Gamma}_{1*}}_{10\times 10} \tilde{\boldsymbol{x}}_{1*,t} + \tilde{\boldsymbol{\varepsilon}}_{w\cdot x,1*,t},\ \ \tilde{\boldsymbol{\varepsilon}}_{w\cdot x,1*,t} \sim \mathcal{N}_{10}(\mathbf{0}, \tilde{\Omega}_{w\cdot x,1*}).
\end{aligned}$$

Similar to Section 3.2, the prior and posterior statistics, i.e. prior mean, posterior mean, posterior standard deviation, posterior median, 2.5% quantile, and 97.5% quantile for the risk premia $\tilde{\boldsymbol{\lambda}}_{1*}$ are provided in Table 8.

5.2 Internal Consistency Condition

Just as in Section 3.3, we can see that 27 out of 29 models in the top model set $\tilde{\mathscr{M}}_*$ satisfy the ICC condition completely. The two exceptions occur for $\mathbb{M}_{16*}$ which is unable to explain IA and $\mathbb{M}_{16*}$ which is unable to explain RMW. In constrast, models with the same risk factor sets as CAPM, Fama and French family, HXZ, SY, and DHS deviate from ICC further, leaving out 13, 12, 10, 9, 5, 5, and 4 non-risk factors unexplained. Moreover, none of models with the same risk factor sets as the top three models in the winners scan can explain PC1, PC2, and PC3.

5.3 Prediction

Similar to Section 3.4, it is important to consider how well the winning model performs out-of-sample and we compute the *predictive likelihood* for a set of future observations $\tilde{\boldsymbol{y}}_{(T+1):(T+12)} = (\tilde{\boldsymbol{f}}_{T+1},...,\tilde{\boldsymbol{f}}_{T+12})$ denote 12 months of out-of-sample data on the winners and principal components. Table 9 reports the log predictive likelihoods for top models in $\tilde{\mathscr{M}}_*$ as well as models with the same risk factor sets as CAPM, Fama and French family models, SY and DHS. We can tell that those top models do not fail out of sample.

6 Economic Performance: Sharpe Ratios

Now suppose that based on the identity of the risk factors in (say) the best model $\tilde{\mathbb{M}}_{1*}$ of the winners plus genuine anomalies model scan, namely, Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, and PC7, we construct an optimal mean-variance portfolio of these risk factors together with a risk-free asset. Similarly, we construct the optimal mean-variance portfolios from the risk factors in $\tilde{\mathbb{M}}_1$, $\mathbb{M}_1$ CAPM, Fama and French family, HXZ, SY, and DHS collections, as well as the risk factors of top models of the winners model scan. This leads to the important question: how do the Sharpe ratios of those different portfolios compare?

We construct these different portfolios in the following manner. Consider model $\mathbb{M}_j$ with associated risk factors $\boldsymbol{x}_j$. Given the data $\boldsymbol{y}_{1:T}$, consider calculating the *predictive* mean of $\boldsymbol{x}_{j,T+1}$

$$\mathbb{E}[\boldsymbol{x}_{j,T+1}|\mathbb{M}_j,\boldsymbol{y}_{1:T}] \triangleq \boldsymbol{x}_{j,T+1|T} = \int \boldsymbol{x}_{j,T+1}\mathscr{N}_{k_{x,j}}(\boldsymbol{x}_{j,T+1}|\boldsymbol{\lambda}_{x,j},\Omega_{x,j})\pi(\boldsymbol{\theta}_j|\mathbb{M}_j,\boldsymbol{y}_{1:T})\mathrm{d}\boldsymbol{\theta}_j\mathrm{d}\boldsymbol{x}_{j,T+1},$$

which by changing the order of the integration can be seen to just equal the posterior mean of $\boldsymbol{\lambda}_{x,j}$

$$\boldsymbol{x}_{j,T+1|T} = \hat{\boldsymbol{\lambda}}_{x,j} \triangleq \int_{\Theta_j} \boldsymbol{\lambda}_{x,j} \pi(\boldsymbol{\theta}_j|\mathbb{M}_j, \boldsymbol{y}_{1:T}) \mathrm{d}\boldsymbol{\theta}_j,$$

and the *predictive* covariance of $\boldsymbol{x}_{j,T+1}$

$$\Omega_{x,j,T+1|T} \triangleq \int (\boldsymbol{x}_{j,T+1} - \boldsymbol{x}_{j,T+1|T})(\boldsymbol{x}_{j,T+1} - \boldsymbol{x}_{j,T+1|T})' \mathcal{N}_{k_{x,j}}(\boldsymbol{x}_{j,T+1}|\boldsymbol{\lambda}_{x,j}, \Omega_{x,j}) \pi(\boldsymbol{\theta}_j|\mathbb{M}_j, \boldsymbol{y}_{1:T}) \mathrm{d}\boldsymbol{\theta}_j \mathrm{d}\boldsymbol{x}_{j,T+1},$$

