USING EXPECTATIONS DATA TO INFER MANAGERIAL OBJECTIVES AND CHOICES

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ABSTRACT

We develop a framework that combines observed market data with self-reported managerial expectations data to jointly estimate the demand function and objective function of the marketing manager at a large university performing arts center. Our methodology helps us to address four critical issues of great concern to many structural econometric models: (1) the endogeneity issue that arises when almost *all* product attributes and marketing policies are correlated with unobserved product quality; (2) the decision-maker may be uncertain or even biased in her assessment of true product quality; (3) the manager may be biased in her beliefs concerning the impact of her actions on outcomes, generating choices that appear non-optimal; (4) the manager may have objectives other than pure static profit maximization. The availability of expectations data in our application allows us to relax strong behavioral assumptions, such as rational expectations and profit maximization, and test the degree of the manager's bias in her beliefs regarding product appeal and advertising effectiveness. Our findings suggest that the manager has "objectively correct" beliefs concerning certain key economic parameters, such as the price elasticity of demand, and the impact of advertising. However, the manager appears to be biased in her beliefs concerning the appeal of certain product attributes, and departs from static profit maximization by exhibiting special preference for promoting "avant-garde" art. This latter finding is consistent with the mission statement of the center.

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I. INTRODUCTION

Structural econometric models have been used in fields such as labor economics, industrial organization, and marketing to recover parameters of interest derived from economic theory concerning the choice behavior of decision-makers. One advantage of the structural approach is the ability to conduct policy simulations using observational data. A second advantage is the ability to test the predictive performance of competing theories of behavior.[1] Despite these advantages, many researchers are sceptical about results derived using structural approaches, due to the strong assumptions concerning behaviour that are often required by the econometric model (and implied by theory). As noted by Manski (2004), economists generally assume that individuals or firms have rational expectations concerning choices so that the researcher can focus on the determinants of revealed preference. However, empirical findings may be quite sensitive to assumptions concerning expectations. For example, a manager may have inside information regarding the latent appeal of her products which is in general unobserved by researchers. Yet, her information may not be perfect and could also be biased; hence, the manager's expectations of product sales may not be consistent with actual sales in data. In addition, the specification of the information set at the time the agent makes a decision is often critical.[2]

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[1] Wolak and Reiss (2005) discuss the merits of structural econometric models for applications in industrial organization.

[2] Manski (1993) shows that assumptions concerning expectations formation can have a substantial impact on the factors that are believed to impact educational attainment.

Another restrictive assumption concerning behavior in the econometric model relates to an agent's objective function. For example, profit maximization is a generally accepted assumption regarding a firm's objective when making pricing or advertising decisions; however, the manager or decision maker inside the firm may have other objectives, particularly if the firm is a non-profit. By imposing assumptions on the manager's expectations or information set and profit maximization objective, the structural econometric model may be mis-specified and hence estimation results of the model may be biased.

A related issue concerns the specification in the econometric model of the information set and beliefs of the agent at the time he or she makes a decision. This is a potential source of endogeneity in the model if the agent's information set and beliefs unobserved to researchers (stochastic demand or productivity shocks in the econometric model) affecting choice are also correlated with outcomes. In this case, one cannot infer the impact of the choice variable on the outcome of interest. For example, when choosing advertising expenditures, a manager may decide to advertise more (less) for products with less (more) latent appeal. Failure to account for the manager's beliefs concerning product appeal may lead to negatively biased estimates of advertising effectiveness – implying a negative relationship between advertising and product sales. As we discussed above, it is also important to account for how the information set evolves, especially at the time the agent makes a decision.

In this paper, we construct a structural model of the advertising decisions of the manager of a large university performing arts center (the "Center"). While the Center may be little intrinsic interest to many researchers, our application contains a number of features of wider importance. At the start of each of the three years we study, the manager reported the amount she planned to spend on advertising each of the 60-70 performances during the season. In addition, she also reported her expectations concerning the number of tickets that would be sold for each performance. Consequently, we are able to relax assumptions concerning expectations in our model and test whether her beliefs are indeed rational, as is assumed in many applications. We then examine how these beliefs affect advertising choices.

