

# Scalable Estimation of Multinomial Response Models with Random Consideration Sets\*

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March 6, 2025

## Abstract

A common assumption in the fitting of unordered multinomial response models for  $J$  mutually exclusive categories is that the responses arise from the same set of  $J$  categories across subjects. However, when responses measure a choice made by the subject, it is more appropriate to condition the distribution of multinomial responses on a subject-specific consideration set, drawn from the power set of  $\{1, 2, \dots, J\}$ . This leads to a mixture of multinomial response models governed by a probability distribution over the  $J^* = 2^J - 1$  consideration sets. We introduce a novel method for estimating such generalized multinomial response models based on the fundamental result that any mass distribution over  $J^*$  consideration sets can be represented as a mixture of products of  $J$  component-specific inclusion-exclusion probabilities. Moreover, under time-invariant consideration sets, the conditional posterior distribution of consideration sets is sparse. These features enable a scalable MCMC algorithm for sampling the posterior distribution of parameters, random effects, and consideration sets. Under regularity conditions, the posterior distributions of the marginal response probabilities and the model parameters satisfy consistency. The methodology is demonstrated in a longitudinal data set on weekly cereal purchases that cover  $J = 101$  brands, a dimension substantially beyond the reach of existing methods.

*Keywords:* Multinomial response, Bayesian computation, Dirichlet process mixture, Markov chain Monte Carlo, Metropolis-Hastings algorithm, Posterior consistency

## 1 Introduction

A common assumption when fitting unordered multinomial response models, whether applied to cross-sectional or longitudinal data, is that the responses stem from the same set of  $J$  mutually exclusive categories across all subjects. However, this assumption may be

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questionable, especially when modeling the choices made by human subjects. For example, in fields such as economics and marketing, it is recognized that individuals may select from only a subset of the available alternatives, termed the “consideration set” (Manski, 1977; Honka et al., 2019). Neglecting this heterogeneity in the consideration sets can result in biased parameter estimates in the model (Bronnenberg and Vanhonacker, 1996; Chiang et al., 1998; Goeree, 2008; Draganska and Klapper, 2011; De los Santos, 2018; Morozov et al., 2021; Crawford et al., 2021). Such biases are problematic because these models are typically employed to understand the impact of covariates on outcomes and inform decision making.

In order to fix ideas, let  $\mathcal{C}_i$  represent the latent consideration set for subject  $i$ . When  $J$  alternatives are available,  $\mathcal{C}_i$  is a subset of  $\{1, \dots, J\}$ , and there are  $J^* = 2^J - 1$  possible consideration sets. A priori,  $\mathcal{C}_i$  is assumed to be drawn from a probability mass function  $\Pr(\mathcal{C}_i = c)$ . When  $J$  is small, the direct approach proposed by Chiang et al. (1998) is effective. In this approach, all possible consideration sets  $1, 2, \dots, J^*$  are enumerated and assigned unknown probabilities  $\pi_1, \pi_2, \dots, \pi_{J^*}$ , which can be estimated using MCMC methods under a Dirichlet prior. However, when  $J$  is large, the model has traditionally been estimated under the assumption that the distribution over consideration sets is determined by  $J$  independent attention probabilities. In this framework, it is assumed that each alternative appears independently in any given consideration set (Ben-Akiva and Boccara, 1995; Goeree, 2008; Manzini and Mariotti, 2014; Kawaguchi et al., 2021; Abaluck and Adams-Prassl, 2021). Specifically, let  $q_{ij}$  denote the probability that subject  $i$  considers the alternative  $j$  for  $j = 1, \dots, J$ . The probability that  $\mathcal{C}_i = c$  given  $\mathbf{q}_i = (q_{i1}, \dots, q_{iJ})'$  is then modeled as:

$$\Pr(\mathcal{C}_i = c \mid \mathbf{q}_i) = \prod_{j \in c} q_{ij} \prod_{j \notin c} (1 - q_{ij}).$$

Although this model is appealing for handling the large  $J$  case, the distribution over con-

sideration sets is unrealistic and leads to model misspecification (Crawford et al., 2021).

In another approach, the consideration sets are modeled as vectors of 0-1 binary variables (Van Nierop et al., 2010). This vector is then modeled by a multivariate probit model (Albert and Chib, 1993, Chib and Greenberg, 1998). Although this can generate correlation of items in consideration sets, inference is challenging because the number of parameters in the correlation matrix of the multivariate probit model increases quadratically in  $J$ .

Given the significant interest in incorporating consideration set heterogeneity in various fields - such as marketing (Van Nierop et al., 2010; Ching et al., 2014; Kawaguchi et al., 2021; Turlo et al., 2025), economics (Goeree, 2008; Ching et al., 2009; Kashaev et al., 2019; Agarwal and Somaini, 2022), transportation science (Swait and Ben-Akiva, 1987; Paleti et al., 2021), and psychology (Traets et al., 2022) - there is a pressing need to develop a scalable estimation approach for estimating such generalized multinomial response models. The importance of accounting for consideration set heterogeneity becomes even more critical as  $J$  increases, which is precisely the case that current methods struggle to address. The method we propose is based on two key components. The first component is a representation of the probability masses  $\pi_1, \pi_2, \dots, \pi_{J^*}$  in terms of a weighted average of products of item-specific inclusion  $q_j$  and exclusion  $1 - q_j$  probabilities, which is based on a result from Dunson and Xing (2009). We refer to this approach as a *mixture of independent consideration models*. To simulate the latent consideration sets, we introduce a straightforward and intuitive Metropolis-Hastings algorithm. It is important to highlight that, in this context, the consideration sets are latent, unlike in Dunson and Xing (2009), where the categorical variables are observed. This difference necessitates additional steps in both the theoretical derivations and the computational procedure. Another crucial feature of the method is the sparsity of the posterior distribution of the consideration sets, which occurs because sets that do not include the actual choices made by a subject must have a posterior probability

of zero (Chiang et al., 1998). The scalability of the proposed approach is demonstrated through an application to marketing data involving  $J = 101$  brands.

We establish two key theoretical results. First, under regularity conditions, as the number of subjects increases, we demonstrate that the posterior distribution of the marginal response probabilities is consistent. Second, under certain additional identification assumptions, the posterior distribution of the model parameters also achieves consistency.

In general, this paper contributes to the expanding literature on high-dimensional demand estimation in statistics and marketing: (Braun and McAuliffe, 2010; Chiong and Shum, 2019; Smith and Allenby, 2019; Loaiza-Maya and Nibbering, 2022; Jiang et al., 2024; Iaria and Wang, 2024; Ershov et al., 2024; Amano et al., 2018). To incorporate latent consideration sets, it is necessary to generalize the standard multinomial response model by conditioning the distribution of responses on a latent subject-specific consideration set, which is drawn from the power set of  $\{1, 2, \dots, J\}$ . This results in a mixture of multinomial models based on a probability distribution over consideration sets. However, the exponential size of this power set renders the estimation of this mixture of multinomial response models computationally infeasible in general. Moreover, the proposed method can be interpreted as a generalized multinomial logit (MNL) model, with “structural zeros” incorporated in the first layer of its hierarchical structure. In the field of biostatistics, methodologies have been extensively explored to estimate microbial compositions that account for the sparsity due to excessive zero counts (e.g. Aitchison, 1982; Martín-Fernández et al., 2015; Liu et al., 2020; Cao et al., 2020; Paulson et al., 2013; Chen and Li, 2016; Tang and Chen, 2019). More recently, Zeng et al. (2023) introduced a zero-inflated probabilistic PCA model designed for high-dimensional, sparse microbiome data sets. Although our paper focuses on a different problem, the proposed method has the potential to be applied in similar contexts, as we discuss in the concluding section.

The remainder of the article is structured as follows. Section 2 introduces the model, while Section 3 presents the theoretical results. Section 4 discusses posterior inference and computational methods. Section 5 reports numerical simulations, and Section 6 applies the methodology to a marketing dataset. Finally, the concluding section explores the broader implications of the proposed framework.

## 2 The approach

Suppose that we have panel (longitudinal) data with  $n$  a priori independent subjects that contains multinomial (polychotomous) responses from a set  $\mathcal{J} = \{1, \dots, J\}$  of  $J$  mutually exclusive nominal categories/items as well as some covariates. Let  $Y_{it} \in \mathcal{J}$  be the measured response for unit  $i$  at time  $t$ , where  $i = 1, \dots, n$  and  $t = 1, \dots, T_i$ . Let  $\mathbf{w}_{it} = \{\mathbf{w}_{ijt}\}_{j \in \mathcal{J}}$ , where  $\mathbf{w}_{ijt}$  is the vector of covariates characterizing the category  $j$  for subject  $i$  at time  $t$ . Each subject  $i$  is associated with a latent consideration set  $\mathcal{C}_i$ , which is a subset of the entire set of alternatives  $\mathcal{J}$ . We model the distribution of the observed outcomes using a hierarchical approach. Specifically, we first specify the marginal distribution of the consideration sets and then define the response distribution conditional on a given consideration set. In this framework, we make the following assumptions.

**Assumption 1:** Consideration sets  $\mathcal{C}_i$  vary over subjects but not over time, and the distribution over consideration sets, denoted by  $\pi_c = \Pr(\mathcal{C}_i = c)$  for  $c \in \mathcal{C}$ , the set of all possible consideration sets minus the empty set, is free of covariates.

The assumption of time invariance is relatively mild and aids in inference. It also plays a role in the identification of model parameters. Covariates can be included in the model for consideration sets, but, as noted by Chiang et al. (1998), a covariate-dependent model is difficult to specify without increasing the risk of model mis-specification.

**Assumption 2:** For each  $j \in \mathcal{J}$ , the responses  $Y_{it}$  of subject  $i$  given  $\mathcal{C}_i$  and random

effects  $\mathbf{b}_i$  are independent over time and follow the multinomial logit model.

Based on Assumptions 1 and 2, the generalized multinomial logit model of interest has the hierarchical form:

$$\text{Stage 1: } \mathcal{C}_i \stackrel{iid}{\sim} \boldsymbol{\pi},$$

$$\text{Stage 2: } \mathbf{b}_i \stackrel{iid}{\sim} \mathcal{N}(\mathbf{0}, \mathbf{D}), \tag{1}$$

$$\text{Stage 3: } \Pr(Y_{it} = j \mid \boldsymbol{\beta}, \mathbf{w}_{it}, \mathcal{C}_i, \mathbf{b}_i) = \begin{cases} \frac{\exp(\mathbf{x}'_{ijt}\boldsymbol{\beta} + \mathbf{z}'_{ijt}\mathbf{b}_i)}{\sum_{\ell \in \mathcal{C}_i} \exp(\mathbf{x}'_{i\ell t}\boldsymbol{\beta} + \mathbf{z}'_{i\ell t}\mathbf{b}_i)} & \text{if } j \in \mathcal{C}_i \\ 0 & \text{otherwise} \end{cases} \quad t = 1, \dots, T_i,$$

for  $i = 1, \dots, n$ , where  $\boldsymbol{\pi} = \{\pi_c : c \in \mathcal{C}, 0 \leq \pi_c \leq 1, \sum_{c \in \mathcal{C}} \pi_c = 1\}$  denotes the collection of probabilities associated with all possible consideration sets, and  $\mathbf{b}_i$  are random effects normally and independently distributed across subjects with zero mean and unknown covariance matrix  $\mathbf{D}$ . The covariates are denoted by  $\mathbf{w}_{it} = \{\mathbf{x}_{ijt}, \mathbf{z}_{ijt}\}_{j \in \mathcal{J}}$ , where  $\mathbf{x}_{ijt} \in \mathbb{R}^{d_x}$  and  $\mathbf{z}_{ijt} \in \mathbb{R}^{d_z}$ . Stage 1 can be interpreted as introducing another layer of random effects, where heterogeneity arises from the random consideration sets.

Letting  $\Pr(\mathbf{Y}_i \mid \boldsymbol{\theta}, \mathbf{w}_i, \mathcal{C}_i = c)$  denote the distribution of outcomes  $\mathbf{Y}_i = (Y_{1i}, \dots, Y_{T_i i})$  of subject  $i$  marginalized over the random effects given covariates  $\mathbf{w}_i = \{\mathbf{w}_{i1}, \dots, \mathbf{w}_{iT_i}\}$ , the distribution of responses takes the finite mixture form:

$$\Pr(\mathbf{Y}_i \mid \boldsymbol{\theta}, \mathbf{w}_i) = \sum_{c \in \mathcal{C}} \pi_c \Pr(\mathbf{Y}_i \mid \boldsymbol{\theta}, \mathbf{w}_i, \mathcal{C}_i = c).$$

This can be seen as a generalized multinomial logit response model.

Assumptions 1 and 2 imply time-invariant consideration sets, conditional independence of responses, and full support of the conditional response probabilities given consideration sets. These conditions, along with additional assumptions detailed below, establish the point identification of the model parameters (Aguiar and Kashaev, 2024) in the model that excludes random effects. Furthermore, in Theorem 2 of Section 3, we show the posterior

consistency of the parameters in this case.

## 2.1 The latent consideration sets

To fix notation, let  $\mathcal{C}$  represent the collection of all possible consideration sets, which corresponds to the power set of  $\mathcal{J} = \{1, \dots, J\}$ , excluding the empty set. The consideration set for subject  $i$  is indicated by  $\mathcal{C}_i = c$ , where  $c \in \mathcal{C}$ . For example, when  $J = 3$ ,  $\mathcal{C} = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \text{ and } \{1, 2, 3\}\}$ , and  $c$  is one of these elements. Furthermore, by  $\mathbf{C}_i = (C_{i1}, \dots, C_{iJ})'$ , we mean a  $J \times 1$  multivariate binary vector where  $C_{ij} = 1$  if category  $j$  is in the consideration set, and 0 otherwise. In the example of  $J = 3$ ,  $\mathcal{C}_i = \{1\}$  is equivalent to  $\mathbf{C}_i = (1, 0, 0)'$  and  $\mathcal{C}_i = \{1, 3\}$  is equivalent to  $\mathbf{C}_i = (1, 0, 1)'$  etc. In the following, we use the two notations interchangeably depending on the context. Researchers sometimes include an outside option in the model that is always considered by each subject. We can incorporate this into our framework by adding a  $(J + 1)$ th category and fixing  $C_{iJ+1} = 1$  for all  $i$ . Our goal is to put a probability distribution on  $\mathcal{C}$  that is rich enough to accommodate dependencies while maintaining scalability.

## 2.2 Dimensionality reduction via tensor decomposition

We now review the factor decomposition technique that we employ to specify the distribution over consideration sets. [Dunson and Xing \(2009\)](#) consider modeling large contingency tables that, for example, represent DNA sequences, each of which is defined as a collection of  $J$  categorical variables, each having  $d_j$  possible values  $j = 1, \dots, J$ , where  $J$  is large. A realization of the contingency table can be expressed as a vector  $(a_1, \dots, a_J)'$ , where  $a_j \in \{1, \dots, d_j\}$  for  $j = 1, \dots, J$ . The true distribution of the contingency tables is a probability tensor  $\boldsymbol{\pi} = \{\pi_{a_1 a_2 \dots a_J}, a_j = 1, \dots, d_j, j = 1, \dots, J\}$ , where  $0 \leq \pi_{a_1 a_2 \dots a_J} \leq 1$  and  $\sum_{a_1=1}^{d_1} \dots \sum_{a_J=1}^{d_J} \pi_{a_1 a_2 \dots a_J} = 1$ . Note that consideration sets can be seen as contingency tables with  $d_j = 2$  for all  $j$ . Generally, there are a large number of elements in the tensor

$\boldsymbol{\pi}$ ,  $d_1 \times \cdots \times d_J$ , when  $J$  is large. [Dunson and Xing \(2009\)](#) show that  $\boldsymbol{\pi}$  can be expressed as a finite mixture of rank 1 tensors. We describe this result for the special case that corresponds to modeling consideration sets.

**Lemma 1** (Exact matching of consideration set probabilities). *Let  $\boldsymbol{\pi}$  be the probability mass distribution over the consideration sets: it is a collection of probabilities  $\{\pi_c = \Pr(\mathcal{C}_i = c) : c \in \mathcal{C}\}$ , where  $0 \leq \pi_c \leq 1$  and  $\sum_{c \in \mathcal{C}} \pi_c = 1$ . Then there are  $K \in \mathbb{Z}^+$ ,  $\boldsymbol{\omega} = (\omega_1, \dots, \omega_K) \in \Delta^{K-1}$ ,  $\mathbf{q}_h = (q_{h1}, \dots, q_{hJ})'$ ,  $h = 1, \dots, K$ ,  $q_{hj} \in [0, 1]$  such that for each  $c \in \mathcal{C}$ ,*

$$\pi_c = \sum_{h=1}^K \omega_h \left\{ \prod_{j \in c} q_{hj} \prod_{j \notin c} (1 - q_{hj}) \right\}. \quad (2)$$

This result states that a mixture of  $K$  independent consideration models can model an arbitrary distribution over the  $J^* = 2^J - 1$  possible consideration sets. Within each component  $h$ , items are included in or excluded from a consideration set  $c$  according to an independent consideration model defined by a vector of attention probabilities  $\mathbf{q}_h = (q_{h1}, \dots, q_{hJ})'$ . Therefore, the number of parameters needed to model the probabilities in  $\boldsymbol{\pi}$  is reduced from  $J^*$  to  $K \times J + (K - 1)$ , which scales linearly with  $J$ .

### 2.3 Infinite mixture of independent consideration models

Building on this result, we model the  $J$ -dimensional latent vectors  $\{\mathcal{C}_i\}$  as a mixture of independent probabilities. Since the number of components  $K$  in (2) is unknown, we follow [Dunson and Xing \(2009\)](#) and use a Dirichlet process (DP) prior ([Ferguson, 1973](#)) to induce an infinite mixture model. One key difference from [Dunson and Xing \(2009\)](#) is that their categorical variables (contingency tables) are observed, while the corresponding consideration sets are latent. This difference leads to differences in the theoretical analysis (Section 3) and in the posterior simulation approach (Section 4).

In our approach, we do not estimate  $K$ . This is because existing methods for consid-



tently estimating  $K$ , such as those proposed by [Kwon and Mbakop \(2021\)](#), may not be applicable when the variables modeled by the mixture are latent. Posterior consistency in our framework only requires that the prior on  $K$  has positive mass for all positive integers. Posterior inferences on model parameters and their functions (e.g., predictions) automatically account for uncertainty regarding the value of  $K$ .

Assume that  $\{\mathbf{C}_i\}$  is i.i.d. with density  $f(\cdot | G) = \int \prod_{j=1}^J q_j^{C_{ij}} (1 - q_j)^{1 - C_{ij}} dG(\mathbf{q})$ . The discrete mixing distribution  $G$  is modeled by a DP prior with a concentration parameter  $\alpha$  and a specified base probability measure  $G_0$  that depends on a hyperparameter  $\underline{\phi}_q$ . Equivalently, by using the stick breaking construction ([Sethuraman, 1994](#)), we have the following representation:  $\mathbf{C}_i$ 's are i.i.d. with the density for the infinite mixture of independent consideration models:

$$\Pr(\mathbf{C}_i = \mathbf{c}_i) = \sum_{h=1}^{\infty} \omega_h \prod_{j=1}^J \{q_{hj}^{c_{ij}} (1 - q_{hj})^{1 - c_{ij}}\}, \quad (3)$$

where  $\mathbf{c}_i = (c_{i1}, \dots, c_{iJ})'$ ,  $\omega_1 = V_1, \omega_h = V_h \prod_{\ell < h} (1 - V_\ell), h = 2, \dots, \infty, V_h \stackrel{iid}{\sim} \text{Beta}(1, \alpha)$ , and  $\mathbf{q}_h \stackrel{iid}{\sim} G_0(\cdot | \underline{\phi}_q), h = 1, \dots, \infty$ , with  $\mathbf{q}_h = (q_{h1}, \dots, q_{hJ})'$  being the vector of attention probabilities specific to the component  $h$ . A priori, the first few weights dominate and cover most of the probability mass, which are then adjusted by the data. Although the model (3) includes infinitely many components, typically only a small number of distinct values for  $\mathbf{q}_h$  are imputed.

For the baseline distribution  $G_0$ , we assume that  $q_{hj} \sim G_{0j}$  independently for  $j = 1, \dots, J$  and  $h = 1, \dots, \infty$ . Specifically, we assume that  $q_{hj} \sim \text{Beta}(\underline{a}_{q_j}, \underline{b}_{q_j})$ , independently over  $j = 1, \dots, J$ , for  $h = 1, \dots, \infty$ , and we define  $\underline{\phi}_q = (\underline{\mathbf{a}}_q, \underline{\mathbf{b}}_q)$  with  $\underline{\mathbf{a}}_q = (\underline{a}_{q1}, \dots, \underline{a}_{qJ})'$  and  $\underline{\mathbf{b}}_q = (\underline{b}_{q1}, \dots, \underline{b}_{qJ})'$ . Note that  $\underline{\phi}_q = (\underline{\mathbf{a}}_q, \underline{\mathbf{b}}_q)$  are the hyperparameters chosen by the user. We discuss this in more detail in the Supplementary Material. We complete the model specification by assuming the prior distribution for the DP concentration parameter

$\alpha \sim \text{Gamma}(\underline{a}_\alpha, \underline{b}_\alpha)$ , where  $(\underline{a}_\alpha, \underline{b}_\alpha)$  are the hyperparameters chosen by the user. For smaller values of  $\alpha$ ,  $\omega_h$  decreases toward zero more rapidly as  $h$  increases, so that the prior favors a sparse representation with most of the weight on a few components. We allow the data to inform about  $\alpha$  and, therefore, an appropriate degree of sparsity.