which by the law of iterated expectations for covariances simplifies to the sum of the posterior mean of $\Omega_{x,j}$ and the posterior variance of $\boldsymbol{\lambda}_{x,j}$:

$$\Omega_{x,j,T+1|T} = \int_{\Theta_j} \Omega_{x,j} \pi(\boldsymbol{\theta}_j|\mathbb{M}_j, \boldsymbol{y}_{1:T}) \mathrm{d}\boldsymbol{\theta}_j + \int_{\Theta_j} (\boldsymbol{\lambda}_{x,j} - \hat{\boldsymbol{\lambda}}_{x,j})(\boldsymbol{\lambda}_{x,j} - \hat{\boldsymbol{\lambda}}_{x,j})' \pi(\boldsymbol{\theta}_j|\mathbb{M}_j, \boldsymbol{y}_{1:T}) \mathrm{d}\boldsymbol{\theta}_j.$$

Of course, both these quantities are straightforwardly estimated from the output of the simulation of the posterior density $\pi(\boldsymbol{\theta}_j|\mathbb{M}_j, \boldsymbol{y}_{1:T})$. Given these predictive moments, with certain calculations, the Sharpe ratio of the optimal mean-variance portfolio of the factors in $\boldsymbol{x}_j$ plus a risk-free asset is available as

$$\mathrm{Sh}_j = \left(\hat{\boldsymbol{\lambda}}'_{x,j} \Omega^{-1}_{x,j,T+1|T} \hat{\boldsymbol{\lambda}}_{x,j} \right)^{\frac{1}{2}}.$$

In Table 10 we report the Sharpe ratios of risk-factor portfolios based on several asset pricing models. In Panel A we consider the risk factor sets of the top models in $\tilde{\mathscr{M}}_*$, in Panel B for the risk factor sets of the top models in $\mathscr{M}_*$, and in Panel C for the $\tilde{\mathbb{M}}_1$, $\mathbb{M}_1$, CAPM, Fama and French family, HXZ, SY, and DHS models.

Looking at the results in Panel B and C, we can see that the Sharpe ratios are much higher for the top models from the winners scan than those from some of the existing asset pricing models. Comparing the results in Panel A and Panel B, we see that the top models from the winners plus genuine anomalies model scan provide even higher Sharpe ratios. Taken together, if we consider the best performing DHS model as the benchmark from Panel C, we can see that the models in $\mathscr{M}_*$ have about 17% higher Sharpe ratios, and the models in $\tilde{\mathscr{M}}_*$ have about 49% higher Sharpe ratios.

Finally, in the winners model scan, all twelve winners can achieve a Sharpe ratio of 0.56, whereas investing in the seven winners of winners in $\mathbb{M}_{2*}$ gives a Sharpe ratio of 0.55. And in the winners plus genuine anomalies model scan, investing in those top risk factor sets produces a Sharpe ratio as high as 0.70 while investing in all twelve winners plus eleven PCs gives 0.71. From these close Sharpe ratios we can make two useful conclusions. First, the portfolios of risk factors from the top models perform as well as those from the complete set of risk factors or; in other words, there is some information redundancy in the potential risk factor set. Second, the marginal likelihood ranking and the Sharpe ratio ranking of models are aligned.

7 Conclusion

Our paper makes a contribution to the literature on Bayesian risk factor selection. Starting from the twelve distinct risk factors in Fama and French (1993, 2015, 2018), Hou, Xue, and Zhang (2015), Stambaugh and Yuan (2017), and Daniel, Hirshleifer, and Sun (2020), we construct and compare 4,095 asset pricing models and find that the top models with the highest posterior model probabilities have six risk factors, superior out-of-sample predictive performance, and higher Sharpe ratios. We show also that both fundamental and behavioral risk factors appear in our top model set, highlighting the importance of behavioral factors in pricing and investment decision making.

We also consider if we can get an improved set of risk factors by formulating models that involve the set of winners and additional factors based on the genuine anomalies, i.e., anomalies that cannot be explained by the winners. The framework we have developed, in which we reduce the 125 anomalies to the set of twenty genuine anomalies before converting these genuine anomalies to the space of principal components, allows us to understand whether the PC factors provide incremental information for pricing assets. An extensive comparison of 8,388,607 asset pricing models, constructed from the twelve winners plus eleven principal components of genuine anomalies, shows that there is much to gain by incorporating information in the genuine anomalies. For example, the Sharp ratios increase by more than 30%. Nonetheless, the risk factors from the winners set are the key risk factors, even though some prove to be redundant.

The general approach that we describe in this paper has wide applications. The idea of combining well vetted factors (the winners) with the PCs of less established factors in a model comparison setup, is likely to prove extremely useful beyond this problem to other asset categories such as bonds, currencies and commodities, and is likely to open up many interesting avenues for further research.

Table 1 Winners Definitions

Factors	Definitions
Mkt	the excess return of the market portfolio
SMB	the return spread between diversified portfolios of small size and big size stocks
HML	the return spread between diversified portfolios of high and low B/M stocks
RMW	the return spread between diversified portfolios of stocks with robust and weak profitability
CMA	the return spread between diversified portfolios of the stocks of low (conservative) and high (aggressive) investment firms
MOM	the momentum factor based on two prior returns
IA	the investment factor based on annual changes in total assets divided by lagged total assets
ROE	the profitability factor based on income before extraordinary items divided by one-quarter-lagged book equity
MGMT	the mispricing factor controlled by management
PERF	the mispricing factor related to performance
PEAD	the short-horizon behavioral factor motivated by investor inattention and evidence of short-horizon under reaction
FIN	the long-horizon behavioral factor exploiting the information in managers' decisions to issue or repurchase equity

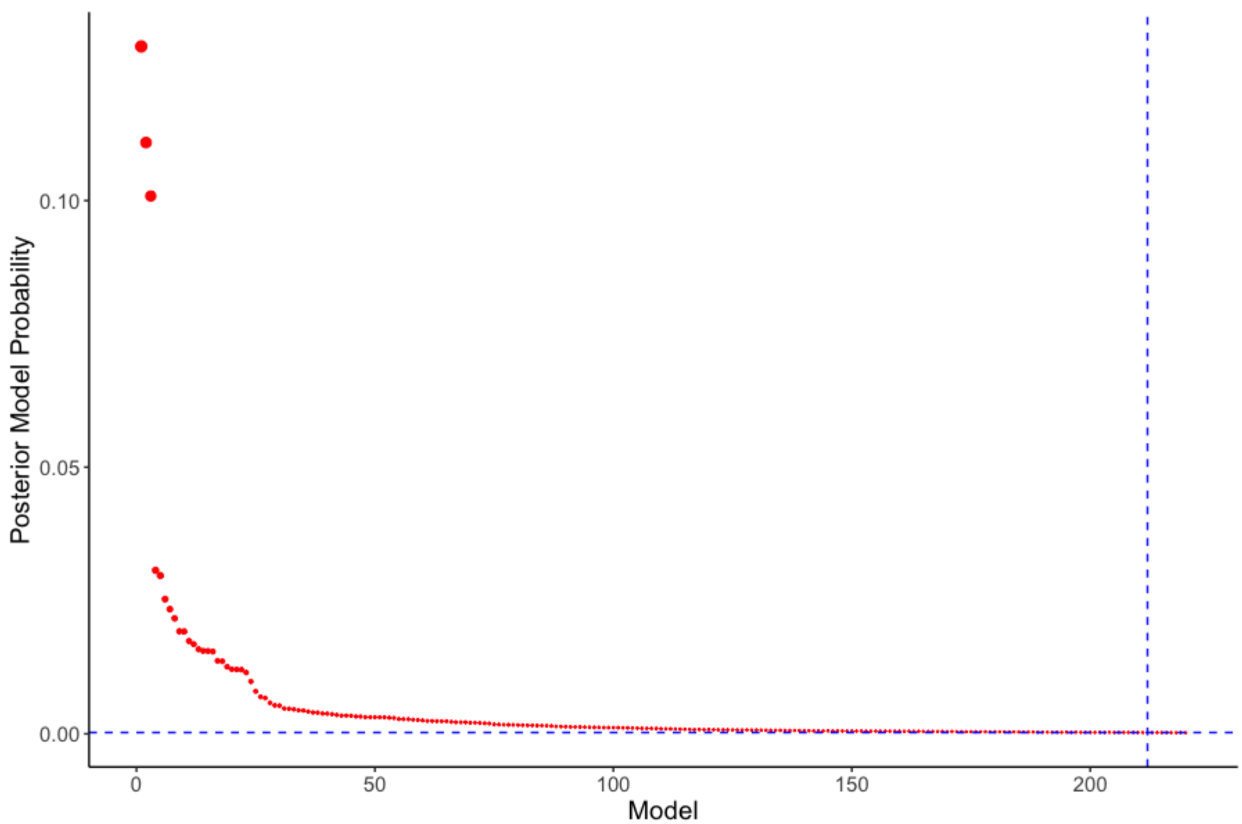

Figure 1 Top 220 Models of the Winners Model Scan