### 3 Theoretical results

We establish two key results. For simplicity, let  $T_i = T, \forall i$  and suppose that  $T \geq 1$  is fixed and  $n \rightarrow \infty$ . In Theorem 1 we show that the posterior of the marginal response probabilities is consistent, and in Theorem 2, we show that the posterior of the model parameters is consistent when  $T$  is large enough and the model does not include random effects.

Let  $\boldsymbol{\theta} = \{\boldsymbol{\beta}, \mathbf{D}\}$  denote the parameters in the response model. Also, recall that the distribution over the consideration sets is denoted by  $\boldsymbol{\pi} = \{\pi_c : c \in \mathcal{C}\}$ , where  $0 \leq \pi_c \leq 1$  and  $\sum_{c \in \mathcal{C}} \pi_c = 1$ . Define the probability that the sequence of items  $\mathbf{y} = (y_1, \dots, y_T)' \in \mathcal{J}^T$  is chosen conditional on covariates  $\mathbf{w}_i = \{\mathbf{w}_{i1}, \dots, \mathbf{w}_{iT}\}$  taking some specific value  $\mathbf{w} = \{\mathbf{w}_1, \dots, \mathbf{w}_T\} \in \mathbb{R}^{TJ(d_x+d_z)}$ :

$$p_{\boldsymbol{\theta}, \boldsymbol{\pi}}(\mathbf{y} | \mathbf{w}) \equiv \sum_{c \in \mathcal{C}} \pi_c \Pr(\mathbf{Y}_i = \mathbf{y} | \boldsymbol{\theta}, \mathbf{w}, c),$$

where the response probability given a consideration set  $c$  is

$$\Pr(\mathbf{Y}_i = \mathbf{y} | \boldsymbol{\theta}, \mathbf{w}, c) = \int \prod_{t=1}^T \Pr(Y_{it} = y_t | \boldsymbol{\beta}, \mathbf{w}_t, \mathcal{C}_i = c, \mathbf{b}_i) \phi(\mathbf{b}_i | \mathbf{0}, \mathbf{D}) d\mathbf{b}_i,$$

where the integrand is defined in (1). The data set contains responses  $\mathbf{y}_i = \{y_{it}\}$  and covariates  $\mathbf{w}_i = \{\mathbf{w}_{it}\}$  and we let  $\mathbf{D}^n = \{(\mathbf{y}_i, \mathbf{w}_i) : i = 1, \dots, n\}$ . The covariates  $\mathbf{w}_i$  are i.i.d. and follow an unknown distribution with density  $g^*$  with support  $\mathcal{W} \subset \mathbb{R}^{TJ(d_x+d_z)}$ . We do not model the covariate distribution. Conditional on covariates, responses are generated from

the collection of the data-generating response probabilities  $\mathbf{p}^* = \{p_{\boldsymbol{\theta}^*, \boldsymbol{\pi}^*}(\mathbf{y}|\mathbf{w})\}_{\mathbf{y} \in \mathcal{J}^T, \mathbf{w} \in \mathcal{W}}$ , where  $\boldsymbol{\theta}^*$  denotes the true response model parameter and  $\boldsymbol{\pi}^* = \{\pi_c^* : c \in \mathcal{C}\}$  denotes the true probability mass function over consideration sets. We emphasize that  $\boldsymbol{\pi}^*$  does not have to be a finite mixture. The joint probability measure implied by  $\mathbf{p}^*$  and  $g^*$  is denoted by  $F_0$ . For  $\varepsilon > 0$ , define a Kullback-Leibler neighborhood of  $\mathbf{p}^*$  as

$$KL_\varepsilon(\mathbf{p}^*) = \left\{ (\boldsymbol{\theta}, \boldsymbol{\pi}) : \int \sum_{\mathbf{y} \in \mathcal{J}^T} \log \left( \frac{p_{\boldsymbol{\theta}^*, \boldsymbol{\pi}^*}(\mathbf{y}|\mathbf{w})}{p_{\boldsymbol{\theta}, \boldsymbol{\pi}}(\mathbf{y}|\mathbf{w})} \right) p_{\boldsymbol{\theta}^*, \boldsymbol{\pi}^*}(\mathbf{y}|\mathbf{w}) g^*(\mathbf{w}) d\mathbf{w} < \varepsilon \right\}.$$

It is essentially a set of  $(\boldsymbol{\theta}, \boldsymbol{\pi})$  that makes  $p_{\boldsymbol{\theta}, \boldsymbol{\pi}}$  close to  $p_{\boldsymbol{\theta}^*, \boldsymbol{\pi}^*}$ .

Given a  $K \in \mathbb{Z}^+$ , define  $\boldsymbol{\phi}_{1:K} = \{\omega_h, \mathbf{q}_h : h = 1, \dots, K\}$ , the collection of all component-specific parameters, where  $\mathbf{q}_h = (q_{h1}, \dots, q_{hJ})'$ . Note that by Lemma 1, there exist  $\{K, \tilde{\boldsymbol{\phi}}_{1:K}\}$ , which may not be unique, such that  $\pi_c^* = \sum_{h=1}^K \tilde{\omega}_h \left\{ \prod_{j \in c} \tilde{q}_{hj} \prod_{j \notin c} (1 - \tilde{q}_{hj}) \right\}$ , for all  $c \in \mathcal{C}$ , and the KL divergence is zero at  $\{\boldsymbol{\theta}^*, K, \tilde{\boldsymbol{\phi}}_{1:K}\}$ . In the following lemma, we establish that the KL divergence can be made arbitrarily small in sufficiently small neighborhoods of  $(\boldsymbol{\theta}^*, \tilde{\boldsymbol{\phi}}_{1:K})$ . Define the model induced probability for a consideration set  $c \in \mathcal{C}$ :  $\pi(c|K, \boldsymbol{\phi}_{1:K}) = \sum_{h=1}^K \omega_h \prod_{j \in c} q_{hj} \prod_{j \notin c} (1 - q_{hj})$ , and the model induced marginal response probability as

$$p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}, K, \boldsymbol{\phi}_{1:K}) = \sum_{c \in \mathcal{C}} \pi(c|K, \boldsymbol{\phi}_{1:K}) \Pr(\mathbf{Y}_i = \mathbf{y} | \boldsymbol{\theta}, \mathbf{w}, c).$$

**Lemma 2.** *Suppose: (i)  $\boldsymbol{\beta}^* \in \text{interior}(\mathcal{B})$ , where  $\mathcal{B}$  is a compact subset of  $\mathbb{R}^{d_x}$  and  $\mathbf{D}^*$  is positive definite, and (ii)  $\mathcal{W}$  is compact. Then  $\forall \varepsilon > 0$ ,  $\exists$  an open neighborhood  $\mathcal{O}$  of  $\boldsymbol{\theta}^*$ ,  $K \in \mathbb{Z}^+$ , and an open neighborhood  $\mathcal{P}^K$  such that for any  $\boldsymbol{\theta} \in \mathcal{O}$  and  $\boldsymbol{\phi}_{1:K} \in \mathcal{P}^K$ ,*

$$\int \sum_{\mathbf{y} \in \mathcal{J}^T} \log \left( \frac{p_{\boldsymbol{\theta}^*, \boldsymbol{\pi}^*}(\mathbf{y}|\mathbf{w})}{p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}, K, \boldsymbol{\phi}_{1:K})} \right) p_{\boldsymbol{\theta}^*, \boldsymbol{\pi}^*}(\mathbf{y}|\mathbf{w}) g^*(\mathbf{w}) d\mathbf{w} < \varepsilon.$$

The proof can be found in the Appendix. Let  $\Pi(\cdot)$  denote the prior for the response model parameter  $\boldsymbol{\theta}$  and the distribution of consideration sets  $\boldsymbol{\pi}$ .

**Theorem 1.** *Suppose conditions (i) and (ii) of Lemma 2. Suppose (iii) for any open neighborhood  $\mathcal{O}$  of  $\boldsymbol{\theta}^*$ , and for any  $K, \boldsymbol{\phi}_{1:K}$ , and an open neighborhood  $\mathcal{P}^K$  of  $\boldsymbol{\phi}_{1:K}$ ,  $\Pi(\boldsymbol{\theta} \in \mathcal{O}, \boldsymbol{\phi}_{1:K} \in \mathcal{P}^K, K) > 0$ . Then, for all weak neighborhoods  $\mathcal{U}$  of  $\boldsymbol{p}^*$ , as  $n \rightarrow \infty$ ,  $\Pi(\mathcal{U} | \mathbf{D}^n) \rightarrow 1$  a.s.  $F_0^\infty$ .*

*Proof of Theorem 1.* By Schwartz’s theorem (Ghosal and van der Vaart, 2017, ch.6), the result follows if we show that  $\Pi(KL_\varepsilon(\boldsymbol{p}^*)) > 0$ . By Lemma 2, there exist open neighborhoods  $\mathcal{O}$  and  $\mathcal{P}^K$  on which the KL divergence can be made sufficiently small. The lemma combined with a prior that places positive mass on open neighborhoods (condition iii) implies that  $\Pi(KL_\varepsilon(\boldsymbol{p}^*)) > 0$ .  $\square$

This result shows that the model-induced response probability in the limit converges to the true data-generating process. A similar result is proved in Dunson and Xing (2009), Theorem 2, but for the case in which the categorical variables are observed and there are no covariates. Because our setup relaxes both of those conditions, we have a more involved proof that involves the KL-divergence (Lemma 2). Norets and Shimizu (2024) also establish a related result for semiparametric dynamic discrete choice models, but our proof strategy is different, due to the random effects, continuous covariates, and a different model. Last, the compactness assumption (ii) is common in Bayesian nonparametric estimation, and condition (iii) of Theorem 1 is satisfied by our DP prior for  $\omega_h$ ’s and the Beta prior for  $q_{hj}$ ’s, following Dunson and Xing (2009).

We now address the possibility that multiple parameter pairs  $(\boldsymbol{\theta}, \boldsymbol{\pi})$  may be consistent with the true response probabilities. This relates to the issue of *partial identification* (Masatlioglu et al., 2012; Cattaneo et al., 2020; Barseghyan et al., 2021a; Lu, 2022), where point identification holds only under specific conditions (Dardanoni et al., 2020; Abaluck and Adams-Prassl, 2021; Barseghyan et al., 2021b). Following Aguiar and

Kashaev (2024), we impose the assumption that the panel is sufficiently *long* and that *random effects are absent*. Under these conditions, we show that the two sources of variation in responses—differences in utility and differences in consideration sets—can be separately identified. Formally, we show in the next theorem that the posterior distribution contracts to within an arbitrarily small ball around  $(\boldsymbol{\beta}^*, \boldsymbol{\pi}^*)$  under the distance function  $d((\boldsymbol{\beta}, \boldsymbol{\pi}), (\boldsymbol{\beta}', \boldsymbol{\pi}')) = \max\{\|\boldsymbol{\pi} - \boldsymbol{\pi}'\|_1, \|\boldsymbol{\beta} - \boldsymbol{\beta}'\|_2\}$ .

**Theorem 2.** *Suppose (i) the model does not contain random effects; (ii) the parameter  $\boldsymbol{\beta}$  belongs to  $\mathcal{B}$ , a compact subset of  $\mathbb{R}^{d_x}$ , with  $\boldsymbol{\beta}^* \in \text{interior}(\mathcal{B})$ ; (iii)  $\mathcal{W}$  is compact; and (iv) for any open neighborhood  $\mathcal{O}$  of  $\boldsymbol{\beta}^*$ , any  $K$ , any  $\boldsymbol{\phi}_{1:K}$ , and any open neighborhood  $\mathcal{P}^K$  of  $\boldsymbol{\phi}_{1:K}$ , it holds that  $\Pi(\boldsymbol{\beta} \in \mathcal{O}, \boldsymbol{\phi}_{1:K} \in \mathcal{P}^K, K) > 0$ . Then, if the number of periods  $T$  satisfies  $\lfloor (T - 3)/2 \rfloor \geq J$ , we have that for all  $\varepsilon > 0$ , as  $n \rightarrow \infty$ ,  $\Pi((\boldsymbol{\beta}, \boldsymbol{\pi}) : d((\boldsymbol{\beta}, \boldsymbol{\pi}), (\boldsymbol{\beta}^*, \boldsymbol{\pi}^*)) < \varepsilon \mid \mathbf{D}^n) \rightarrow 1$  a.s.  $F_0^\infty$ .*

*Proof of Theorem 2.* The proof is by Schwartz’s theorem. The identification assumption together with Assumptions 1-2 ensures that  $p_{\boldsymbol{\beta}, \boldsymbol{\pi}} \neq p_{\boldsymbol{\beta}', \boldsymbol{\pi}'}$  whenever  $(\boldsymbol{\beta}, \boldsymbol{\pi}) \neq (\boldsymbol{\beta}', \boldsymbol{\pi}')$  (Aguiar and Kashaev, 2024). Identifiability, continuity of  $p_{\boldsymbol{\beta}, \boldsymbol{\pi}}$  in  $(\boldsymbol{\beta}, \boldsymbol{\pi})$  for the total variation norm (Lemma SA.3), and compactness of the parameter space ensure the existence of consistent tests (Van der Vaart, 2000, Lemma 10.6). The approximation result (Lemma 2) without random effects can be established as a special case, and together with the regularity conditions on the prior distribution, the KL-support condition holds.  $\square$

We remark that in Theorem 2 we suppose a model without random effects, though we use random effects in our modeling. The complication in having both is that latent consideration sets in our model operate similarly to random effects and introduce dependence across time. Disentangling these two sources of dependence at a theoretical level requires a stronger condition on  $T$ , though the precise details are not straightforward to establish. We

leave this extension for future work. Nonetheless, the numerical experiments in the Supplementary Material indicate that the convergence described in the theorem holds more generally, as we observe convergence to the true values even in the presence of random effects.

## 4 Inference

Let  $\mathbf{Y}_i = (Y_{i1}, \dots, Y_{iT_i})'$  and  $\mathbf{y}_i = (y_{i1}, \dots, y_{iT_i})'$  be the sequence of random responses made by unit  $i$  over  $T_i$  periods and its observed counterpart. Define

$$p(\mathbf{Y}_i = \mathbf{y}_i | \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{w}_i, \mathbf{C}_i) = \prod_{t=1}^{T_i} \Pr(Y_{it} = y_{it} | \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{w}_{it}, \mathcal{C}_i), \quad (4)$$

where  $\mathbf{w}_i = \{\mathbf{w}_{i1}, \dots, \mathbf{w}_{iT_i}\}$  and  $\Pr(Y_{it} = y_{it} | \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{w}_{it}, \mathcal{C}_i)$  is

$$\Pr(Y_{it} = j | \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{w}_{it}, \mathcal{C}_i) = \frac{\exp(\mathbf{x}'_{ijt}\boldsymbol{\beta} + \mathbf{z}'_{ijt}\mathbf{b}_i)}{\sum_{\ell \in \mathcal{C}_i} \exp(\mathbf{x}'_{i\ell t}\boldsymbol{\beta} + \mathbf{z}'_{i\ell t}\mathbf{b}_i)} \text{ if } j \in \mathcal{C}_i, \text{ and } 0 \text{ otherwise.} \quad (5)$$

Note that  $\mathbf{C}_i$  is the conditioning variable on the left side of (4), while  $\mathcal{C}_i$  is on the right side. Although the two objects represent the same information, the  $J$ -dimensional vector  $\mathbf{C}_i$  is easier to use when we discuss posterior sampling of individual consideration sets. Hence, we use  $\mathbf{C}_i$  to define the individual's contribution to the likelihood. Let  $\mathbf{Y} = \{\mathbf{Y}_1, \dots, \mathbf{Y}_n\}$  and  $\mathbf{y} = \{\mathbf{y}_1, \dots, \mathbf{y}_n\}$  denote the random and observed sequences of the responses made by all units, and let  $\mathbf{W} = \{\mathbf{w}_1, \dots, \mathbf{w}_n\}$  be the observed covariates. Then the likelihood conditional on the common fixed-effects  $\boldsymbol{\beta}$ , the random effects  $\mathbf{b} = (\mathbf{b}_1, \dots, \mathbf{b}_n)'$ , the covariates  $\mathbf{W}$ , and the latent consideration sets  $\mathbf{C} = (\mathbf{C}_1, \dots, \mathbf{C}_n)$  is given by

$$p(\mathbf{Y} = \mathbf{y} | \boldsymbol{\beta}, \mathbf{b}, \mathbf{W}, \mathbf{C}) = \prod_{i=1}^n p(\mathbf{Y}_i = \mathbf{y}_i | \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{w}_i, \mathbf{C}_i). \quad (6)$$

We complete the model by specifying standard prior distributions for the parameters in the response model:  $\boldsymbol{\beta} \sim \mathcal{N}_{d_x}(\mathbf{0}, \mathbf{V}_\beta)$  and  $\mathbf{D}^{-1} \sim \text{Wishart}(\underline{v}, \mathbf{R})$ , independently, a normal distribution for  $\boldsymbol{\beta}$ , and an inverse Wishart distribution for  $\mathbf{D}$  with degrees-of-freedom

parameter  $\underline{v}$  and scale matrix  $\underline{\mathbf{R}}$ . The hyperparameters  $(\underline{\mathbf{V}}_\beta, \underline{v}, \underline{\mathbf{R}})$  are chosen by the user.

## 4.1 Posterior distribution

For the mixture model on the latent consideration sets  $\mathbf{C} = (\mathbf{C}_1, \dots, \mathbf{C}_n)$ , let  $S_i \in \{1, 2, \dots\}$  be the latent cluster assignment such that  $C_{ij}|S_i = h \sim \text{Bernoulli}(q_{hj})$ , independently  $j = 1, \dots, J$ , for  $i = 1, \dots, n$ . We have the latent consideration sets  $\mathbf{C}$ , the common fixed-effects  $\beta$ , the random effects  $\mathbf{b}$ , the corresponding covariance matrix  $\mathbf{D}$ , the DP parameters  $\mathbf{V} = (V_1, V_2, \dots)$  as well as  $\mathbf{Q} = (\mathbf{q}_1, \mathbf{q}_2, \dots)$ , the DP cluster assignment variables  $\mathbf{S} = (S_1, \dots, S_n)$ , and the DP concentration parameter  $\alpha$ . Let  $\pi(\cdot)$  denote the prior density. Then, from the Bayes theorem, the posterior density of interest is

$$p(\mathbf{C}, \mathbf{S}, \mathbf{V}, \mathbf{Q}, \alpha, \beta, \mathbf{b}, \mathbf{D} | \mathbf{y}, \mathbf{W}) \propto p(\mathbf{y} | \beta, \mathbf{b}, \mathbf{W}, \mathbf{C}) \cdot p(\beta, \mathbf{b}, \mathbf{D}) \cdot p(\mathbf{C}, \mathbf{S}, \mathbf{Q}, \mathbf{V}, \alpha), \quad (7)$$

where the first term is given by (6) and only the last term is associated with the DP prior.

We sample from the posterior distribution using a tailored Markov Chain Monte Carlo (MCMC) algorithm. The method is designed for scalability and consists of simple and intuitive steps. Posterior inference is then based on the sampled values

$$\{V_h^{(g)}, \mathbf{q}_h^{(g)}, S_i^{(g)}, \alpha^{(g)}, \mathbf{C}_i^{(g)}, \beta^{(g)}, \mathbf{b}_i^{(g)}, \mathbf{D}^{(g)}\}, \quad g = 1, \dots, G, \quad (8)$$

where  $G$  is the number of MCMC draws beyond a suitable burn-in period.

## 4.2 Simulation of consideration sets

We now focus on sampling the conditional distribution of consideration sets. The other steps in the MCMC simulation follow from standard calculations and are given in the Supplementary Material. From Equation (7), the full conditional distribution of  $\mathbf{C}_i$  is

$$\pi(\mathbf{C}_i | \beta, \mathbf{b}_i, \mathbf{q}_{S_i}, S_i, \mathbf{y}_i, \mathbf{w}_i) \propto p(\mathbf{Y}_i = \mathbf{y}_i | \beta, \mathbf{b}_i, \mathbf{w}_i, \mathbf{C}_i) \cdot \prod_{j=1}^J q_{S_i j}^{C_{ij}} (1 - q_{S_i j})^{1 - C_{ij}}, \quad (9)$$

where the proportionality sign is with respect to  $\mathbf{C}_i$ , and the first term is defined in (4). Importantly, consideration sets that exclude any observed response made by subject  $i$  receive zero posterior probability (see Table 1 for an example). This is because the first term on the left-hand side of (9) is zero for these consideration sets. This desirable feature of our approach is based on Chiang et al. (1998). In contrast, in many existing methods, every consideration set receives a strictly positive probability, as pointed out by Crawford et al. (2021). Now, due to the independence structure in (9) over  $j = 1, \dots, J$ ,

$$\pi(C_{ij} | \mathbf{C}_i \setminus \{j\}, \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{q}_{S_i}, S_i, \mathbf{y}_i, \mathbf{w}_i) \propto p(\mathbf{Y}_i = \mathbf{y}_i | \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{w}_i, \mathbf{C}_i) \cdot q_{S_{ij}}^{C_{ij}} (1 - q_{S_{ij}})^{1 - C_{ij}},$$

where  $\mathbf{C}_i \setminus \{j\}$  denotes  $\mathbf{C}_i$  without the coordinate  $j$ . To sample from this distribution, we employ the Metropolis-Hastings (M-H) algorithm (Chib and Greenberg, 1995). An effective implementation of this approach is detailed in Algorithm 1.

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**Algorithm 1:** M-H step for Sampling Consideration Sets

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**Input:** The current draws at the  $g$ th iteration

$$\{\mathbf{C}_i^{(g)}\}, \{\mathbf{q}_h^{(g)}\}, \{S_i^{(g)} = h\}, \boldsymbol{\beta}^{(g)}, \{\mathbf{b}_i^{(g)}\}$$

**Output:** The updated consideration sets  $\{\mathbf{C}_i^{(g+1)}\}$

**for**  $i \in \{1, \dots, n\}$  **do**

**for**  $j \in \{1, \dots, J\}$  **do**

        1) Propose  $\tilde{C}_{ij} \sim \text{Bernoulli}(q_{hj}^{(g)})$  and define

$$\mathbf{C}_i^{(1)} = (C_{i1}^{(g+1)}, \dots, C_{ij-1}^{(g+1)}, \tilde{C}_{ij}, C_{ij+1}^{(g)}, \dots, C_{iJ}^{(g)})'$$

        2) Accept  $\tilde{C}_{ij}$  with probability

$$\min \left\{ \frac{p(\mathbf{Y}_i = \mathbf{y}_i | \boldsymbol{\beta}^{(g)}, \mathbf{b}_i^{(g)}, \mathbf{w}_i, \mathbf{C}_i^{(1)})}{p(\mathbf{Y}_i = \mathbf{y}_i | \boldsymbol{\beta}^{(g)}, \mathbf{b}_i^{(g)}, \mathbf{w}_i, \mathbf{C}_i^{(0)})}, 1 \right\},$$

        where  $\mathbf{C}_i^{(0)} = (C_{i1}^{(g+1)}, \dots, C_{ij-1}^{(g+1)}, C_{ij}^{(g)}, C_{ij+1}^{(g)}, \dots, C_{iJ}^{(g)})'$ .

        Otherwise, set  $C_{ij}^{(g+1)} = C_{ij}^{(g)}$

---

In Step 1 of Algorithm 1, we generate a proposal from a one-dimensional Bernoulli distribution. In Step 2, given the current state  $\mathbf{C}_i^{(0)}$  and the proposed state  $\mathbf{C}_i^{(1)}$ , the



acceptance probability is computed as the ratio of the likelihood contributions for subject  $i$ . This Metropolis-Hastings step is valid because the likelihood  $p(\mathbf{Y}_i = \mathbf{y}_i \mid \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{w}_i, \mathbf{C}_i)$  is uniformly bounded. See [Chib and Greenberg \(1995\)](#) (p. 330, the third algorithm) for more discussion. In practice, we update the states in a random order within each MCMC iteration. In addition, the computational burden is minimized by parallelizing the loop on the  $n$  subjects.

Finally, the proposed Metropolis-Hastings step exhibits an important sparsity property. Suppose that an alternative  $j$  was not chosen by the subject  $i$  in any period (otherwise, it must be in the consideration set for  $i$  and  $C_{ij} = 1$ ). Depending on the current  $C_{ij}^{(g)}$ , and the proposed  $\tilde{C}_{ij}$ , there are four possible moves in the M-H step. First, if  $\tilde{C}_{ij} = C_{ij}^{(g)} = 1$  or  $\tilde{C}_{ij} = C_{ij}^{(g)} = 0$ , then the proposed value is accepted with probability one. Second, if  $\tilde{C}_{ij} = 0$  and  $C_{ij}^{(g)} = 1$ , then the proposed value is also accepted with probability one. In other words, the algorithm “prefers” a smaller consideration set. This sparsity-inducing property is proven below. Lastly, when the proposed consideration set adds an alternative  $j$  that is not in the current consideration set, that is,  $\tilde{C}_{ij} = 1$  and  $C_{ij}^{(g)} = 0$ , the acceptance probability is between 0 and 1 and is determined by the likelihood ratio.

**Proposition 1** (Sparsity-inducing property). *Consider the M-H step described in Algorithm 1. Let  $j$  be an alternative that is not observed to be chosen by the subject  $i$ . If the step proposes to exclude  $j$  from the consideration set of  $i$ , it is accepted with probability 1.*

*Proof.* Let the consideration set for the  $i$ th subject at iteration  $g$  be  $\mathcal{C}_i^{(g)}$ . Suppose that a category  $j \in \mathcal{C}_i^{(g)}$  is proposed to be removed so that  $\tilde{\mathcal{C}}_i = \mathcal{C}_i^{(g)} \setminus \{j\}$ . The acceptance probability is

$$\min \left\{ \frac{p(\mathbf{Y}_i = \mathbf{y}_i \mid \boldsymbol{\beta}^{(g)}, \mathbf{b}_i^{(g)}, \mathbf{w}_i, \tilde{\mathcal{C}}_i)}{p(\mathbf{Y}_i = \mathbf{y}_i \mid \boldsymbol{\beta}^{(g)}, \mathbf{b}_i^{(g)}, \mathbf{w}_i, \mathcal{C}_i^{(g)})}, 1 \right\} = \min \left\{ \frac{\prod_t \sum_{\ell \in \mathcal{C}_i^{(g)}} \exp(V_{i\ell t})}{\prod_t \sum_{\ell \in \tilde{\mathcal{C}}_i} \exp(V_{i\ell t})}, 1 \right\} = 1,$$

where  $V_{ijt} = \mathbf{x}'_{ijt}\boldsymbol{\beta}^{(g)} + \mathbf{z}'_{ijt}\mathbf{b}_i^{(g)}$ , and the last equality is due to the fact that the ratio is larger than 1. Hence,  $\tilde{\mathcal{C}}_i$  is accepted with probability 1.  $\square$

### 4.3 Numerical illustration

We illustrate posterior probabilities of consideration sets on synthetic panel data with  $n = 100$  subjects observed over  $T \in \{1, 2, \dots, 15\}$  time periods. We let  $J = 4$  and give the  $2^J - 1 = 15$  consideration sets in the first column of Table 1. In the table we report the posterior probabilities of each possible consideration set for a randomly chosen subject  $i$  whose true consideration set is  $\mathcal{C}_i^* = \{1, 3, 4\}$ .

Table 1: Posterior probabilities of consideration sets for unit  $i$

	$T=1$	$T=2$	$T=3$	$T=4$	$T=5$	$T=6$	$T=7$	$T=8$	$T=9$	$T=10$	$T=11$	$T=12$	$T=13$	$T=14$	$T=15$
{1}	0.059	0.602	0	0	0	0	0	0	0	0	0	0	0	0	0
{2}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
{3}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
{4}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
{1, 2}	0.199	0.15	0	0	0	0	0	0	0	0	0	0	0	0	0
{1, 3}	0.046	0.033	0	0	0	0	0	0	0	0	0	0	0	0	0
{1, 4}	0.065	0.12	0.691	0.738	0.746	0.78	0.834	0.864	0.884	0.844	0	0	0	0	0
{2, 3}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
{2, 4}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
{3, 4}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
{1, 2, 3}	0.113	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0
{1, 2, 4}	0.243	0.048	0.164	0.138	0.113	0.119	0.058	0.052	0.042	0.035	0	0	0	0	0
{1, 3, 4}	0.048	0.028	0.118	0.1	0.114	0.091	0.101	0.074	0.072	0.114	0.969	0.99	0.98	0.993	0.992
{2, 3, 4}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
{1, 2, 3, 4}	0.227	0.009	0.027	0.024	0.027	0.01	0.007	0.01	0.002	0.007	0.031	0.01	0.02	0.007	0.008
$y_{iT}$	1	1	4	1	4	1	4	4	1	1	3	3	1	1	4
Acc.Rate.	0.876	0.77	0.647	0.618	0.641	0.64	0.598	0.591	0.605	0.588	0.662	0.685	0.693	0.696	0.709

The results are based on a synthetic panel data with  $J = 4$  and  $n = 100$ . The true consideration set is  $\mathcal{C}_i^* = \{1, 3, 4\}$ . The row  $y_{iT}$  shows the actual response made by subject  $i$  at time  $T$ . Acc. Rate denotes the acceptance rate of consideration sets in the M-H step.

The first column ( $T = 1$ ) shows the results for the initial period given the observed outcome of 1. Consideration sets that do not include item 1 have a posterior probability of zero. As  $T$  increases, the posterior concentrates on the true consideration set  $\{1, 3, 4\}$ .

## 5 Monte Carlo Simulation

We demonstrate the sampling performance of the proposed approach through simulation studies containing  $J = 4$  and  $J = 101$  alternatives. The goal is to empirically validate the findings of Theorem 2 and demonstrate that the proposed approach can effectively

assess consideration dependence. In the Supplementary Material, we conduct additional experiments under autocorrelated covariates, random effects, and time-varying true consideration sets. In general, the experiments show posterior consistency in the estimation of  $\theta = (\beta, \mathbf{D})$  and  $\pi$ , and that the restrictive approach with  $K = 1$  produces larger mean-square-errors and biases.

### 5.1 $J = 4$

We let  $T_i = T$  for all  $i$ . In one case we set  $T = 5$  and in the other  $T = 15$ . The latter satisfies the length condition of Theorem 2. In simulating the data, we first specify the distribution of the consideration sets  $\pi^* = \{\pi_c^* = \Pr(\mathcal{C}_i = c) : c \in \mathcal{C}\}$ . We induce dependence in product consideration by letting the first two and last two products have a relatively high probability of being considered together:  $\pi_{\{1,2\}}^* = \pi_{\{3,4\}}^* = 0.25$ . As motivation, the first two products might represent non-vegetarian options, and the last two vegetarian. The other 13 consideration sets  $c \in \mathcal{C}$  are given a probability of 0.0385. Figure 1 shows  $\pi^*$  in red. Given this  $\pi^*$ , we generate the true consideration sets  $\mathcal{C}_i^*$ , for  $i = 1, \dots, n$ . We then generate outcomes from the logit model with  $V_{ijt} = \delta_j^* + \beta^* x_{ijt}$ , letting  $(\delta_1^*, \delta_2^*, \delta_3^*, \delta_4^*)' = (1.0, 0.5, -1.0, 0)'$  and  $\beta^* = 1$ , and  $x_{ijt} \stackrel{iid}{\sim} N(0, 1)$ .

We compare the performance between the proposed infinite mixture of independent consideration models ( $K = \infty$ ) and the model that assumes independent consideration ( $K = 1$ ) over 50 replicated data sets. The results are given in Table 2 where we report the mean-square-error (MSE), posterior standard deviation, and empirical coverage of the equal-tailed 95% credible intervals and the computational time. As  $n$  increases, the posterior of the response parameter  $\beta = (\delta_1, \delta_2, \delta_3, \beta)$  and the distribution of the consideration sets  $\pi$  is close to the true values, even when  $T = 5$ . In contrast, when  $K = 1$ , we do not observe sufficient evidence of posterior consistency. In particular, MSEs are an order of

magnitude larger in some cases than those under  $K = \infty$ , due to misspecification. When  $T$  increases to 15, the MSEs/SDs become smaller for both  $K = \infty$  and  $K = 1$ , but there are distortions in the coverage for  $K = 1$ .

Table 2: Simulation results with  $J = 4$

$(K, T)$	$n$	$\beta$	$\delta_1$	$\delta_2$	$\delta_3$	$\pi$	Time
$(\infty, 5)$	50	0.029 (0.16 ; 0.92)	0.28 (0.42 ; 0.90)	0.299 (0.43 ; 0.90)	0.098 (0.34 ; 0.96)	0.46 (0.03)	1.68
	200	0.006 (0.08 ; 0.96)	0.047 (0.22 ; 0.98)	0.041 (0.22 ; 0.98)	0.034 (0.17 ; 0.94)	0.266 (0.01)	5.7
$(\infty, 15)$	50	0.007 (0.09 ; 0.94)	0.033 (0.2 ; 0.98)	0.037 (0.2 ; 0.94)	0.028 (0.18 ; 0.98)	0.36 (0.01)	2.19
	200	0.002 (0.04 ; 0.92)	0.011 (0.1 ; 0.92)	0.01 (0.1 ; 0.92)	0.007 (0.09 ; 0.92)	0.191 (0.01)	7.54
$(1, 5)$	50	0.024 (0.15 ; 0.96)	0.787 (0.4 ; 0.46)	0.927 (0.4 ; 0.42)	0.094 (0.33 ; 0.94)	0.728 (0.03)	1.4
	200	0.01 (0.07 ; 0.8)	0.894 (0.19 ; 0.00)	0.949 (0.2 ; 0.02)	0.032 (0.17 ; 0.94)	0.721 (0.01)	4.63
$(1, 15)$	50	0.007 (0.09 ; 0.94)	0.068 (0.21 ; 0.86)	0.073 (0.21 ; 0.90)	0.032 (0.18 ; 0.98)	0.402 (0.01)	1.98
	200	0.002 (0.04 ; 0.94)	0.029 (0.11 ; 0.76)	0.023 (0.11 ; 0.78)	0.011 (0.09 ; 0.88)	0.261 (0.01)	6.17

For  $\beta$  and  $\delta$ , we show the average of the mean squared errors (MSEs), using the posterior means as point estimators. In parentheses, the average posterior standard deviations (SDs) and empirical coverages of 95% credible intervals are shown. For  $\pi$ , we show the average of  $L_1$  norm between the posterior mean and  $\pi^*$  with the posterior SDs in the parentheses. Time is the average seconds taken for sampling 1,000 MCMC draws in Matlab on a desktop with a 4.9GHz processor and 64GB RAM.

The vertical axes of Figure 1 list the 15 consideration sets, with the true distribution of the consideration sets,  $\pi^*$ , highlighted in red. Each panel of the figure displays the posterior mean (solid with dots, blue) along with the 95% credible intervals (dashed, blue), based on one realized data set. The first two panels illustrate that under the proposed approach ( $K = \infty$ ), as the sample size  $n$  increases, the discrepancy between the posterior mean and the true distribution diminishes. In contrast, the right two panels show that when  $K = 1$ , even as  $n$  increases, the posterior does not adequately converge to the truth. This is because the model does not account for the true consideration dependence.

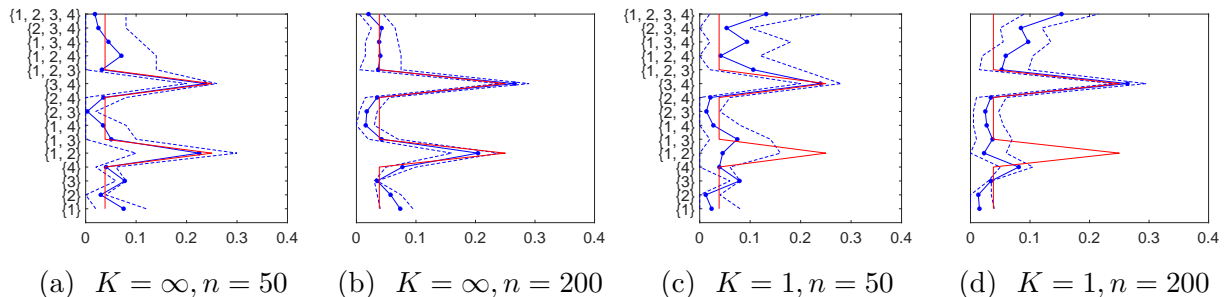


Figure 1: The true distribution over consideration sets (solid, red), posterior mean (solid with dots, blue), 95% equal-tailed credible interval (dashed, blue). Each plot is based on one realization of simulated data.  $J = 4, T = 5$ .

## 5.2 $J = 100$

We now consider a high-dimensional scenario with  $J = 100$ . One mechanism by which the dependence of consideration among categories can be induced is through multiple latent subpopulations of subjects having different probabilities of consideration. Within a subpopulation, considerations are independent across categories. However, marginalizing out the latent subpopulation indicator, one obtains dependence in those category considerations. We generate the data with two subpopulations. To generate the true consideration set of a given subject, we used a Bernoulli distribution with attention probability 0.05 for each category except for categories 10, 30, 50, 70, and 90 for the first subpopulation ( $i = 1, \dots, 50$ ) where the Bernoulli attention probability was set to 0.8. For the remaining subjects in the second subpopulation ( $i = 51, \dots, 100$ ), the Bernoulli probability was set at 0.05 except for categories 20, 40, 60, 80, and 100 where the probability was set to 0.8. Conditional on the true consideration sets, we generated the responses as in the case with  $J = 4$ .

Because in this case there are  $2^{100} - 1$  support points in the distribution of the consideration sets, it is not possible to show the entire distribution as in the case of  $J = 4$  before. We focus on the estimation results regarding subject 1 whose true consideration set contains 7 categories:  $\mathcal{C}_1^* = \{10, 30, 50, 70, 75, 88, 90\}$ . The upper panels of Figure 2 show the posterior probabilities that  $\mathcal{C}_1$  include each of the categories (filled square/circle if the particular category in the true consideration set has/has not been chosen by  $T$ ; unfilled circle if it is outside the true consideration set). The lower panels of Figure 2 are the  $n \times n$  “similarity matrices”. These give the posterior probability that a given subject in a particular row  $k$  is in the same cluster as another subject at a specific column  $i$  which is computed as the posterior probability of the event  $\{S_k = S_i\}$ . This probability ranges

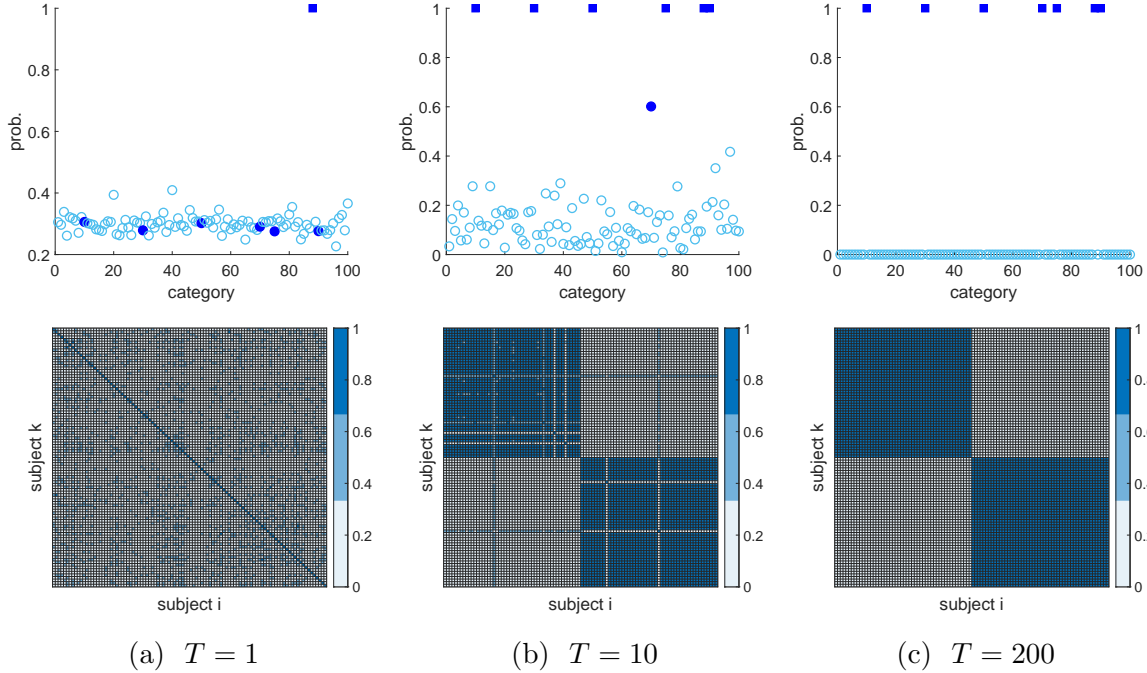


Figure 2: Upper panels: Posterior mean of  $1\{j \in \mathcal{C}_1\}$  for each item  $j = 1, \dots, J$ . Filled square(circle) if the category  $j$  within the true consideration set  $\mathcal{C}_1^* = \{10, 30, 50, 70, 75, 88, 90\}$  has(not) been chosen by  $T$ . Unfilled circle if it is outside  $\mathcal{C}_1^*$ . Lower panels: similarity matrices. The results are based on the simulated data with  $J = 100$ ,  $T = \{1, 10, 200\}$ , and  $n = 100$ . The time (seconds) taken for sampling 1,000 MCMC draws in Matlab on a desktop with a 4.9GHz processor and 64GB RAM are 53.5, 97.1, and 1788.0, respectively.

from zero (light blue) to one (dark blue).

We again observe posterior concentration toward the true consideration set as  $T$  increases. Even with a relatively short  $T = 10$ , category 70, which subject 1 has not yet chosen, has a relatively high posterior probability of being an element of the subject's consideration set. This occurs because subject 1 is clustered with others whose observed choices include category 70. By  $T = 10$ , the estimated similarity matrix closely aligns with the true clustering structure, leading to consistent consideration set estimates within each cluster. For  $T = 200$ , a span that approximately satisfies the identification condition in Theorem 2, the clustering structure is recovered with high accuracy.

## 6 Application to Cereal Consumption in Midwest

In this section, we apply our approach to a manually constructed longitudinal data set that includes  $J = 101$  cereal brands, a size that is significantly beyond the feasibility of existing methods. For comparison,  $J$  was 4 in [Chiang et al. \(1998\)](#), 10 in [Van Nierop et al. \(2010\)](#), and 5 in [Aguiar and Kashaev \(2024\)](#). We constructed the data set by integrating Nielsen Consumer Panel data with Retail Scanner Data, focusing on weekly shopping trips in 2019 in stores operated by a single anonymous retailer primarily based in the United States Midwest. Although data from 2020 are available, we chose to use the most recent pre-pandemic year to avoid potential biases introduced by pandemic-related shopping behavior. This particular retailer was selected because it consistently stocked more than 100 cereal brands throughout the sample period. Furthermore, we limited our analysis to a single retailer to prevent inconsistencies in brand definitions between different retailers, which would have required speculative alignment of brand names from various sources. The final data set includes  $J = 101$  brands and  $n = 1880$  households, covering 25,849 purchases in 239 stores during the 52-week period in 2019. See [Figure 3](#), for the locations of these stores with relative purchase volumes, and [Table 4](#), for the list of the brands. The average number of shopping trips per household ( $T_i$ ) is 13.7, and the price  $P_{ijt}$  of each brand  $j \in 1, \dots, J$  is represented by a size-weighted price index constructed from prices at the UPC level. For the analysis, we used the first 10 months of data for the estimation and reserved the last two months for the prediction outside the sample. Further details on data preparation are provided in the Supplementary Material.