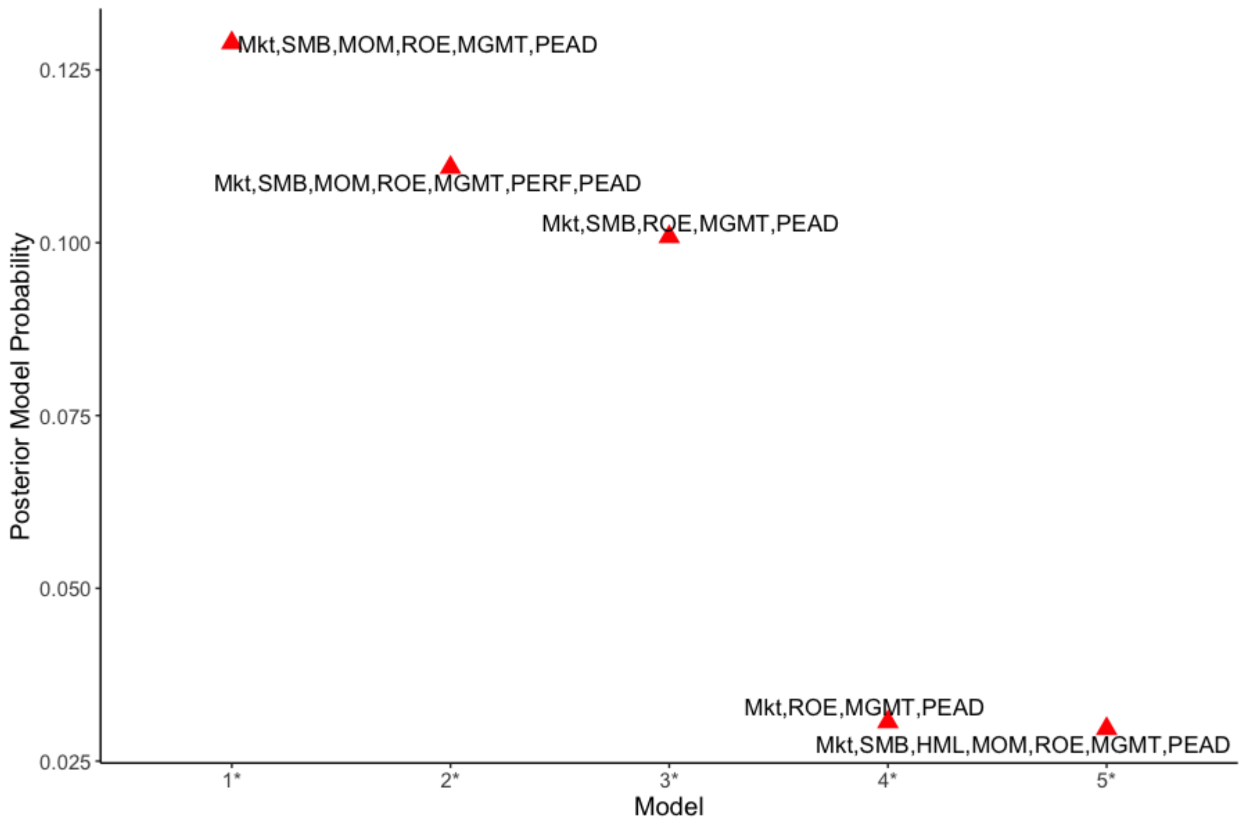

Figure 2 Top 5 Models of the Winners Model Scan

Table 2 Marginal Likelihoods and the Ratios of Posterior Model Probability and Prior Model Probability of Selected Models in the Winners Model Space $\mathscr{M}$

Results from the comparison of the $\tilde{J} = 4,095$ models. Panel A has the results for the top three models, and Panel B for $\tilde{\mathbb{M}}_1$ and models with the same risk factor sets as CAPM, FF3, FF5, FF6, HXZ, SY, and DHS.

Risk factors	$\log m_j(\boldsymbol{y}_{1:T}\vert\mathbb{M}_j)$	$\frac{\Pr(\mathbb{M}_j\vert\boldsymbol{y}_{1:T})}{\Pr(\mathbb{M}_j)}$
Panel A: Top three models		
Mkt, SMB, MOM, ROE, MGMT, PEAD	14186.43	527.89
Mkt, SMB, MOM, ROE, MGMT, PERF, PEAD	14186.28	454.09
Mkt, SMB, ROE, MGMT, PEAD	14186.18	413.01
Panel B: $\tilde{\mathbb{M}}_1$ and models with the same risk factor sets as CAPM, FF3, FF5, FF6, HXZ, SY, and DHS		
12 winners	1.66×10^{-1}	
Mkt	14140.85	8.44×10^{-18}
Mkt, SMB, HML	14140.32	4.98×10^{-18}
Mkt, SMB, HML, RMW, CMA	14152.79	1.30×10^{-12}
Mkt, SMB, HML, RMW, CMA, MOM	14154.45	6.87×10^{-12}
Mkt, SMB, IA, ROE	14164.47	1.54×10^{-7}
Mkt, SMB, MGMT, PERF	14173.32	1.07×10^{-3}
Mkt, PEAD, FIN	14178.86	2.73×10^{-1}