Conditional on the consideration set  $\{\mathcal{C}_i\}$ , in the most general version of the model, we enter the fixed effects and random effects in the MNL model  $V_{ijt} = \delta_j + P_{ijt}(\beta + b_i)$ , where  $i \in \{1, \dots, 1880\}$  indexes households, and  $t \in \{1, \dots, T_i\}$  indexes purchase occasions. In



Figure 3: Locations of the 239 stores under the chosen retailer. Circle sizes correspond to purchases (percentages).

this model,  $\delta_j$  represents the brand-specific fixed effect for brand  $j$ , with the normalization  $\delta_J = 0$ . The parameter  $\beta$  is the common fixed effect, and  $b_i \sim \mathcal{N}(0, D)$  is the random effect for household  $i$ . We consider four variants of the MNL, differentiated by the inclusion of random effects and/or consideration set heterogeneity, as detailed in models (1)–(4) of Table 3. In addition, models (5) and (6) assume an independent consideration structure (i.e.,  $K = 1$ ). Each of these cases is estimated using the simulation method developed in Section 4, by omitting the components not present in the full hierarchical model (MNL-RC).

## 6.1 Empirical Results

We obtained 20,000 MCMC draws for each of the six models in Matlab on a desktop with a 4.9GHz processor and 64GB RAM. The average of the inefficiency factors is around 7.38 with standard deviation 2.41, indicating that the MCMC output mixes well. Broadly speaking, the estimated parameters of the response model from the approaches (1)–(4) shown in Table 3 are similar to those in the literature. For instance, when consideration set heterogeneity is incorporated, the magnitude of the slope parameter  $\beta$  on price increases



and the number of significant brand-specific terms  $\delta_j$ 's decreases (See Table 4 for the list of estimated  $\delta_j$ 's under the MNL\_RC model). These patterns are consistent with previous studies based on smaller models, including [Stopher \(1980\)](#), [Swait and Ben-Akiva \(1986\)](#), [Chiang et al. \(1998\)](#), and [Van Nierop et al. \(2010\)](#). However, without the scalable fitting methodology developed in this paper, it was unclear if those patterns would persist in a model of the scale we have estimated.

Moreover, when we control for consideration sets, the posterior mean of  $D^{1/2}$  decreases, which aligns with the findings in [Chiang et al., 1998](#), [Morozov et al., 2021](#) that random effect heterogeneity is overestimated in models that omit consideration set heterogeneity.

Table 3: Estimation results

	(1) MNL		(2) MNL_R		(3) MNL_C		(4) MNL_RC		(5) MNL_C_K1		(6) MNL_RC_K1	
	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
<i>Random effects on price</i>												
$\beta$	-0.69***	0.02	-0.77***	0.03	-0.73***	0.02	-0.82***	0.04	-0.73***	0.02	-0.85***	0.04
$D^{1/2}$	—	—	0.99	0.02	—	—	0.97	0.03	—	—	1.06	0.04
<i>Brand-specific fixed-effects</i>												
# of "significant" params.	97		98		75		67		73		69	
<i>Computational time</i>												
min. per 1,000 MCMC iters.	66		67		120		124		118		122	
<i>Test for indep. consid.</i>												
random effects	No		Yes		No		Yes		No		Yes	
Consideration sets	No		No		$K = \infty$		$K = \infty$		$K = 1$		$K = 1$	

$K = \infty$  ( $K = 1$ ) refers to the proposed infinite mixture of independent consideration models (the model under the independent consideration). The first panel shows posterior means of the mean  $\beta$  of the random effects on price and the standard deviations  $D^{1/2}$  with their posterior standard deviations. Three stars indicate that the corresponding 99% credible interval does not include 0. The second panel shows the number of brand-specific fixed effects whose 95% posterior credible intervals do not include 0 (out of 100 terms). See Table 4 for the estimated  $\delta_j$ 's under MNL\_RC. The third panel shows computational time (minutes) on a desktop with a 4.9GHz processor and 64GB RAM. The fourth panel shows the results for the test for independent consideration with the null hypothesis  $H_0$  : independent consideration, which we discuss in the Supplementary Material in detail. The results are based on 20,000 posterior draws. We discard the first 6,000 draws as a burn-in sample and use the remaining 14,000 draws for the analysis. The average of the inefficiency factors is around 7.38 with standard deviation 2.41.

Under the independent consideration assumption ( $K = 1$ ), i.e., (5) and (6), the estimated parameters are similar to the proposed flexible approach i.e., (3) and (4) except that the estimated  $D^{1/2}$  under (6) is slightly larger than (2), which contradicts with the previous studies. In general, it is possible that the obtained estimates under  $K = 1$  are biased, as shown in simulation studies in Section 5. We conduct the test for independent consideration, which is introduced and studied in Supplementary Material. Under both (3)

and (4), the estimated posterior probability of the alternative hypothesis (dependent consideration) is very close to one, and we conclude that the considerations of cereal products in this particular market are dependent.

Table 3 also shows the computational time per 1,000 MCMC draws. The extra burden of estimating latent consideration sets using our proposed approach is reasonable. For instance, when consideration sets are estimated along with random effects, the computational time roughly doubles (67 mins. for MNL\_R and 124 mins. for MNL\_RC). Not surprisingly, compared to the fully flexible estimator, the estimators that assume independent consideration take less computational time but only slightly.

## 6.2 Estimated parameters in the mixture model

We begin by reporting in Table 4 the posterior mean and standard deviation (s.d.) of the 100 brand fixed effect parameters. The 95% posterior credibility intervals of most of these brand-specific intercepts exclude zero indicating that these brands are endowed with significant brand equity. We next investigate the clustering of households according to the proposed mixture model. The posterior mode of the number of nonempty clusters under the full-specification (MNL\_RC) is six. The Supplementary Material shows further estimation results on the number of clusters and the DP concentration parameter  $\alpha$ .

To understand how households are clustered, we computed the posterior mean of the event that a given pair of households  $(i, k)$  are clustered together i.e.  $\{S_i = S_k\}$ . This results in a  $n \times n$  similarity matrix, which can be found in the Supplementary Material.

An examination of how households are clustered reveals interesting points. Take household A as an example whose actual choices consist of  $\{4, 37, 62, 64, 73\}$ . Define an estimator  $\hat{C}_i$  of the consideration set for household  $i$  as the set of brands  $j$  whose posterior probability that  $C_{ij} = 1$  is greater than 0.2658, the prior median of  $q_{hj}$ . This results in the

Table 4: MNL\_RC Model on Cereal Market data: Estimates of the 100 brand fixed-effects

brand	mean	s.d	brand	mean	s.d
1 BEAR NAKED FIT GRN	-0.02	0.29	51 KELLOGGS FROOT LOOPS	-0.27*	0.07
2 BEAR NAKED GRN	0.48*	0.19	52 KELLOGGS FROOT LOOPS MARSHMALLOW	-1.38*	0.18
3 BETTER OATS	-0.8*	0.21	53 KELLOGGS FROSTED FLAKES	0.19*	0.07
4 CREAM OF WHEAT	0.24*	0.11	54 KELLOGGS FROSTED MINIWHEATS	0.61*	0.06
5 CTL BR	-0.34*	0.06	55 KELLOGGS FROSTED MINIWHT LITTLE BTS	-0.41*	0.08
6 GENERAL MILLS APPLE CINNAMON CHEERIOS	-1.09*	0.15	56 KELLOGGS KRAVE	0.64*	0.09
7 GENERAL MILLS BLUEBERRY CHEX	-0.27	0.14	57 KELLOGGS RAISIN BRAN	-0.02	0.07
8 GENERAL MILLS BREAKFAST PACK	-1.62*	0.52	58 KELLOGGS RAISIN BRAN CRUNCH	0.08	0.07
9 GENERAL MILLS CHEERIOS	0.2*	0.06	59 KELLOGGS RICE KRISPIES	-0.32*	0.08
10 GENERAL MILLS CHEERIOS OAT CRUNCH CNMN	-0.38*	0.11	60 KELLOGGS RICE KRISPIES TREATS	0.11	0.32
11 GENERAL MILLS CHOCOLATE CHEERIOS	-1.53*	0.22	61 KELLOGGS SPECIAL K	-0.49*	0.17
12 GENERAL MILLS CHOCOLATE CHEX	-0.43*	0.13	62 KELLOGGS SPECIAL K CHOCOLATY DELGHT	0.18	0.11
13 GENERAL MILLS CHOCOLATE PNUT BTR CHEERIO	-1.01*	0.19	63 KELLOGGS SPECIAL K CINNAMON PECAN	-0.57*	0.17
14 GENERAL MILLS CINNAMON CHEX	-1.14*	0.18	64 KELLOGGS SPECIAL K FRUIT & YOGURT	0.25*	0.12
15 GENERAL MILLS CINNAMON TOAST CRUNCH	0.06	0.06	65 KELLOGGS SPECIAL K PROTEIN	-0.13	0.11
16 GENERAL MILLS CINNAMON TOAST CRUNCH CHR5	-1.51*	0.18	66 KELLOGGS SPECIAL K RED BERRY	0.24*	0.08
17 GENERAL MILLS COCOA PUFFS	-0.55*	0.09	67 KELLOGGS SPECIAL K VANILLA ALMOND	0.05	0.12
18 GENERAL MILLS COOKIECRISP	-0.96*	0.16	68 KELLOGGS STBY KRISPIES US OLYMPC TM	-1.5*	0.19
19 GENERAL MILLS CORN CHEX	-0.36*	0.11	69 MOM BERRY COLOSSAL CRN	-0.67*	0.33
20 GENERAL MILLS FIBER ONE	0.03	0.23	70 MOM CINNAMON TOASTERS	-0.03	0.28
21 GENERAL MILLS FIBER ONE HONEY CLUSTERS	0.42*	0.21	71 MOM COCOA DYNOBITES	-0.2	0.27
22 GENERAL MILLS FROSTED CHEERIOS	-1.67*	0.3	72 MOM FROSTED FLAKES	-0.25	0.31
23 GENERAL MILLS GOLDEN GRAHAMS	-0.06	0.08	73 MOM FROSTED MINI SPOONERS	0.72*	0.29
24 GENERAL MILLS HONEY NUT CHEERIOS	0.26*	0.06	74 MOM FRUITY DYNOBITES	0.01	0.25
25 GENERAL MILLS HONEY NUT CHEX	-0.93*	0.17	75 MOM GOLDEN PUFFS	0.37*	0.18
26 GENERAL MILLS LUCKY CHARMS	0.21*	0.06	76 MOM TOOTIE FRUITIES	-0.28	0.26
27 GENERAL MILLS MPL CHEERIOS CLC DSS FNDTN	-0.63*	0.11	77 POST COCOA PEBBLES	-0.42*	0.15
28 GENERAL MILLS MULTIGRAIN CHEERIOS	0.1	0.08	78 POST FRUITY PEBBLES	-0.26*	0.1
29 GENERAL MILLS NATURE VALLEY GRN PROTEIN	-0.35	0.37	79 POST GOLDEN CRISP	-1.35*	0.18
30 GENERAL MILLS RAISIN NUT BRAN	0.26	0.17	80 POST GRAPENUTS	-0.21	0.21
31 GENERAL MILLS REESE'S PUFFS	0.31*	0.07	81 POST GRAPENUTS FLAKES	0.31	0.29
32 GENERAL MILLS RICE CHEX	-0.33*	0.1	82 POST HONEY BUNCHES OF OATS	0.37*	0.07
33 GENERAL MILLS VANILLA CHEX	-0.74*	0.16	83 POST HONEY BUNCHES OF OATS GRN	-2.15*	0.73
34 GENERAL MILLS VERY BERRY CHEERIOS	-0.86*	0.15	84 POST HONEYCOMB	-0.97*	0.14
35 GENERAL MILLS WHEAT CHEX	-0.56*	0.16	85 POST OREO OS	-1.63*	0.36
36 GENERAL MILLS WHEATIES	0.16	0.18	86 POST RAISIN BRAN	-0.5*	0.22
37 KASHI CINNAMON HARVEST	-0.55*	0.29	87 POST SELECTS GREAT GRAINS	-0.14	0.12
38 KASHI GO LEAN	-0.87*	0.16	88 POST SHRD WHT 'N BRN SP SZ	0.46*	0.16
39 KASHI GO LEAN CRUNCH!	-1.36*	0.28	89 POST SHREDDED WHEAT	-0.92*	0.35
40 KASHI ORGANIC BLUEBERRY CLST	-1.4*	0.28	90 QUAKER	-0.05	0.06
41 KELLOGGS AL JS CN PS FRFL FTLF CKSP	-3.02*	0.53	91 QUAKER CAP'N CRN	-0.76*	0.13
42 KELLOGGS ALLBRAN	-0.53	0.3	92 QUAKER CAP'N CRN CRN BRY	-0.89*	0.12
43 KELLOGGS ALLBRAN COMPLETE WHT FLK	0.59	0.45	93 QUAKER CINNAMON LIFE	-0.47*	0.09
44 KELLOGGS APPLE JACKS	-0.25*	0.09	94 QUAKER GRN	-0.91	0.67
45 KELLOGGS CHOCOLT FR5 FLKS TN TH TGR	-1.68*	0.26	95 QUAKER LIFE	-0.58*	0.1
46 KELLOGGS COCOA KRISPIES	-0.64*	0.12	96 QUAKER OATMEAL SQUARES	-0.1	0.11
47 KELLOGGS CORN FLAKES	-0.37*	0.1	97 QUAKER OVERNIGHT OATS	-2.33*	0.26
48 KELLOGGS CORN POPS	-0.5*	0.12	98 QUAKER PROTEIN	-0.49*	0.15
49 KELLOGGS CRACKLIN' OAT BRAN	0.49	0.24	99 QUAKER REAL MEDLEYS	-1.06*	0.22
50 KELLOGGS CRISPIX	-0.09	0.1	100 QUAKER SELECT STARTS	-0.88*	0.14

The stars indicate that the corresponding 95% credible interval does not include 0. The "other" option -specific fixed-effect is normalized to 0 for identification. The results are obtained under the MNL\_RC model.

estimated set  $\hat{C}_A = \{4, 26, 37, 59, 60, 61, 62, 64, 68, 73, 79, 101\}$ . The upper panel of Table 5 lists the three households with the highest posterior similarity to subject A. There are several observations.

First, the actual choices of the households tend to overlap within a cluster; each household purchased at least one of brands 5, 73, or 89. Second, the estimated consideration sets  $\hat{C}_k$  are similar between households in a cluster. For example, household A did not choose brands 26, 79, and 101, but other households did, and they are in  $\hat{C}_A$ . Third, the stronger the purchase overlap, the higher the chance of being in the same cluster. The lower panel of Table 5 shows the results for household B. In this cluster, brands 79 and 81 were purchased by all the four households, brands 26, 51, 68, and 101 were each purchased by

Table 5: Households clustered with  $i \in \{A, B\}$ .

$k$	Similarity	Chosen brands	$\hat{C}_k$
<i>Household <math>i = A</math></i>			
$k = A$	1.00	{4, 37, 62, 64, 73}	{4, 26, 37, 59, 60, 61, 62, 64, 68, 73, 79, 101}
$k = 1357$	0.36	{8, 11, 73, 89}	{3, 8, 11, 26, 59, 60, 61, 68, 73, 79, 89, 101}
$k = 226$	0.36	{5, 25, 33, 39, 53, 63, 77, 79, 81, 89, 92}	{3, 5, 25, 33, 39, 53, 60, 63, 68, 77, 79, 81, 89, 92, 101}
$k = 105$	0.35	{3, 5, 26, 101}	{3, 5, 25, 26, 59, 60, 61, 63, 66, 68, 73, 79, 101}
<i>Household <math>i = B</math></i>			
$k = B$	1.00	{25, 26, 49, 77, 79, 81, 101}	{3, 25, 26, 49, 59, 60, 63, 68, 73, 77, 79, 81, 101}
$k = 1689$	0.71	{7, 33, 49, 51, 68, 69, 79, 81, 96, 101}	{3, 7, 33, 49, 51, 68, 69, 79, 81, 96, 101}
$k = 481$	0.65	{5, 7, 8, 12, 25, 26, ..., 51, ..., 68, 69, 73, 77, 78, 79, 81, 89, 90, 101}	{5, 7, 8, 12, 25, 26, ..., 51, ..., 68, 69, 73, 77, 78, 79, 81, 89, 90, 101}
$k = 1019$	0.62	{12, 26, 27, 51, 60, 67, 68, 72, 79, 80, 81, 86}	{3, 12, 25, 26, 27, 51, 59, 60, 63, 67, 68, 72, 73, 79, 80, 81, 86, 101}

Similarity is defined as the posterior mean of  $1\{S_k = S_i\}$ , which corresponds to the  $i$ th row (equivalently  $i$ th column) of the similarity matrix presented in the Supplementary Material. The estimated consideration set  $\hat{C}_i$  is defined as the set of brands  $j$  whose posterior probability that  $C_{ij} = 1$  is greater than 0.2658, the prior median of  $q_{hj}$ . The result is from the MNL\_RC model.

three households, and we see higher similarity scores ( $\geq 0.60$ ). In this way, our algorithm discovers the probabilistic grouping patterns in the choice data.

### 6.3 Price sensitivity of demand

To analyze household shopping behavior, we randomly select 100 units and report in Figure 4 the percentage decrease in aggregate demand when the price of a brand increases

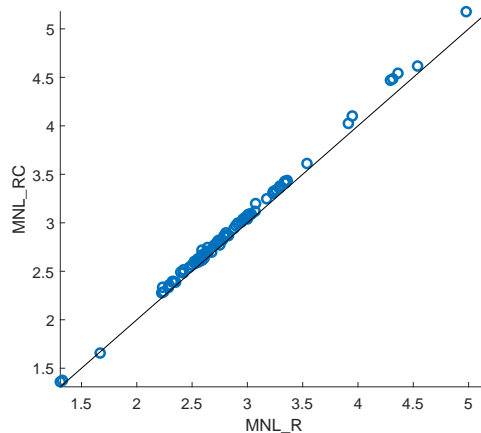


Figure 4: Price sensitivity of aggregate demand based on a random set of 100 households. Each circle represents the percentage decrease in demand of brand  $j$  when its price increases by 1%,  $j = 1, \dots, J$  under the MNL\_R and MNL\_RC models. The 45-degree line is plotted as a solid line.

by 1% under the MNL\_R and MNL\_RC models. For all but one brand, this sensitivity is higher under consideration set heterogeneity, in conformity with previous findings that were derived in a small  $J$  setting.

## 6.4 Predictive performance

We next assess the predictive performance of the proposed model using the last two months of data as an out-of-sample period. Let  $\mathcal{O} \subset \{1, \dots, n\}$  denote the set of subjects who made purchases in the out-of-sample period. This set contains 1079 subjects. For each  $i \in \mathcal{O}$ , we predict  $\mathbf{Y}_i^f = \{Y_{iT_i+s} : s = 1, \dots, h_i\}$ , given the covariates  $\mathbf{w}_i^f = \{\mathbf{w}_{iT_i+s} : s = 1, \dots, h_i\}$ , where  $h_i$  denotes the forecast horizon for the subject  $i$ . Let  $\mathbf{y}_i^f = \{y_{iT_i+s} : s = 1, \dots, h_i\}$  be the actual set of responses for the subject  $i \in \mathcal{O}$ . Then, as a measure of predictive performance, we calculate the predictive likelihoods

$$\begin{aligned} p(\mathbf{y}_i^f | \mathbf{y}, \mathbf{w}, \mathbf{w}_i^f) &= \int \prod_{s=1}^{h_i} \Pr(Y_{iT_i+s} = y_{iT_i+s} | \boldsymbol{\delta}, \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{w}_{iT_i+s}, \mathcal{C}_i) d\pi(\boldsymbol{\delta}, \boldsymbol{\beta}, \{\mathbf{b}_i\}, \{\mathcal{C}_i\} | \mathbf{y}, \mathbf{w}) \\ &\approx \frac{1}{G} \sum_{g=1}^G \prod_{s=1}^{h_i} \Pr(Y_{iT_i+s} = y_{iT_i+s} | \boldsymbol{\delta}^{(g)}, \boldsymbol{\beta}^{(g)}, \mathbf{b}_i^{(g)}, \mathbf{w}_{iT_i+s}, \mathcal{C}_i^{(g)}), i \in \mathcal{O}, \end{aligned}$$

where the response probability conditional on a consideration set is given in (5). Figure 5 gives the log-predictive likelihood for each household under the (MNL\_R) and (MNL\_RC) models. The higher predictive likelihood under the latter model shows that including consideration set heterogeneity tends to improve predictive performance. More details about this are given in the Supplementary Material.

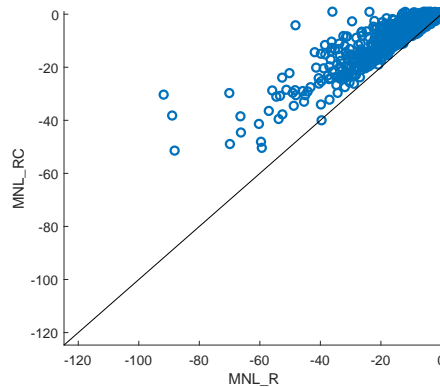


Figure 5: Log-predictive likelihoods (circles) for the 1079 households that made purchases in the out-of-sample period. The x-coordinate of each circle is the log-predictive likelihood under MNL\_R, and the y-coordinate is under MNL\_RC. The 45-degree line is plotted as a solid line.

## 7 Discussion

In this concluding section, we discuss the broader relevance of the work, especially to the modeling of excess zeros in high-dimensional sparse microbiome data sets. In a microbiome dataset with  $n$  samples and  $J$  taxa, let  $x_{ij}$  denote the measured count for taxon  $j$  in sample  $i$ , and  $T_i = \sum_{j=1}^J x_{ij}$  represent the total count for all taxa in the  $i$ th sample, where  $i = 1, \dots, n$  and  $j = 1, \dots, J$ . Typically it is assumed that the vector  $\mathbf{x}_i = (x_{i1}, \dots, x_{iJ})'$  follows a multinomial distribution with index  $T_i$  and a vector of probabilities  $\boldsymbol{\rho}_i = (\rho_{i1}, \dots, \rho_{iJ})'$ , where  $0 < \rho_{ij} < 1$  and  $\sum_{j=1}^J \rho_{ij} = 1$ . In our notation,  $Y_{it}$  denotes the selected item and relates to  $\mathbf{x}_i$  through  $x_{ij} = \sum_{t=1}^{T_i} 1(Y_{it} = j)$ . To address the high dimensionality and sparsity of such datasets, [Zeng et al. \(2023\)](#) propose the following hierarchical model translated in our terminology as

$$C_{ij} \stackrel{\text{ind}}{\sim} \text{Bernoulli}(q_j), \quad f_{i1}, \dots, f_{ik} \stackrel{\text{ind}}{\sim} \mathcal{N}(0, 1),$$

$$\mathbf{x}_i | \boldsymbol{\rho}_i, T_i \stackrel{\text{ind}}{\sim} \text{MN}(\boldsymbol{\rho}_i, T_i), \quad \rho_{ij} = \frac{C_{ij} \exp(\beta_{0j} + \mathbf{f}'_i \boldsymbol{\beta}_j)}{\sum_{\ell=1}^J C_{i\ell} \exp(\beta_{0\ell} + \mathbf{f}'_i \boldsymbol{\beta}_\ell)},$$

where  $1 - C_{ij}$  are latent indicators for excess zeros, and  $q_j$  are the corresponding probabilities. The  $\mathbf{f}_i$  are latent factors, and the  $\boldsymbol{\beta}_j$  denote the loadings of the associated factors. Note that the excess zeros across taxa are independent in this modeling. In other words, their model corresponds to the independent consideration model that we review in the introduction. Our modeling can be used if excess zeros exhibit complex dependency patterns across taxa. In this more general setting,  $\mathbf{C}_i = (C_{i1}, \dots, C_{iJ})'$  are correlated vectors and i.i.d. with the density for the infinite mixture of independent consideration models (3). Specifically, we would have  $\Pr(\mathbf{C}_i = \mathbf{c}_i) = \sum_{h=1}^{\infty} \omega_h \prod_{j=1}^J \{q_{hj}^{c_{ij}} (1 - q_{hj})^{1-c_{ij}}\}$ . We can adapt the MCMC framework of this paper to estimate this model.

Another key issue in practice is variable selection when many subject-level covariates

are available. This challenge can be addressed using shrinkage priors. A natural extension of our framework involves modeling consideration sets that change at one or two points in time due to learning from past choices. This would require incorporating the learning process into the model, and modifying the theoretical analysis accordingly. We leave this promising direction for future work. Matlab code for estimating our model is available.

## A Proof of Lemma 2

*Proof of Lemma 2.* Recall that  $p_{\theta, \pi}(\mathbf{y}|\mathbf{w}) \equiv \sum_{c \in \mathcal{C}} \pi_c \Pr(\mathbf{Y}_i = \mathbf{y} | \boldsymbol{\theta}, \mathbf{w}, c)$ . For any  $\mathbf{y} \in \mathcal{J}^T$ , if  $p_{\theta^*, \pi^*}(\mathbf{y}|\mathbf{w}) = 0$ , the integrand in the KL divergence is  $\log(0)0$  which is defined to be zero. Therefore, without loss of generality, suppose that for all  $\mathbf{y}$ , there is  $c_y \in \mathcal{C}$  that contains all the elements of  $\mathbf{y}$  and have  $\pi_{c_y}^* > 0$ . By Lemma 1, we can find a finite mixture of independent consideration models that exactly matches the true distribution of consideration sets; i.e.  $\exists(K, \tilde{\boldsymbol{\phi}}_{1:K})$  such that  $\pi_c^* = \sum_{h=1}^K \tilde{\omega}_h \prod_{j \in c} \tilde{q}_{hj} \prod_{j \notin c} (1 - \tilde{q}_{hj})$  for each  $c \in \mathcal{C}$ . Hence we have

$$\begin{aligned} & \int \sum_{\mathbf{y}} \log \left( \frac{p_{\theta^*, \pi^*}(\mathbf{y}|\mathbf{w})}{p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}, K, \boldsymbol{\phi}_{1:K})} \right) p_{\theta^*, \pi^*}(\mathbf{y}|\mathbf{w}) g^*(\mathbf{w}) d\mathbf{w} \\ &= \int \sum_{\mathbf{y}} \left\{ \log \left( \frac{p_{\theta^*, \pi^*}(\mathbf{y}|\mathbf{w})}{p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}^*, K, \tilde{\boldsymbol{\phi}}_{1:K})} \right) + \log \left( \frac{p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}^*, K, \tilde{\boldsymbol{\phi}}_{1:K})}{p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}, K, \boldsymbol{\phi}_{1:K})} \right) \right\} p_{\theta^*, \pi^*}(\mathbf{y}|\mathbf{w}) g^*(\mathbf{w}) d\mathbf{w}, \end{aligned}$$

and the first term in the brackets is zero. Hence, it suffices to show that the integral of the second term is continuous in  $(\boldsymbol{\theta}, \boldsymbol{\phi}_{1:K})$  at  $(\boldsymbol{\theta}^*, \tilde{\boldsymbol{\phi}}_{1:K})$ . In the Supplementary Material, we prove that the response probability is continuous in  $\boldsymbol{\phi}_{1:K}$  (Lemma SB1) and it is continuous also in  $\boldsymbol{\theta}$  (Lemma SB2). Let  $(\boldsymbol{\theta}^m, \boldsymbol{\phi}_{1:K}^m)$  be a sequence of parameter values converging to  $(\boldsymbol{\theta}^*, \tilde{\boldsymbol{\phi}}_{1:K})$ . Then

$$\lim_{m \rightarrow \infty} \log \left( \frac{p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}^*, K, \tilde{\boldsymbol{\phi}}_{1:K})}{p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}^m, K, \boldsymbol{\phi}_{1:K}^m)} \right) = 0.$$

The result will follow from the dominated convergence theorem if there is an integrable (with respect to  $p_{\theta^*, \pi^*}(\mathbf{y}|\mathbf{w})g^*(\mathbf{w})$ ) upper bound of  $|\log p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}^m, K, \boldsymbol{\phi}_{1:K}^m)|$ . Note that

$$\begin{aligned} p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}^m, K, \boldsymbol{\phi}_{1:K}^m) &= \sum_{c \in \mathcal{C}} \pi(c|K, \boldsymbol{\phi}_{1:K}^m) \Pr(\mathbf{Y}_i = \mathbf{y}|\boldsymbol{\theta}^m, \mathbf{w}, c) \\ &\geq \pi(c_y|K, \boldsymbol{\phi}_{1:K}^m) \Pr(\mathbf{Y}_i = \mathbf{y}|\boldsymbol{\theta}^m, \mathbf{w}, c_y), \end{aligned}$$

where  $\pi(c_y|K, \boldsymbol{\phi}_{1:K}^m) = \sum_{h=1}^K \omega_h^m \prod_{\ell \in c_y} q_{h\ell}^m \prod_{\ell \notin c_y} (1 - q_{h\ell}^m)$ . First, since  $\boldsymbol{\phi}_{1:K}^m \rightarrow \tilde{\boldsymbol{\phi}}_{1:K}$  and  $\pi_{c_y}^* = \sum_{h=1}^K \tilde{\omega}_h \prod_{\ell \in c_y} \tilde{q}_{h\ell} \prod_{\ell \notin c_y} (1 - \tilde{q}_{h\ell}) > 0$ , the first term is bounded below by some  $\ell_1(\mathbf{y}) > 0$  for sufficiently large  $m$ . Second, since  $\boldsymbol{\theta}^m \rightarrow \boldsymbol{\theta}^*$  and  $\Pr(\mathbf{Y}_i = \mathbf{y}|\boldsymbol{\theta}^*, \mathbf{w}, c_y) > 0$  (as  $\boldsymbol{\beta}^*$  is in a compact set,  $\mathbf{D}^*$  is positive definite, and  $\mathcal{W}$  is compact),  $\Pr(\mathbf{Y}_i = \mathbf{y}|\boldsymbol{\theta}^m, \mathbf{w}, c_y)$  is bounded below by some  $\ell_2(\mathbf{y}, \mathbf{w}) > 0$  for sufficiently large  $m$ . Finally,  $1 \geq p(\mathbf{y}|\mathbf{w}; \boldsymbol{\theta}^m, K, \boldsymbol{\phi}_{1:K}^m) \geq \inf_{\mathbf{w} \in \mathcal{W}} \min_{\mathbf{y} \in \mathcal{J}^T} \ell_1(\mathbf{y}) \ell_2(\mathbf{y}, \mathbf{w}) > 0$ , for all  $(\mathbf{y}, \mathbf{w}) \in \mathcal{J}^T \times \mathcal{W}$ .  $\square$

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## Acknowledgment

We benefited from useful comments from Dimitris Korobilis, Paola Manzini, Francesca Molinari, Nail Kashaev, Alessandro Iaria, Ao Wang, Elena Manresa, Martin Weidner, Laura Liu, Asim Ansari, Andriy Norets, Toru Kitagawa, Frank Schorfheide, Zhentong Lu, Victor Aguiar, Yao Luo, Victor Aguirregabiria, Andrew Ching, Kosuke Uetake, Matthijs Wildenbeest, Mitsuru Igami, and the participants at the economics and statistics seminars at the University of Alberta, the 2023 NBER-NSF Seminar in Bayesian Econometrics and Statistics (SBIES) at the Federal Reserve Bank of Philadelphia, the 2023 INFORMS Annual Meeting in Phoenix, and the 2024 International Industrial Organization Conference in Boston. We gratefully acknowledge financial support from the Canadian Social Sciences and Humanities Research Council Insight Development Grant. Disclaimer: Researchers' own analyses calculated (or derived) based in part on data from Nielsen Consumer LLC and marketing databases provided through the NielsenIQ Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business. The conclusions drawn from the NielsenIQ data are those of the researchers and do not reflect the views of NielsenIQ. NielsenIQ is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

## SUPPLEMENTARY MATERIAL

Section [SA](#) provides intermediate results used to prove Theorems 1 and 2 of the main paper, and their proofs. Section [SB](#) presents the conditional posterior distributions of the parameters other than the consideration sets. Section [SC](#) illustrates the impact of the prior choice for the attention probabilities on the prior on the distribution of consideration sets. Section [SD](#) shows additional simulation results under  $J = 4$ . Section [SE](#) provides additional results from the empirical application.

### SA Intermediate theoretical results and proofs

#### SA.1 Intermediate results in the proof of Lemma 2

The following two lemmas are used to prove Lemma 2. They state that the marginal response probability is continuous with respect to the mixture parameters as well as the parameters in the response model. We prove the intermediate results for the case of  $T = 1$ . The extensions to the  $T > 1$  case can be done similarly but at the expense of proof simplicity.

**Lemma SA.1** (Continuity of response probabilities wrt mixture parameters). *Let  $\boldsymbol{\theta}$  and*

*$\mathbf{w} \in \mathcal{W}$ . Then for each  $j \in \mathcal{J}$ ,  $\forall \varepsilon > 0$  and  $\boldsymbol{\phi}_{1:K}^{(1)}$ ,  $\exists \delta > 0$  such that for any  $\boldsymbol{\phi}_{1:K}^{(2)}$  satisfying*

*$\sum_{j=1}^J |q_{hj}^{(1)} - q_{hj}^{(2)}| < \delta$  and  $|\omega_h^{(1)} - \omega_h^{(2)}| < \delta$ , for  $h = 1, \dots, K$ , we have*

$$\left| p(j|\mathbf{w}; \boldsymbol{\theta}, K, \boldsymbol{\phi}_{1:K}^{(1)}) - p(j|\mathbf{w}; \boldsymbol{\theta}, K, \boldsymbol{\phi}_{1:K}^{(2)}) \right| < \varepsilon.$$

*Proof of Lemma [SA.1](#).* We have

$$\left| p(j|\mathbf{w}; \boldsymbol{\theta}, K, \boldsymbol{\phi}_{1:K}^{(1)}) - p(j|\mathbf{w}; \boldsymbol{\theta}, K, \boldsymbol{\phi}_{1:K}^{(2)}) \right| \leq \sum_{c \in \mathcal{C}} \left| \pi(c|K, \boldsymbol{\phi}_{1:K}^{(1)}) - \pi(c|K, \boldsymbol{\phi}_{1:K}^{(2)}) \right| \Pr(Y_{it} = j | \boldsymbol{\theta}, \mathbf{w}_t, c)$$

where  $\Pr(Y_{it} = j | \boldsymbol{\theta}, \mathbf{w}_t, c) \leq 1$ . The term in the absolute value is

$$\begin{aligned} & \left| \sum_{h=1}^K \omega_h^{(1)} \prod_{j \in c} q_{hj}^{(1)} \prod_{j \notin c} (1 - q_{hj}^{(1)}) - \sum_{h=1}^K \omega_h^{(2)} \prod_{j \in c} q_{hj}^{(2)} \prod_{j \notin c} (1 - q_{hj}^{(2)}) \pm \sum_{h=1}^K \omega_h^{(1)} \prod_{j \in c} q_{hj}^{(2)} \prod_{j \notin c} (1 - q_{hj}^{(2)}) \right| \\ & \leq \sum_{h=1}^K \omega_h^{(1)} \underbrace{\left| \prod_{j \in c} q_{hj}^{(1)} \prod_{j \notin c} (1 - q_{hj}^{(1)}) - \prod_{j \in c} q_{hj}^{(2)} \prod_{j \notin c} (1 - q_{hj}^{(2)}) \right|}_I + \sum_{h=1}^K \left| \omega_h^{(1)} - \omega_h^{(2)} \right| \end{aligned}$$

The term  $I$  equals to

$$\begin{aligned} & \prod_{j \in c} q_{hj}^{(1)} \prod_{j \notin c} (1 - q_{hj}^{(1)}) - \prod_{j \in c} q_{hj}^{(2)} \prod_{j \notin c} (1 - q_{hj}^{(2)}) \pm \prod_{j \in c} q_{hj}^{(2)} \prod_{j \notin c} (1 - q_{hj}^{(1)}) \\ & = \left( \prod_{j \in c} q_{hj}^{(1)} - \prod_{j \in c} q_{hj}^{(2)} \right) \prod_{j \notin c} (1 - q_{hj}^{(1)}) + \prod_{j \in c} q_{hj}^{(2)} \left( \prod_{j \notin c} (1 - q_{hj}^{(1)}) - \prod_{j \notin c} (1 - q_{hj}^{(2)}) \right), \end{aligned}$$

and hence the absolute value of  $I$  is bounded by the sum of the two terms:  $\left| \prod_{j \in c} q_{hj}^{(1)} - \prod_{j \in c} q_{hj}^{(2)} \right|$

and  $\left| \prod_{j \notin c} (1 - q_{hj}^{(1)}) - \prod_{j \notin c} (1 - q_{hj}^{(2)}) \right|$ . It is easy to show that the former is bounded by  $c_1 \sum_{j \in c} |q_{hj}^{(1)} - q_{hj}^{(2)}|$  and the latter is bounded by  $c_2 \sum_{j \notin c} |q_{hj}^{(1)} - q_{hj}^{(2)}|$  for some  $c_1, c_2 > 0$ . So,  $|I| \leq c_3 \sum_{j=1}^J |q_{hj}^{(1)} - q_{hj}^{(2)}|$  for some  $c_3 > 0$ .  $\square$

**Lemma SA.2** (Continuity of response probabilities wrt  $\theta$ ). *Suppose  $\mathcal{W}$  is compact. Let*

*( $K, \boldsymbol{\phi}_{1:K}$ ) and  $\mathbf{w} \in \mathcal{W}$ . Then for each  $j \in \mathcal{J}$ ,  $\forall \varepsilon > 0$  and  $\boldsymbol{\theta}^{(1)} = \{\boldsymbol{\beta}^{(1)}, \mathbf{D}^{(1)}\}$ ,  $\exists \delta > 0$  such*

*that for any  $\boldsymbol{\theta}^{(2)} = \{\boldsymbol{\beta}^{(2)}, \mathbf{D}^{(2)}\}$  satisfying  $\|\boldsymbol{\beta}^{(1)} - \boldsymbol{\beta}^{(2)}\| < \delta$  and*

$$\sqrt{\text{tr}(\mathbf{D}^{(1)-1} \mathbf{D}^{(2)} - \mathbf{I}) - \log \det(\mathbf{D}^{(1)} \mathbf{D}^{(2)-1})} < \delta,$$

$$|p(j | \mathbf{w}; \boldsymbol{\theta}^{(1)}, K, \boldsymbol{\phi}_{1:K}) - p(j | \mathbf{w}; \boldsymbol{\theta}^{(2)}, K, \boldsymbol{\phi}_{1:K})| < \varepsilon.$$

*Proof of Lemma SA.2.* Recall that for  $j \in c$ ,

$$\Pr(Y = j | \boldsymbol{\theta}, \mathbf{w}, c) = \int k_j(\mathbf{w}, \boldsymbol{\beta}, \mathbf{b}) \phi(\mathbf{b} | \mathbf{0}, \mathbf{D}) d\mathbf{b},$$

where we introduced the shorthand notation for the kernel

$$k_j(\mathbf{w}, \boldsymbol{\beta}, \mathbf{b}) = \frac{e^{\mathbf{x}'_j \boldsymbol{\beta} + \mathbf{z}'_j \mathbf{b}}}{\sum_{\ell \in c} e^{\mathbf{x}'_\ell \boldsymbol{\beta} + \mathbf{z}'_\ell \mathbf{b}}},$$

where we suppressed the subscripts with respect to the units  $i$  for simplicity of notation.

We have

$$\begin{aligned} |p(j|\mathbf{w}; \boldsymbol{\theta}^{(1)}, K, \boldsymbol{\phi}_{1:K}) - p(j|\mathbf{w}; \boldsymbol{\theta}^{(2)}, K, \boldsymbol{\phi}_{1:K})| &\leq \sum_{c \in \mathcal{C}} \pi(c|K, \boldsymbol{\phi}_{1:K}) |\Pr(j|\boldsymbol{\theta}^{(1)}, \mathbf{w}, c) - \Pr(j|\boldsymbol{\theta}^{(2)}, \mathbf{w}, c)| \\ &= \sum_{c: j \in c} \pi(c|K, \boldsymbol{\phi}_{1:K}) |\Pr(j|\boldsymbol{\theta}^{(1)}, \mathbf{w}, c) - \Pr(j|\boldsymbol{\theta}^{(2)}, \mathbf{w}, c)|, \end{aligned}$$

where if there is no  $c \in \mathcal{C}$  such that  $j \in c$  and  $\pi(c|K, \boldsymbol{\phi}_{1:K}) > 0$ , the claim is trivially true.

Now,

$$|\Pr(j|\boldsymbol{\theta}^{(1)}, \mathbf{w}, c) - \Pr(j|\boldsymbol{\theta}^{(2)}, \mathbf{w}, c)| \leq |\Pr(j|\{\boldsymbol{\beta}^{(1)}, \mathbf{D}^{(1)}\}, \mathbf{w}, c) - \Pr(j|\{\boldsymbol{\beta}^{(2)}, \mathbf{D}^{(1)}\}, \mathbf{w}, c)| \quad (\text{SA.1})$$

$$+ |\Pr(j|\{\boldsymbol{\beta}^{(2)}, \mathbf{D}^{(1)}\}, \mathbf{w}, c) - \Pr(j|\{\boldsymbol{\beta}^{(2)}, \mathbf{D}^{(2)}\}, \mathbf{w}, c)|. \quad (\text{SA.2})$$

To bound (SA.1), note that for any  $\rho > 0$ , one can find  $M_\rho > 0$  such that  $\int 1\{\|\mathbf{b}\| > M_\rho\} \phi(\mathbf{b}|\mathbf{0}, \mathbf{D}^{(1)}) d\mathbf{b} < \rho$ . The term (SA.1) equals to

$$\begin{aligned} &\left| \int (k_j(\mathbf{w}, \boldsymbol{\beta}^{(1)}, \mathbf{b}) - k_j(\mathbf{w}, \boldsymbol{\beta}^{(2)}, \mathbf{b})) \phi(\mathbf{b}|\mathbf{0}, \mathbf{D}^{(1)}) d\mathbf{b} \right| \\ &\leq \int_{\|\mathbf{b}\| \leq M_\rho} |k_j(\mathbf{w}, \boldsymbol{\beta}^{(1)}, \mathbf{b}) - k_j(\mathbf{w}, \boldsymbol{\beta}^{(2)}, \mathbf{b})| \phi(\mathbf{b}|\mathbf{0}, \mathbf{D}^{(1)}) d\mathbf{b} \\ &\quad + \int_{\|\mathbf{b}\| > M_\rho} |k_j(\mathbf{w}, \boldsymbol{\beta}^{(1)}, \mathbf{b}) - k_j(\mathbf{w}, \boldsymbol{\beta}^{(2)}, \mathbf{b})| \phi(\mathbf{b}|\mathbf{0}, \mathbf{D}^{(1)}) d\mathbf{b}. \end{aligned}$$

Since  $k_j(\mathbf{w}, \boldsymbol{\beta}, \mathbf{b})$  has a bounded first derivative with respect to  $\boldsymbol{\beta}$  for  $\|\mathbf{b}\| \leq M_\rho$  and under a compact  $\mathcal{W}$ , there is some  $c_1 > 0$  such that the first term above is bounded by  $c_1 \|\boldsymbol{\beta}^{(1)} - \boldsymbol{\beta}^{(2)}\|$ . The second term is bounded by  $2 \int 1\{\|\mathbf{b}\| > M_\rho\} \phi(\mathbf{b}|\mathbf{0}, \mathbf{D}^{(1)}) d\mathbf{b} < 2\rho$ , which can be made smaller than  $\|\boldsymbol{\beta}^{(1)} - \boldsymbol{\beta}^{(2)}\|$ . Hence, (SA.1) is bounded by  $c_2 \|\boldsymbol{\beta}^{(1)} - \boldsymbol{\beta}^{(2)}\|$

for some constant  $c_2 > 0$ .