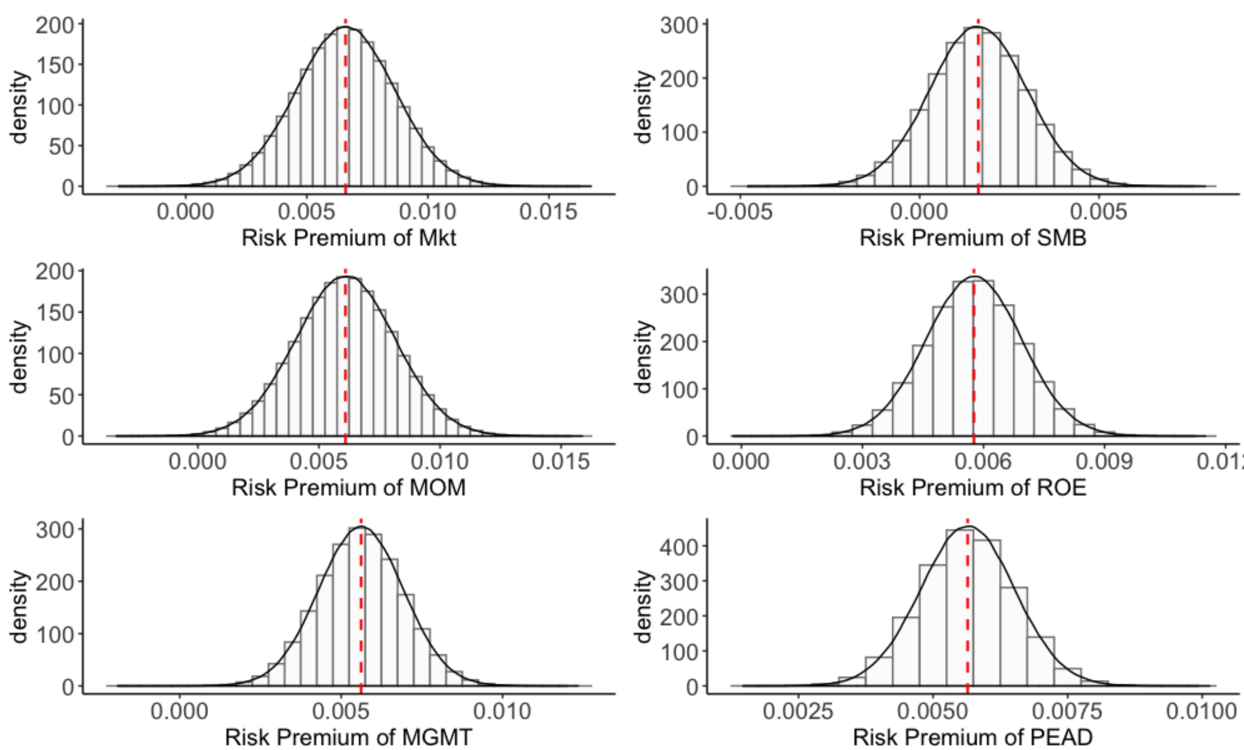

Figure 3 Posterior Distributions of the Risk Premia Parameters of the Best of the Winners Model Scan $\mathbb{M}_{1*}$

Table 3 Prior and Posterior Statistics of the Risk Premia Parameters of the Best of the Winners Model Scan $\mathbb{M}_{1*}$

Prior and posterior statistics, i.e. prior mean, posterior mean, posterior standard deviation, posterior median, 2.5% quantile, and 97.5% quantile for the risk premia $\boldsymbol{\lambda}_{x,1*}$ of the best model $\mathbb{M}_1^*$, which has Mkt, SMB, MOM, ROE, MGMT, and PEAD as risk factors.

	Prior mean	Posterior mean	Posterior sd	Posterior median	2.5% Quantile	97.5% Quantile
Mkt	0.0017	0.0066	0.0020	0.0066	0.0026	0.0106
SMB	0.0130	0.0016	0.0013	0.0016	-0.0010	0.0043
MOM	0.0044	0.0061	0.0021	0.0061	0.0020	0.0101
ROE	0.0041	0.0058	0.0012	0.0058	0.0034	0.0081
MGMT	0.0084	0.0056	0.0013	0.0056	0.0030	0.0082
PEAD	0.0085	0.0056	0.0009	0.0056	0.0039	0.0074

Table 4 Predictive Likelihoods for the Winners Model Scan

Predictive likelihoods of selected asset pricing models in winners model scan.

Risk factors	$\log m_j(\boldsymbol{y}_{(T+1):(T+12)}\|\mathbb{M}_j)$
Panel A: Top three models	
Mkt, SMB, MOM, ROE, MGMT, PEAD	383.48
Mkt, SMB, MOM, ROE, MGMT, PERF, PEAD	383.65
Mkt, SMB, ROE, MGMT, PEAD	383.55
Panel B: Models with the same risk factor sets as CAPM, FF3, FF5, FF6, HXZ, SY, and DHS	
Mkt	382.46
Mkt, SMB, HML	381.76
Mkt, SMB, HML, RMW, CMA	381.67
Mkt, SMB, HML, CMA, RMW, MOM	381.83
Mkt, SMB, IA, ROE	381.87
Mkt, SMB, MGMT, PERF	382.89
Mkt, FIN, PEAD	382.76