The term (SA.2) equals to

$$\begin{aligned} & \left| \int k_j(\mathbf{w}, \boldsymbol{\beta}^{(2)}, \mathbf{b}) (\phi(\mathbf{b}|\mathbf{0}, \mathbf{D}^{(1)}) - \phi(\mathbf{b}|\mathbf{0}, \mathbf{D}^{(2)})) d\mathbf{b} \right| \\ & \leq \int |\phi(\mathbf{b}|\mathbf{0}, \mathbf{D}^{(1)}) - \phi(\mathbf{b}|\mathbf{0}, \mathbf{D}^{(2)})| d\mathbf{b} \\ & \leq \sqrt{\text{tr}(\mathbf{D}^{(1)-1} \mathbf{D}^{(2)} - \mathbf{I}) - \log \det(\mathbf{D}^{(1)} \mathbf{D}^{(2)-1})}, \end{aligned}$$

where the last inequality is due to a known bound on the total variation distance between normal distributions with a same mean vector but different covariance matrices (Devroye et al., 2018).

□

## SA.2 Intermediate results in the proof of Theorem 2

The next lemma shows that the response probabilities are continuous for the total variation distance defined as

$$d_{TV}(\mathbf{p}_{\boldsymbol{\beta}, \boldsymbol{\pi}}, \mathbf{p}_{\boldsymbol{\beta}', \boldsymbol{\pi}'}) = \int \sum_{\mathbf{y} \in \mathcal{J}^T} |p_{\boldsymbol{\beta}, \boldsymbol{\pi}}(\mathbf{y}|\mathbf{w})g^*(\mathbf{w}) - p_{\boldsymbol{\beta}', \boldsymbol{\pi}'}(\mathbf{y}|\mathbf{w})g^*(\mathbf{w})| d\mathbf{w}.$$

**Lemma SA.3** (Continuity of response probabilities). *Let  $\varepsilon > 0$ . Then there is  $\delta > 0$  such that  $d((\boldsymbol{\beta}, \boldsymbol{\pi}), (\boldsymbol{\beta}', \boldsymbol{\pi}')) < \delta$  implies that  $d_{TV}(\mathbf{p}_{\boldsymbol{\beta}, \boldsymbol{\pi}}, \mathbf{p}_{\boldsymbol{\beta}', \boldsymbol{\pi}'}) < \varepsilon$ .*

*Proof of Lemma SA.3.*

$$\begin{aligned} & |p_{\boldsymbol{\beta}, \boldsymbol{\pi}}(\mathbf{y}|\mathbf{w}) - p_{\boldsymbol{\beta}', \boldsymbol{\pi}'}(\mathbf{y}|\mathbf{w})| \leq |p_{\boldsymbol{\beta}, \boldsymbol{\pi}}(\mathbf{y}|\mathbf{w}) - p_{\boldsymbol{\beta}', \boldsymbol{\pi}}(\mathbf{y}|\mathbf{w})| + |p_{\boldsymbol{\beta}', \boldsymbol{\pi}}(\mathbf{y}|\mathbf{w}) - p_{\boldsymbol{\beta}', \boldsymbol{\pi}'}(\mathbf{y}|\mathbf{w})| \\ & \leq \sum_c \pi_c \left| \prod_{t=1}^T \Pr(Y_{it} = y_t | \boldsymbol{\beta}, \mathbf{w}_t, c) - \prod_{t=1}^T \Pr(Y_{it} = y_t | \boldsymbol{\beta}', \mathbf{w}_t, c) \right| \\ & \quad + \sum_c |\pi_c - \pi'_c| \prod_{t=1}^T \Pr(Y_{it} = y_t | \boldsymbol{\beta}', \mathbf{w}_t, c) \\ & \leq \gamma_1 \|\boldsymbol{\beta} - \boldsymbol{\beta}'\|_2 + \gamma_2 \|\boldsymbol{\pi} - \boldsymbol{\pi}'\|_1, \end{aligned}$$



for some positive constants  $\gamma_1$  and  $\gamma_2$ . □

## SB Conditional posterior distributions

For the mixture model on the latent consideration sets  $\mathbf{C} = (\mathbf{C}_1, \dots, \mathbf{C}_n)$ , let  $S_i \in \{1, 2, \dots\}$  be the latent cluster assignment such that  $C_{ij}|S_i = h \sim \text{Bernoulli}(q_{hj})$ , independently  $j = 1, \dots, J$ , for  $i = 1, \dots, n$ . We have the latent consideration sets  $\mathbf{C}$ , the common fixed-effects  $\boldsymbol{\beta}$ , the random effects  $\mathbf{b}$ , the corresponding covariance matrix  $\mathbf{D}$ , the DP parameters  $\mathbf{V} = (V_1, V_2, \dots)$  as well as  $\mathbf{Q} = (\mathbf{q}_1, \mathbf{q}_2, \dots)$ , the DP cluster assignment variables  $\mathbf{S} = (S_1, \dots, S_n)$ , and the DP concentration parameter  $\alpha$ . Then, from the Bayes theorem, we define the posterior density of interest to be

$$\begin{aligned} p(\mathbf{C}, \mathbf{S}, \mathbf{V}, \mathbf{Q}, \alpha, \boldsymbol{\beta}, \mathbf{b}, \mathbf{D} | \mathbf{y}, \mathbf{W}) &\propto p(\mathbf{y} | \boldsymbol{\beta}, \mathbf{b}, \mathbf{W}, \mathbf{C}) \cdot p(\boldsymbol{\beta}, \mathbf{b}, \mathbf{D}) \cdot p(\mathbf{C}, \mathbf{S}, \mathbf{Q}, \mathbf{V}, \alpha) \\ &= p(\mathbf{y} | \boldsymbol{\beta}, \mathbf{b}, \mathbf{W}, \mathbf{C}) \cdot \pi(\boldsymbol{\beta}) p(\mathbf{b} | \mathbf{D}) \pi(\mathbf{D}) \cdot p(\mathbf{C}, \mathbf{S}, \mathbf{Q}, \mathbf{V}, \alpha), \end{aligned} \quad (\text{SB.1})$$

where the first term is the likelihood function and  $\pi(\cdot)$  denotes the prior density. Only the last term in (SB.1) is associated with the DP model and

$$\begin{aligned} p(\mathbf{C}, \mathbf{S}, \mathbf{Q}, \mathbf{V}, \alpha) &\propto p(\mathbf{C} | \mathbf{Q}, \mathbf{S}) p(\mathbf{Q}, \mathbf{V}, \mathbf{S}, \alpha) \\ &\propto \left[ \prod_{i=1}^n p(\mathbf{C}_i | \mathbf{q}_{S_i}) p(S_i | \mathbf{V}) \right] \cdot \left[ \prod_{h=1}^{\infty} p(V_h | \alpha) p(\mathbf{q}_h | \underline{\boldsymbol{\phi}}_q) \right] \cdot \pi(\alpha), \end{aligned} \quad (\text{SB.2})$$

where  $p(\mathbf{C}_i | \mathbf{q}_{S_i})$  is the product of densities for the independent Bernoulli distributions  $\text{Bernoulli}(q_{S_i j})$   $j = 1, \dots, J$ ,  $p(S_i | \mathbf{V}) = \omega_{S_i}$ ,  $p(V_h | \alpha)$  is the density of  $\text{Beta}(1, \alpha)$ ,  $p(\mathbf{q}_h | \underline{\boldsymbol{\phi}}_q)$  is the product of densities for the independent Beta distributions  $\text{Beta}(\underline{a}_{q_j}, \underline{b}_{q_j})$   $j = 1, \dots, J$ , and  $\pi(\alpha)$  is the prior density for  $\alpha$ . We apply the slice sampling approach (Walker, 2007) by augmenting the joint distribution with a sequence of auxiliary random variables  $\mathbf{u} =$

$(u_1, \dots, u_n)$  that follow the uniform distribution on  $(0, 1)$ ,  $u_i \sim \mathcal{U}(0, 1)$ ,  $i = 1, \dots, n$ :

$$p(\mathbf{C}, \mathbf{S}, \mathbf{Q}, \mathbf{V}, \mathbf{u}, \alpha) \propto \left[ \prod_{i=1}^n p(\mathbf{C}_i | \mathbf{q}_{S_i}) I(u_i \leq \omega_{S_i}) \right] \cdot \left[ \prod_{h=1}^{\infty} p(V_h | \alpha) p(\mathbf{q}_h | \underline{\phi}_q) \right] \cdot \pi(\alpha). \quad (\text{SB.3})$$

It is easy to show that we can recover (SB.2) by integrating out  $\mathbf{u}$  from (SB.3). However, by introducing  $\mathbf{u}$ , one only has to choose labels  $S_i$  in the finite set  $\{h : \omega_h \geq u_i\}$ . See the Supplementary Material for discussion on hyperparameter selections.

Our MCMC algorithm proceeds by cycling through various conditional distributions, where these distributions are conditioned on the most recent values of the remaining unknowns. Specifically, given the current draw at the  $g$ th iteration  $\{u_i^{(g)}\}, \{V_h^{(g)}\}, \{\mathbf{q}_h^{(g)}\}, \{S_i^{(g)}\}, \alpha^{(g)}, \{\mathbf{C}_i^{(g)}\}, \boldsymbol{\delta}^{(g)}, \boldsymbol{\beta}^{(g)}, \{\mathbf{b}_i^{(g)}\}$ , and  $\mathbf{D}^{(g)}$ , the next draw in the sequence is obtained by simulating

$$\begin{aligned} & \boldsymbol{\beta}^{(g+1)} \text{ from } \boldsymbol{\beta} | \{y_{it}\}, \{\mathbf{C}_i^{(g)}\}, \boldsymbol{\delta}^{(g)}, \{\mathbf{b}_i^{(g)}\}, \\ & \{\mathbf{b}_i^{(g+1)}\} \text{ from } \{\mathbf{b}_i\} | \{y_{it}\}, \{\mathbf{C}_i^{(g)}\}, \boldsymbol{\delta}^{(g)}, \boldsymbol{\beta}^{(g+1)}, \mathbf{D}^{(g)}, \\ & \mathbf{D}^{(g+1)} \text{ from } \mathbf{D} | \{\mathbf{b}_i^{(g+1)}\}, \\ & \boldsymbol{\delta}^{(g+1)} \text{ from } \boldsymbol{\delta} | \{y_{it}\}, \{\mathbf{C}_i^{(g)}\}, \boldsymbol{\beta}^{(g+1)}, \{\mathbf{b}_i^{(g+1)}\}, \\ & \{\mathbf{C}_i^{(g+1)}\} \text{ from } \{\mathbf{C}_i\} | \{y_{it}\}, \boldsymbol{\delta}^{(g+1)}, \boldsymbol{\beta}^{(g+1)}, \{\mathbf{b}_i^{(g+1)}\}, \{\mathbf{q}_h^{(g)}\}, \{S_i^{(g)}\}, \\ & \{V_h^{(g+1)}\} \text{ from } \{V_h\} | \{S_i^{(g)}\}, \alpha^{(g)}, \\ & \{\mathbf{q}_h^{(g+1)}\} \text{ from } \{\mathbf{q}_h\} | \{\mathbf{C}_i^{(g+1)}\}, \{S_i^{(g)}\}, \\ & \{u_i^{(g+1)}\} \text{ from } \{u_i\} | \{S_i^{(g)}\}, \{V_h^{(g+1)}\}, \\ & \{S_i^{(g+1)}\} \text{ from } \{S_i\} | \{u_i^{(g+1)}\}, \{\mathbf{q}_h^{(g+1)}\}, \{V_h^{(g+1)}\}, \{\mathbf{C}_i^{(g+1)}\}, \\ & \alpha^{(g+1)} \text{ from } \alpha | \{V_h^{(g+1)}\}, \{S_i^{(g+1)}\}. \end{aligned}$$

Repeating this procedure  $G$  times (beyond a suitable burn-in) produces a sample from the posterior distribution.

The main paper illustrates how the consideration sets are simulated. In this section, we show the conditional posterior distributions of the remaining parameters. Let  $K^* = \min\{h : \sum_{\ell=1}^h \omega_h > 1 - u^*\}$ , where  $u^* = \min(u_1, \dots, u_n)$ . Define  $n_h = \sum_{i=1}^n I(S_i = h)$ . Let the dot  $\bullet$  denote all other parameters and the data.

### SB.1 Simulation of $q_h$

From (SB.2), we have that

$$p(\mathbf{q}_h | \bullet) \propto p(\mathbf{q}_h | \underline{\phi}_q) \cdot \prod_{i:S_i=h} \prod_{j=1}^J q_{hj}^{C_{ij}} (1 - q_{hj})^{1-C_{ij}},$$

where  $p(\mathbf{q}_h | \underline{\phi}_q)$  is the product of densities for Beta distributions  $Beta(\underline{a}_{q_j}, \underline{b}_{q_j})$ , independently over  $j = 1, \dots, J$ . Then

$$q_{hj} | \bullet \sim \text{Beta} \left( \underline{a}_{q_j} + \sum_{i:S_i=h} C_{ij}, \underline{b}_{q_j} + \sum_{i:S_i=h} (1 - C_{ij}) \right),$$

independently over  $j = 1, \dots, J$  for  $h = 1, 2, \dots, K^*$ . If component  $h \leq K^*$  does not contain any observations, then the corresponding  $\mathbf{q}_h$  is drawn from the prior.

### SB.2 Simulation of $V_h$

From (SB.2), the conditional distribution of  $\mathbf{V}$  is independent and the marginal conditional distributions are

$$V_h | \bullet \sim \text{Beta} \left( 1 + n_h, \alpha + \sum_{\ell > h} n_\ell \right),$$

for  $h = 1, 2, \dots, K^*$ . If component  $h \leq K^*$  is empty, then the corresponding  $V_h$  is drawn from the prior.

### SB.3 Simulation of $u_i$

From (SB.3), it is easy to see that

$$u_i | \bullet \stackrel{ind}{\sim} \mathcal{U}[0, \omega_{S_i}], \quad i = 1, \dots, n.$$

## SB.4 Simulation of $S_i$

From (SB.3), we can see that for  $h = 1, 2, \dots, K^*$ ,

$$Pr(S_i = h | \mathbf{C}, \mathbf{u}, \mathbf{V}, \mathbf{Q}) = \frac{I(u_i \leq \omega_h) \prod_{j=1}^J q_{hj}^{C_{ij}} (1 - q_{hj})^{1-C_{ij}}}{\sum_{\ell} I(u_i \leq \omega_{\ell}) \prod_{j=1}^J q_{\ell j}^{C_{ij}} (1 - q_{\ell j})^{1-C_{ij}}}.$$

Note that  $Pr(S_i = h | \bullet) = 0$  for  $h > K^*$ .

## SB.5 Simulation of $\alpha$

The conditional posterior of  $\alpha$  is

$$p(\alpha | \bullet) \propto p(\mathbf{S} | \alpha) \pi(\alpha).$$

Following Escobar and West (1995), this distribution is sampled by first generating  $\eta$  conditional on  $\alpha$  from the Beta distribution

$$\eta | \alpha, \mathbf{S} \sim \text{Beta}(\alpha + 1, n),$$

and then sampling  $\alpha$  conditional on  $\eta$  from the Gamma mixture

$$\begin{aligned} p(\alpha | \eta, \mathbf{S}) &= \frac{\underline{a}_{\alpha} + G - 1}{\underline{a}_{\alpha} + G - 1 + n(\underline{b}_{\alpha} - \log(\eta))} \text{Gamma}(\underline{a}_{\alpha} + G, \underline{b}_{\alpha} - \log(\eta)) \\ &+ \frac{n(\underline{b}_{\alpha} - \log(\eta))}{\underline{a}_{\alpha} + G - 1 + n(\underline{b}_{\alpha} - \log(\eta))} \text{Gamma}(\underline{a}_{\alpha} + G - 1, \underline{b}_{\alpha} - \log(\eta)), \end{aligned}$$

where  $G$  is the total number of existing clusters.

## SB.6 Simulation of $\beta$

From Bayes theorem,

$$\pi(\beta | \bullet) \propto \pi(\beta) \cdot \prod_{i=1}^n \prod_{t=1}^{T_i} \Pr(Y_{it} = y_{it} | \delta, \beta, \mathbf{b}_i, \mathbf{w}_{it}, \mathcal{C}_i),$$

where  $\Pr(Y_{it} = y_{it} | \delta, \beta, \mathbf{b}_i, \mathbf{w}_{it}, \mathcal{C}_i) = \frac{\exp(V_{iy_{it}t})}{\sum_{\ell \in \mathcal{C}_i} \exp(V_{i\ell t})}$  and  $V_{ijt} = \delta_j + \mathbf{x}'_{ijt} \beta + \mathbf{z}'_{ijt} \mathbf{b}_i$ .

We use a tailored Metropolis–Hastings (M-H) algorithm to sample  $\boldsymbol{\beta}$  (Chib and Greenberg, 1995). Define the conditional log-likelihood of  $\boldsymbol{\beta}$  given  $\boldsymbol{\delta}$ ,  $\{\mathbf{b}_i\}$ , and  $\{\mathcal{C}_i\}$ :  $\log L(\boldsymbol{\beta}|\bullet) = \sum_{i=1}^n \sum_{t=1}^{T_i} \log \Pr(Y_{it} = y_{it}|\boldsymbol{\delta}, \boldsymbol{\beta}, \mathbf{b}_i, \mathcal{C}_i)$ . At iteration  $g$ , let  $\boldsymbol{\beta}^{(g)}$  be the value of  $\boldsymbol{\beta}$ . A candidate value is drawn as

$$\tilde{\boldsymbol{\beta}} \sim N_{d_x} \left( \hat{\boldsymbol{\beta}}, \hat{\mathbf{V}}_{\boldsymbol{\beta}} \right),$$

where

$$\hat{\boldsymbol{\beta}} = \arg \max_{\boldsymbol{\beta}} \log L(\boldsymbol{\beta}|\bullet)\pi(\boldsymbol{\beta}), \quad \hat{\mathbf{V}}_{\boldsymbol{\beta}}^{-1} = - \left. \frac{\partial^2}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}'} \log L(\boldsymbol{\beta}|\bullet)\pi(\boldsymbol{\beta}) \right|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}},$$

which is accepted with probability

$$\min \left\{ \frac{\pi(\tilde{\boldsymbol{\beta}}|\bullet)\phi(\boldsymbol{\beta}^{(g)}|\hat{\boldsymbol{\beta}}, \hat{\mathbf{V}}_{\boldsymbol{\beta}})}{\pi(\boldsymbol{\beta}^{(g)}|\bullet)\phi(\tilde{\boldsymbol{\beta}}|\hat{\boldsymbol{\beta}}, \hat{\mathbf{V}}_{\boldsymbol{\beta}})}, 1 \right\},$$

where  $\phi(\cdot)$  denotes the density of normal distribution. The conditional posterior mode  $\hat{\boldsymbol{\beta}}$  is computed using the Newton-Raphson method. The likelihood is known to be concave with respect to  $\boldsymbol{\beta}$  under the Gumbel error distribution, so the convergence to  $\hat{\boldsymbol{\beta}}$  is fast and only requires a few iterations in many cases. In the empirical application, we multiply the variance of the proposal distribution by  $10^{-2}$  in order to achieve desirable acceptance rates.

## SB.7 Simulation of $b_i$

The full conditional of  $\mathbf{b}_i$  (for each  $i$ ) is proportional to

$$\pi(\mathbf{b}_i|\bullet) \propto \phi(\mathbf{b}_i|\mathbf{0}, \mathbf{D}) \cdot \prod_{t=1}^{T_i} \Pr(Y_{it} = y_{it}|\boldsymbol{\delta}, \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{w}_{it}, \mathcal{C}_i).$$

We use a symmetric random-walk M-H to draw from the conditional distribution. Define the conditional log-likelihood of  $\mathbf{b}_i$  given  $\boldsymbol{\delta}$ ,  $\boldsymbol{\beta}$ , and  $\{\mathcal{C}_i\}$ :  $\log L(\mathbf{b}_i|\bullet) = \sum_{t=1}^{T_i} \log \Pr(Y_{it} = y_{it}|\boldsymbol{\delta}, \boldsymbol{\beta}, \mathbf{b}_i, \mathbf{w}_{it}, \mathcal{C}_i)$ . At iteration  $g$ , let  $\mathbf{b}_i^{(g)}$  be the value of  $\mathbf{b}_i$ . A candidate value is drawn as

$$\tilde{\mathbf{b}}_i \sim N_{d_z} \left( \mathbf{b}_i^{(g)}, \mathbf{D}^{(g)} \right),$$

which is accepted with probability

$$\min \left\{ \frac{\pi(\tilde{\mathbf{b}}_i|\bullet)}{\pi(\mathbf{b}_i^{(g)}|\bullet)}, 1 \right\}.$$

The updating step for  $b_i$  is independent over  $i$ , so it can be easily parallelized in a modern computer.

## SB.8 Simulation of $D$

We simulate  $D$  by first simulating  $D^{-1}$  and then taking the inverse of the simulated draw. This is because it can be shown that

$$D^{-1}|\bullet \sim \text{Wishart} \left( \underline{v} + n, \left[ \mathbf{R}^{-1} + \sum_{i=1}^n \mathbf{b}_i \mathbf{b}'_i \right]^{-1} \right).$$

## SB.9 Simulation of $\delta$

In principle, we could treat  $\delta$  as a part of  $\beta$  and sample from the conditional distribution altogether using a tailored M-H algorithm. However, the involved optimization step could be slow when  $J$  is large, which is exactly our focus of the current paper. Hence, we sample  $\delta$  separately from  $\beta$ . Specifically, we use a tailored Metropolis–Hastings (M-H) algorithm to sample  $\delta_k$  for  $k = 1, \dots, J - 1$ , one after another.

From Bayes theorem,

$$\pi(\delta_k | \delta_{\setminus k}, \beta, \mathbf{b}, \mathbf{W}, \mathbf{C}) \propto \pi(\delta_k) \cdot \prod_{i=1}^n \prod_{t=1}^{T_i} \Pr(Y_{it} = y_{it} | \delta, \beta, \mathbf{b}_i, \mathbf{w}_{it}, \mathcal{C}_i),$$

where  $\delta_{\setminus k}$  denotes  $\delta$  except for the  $k$ th element. Define the conditional log-likelihood of  $\delta_k$  given  $\delta_{\setminus k}$ ,  $\beta$ ,  $\{\mathbf{b}_i\}$ , and  $\{\mathcal{C}_i\}$ :  $\log L(\delta_k|\bullet) = \sum_{i=1}^n \sum_{t=1}^{T_i} \log \Pr(Y_{it} = y_{it} | \delta, \beta, \mathbf{b}_i, \mathbf{w}_{it}, \mathcal{C}_i)$ . At iteration  $g$ , let  $\delta_k^{(g)}$  be the value of  $\delta_k$ . A candidate value is drawn as

$$\tilde{\delta}_k \sim N_1 \left( \hat{\delta}_k, \hat{\sigma}_{\delta_k}^2 \right),$$

where

$$\hat{\delta}_k = \arg \max_{\delta_k} \log L(\delta_k | \bullet) \pi(\delta_k), \quad \hat{\sigma}_{\delta_k}^{-2} = - \frac{\partial^2}{\partial \delta_k^2} \log L(\delta_k | \bullet) \pi(\delta_k) \Big|_{\delta_k = \hat{\delta}_k},$$

which is accepted with probability

$$\min \left\{ \frac{\pi(\tilde{\delta}_k | \bullet) \phi(\delta_k^{(g)} | \hat{\delta}_k, \hat{\sigma}_{\delta_k}^2)}{\pi(\delta_k^{(g)} | \bullet) \phi(\tilde{\delta}_k | \hat{\delta}_k, \hat{\sigma}_{\delta_k}^2)}, 1 \right\}.$$

We randomize the order of updating  $\delta_k$ ,  $k = 1, \dots, J - 1$ .

## SC Prior on the distribution of attention probabilities

### SC.1 Remarks on hyperparameters

In the fitting, we set the parameters of the prior as follows: for the product-specific fixed-effects,  $\delta_j \sim N(0, 2)$  independently for  $j = 1, \dots, J$ , for the common fixed-effect,  $\beta_k \sim N(0, 3)$  independently for  $k = 1, \dots, d_x$ , for the variance of the random effects,  $\mathbf{D}^{-1} \sim \text{Wishart}(9, (1/9)\mathbf{I}_{d_z})$ , and for the DP concentration parameter,  $\alpha \sim \text{Gamma}(\underline{a}_\alpha, \underline{b}_\alpha) = (1/4, 1/4)$  to produce  $\Pr(H_0 : \omega^* > 1 - \varepsilon) \approx 0.5$ . The prior of the attention probabilities is  $q_{hj} \sim \text{Beta}(\underline{a}_{q_j}, \underline{b}_{q_j})$ , independently over  $j = 1, \dots, J$  for  $h = 1, \dots, \infty$ . The choice of hyperparameters,  $(\underline{a}_{q_j}, \underline{b}_{q_j})$ , is important, as it controls the sparsity of the consideration sets. We set  $(\underline{a}_{q_j}, \underline{b}_{q_j}) = (s \cdot r, s \cdot (1 - r))$ , where  $s > 0$  and  $r$  is a small prior expectation of  $q_{hj}$  (that is,  $r < 0.5$ ), for example,  $r = \frac{r_0}{J}$ , where  $r_0$  is a positive integer. We call this a sparsity-supporting prior because the prior probability is smaller for consideration sets with larger cardinality.

### SC.2 Illustration

When  $J$  is small, we can examine the impact of the hyperparameters on the implied prior probability distribution on consideration sets by simulating from the prior. First, fix

a large positive integer  $K$ . Second, generate draws from the prior by drawing

$$\begin{aligned} \alpha &\sim \text{Gamma}(\underline{a}_\alpha, \underline{b}_\alpha), \\ V_h | \alpha &\stackrel{\text{ind}}{\sim} \text{Beta}(1, \alpha) \text{ for } h = 1, \dots, K, \\ \omega_h &= V_h \prod_{\ell > h} (1 - V_\ell) \text{ for } h = 1, \dots, K, \\ q_{hj} &\stackrel{\text{ind}}{\sim} \text{Beta}(\underline{a}_{q_j}, \underline{b}_{q_j}) \text{ for } j = 1, \dots, J, \quad h = 1, \dots, K. \end{aligned}$$

Finally, given these draws, calculate the probability of each possible consideration set using the representation in Lemma 1; that is,

$$\pi_c = \sum_{h=1}^K \omega_h \left\{ \prod_{j \in c} q_{hj} \prod_{j \notin c} (1 - q_{hj}) \right\}.$$

For example, when  $J = 4$ ,  $\Pr(\mathcal{C}_i = \{2, 4\}) = \sum_{h=1}^K \omega_h \{q_{h2}q_{h4}(1 - q_{h1})(1 - q_{h3})\}$ .

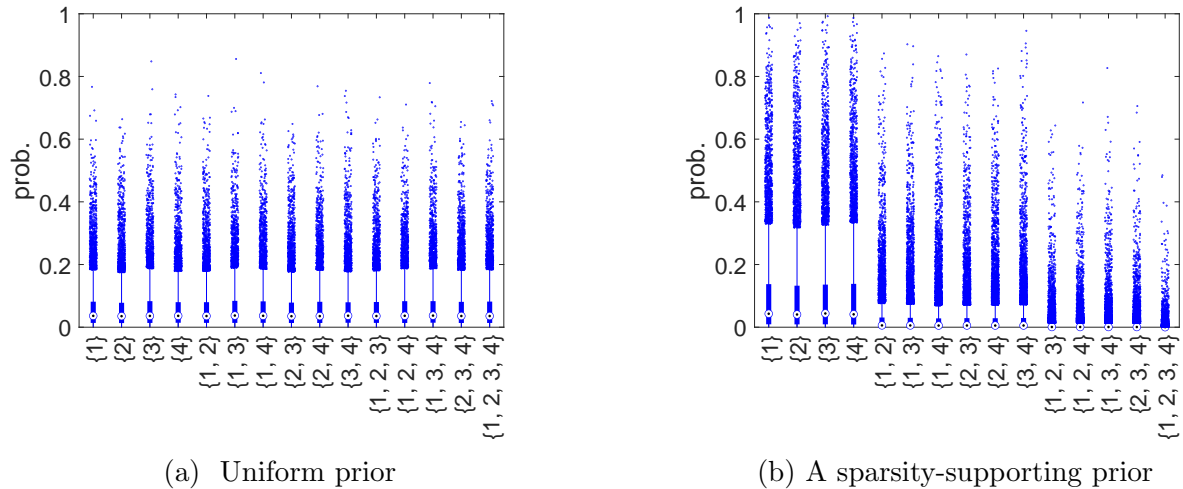


Figure SC.1: Implied prior distribution over consideration sets (box plots) for two different priors on  $q_{hj}$ . Uniform prior (a) with  $(\underline{a}_{q_j}, \underline{b}_{q_j}) = (1, 1)$  and sparsity supporting prior (b) with  $(\underline{a}_{q_j}, \underline{b}_{q_j}) = (sr, s(1 - r))$  with  $r = \frac{1}{J}$ ,  $s = 1$ .  $K = 20$ .  $(\underline{a}_\alpha, \underline{b}_\alpha) = (1/4, 1/4)$ . and 10,000 draws from the prior.

Panel (a) in Figure SC.1 shows the implied prior distribution over the consideration sets under the uniform prior on  $q_{hj}$  when  $J = 4$ . Under the uniform prior, the prior expectation of  $q_{hj} = 0.5$ , so the prior is the same across all consideration sets and is centered around



$0.5^4 = 0.0625$ . Panel (b) gives results under our sparsity-supporting prior. In this case, the prior distribution shrinks to 0 as the cardinality of the consideration set increases.

The preceding shows that the prior on the attention probabilities  $\{q_{hj}\}$  induces quite different prior distributions on consideration sets. As the number of consideration sets increase exponentially in  $J$ , it is crucial to apply regularization to the parameter space. Our sparsity-supporting prior promotes this regularization. It favors smaller consideration sets, while maintaining positive probabilities on larger sets.

## SD Additional material for the simulation

Section [SD.1](#) introduces a test for dependence of considerations and illustrates its performance. Sections [SD.2](#), [SD.3](#), and [SD.4](#) show the simulation studies under random effects, auto-correlated covariate, and time-varying consideration sets, respectively.

### SD.1 Testing for dependent consideration

From the MCMC output, it is possible to assess the degree of consideration dependence using the method proposed by [Dunson and Xing \(2009\)](#), but now applied to the latent consideration sets. The null hypothesis tests for independent consideration, formulated as  $H_0 : \omega_1 = 1$ . We utilize the interval null of  $H_0 : \omega^* > 1 - \varepsilon$  with  $\omega^* = \max\{\omega_k : k = 1, \dots, k^*\}$  and  $\varepsilon > 0$  is a small value. The Bayes factor in favor of the alternative hypothesis,  $H_1 : \omega^* \leq 1 - \varepsilon$ , is defined as  $\frac{\Pr(H_1|\mathbf{D}^n)\Pr(H_1)}{\Pr(H_0|\mathbf{D}^n)\Pr(H_0)}$ , which can be estimated using  $\hat{\Pr}(H_1|\mathbf{D}^n)$ , the portion of the posterior sample such that  $\omega^* \leq 1 - \varepsilon$ , and  $\hat{\Pr}(H_0|\mathbf{D}^n) = 1 - \hat{\Pr}(H_1|\mathbf{D}^n)$ . In the simulations and the application,  $\underline{a}_\alpha = \underline{b}_\alpha = 1/4$  is fixed to produce  $\Pr(H_0) \approx 0.5$ .

We use the current data-generating process as the first case (dependent consideration). In the second case, the consideration is independent. We generate  $C_{ij} \stackrel{iid}{\sim} \text{Bernoulli}(\gamma_j)$ , for  $j = 1, 2, 3$  with  $(\gamma_1, \gamma_2, \gamma_3) = (0.2, 0.15, 0.35)$  and fix  $C_{i4} = 1$ , for  $i = 1, \dots, n$ . Figure [SD.1](#) (a) provides a histogram showing the estimated posterior probabilities of  $H_1$  under

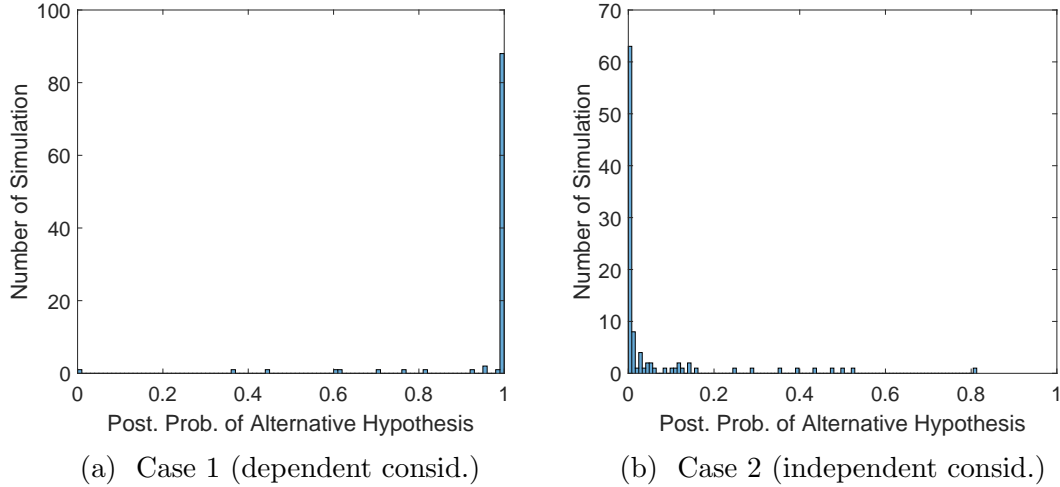


Figure SD.1: Histograms of estimated posterior probabilities of  $H_1$  in each of the 100 simulations under (a) case 1 (dependent consideration -  $H_1$  is true) and (b) case 2 (independent consideration -  $H_0$  is true).  $\varepsilon = 0.1, n = 50, T = 5$ .

the first case ( $H_1$  is true) across the 100 simulated data sets using  $\varepsilon = 0.1$ . The method appropriately assigns a value close to one to  $\Pr(H_1|\mathbf{D}^n)$  in most cases, with only 3/100 having an estimated  $\Pr(H_1|\mathbf{D}^n) < 0.5$ . Figure SD.1 (b) provides the results for case 2. The posterior probability assigned to  $H_1$  is close to zero for most simulations. We find similar results with random effects, as shown below.

## SD.2 Simulation results with random effects

We repeat the simulation study now with preference heterogeneity. We generate the consideration sets as before, but now use the random effects logit with the specification  $V_{ijt} = \delta_j^* + (\beta^* + b_i)x_{ijt}$ , where  $b_i \sim N(0, D^*)$  with  $D^* = 1$ . We fit the random effects logit with the proposed flexible approach for the distribution of consideration sets. The results are presented in Table SD.1. We see that as  $n$  increases, the MSEs/bias/standard deviation tend to decrease. However, this is not the case when the independent consideration is imposed, i.e.  $K = 1$ . Also, MSEs/biases are larger in general than for the proposed flexible approach. In addition, there are distortions of the coverage of the credible intervals under  $K = 1$ . Figure SD.2 shows that for  $K = \infty$ , we see the posterior on  $\boldsymbol{\pi}$  approaches to the

truth while it does not under  $K = 1$ , due to the mis-specification.

Table SD.1: Simulation results for  $J = 4$  with random effects

$(K, T)$	$n$	$\beta$	$\delta_1$	$\delta_2$	$\delta_3$	$D^{1/2}$	$\pi$	Time
$(\infty, 5)$	50	0.135 (0.136 ; 0.44)	0.229 (0.418 ; 0.92)	0.249 (0.421 ; 0.92)	0.149 (0.321 ; 0.92)	0.722 (0.064 ; 0.06)	0.444 (0.03)	2.36
	500	0.009 (0.079 ; 0.90)	0.019 (0.151 ; 0.96)	0.021 (0.149 ; 0.96)	0.014 (0.123 ; 0.90)	0.017 (0.093 ; 0.84)	0.211 (0.01)	24.86
$(\infty, 15)$	50	0.037 (0.159 ; 0.86)	0.046 (0.211 ; 0.96)	0.039 (0.207 ; 0.96)	0.026 (0.182 ; 0.96)	0.091 (0.154 ; 0.72)	0.358 (0.01)	3.11
	500	0.003 (0.059 ; 0.96)	0.005 (0.067 ; 0.96)	0.004 (0.067 ; 0.96)	0.004 (0.058 ; 0.98)	0.004 (0.055 ; 0.92)	0.129 (0)	37.7
$(1, 5)$	50	0.156 (0.13 ; 0.30)	0.732 (0.365 ; 0.48)	0.925 (0.369 ; 0.44)	0.142 (0.318 ; 0.86)	0.733 (0.058 ; 0.01)	0.744 (0.03)	2.14
	500	0.032 (0.068 ; 0.36)	1.027 (0.129 ; 0.01)	1.145 (0.13 ; 0.01)	0.016 (0.119 ; 0.90)	0.035 (0.082 ; 0.50)	0.719 (0.01)	21.85
$(1, 15)$	50	0.037 (0.159 ; 0.84)	0.082 (0.223 ; 0.86)	0.074 (0.22 ; 0.88)	0.032 (0.186 ; 0.96)	0.09 (0.149 ; 0.68)	0.393 (0.01)	2.86
	500	0.003 (0.058 ; 0.96)	0.022 (0.072 ; 0.60)	0.02 (0.071 ; 0.62)	0.007 (0.06 ; 0.82)	0.003 (0.054 ; 0.92)	0.207 (0.01)	33.89

For  $\beta$ ,  $\delta$ , and  $D^{1/2}$ , we show the average of the mean squared errors (MSEs), using the posterior means as point estimators. In parentheses, the average posterior standard deviations (SDs) and empirical coverages of 95% credible intervals are shown. For  $\pi$ , we show the average of  $L_1$  norm between the posterior mean and  $\pi^*$  with the posterior SDs in the parentheses. Time is the average seconds taken for sampling 1,000 MCMC draws in Matlab on a desktop with a 4.9GHz processor and 64GB RAM.

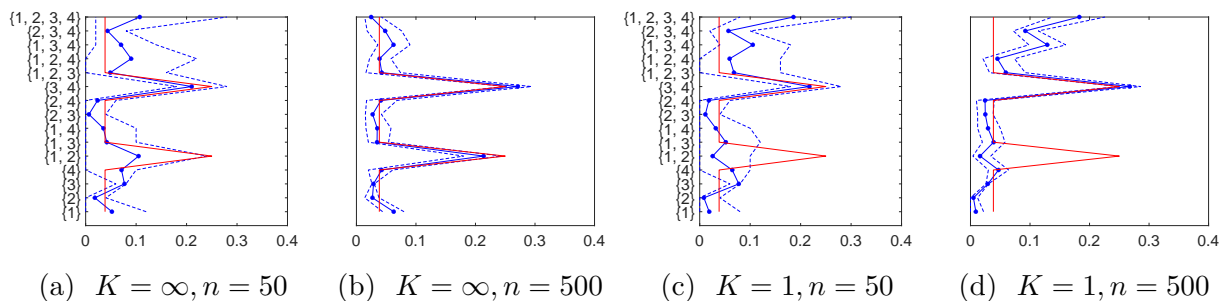


Figure SD.2: The true distribution over consideration sets (solid, red), posterior mean (solid with dots, blue), 95% equal-tailed credible interval (dashed, blue). Each plot is based on one realization of simulated data.  $J = 4$ ,  $T = 5$ , with random effects.

Figure SD.3 (a) shows a histogram of the estimated posterior probability of  $H_1$  (dependent consideration) when  $H_1$  is true. The method appropriately assigns values close to one for the majority of the simulations. Figure SD.3 (b) shows the result when  $H_0$  is true. The posterior probability assigned to  $H_1$  is close to zero for the majority of the simulations.

In summary, even with random effects, our proposed method can deliver consistent estimates of the preference parameters i.e.  $\beta$  and  $D$  as well as the distribution of consideration sets  $\pi$ . In addition, our method can be used to test whether latent consideration is independent or dependent.

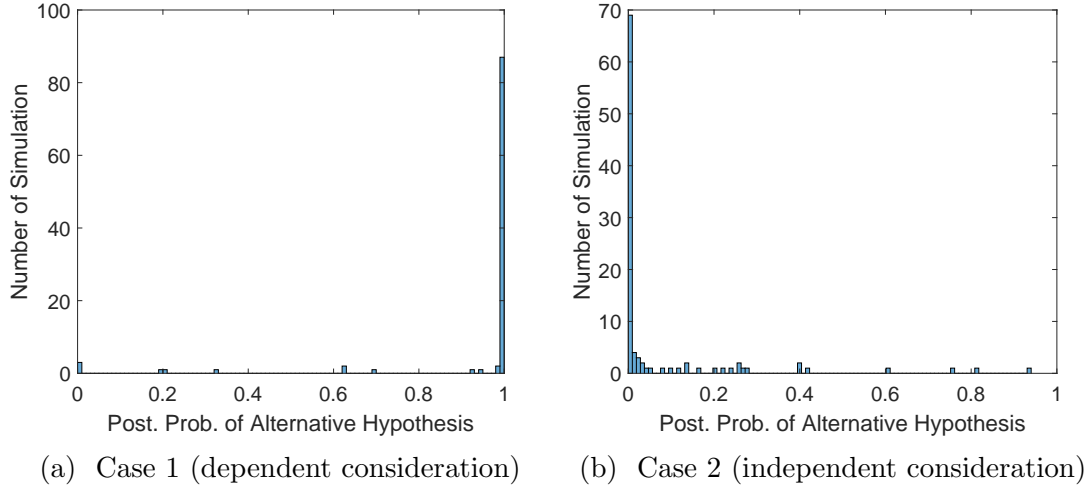


Figure SD.3: Histograms of estimated posterior probabilities of  $H_1$  in each of the 100 simulations under (a) case 1 (dependent consideration -  $H_1$  is true) and (b) case 2 (independent consideration -  $H_0$  is true).  $\varepsilon = 0.1, n = 50. T = 5$ .