Table 5 Surviving Anomalies Explanations

Anomalies	Explanations
acc	annual income before extraordinary items minus operating cash flows divided by average total assets
age	number of years since first Compustat coverage
currat	current assets / current liabilities
hire	percent change in number of employees
lev	total liabilities divided by fiscal year-end market capitalization
quick	(current assets - inventory) / current liabilities
salecash	annual sales divided by cash and cash equivalents
sgr	annual percent change in sales (sale)
Em	enterprise value divided by operating income before depreciation (Compustat annual item OIBDP)
Lbp	leverage component of book to price
dFin	the change in net financial assets
Cop	cash-based operating profitability
Lfe	labor force efficiency
SA	SA index measuring financial constraint
sue	the high-minus-low earnings surprise
cash	cash and cash equivalents divided by average total assets
OLAQ	quarterly operating profits-to-lagged assets
CLAQ	quarterly cash-based operating profits-to-lagged assets
TBIQ	quarterly taxable income-to-book income
BLQ	quarterly book leverage

Table 6 Importance of Principal Components

	PC1	PC2	PC3	PC4	PC5
Standard deviation	0.1143	0.0785	0.0441	0.0415	0.0408
Proportion of Variance	0.3953	0.1866	0.0590	0.0521	0.0503
Cumulative Proportion	0.3953	0.5819	0.6409	0.6930	0.7433
	PC6	PC7	PC8	PC9	PC10
Standard deviation	0.0366	0.0331	0.0298	0.0282	0.0274
Proportion of Variance	0.0405	0.0332	0.0269	0.0241	0.0227
Cumulative Proportion	0.7837	0.8170	0.8438	0.8680	0.8907
	PC11	PC12	PC13	PC14	PC15
Standard deviation	0.0257	0.0231	0.0218	0.0201	0.0194
Proportion of Variance	0.0200	0.0162	0.0144	0.0122	0.0113
Cumulative Proportion	0.9106	0.9268	0.9412	0.9535	0.9648
	PC16	PC17	PC18	PC19	PC20
Standard deviation	0.0172	0.0162	0.0158	0.0142	0.0125
Proportion of Variance	0.0090	0.0079	0.0075	0.0061	0.0047
Cumulative Proportion	0.9737	0.9817	0.9892	0.9953	1.0000