### SD.3 Simulation results with auto-correlated covariate

We generate a set of auto-correlated covariates as follows:  $x_{ijt} = \rho x_{ijt-1} + N(0, 1)$  for  $t = 1, \dots, T$  with  $\rho = 0.9$  and  $x_{ij1} \sim N(0, 1)$ . The rest of the simulation design is the same as in Section 5.1 of the paper. Table SD.2 shows the results. Overall, the results are similar to the case with no correlation.

Table SD.2: Simulation results with  $J = 4$  (Auto-correlated covariate)

$(K, T)$	$n$	$\beta$		$\delta_1$		$\delta_2$		$\delta_3$		$\pi$		Time
$(\infty, 5)$	50	0.026	(0.15 ; 0.90)	0.211	(0.45 ; 0.94)	0.17	(0.45 ; 0.98)	0.107	(0.37 ; 0.98)	0.425	(0.03)	1.65
	200	0.004	(0.07 ; 0.98)	0.072	(0.23 ; 0.86)	0.07	(0.23 ; 0.90)	0.032	(0.18 ; 0.98)	0.266	(0.02)	5.73
$(\infty, 15)$	50	0.006	(0.08 ; 0.96)	0.073	(0.24 ; 0.92)	0.058	(0.24 ; 0.94)	0.041	(0.21 ; 0.96)	0.373	(0.02)	2.26
	200	0.002	(0.04 ; 0.94)	0.012	(0.12 ; 0.96)	0.011	(0.12 ; 0.96)	0.008	(0.1 ; 1.00)	0.201	(0.01)	7.64
$(1, 5)$	50	0.03	(0.15 ; 0.92)	0.507	(0.46 ; 0.78)	0.463	(0.46 ; 0.76)	0.138	(0.38 ; 0.98)	0.624	(0.03)	1.41
	200	0.005	(0.07 ; 0.96)	0.493	(0.24 ; 0.38)	0.488	(0.24 ; 0.38)	0.038	(0.19 ; 0.90)	0.548	(0.01)	4.68
$(1, 15)$	50	0.006	(0.08 ; 0.96)	0.097	(0.25 ; 0.88)	0.074	(0.24 ; 0.94)	0.042	(0.21 ; 0.94)	0.422	(0.02)	2
	200	0.002	(0.04 ; 0.92)	0.019	(0.12 ; 0.90)	0.019	(0.12 ; 0.94)	0.008	(0.1 ; 1.00)	0.277	(0.01)	6.38

For  $\beta$  and  $\delta$ , we show the average of the mean squared errors (MSEs), using the posterior means as point estimators. In parentheses, the average posterior standard deviations (SDs) and empirical coverages of 95% credible intervals are shown. For  $\pi$ , we show the average of  $L_1$  norm between the posterior mean and  $\pi^*$  with the posterior SDs in the parentheses. Time is the average seconds taken for sampling 1,000 MCMC draws in Matlab on a desktop with a 4.9GHz processor and 64GB RAM.

### SD.4 Simulation results with time-varying consideration sets

As described in the paper, the assumption of time invariant consideration sets facilitates theoretical study and computation. However, the actual consideration sets might have some

dynamics over time. In this simulation, we study how sensitive our proposed method is with respect to a violation of the time invariance assumption.

In order to generate time-varying consideration sets, we first draw the consideration sets in the first period as before from  $\pi^*$ . They remain unchanged in period 2. At period  $t = 3$ , the units learn about items outside their consideration sets and 50% of them add a new item randomly to the sets. The consideration sets are unchanged after this period.

The rest of the simulation design is the same as in Section 5.1 of the paper. Table SD.3 shows the simulation result. Crawford et al. (2021) show that incorrectly adding items to the consideration sets lead to biased estimates, which is reflected in the increased MSEs, especially when  $n$  is large, compared to Table 2 of the paper where the time invariance assumption holds. In addition, there are distortions of the coverage.

We still see that the MSE/SD decrease in  $n$  under the relatively large  $T = 15$ . This is because the sample with  $T = 15$  has more periods with stable consideration sets than  $T = 5$ . Thus, although the violation of the time-invariant consideration sets leads to issues known in the literature, and we are not an exception, our approach delivers reasonable result when there are enough periods during which the invariant assumption holds.

Table SD.3: Simulation results with  $J = 4$  (Time-varying consideration sets)

$(K, T)$	$n$	$\beta$	$\delta_1$	$\delta_2$	$\delta_3$	Time
$(\infty, 5)$	50	0.032 ( 0.13 ; 0.8 )	0.216 ( 0.33 ; 0.84 )	0.151 ( 0.34 ; 0.92 )	0.182 ( 0.34 ; 0.94 )	1.59
	200	0.023 ( 0.06 ; 0.48 )	0.196 ( 0.16 ; 0.3 )	0.153 ( 0.17 ; 0.48 )	0.046 ( 0.17 ; 0.9 )	5.57
$(\infty, 15)$	50	0.01 ( 0.07 ; 0.82 )	0.032 ( 0.15 ; 0.92 )	0.039 ( 0.15 ; 0.88 )	0.026 ( 0.18 ; 0.9 )	2.15
	200	0.004 ( 0.04 ; 0.76 )	0.016 ( 0.08 ; 0.7 )	0.011 ( 0.08 ; 0.88 )	0.009 ( 0.09 ; 0.96 )	7.16

For  $\beta$  and  $\delta$ , we show the average of the mean squared errors (MSEs), using the posterior means as point estimators. In parentheses, the average posterior standard deviations (SDs) and empirical coverages of 95% credible intervals are shown. For  $\pi$ , we show the average of  $L_1$  norm between the posterior mean and  $\pi^*$  with the posterior SDs in the parentheses. Time is the average seconds taken for sampling 1,000 MCMC draws in Matlab on a desktop with a 4.9GHz processor and 64GB RAM.

## SE Additional material for the application

### SE.1 Data description

We combine two sources of the data sets obtained from Nielsen, a store data and a purchase data, in order to prepare a panel data set. The preference and consideration patterns might have affected during the pandemic, so we chose the year 2019, which is the earliest year available before the pandemic. In the store data, we first choose a retailer, whose identity is not revealed in the Nielsen data, which consistently had over 100 cereal brands available at the majority of its stores. There are 239 stores under this retailer, operating mainly in the Midwest of the United States. See Figure 3 for the locations of the stores and percentages of the purchases. The store data contains product information at UPC (universal product code) level such as price and size (ounce). A ‘brand’ can consist of multiple UPCs. Brand-level prices are defined as size-weighted averages of UPC prices. We first pick the top 135 cereal brands in terms of the availability at these stores, which are responsible for over 90 percentages of the purchases in the purchase data at these stores in 2019. In more than 95% of the store-week combinations, the price information of the 135 brands is available, but if it is missing, we impute the value with the average of the prices of the same brand at the other stores in the same week. We then defined the top 100 to be the inside options and the rest to be the ‘other’ option. In the purchase data, we removed the households who made less than 3 units of cereal. When households purchased multiple units of cereal at one shopping trip, we treat them as separate purchases. This leaves us a sample with  $J = 101$  brands (see Appendix for a complete list of the brand names). The data contains  $n = 1880$  households, 25849 purchases at 239 stores of the same retailer throughout 52 weeks.

## SE.2 Hyperparameters

We set the hyper prior parameters as follows: a sparsity-supporting prior for the attention probabilities  $q_{hj} \sim \text{Beta}(\underline{a}_{q_j}, \underline{b}_{q_j})$ , independently over  $j = 1, \dots, J$  for  $h = 1, \dots, \infty$ , with  $(\underline{a}_{q_j}, \underline{b}_{q_j}) = (s \cdot r, s \cdot (1 - r))$ ,  $r = \frac{r_0}{J}$  with  $s = 5$  and  $r_0 = 30$ , which implies that the prior mean of  $q_{hj}$  is about 0.44. For the DP concentration parameter,  $\alpha \sim \text{Gamma}(1/4, 2)$ . The priors for  $\delta$  and  $\beta$  are independent normal distributions with zero mean and variance 3. The prior for  $D$  is an inverse-Wishart distribution with hyper-parameters  $(\underline{v}, \underline{R}) = (9, (1/9)I)$ .

## SE.3 Additional estimation results and discussion

### SE.3.1 Estimated parameters in the response model

**Brand-specific fixed-effects.** The estimated brand-specific fixed-effects are shown in Table 4 of the paper. The number of brand-specific fixed-effects whose 95% credible intervals do not include 0 is larger for MNL than MNL\_C and for MNL\_R than MNL\_RC. This phenomenon was also observed by [Chiang et al. \(1998\)](#). To explain this, we note that under MNL\_C and MNL\_RC, the estimated consideration sets  $\{\mathcal{C}_i\}$  are much smaller than the set with all brands. If, for example, there is a brand that is almost never chosen by any household, the estimated  $\{\mathcal{C}_i\}$  tends to exclude such a brand. The standard logit model does not account for such nonconsideration and instead assumes that every household considers all brands. As a result, the magnitudes (absolute value) of brand-specific fixed effects tend to be overestimated. Under the full specification, for 67 out of 100 of them, the corresponding 95% credible interval does not include 0. Note that we fixed  $\delta_J = 0$  for identification.

### SE.3.2 Estimated parameters in the mixture model

In the following, we present additional estimation results concerning the parameters in the mixture model under the MNL\_RC specification. Figure [SE.1](#) compares the prior

and posterior densities of the DP concentration parameter  $\alpha$ . The vague prior density  $\alpha \sim \text{Gamma}(1/4, 1/4)$ , suggested by [Dunson and Xing \(2009\)](#), is shown as the dashed line and the posterior as the solid line.

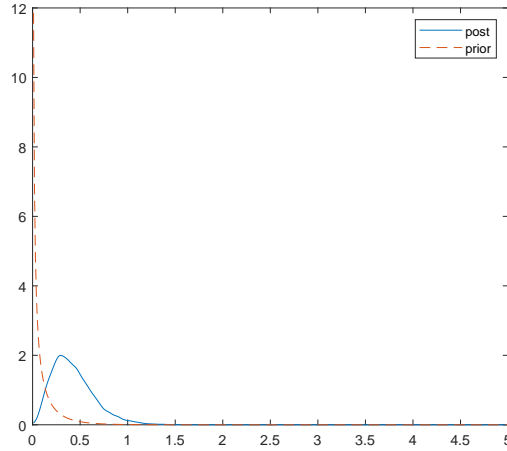


Figure SE.1: pdf's of  $\alpha$ : prior (dashed) and posterior (solid) from the empirical application. MNL\_RC.

Figure [SE.2](#) shows the posterior probability mass function of the non-empty mixture components. The posterior mode of the number of non-empty components is six.

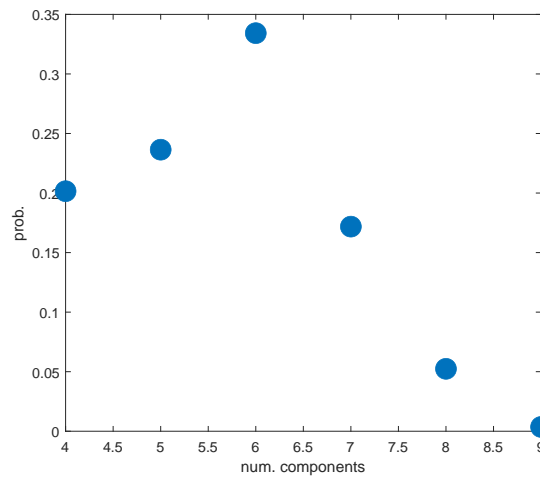


Figure SE.2: Posterior probability mass function of the number of nonempty components from the empirical application. MNL\_RC.

The similarity matrix is shown in [Figure SE.3](#). Each entry of the matrix shows the



posterior probability that a given pair of households  $(k, i)$  are clustered together i.e.  $S_k = S_i$ , ranging from zero (light blue) to one (dark blue).

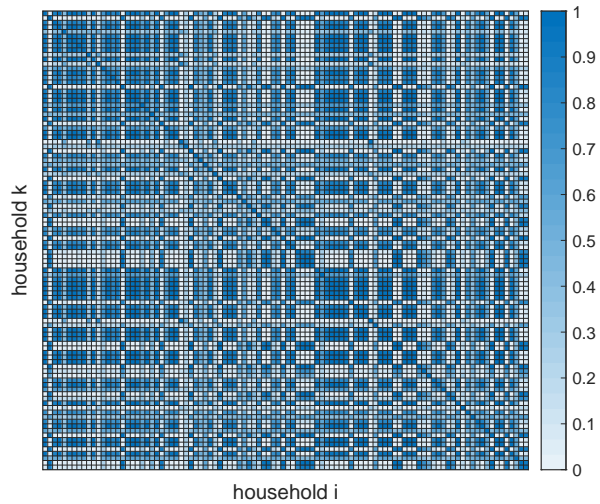


Figure SE.3: The similarity matrix of a sample of 100 households (out of 1880).

### SE.3.3 Additional material on the prediction

To investigate why MNL\_RC outperforms MNL\_R in prediction, we compare the predictive response probabilities between the two models. For each  $i \in \mathcal{O}$ , we can compute the marginal posterior of  $\Pr(Y_{iT_{i+s}} = j)$ , for each alternative  $j$  and forecasting horizon  $s = 1, \dots, h_i$ . Figure SE.4 presents the estimated response probabilities for the household  $i = 3$  in the first out-of-sample period,  $s = 1$ . This household repeatedly purchased brands 13, 45, 57, and 101 in the estimation sample:  $\{45, 13, 13, 101, 13, 13, 13, 57, 57, 57, 57\}$ , in the order of the purchases. In the first out-of-sample week, the household purchased brand 13. The figure shows the 90% credible intervals (vertical bars) as well as the mean of the estimated response probabilities (circles). Clearly, the estimated response probabilities are much sparser for MNL\_RC (lower panel) than MNL\_R (upper). The traditional MNL approach necessarily implies a positive probability for every alternative. In contrast,

the consideration set model allows many alternatives to actually receive zero predictive probabilities. Thus, incorporating consideration set heterogeneity can improve predictive performance due to the sparsity in the predictive response probabilities when the time-invariant consideration set assumption is appropriate, which seems to be the case in this data set.

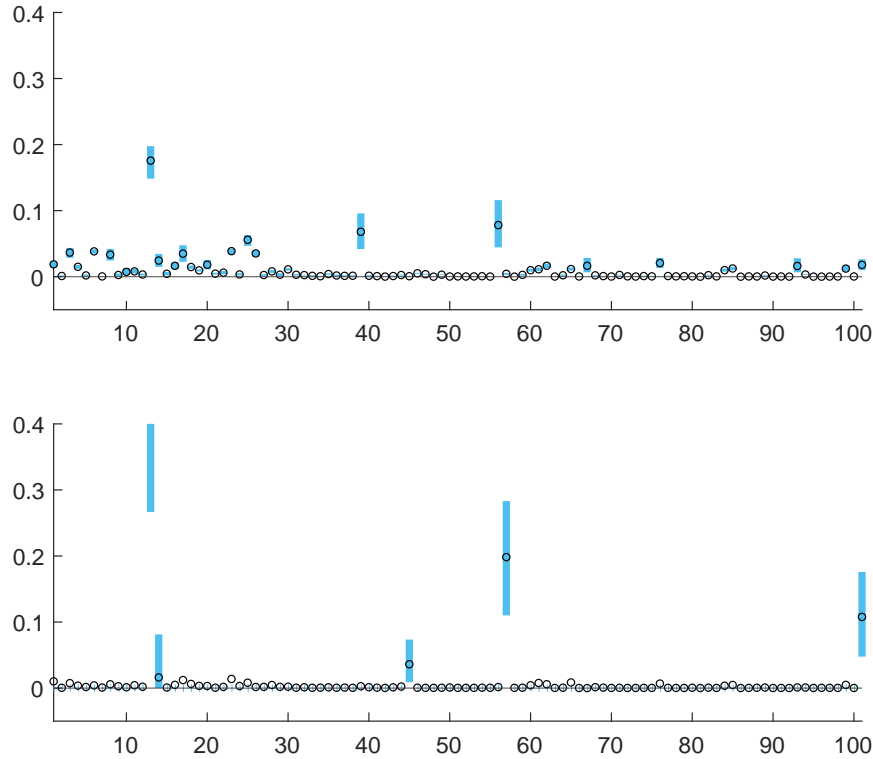


Figure SE.4: Estimated predictive response probabilities  $\Pr(Y_{i,T_{i+1}} = j)$  for  $i = 3$ . 90% credible intervals (bars) and means (circles). The horizontal axis represents the brands  $j \in \{1, \dots, 101\}$ . The actual out-of-sample purchase was brand 13.

### SE.3.4 Estimated consideration dependence

We conduct the test for independent consideration introduced in Section 6. The estimated posterior probability of the alternative hypothesis is very close to one i.e.  $\Pr(H_1 | \mathbf{D}^n) \approx 1$ , and we conclude that the considerations of cereal products in this particular market are de-

pendent. Furthermore, to investigate brand pair-level dependence, consider a hypothetical consumer  $i$  whose consideration set is drawn from the true unknown distribution. Define the marginal probability that brand  $j$  is (and not) considered:  $\pi_1^{(j)} = \Pr(C_{ij} = 1)$  and  $\pi_0^{(j)} = \Pr(C_{ij} = 0)$ . Also define the probability that a pair of brands  $(j, \ell)$  is considered jointly as  $\pi_{11}^{(j, \ell)} = \Pr(C_{ij} = 1 \text{ and } C_{i\ell} = 1)$ , and similarly define the probabilities for the remaining three cases:  $\pi_{01}^{(j, \ell)} = \Pr(C_{ij} = 0 \text{ and } C_{i\ell} = 1)$ ,  $\pi_{10}^{(j, \ell)} = \Pr(C_{ij} = 1 \text{ and } C_{i\ell} = 0)$ , and  $\pi_{00}^{(j, \ell)} = \Pr(C_{ij} = 0 \text{ and } C_{i\ell} = 0)$ . We employ the model-based Cramer's V statistics as a measure of consideration dependence between brands  $j$  and  $\ell$  as:  $\rho_{j, \ell}^2 = \sum_{s=0}^1 \sum_{m=0}^1 \left( \pi_{(s,m)}^{(j, \ell)} - \pi_{(s)}^{(j)} \pi_{(m)}^{(\ell)} \right)^2 / \pi_{(s)}^{(j)} \pi_{(m)}^{(\ell)}$ , which ranges from 0 to 1, and  $\rho_{j, \ell}^2 \approx 0$  indicates that the consideration of the two brands  $(j, \ell)$  is nearly independent. These probabilities are approximated as functions of the model parameters, for example,  $\pi_1^{(j)} = \sum_{h=1}^{k^*} \omega_h q_{hj}$ ,  $\pi_0^{(j)} = \sum_{h=1}^{k^*} \omega_h (1 - q_{hj})$ , and  $\pi_{10}^{(j, \ell)} = \sum_{h=1}^{k^*} \omega_h q_{hj} (1 - q_{h\ell})$ , and so on. Figure SE.5a shows the posterior means of  $\{\rho_{j, \ell}\}$ . Figure SE.5b shows the brand pairs  $(j, \ell)$  for which the posterior probability that  $\rho_{j, \ell} > 0.1$  is greater than 0.95. Based on this criteria, we identified 72 brand pairs (shown in black).

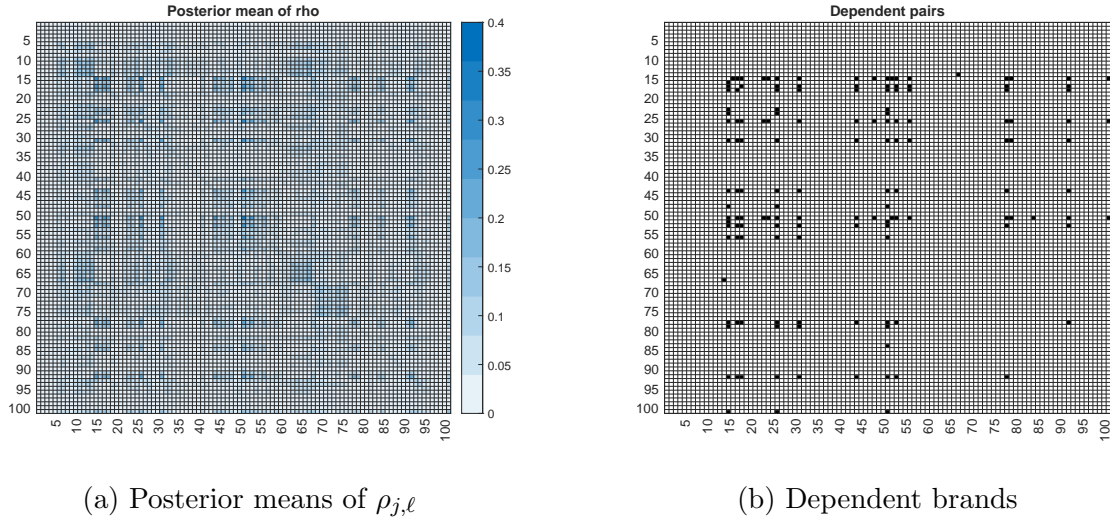


Figure SE.5: Consideration dependence in the 2019 Midwest cereal consumption data.