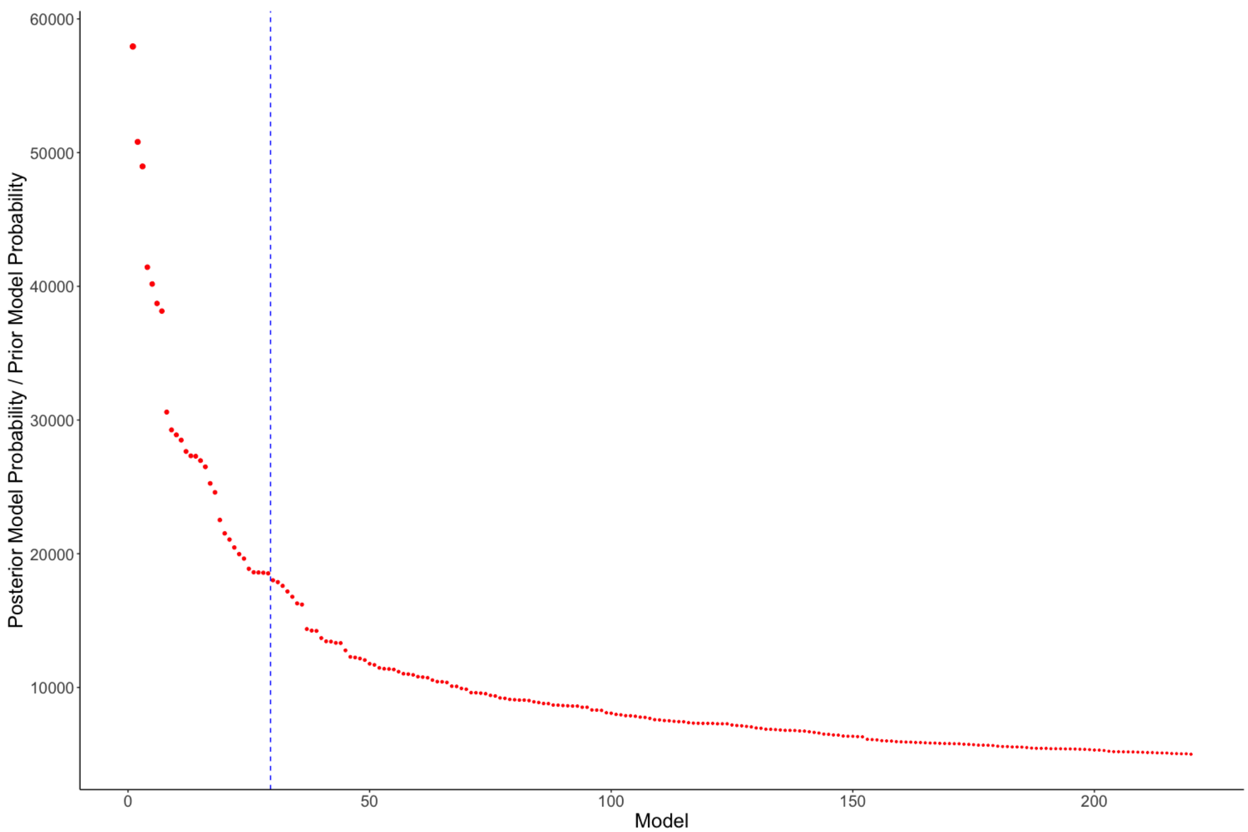

Figure 4 Top 220 Models of the Winners Plus Genuine Anomalies Model Scan

Table 7 Marginal Likelihoods and Ratios of Posterior Model Probability and Prior Model Probability of Selected Models in the Winners Plus Genuine Anomalies Model Space $\tilde{\mathscr{M}}$

Risk Factors	$\log m_j(\tilde{\boldsymbol{y}}_{1:T}\vert\tilde{\mathbb{M}}_j)$	$\frac{\Pr(\tilde{\boldsymbol{y}}_{1:T}\vert\tilde{\mathbb{M}}_j)}{\Pr(\tilde{\mathbb{M}}_j)}$
Panel A: Top models in $\tilde{\mathscr{M}}_*$		
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	24621.85	57939.28
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC7	24621.72	50803.12
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	24621.68	48969.29
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5	24621.51	41429.51
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC3, PC4, PC5, PC7	24621.48	40171.69
Mkt, RMW, CMA, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	24621.44	38720.10
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5	24621.43	38145.58
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC3, PC4, PC5	24621.21	30594.32
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC4, PC5	24621.16	29262.14
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC7, PC9	24621.15	28892.28
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC4, PC5, PC7	24621.14	28496.78
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC4, PC5, PC7	24621.11	27648.27
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC4, PC5	24621.10	27321.30
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	24621.09	27292.03
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5	24621.08	26966.60
Mkt, MOM, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	24621.07	26499.81
Mkt, RMW, CMA, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5	24621.02	25260.55
Mkt, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	24620.99	24590.60
Mkt, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	24620.90	22526.00
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7, PC9	24620.86	21524.20
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC9	24620.84	21061.93
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC7	24620.81	20467.26
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, PC1, PC4, PC5, PC7	24620.78	19969.41
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7, PC9	24620.77	19636.23
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, PC1, PC4, PC5	24620.73	18874.75
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5	24620.71	18612.73
Mkt, CMA, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	24620.71	18597.04
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, PC1, PC3, PC4, PC5, PC7	24620.71	18577.15
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5	24620.71	18531.14
Panel B: $\tilde{\mathbb{M}}_1$ and models with the same risk factor sets as CAPM, FF3, FF5, FF6, HXZ, SY, and DHS		
12 winners and 11 PCs	24606.63	1.71×10^{-9}
Mkt	24560.93	2.42×10^{-29}
Mkt, SMB, HML	24560.17	1.13×10^{-29}
Mkt, SMB, HML, RMW, CMA	24572.01	1.57×10^{-24}
Mkt, SMB, HML, RMW, CMA, MOM	24573.47	6.74×10^{-24}
Mkt, SMB, IA, ROE	24583.58	1.66×10^{-19}
Mkt, SMB, MGMT, PERF	24592.21	9.25×10^{-16}
Mkt, PEAD, FIN	24597.68	2.20×10^{-13}
Panel C: Models with the same risk factor sets as the top three models in the winners model scan		
Mkt, SMB, MOM, ROE, MGMT, PEAD	24604.69	2.44×10^{-10}
Mkt, SMB, MOM, ROE, MGMT, PERF, PEAD	24604.41	1.84×10^{-10}
Mkt, SMB, ROE, MGMT, PEAD	24604.60	2.22×10^{-10}

Table 8 Prior and Posterior Statistics of the Risk Premia Parameters of the Best of the Winners Plus Genuine Model Scan $\tilde{\mathbb{M}}_{1*}$

Prior and posterior statistics, i.e. prior mean, posterior mean, posterior standard deviation, posterior median, 2.5% quantile, and 97.5% quantile for the risk premia $\tilde{\boldsymbol{\lambda}}_{x,1*}$ of the best model $\mathbb{M}_1^*$, which has Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, and PC7 as risk factors.

	Prior mean	Posterior mean	Posterior sd	Posterior median	2.5% Quantile	97.5% Quantile
Mkt	0.0017	0.0066	0.0021	0.0066	0.0025	0.0106
RMW	-0.0015	0.0036	0.0011	0.0036	0.0014	0.0057
MOM	0.0044	0.0061	0.0021	0.0061	0.0020	0.0102
IA	0.0075	0.0033	0.0009	0.0033	0.0016	0.0050
ROE	0.0041	0.0058	0.0012	0.0058	0.0034	0.0081
MGMT	0.0084	0.0056	0.0013	0.0056	0.0030	0.0082
PEAD	0.0085	0.0056	0.0009	0.0056	0.0039	0.0074
FIN	0.0077	0.0070	0.0018	0.0070	0.0034	0.0106
PC1	0.0044	-0.0006	0.0056	-0.0006	-0.0117	0.0103
PC3	0.0191	0.0082	0.0020	0.0082	0.0043	0.0121
PC4	0.0221	0.0112	0.0018	0.0112	0.0076	0.0147
PC5	0.0009	0.0013	0.0019	0.0013	-0.0024	0.0051
PC7	-0.0005	-0.0011	0.0015	-0.0011	-0.0040	0.0018

Table 9 Predictive Likelihoods for the Winners Plus Genuine Anomalies Model Scan
Predictive likelihoods of selected models in the winners plus genuine anomalies model scan.

Risk factors	$\log m_j(\tilde{\boldsymbol{y}}_{(T+1):(T+12)}\|\tilde{\mathbb{M}}_j)$
Panel A: Top models in $\tilde{\mathscr{M}}_*$	
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	639.27
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC7	639.36
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	639.46
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5	639.01
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC3, PC4, PC5, PC7	639.86
Mkt, RMW, CMA, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	639.09
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5	639.08
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC3, PC4, PC5	639.59
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC4, PC5	639.36
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC7, PC9	639.32
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC4, PC5, PC7	639.65
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC4, PC5, PC7	639.16
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC4, PC5	638.86
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	639.35
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5	639.23
Mkt, MOM, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	639.53
Mkt, RMW, CMA, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5	638.82
Mkt, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	639.48
Mkt, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	639.41
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7, PC9	639.43
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC9	639.08
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC7	639.25
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, PC1, PC4, PC5, PC7	639.71
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7, PC9	639.19
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, PC1, PC4, PC5	639.42
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5	638.95
Mkt, CMA, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	639.26
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, PC1, PC3, PC4, PC5, PC7	639.90
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5	639.09
Panel B: $\tilde{\mathbb{M}}_1$ and models with risk factor sets same as CAPM, FF3, FF5, FF6, HXZ, SY, and DHS	
12 winners and 11 PCs	638.61
Mkt	640.03
Mkt, SMB, HML	639.35
Mkt, SMB, HML, RMW, CMA	639.27
Mkt, SMB, HML, CMA, RMW, MOM	639.43
Mkt, SMB, IA, ROE	639.48
Mkt, SMB, MGMT, PERF	640.48
Mkt, FIN, PEAD	640.35

Table 10 Sharpe Ratios

Sharpe ratios for the risk factor sets of selected asset pricing models based on $G = 100,000$.

Risk factors	Sharpe ratios
Panel A: Risk factor sets of the top models in $\tilde{\mathscr{M}}_*$	
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	0.69
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC7	0.68
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	0.69
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5	0.68
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC3, PC4, PC5, PC7	0.68
Mkt, RMW, CMA, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	0.69
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5	0.67
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC3, PC4, PC5	0.67
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC4, PC5	0.66
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC7, PC9	0.69
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, PC1, PC4, PC5, PC7	0.67
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC4, PC5, PC7	0.68
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC4, PC5	0.67
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	0.70
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5	0.68
Mkt, MOM, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	0.68
Mkt, RMW, CMA, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5	0.68
Mkt, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	0.67
Mkt, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	0.68
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5, PC7, PC9	0.70
Mkt, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC9	0.68
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5, PC7	0.69
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, PC1, PC4, PC5, PC7	0.68
Mkt, RMW, MOM, IA, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7, PC9	0.70
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, PC1, PC4, PC5	0.67
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC4, PC5	0.68
Mkt, CMA, MOM, ROE, MGMT, PEAD, FIN, PC1, PC3, PC4, PC5, PC7	0.68
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, PC1, PC3, PC4, PC5, PC7	0.68
Mkt, RMW, MOM, IA, ROE, MGMT, PERF, PEAD, FIN, PC1, PC3, PC4, PC5	0.69
Panel B: Risk factor sets of the top three models in the winners model scan	
Mkt, SMB, MOM, ROE, MGMT, PEAD	0.54
Mkt, SMB, MOM, ROE, MGMT, PERF, PEAD	0.55
Mkt, SMB, ROE, MGMT, PEAD	0.53
Panel C: Risk factor sets of $\tilde{\mathbb{M}}_1$, $\mathbb{M}_1$, CAPM, FF3, FF5, FF6, HXZ, SY, and DHS models	
12 winners and 11 PCs	0.71
12 winners	0.56
Mkt	0.15
Mkt, SMB, HML	0.20
Mkt, SMB, HML, RMW, CMA	0.34
Mkt, SMB, HML, CMA, RMW, MOM	0.36
Mkt, SMB, IA, ROE	0.40
Mkt, SMB, MGMT, PERF	0.45
Mkt, FIN, PEAD	0.47

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