Our results highlight the value of the subjective expectations data in this setting. Simple OLS regressions of ticket sales on advertising and prices imply that more advertising leads to reduced ticket sales and higher prices sell more tickets! Using the expectations data to infer the manager's prior beliefs concerning the latent appeal of each show, our structural estimates indicate that the advertising response is positive and significant. In addition, we are able to test whether the manager's expectations concerning both the impact of advertising on ticket sales and the price elasticity of demand are indeed rational. We find that most of the manager's prior beliefs are unbiased, as is typically assumed in many structural models.

A key mission of the non-profit Center is to provide avant-garde art to the local community. Using our structural model, we are able to test the extent to which the manager follows this mission, an objective outside pure (static) profit maximization. In particular, the manager appears to spend too much on advertising for avant-garde artists if she was simply attempting to maximize profits. Our data on her expectations allow us to decompose this into two parts. First, as opposed to the rational expectations assumption, we find that she is over-optimistic concerning the appeal of avant-garde shows to consumers, leading to over-expenditure on advertising. Second, her objectives coincide with the Center's mission statement: advertising for avant-garde shows directly increases her utility relative to spending on "normal" performances.

The remainder of the paper proceeds as follows. Section II describes the relevant operations of the performing arts center and the timing of the marketing manager's advertising decisions. Section III develops the framework for incorporating subjective expectations data into a structural econometric model of advertising choice, and describes our two step estimation process for recovering the parameters of the market demand and managerial objective functions. We discuss the results in Section IV, and finally conclude in Section V.

II. THE EMPIRICAL SETTING

We analyze the advertising decisions of the marketing manager for one of the largest university-based performing arts centers (termed the "Center" in the paper) in the United States. The Center presents approximately 60 music, dance, and theatrical events each year, with each event usually running from one to five performances. Unlike commercial presenters, it is a non-profit organization whose mission is to bring to the local community, and especially the university community, performers who reflect a wide range of cultural and artistic backgrounds. In particular, a key objective is to provide avant-garde art to the community. Consequently, while some performances are by popular artists (e.g., Keith Jarrett, David Sedaris, Mikhail Baryshnikov), others are by relatively unknown artists who are on the very cutting edge of experimental performance art (e.g., La La La Human Steps, Umabatha, Vietnamese Water Puppets). The Center can be considered a local monopoly: although there are a large number of entertainment options in the community, the Center is the only major presenter of avant-garde artists within an easy driving distance of the affluent area of the city in which it is located.

The Center Director hires the artists and books the venue. The Center operates both large and small performance venues, and artists are booked into these venues depending in part on expected ticket sales (based on input from the Marketing Department). The Marketing Department is responsible for generating ticket sales, which account for roughly two-thirds of the Center's operating budget. The marketing manager sets prices for both individual shows and performance series. These series arranged by genre and generally feature both well-known and lesser-known artists. Most of the Marketing Department's budget, which is set at the beginning of the year, is spent on advertising in print (the local major newspaper and the campus newspaper), radio, and direct mail. Ticket packages are offered during the pre-season. After the season begins, each event is advertised individually, starting about a month before the show opens.

To model the decision-making of the Center's marketing manager, it is important to specify the sequence of options available to her when advertising each show. The timing these decisions and their outcomes can be divided into three periods:

Period 0: Before the season begins, the marketing manager decides on the ticket prices of individual shows and Series ticket packages. Once set, these prices do not change over the course of the season, and the Center does not offer discounts for poorly selling shows (there are student discounts but they are a small proportion of total ticket sales). Consequently, after period 0, the only strategic option available to the manager to increase demand for a particular show is the level of advertising for that show. As part of the venue booking process, the manager generates and reports a forecast of the ticket sales for each performance that is used in deciding which hall to allocate to each show. The manager also uses heuristic "rules-of-thumb" to form expectations concerning advertising expenditures and to decide the preliminary advertising budget for each performance.[3] Her expected ticket sales are a function of venue, time of year (university semester), day and time of week of the performance (weekends, weekdays, daytime, evening), Series, genre (traditional, family, avant-garde), price, the expected advertising expenditure, and the manager's beliefs concerning the latent attractiveness of the show before any ticket is sold.

Period 1: At the beginning of period 1, the Center mails circulars describing the upcoming season. Over the course of the period, individuals purchases tickets for each performance (both as part of a series package and individually). No advertising for individual shows is conducted. Note that for shows occurring early (late) in the season, period 1 may be fairly short (long). Overall, approximately 36% of tickets sales occur in period 1. At the end of period 1, roughly one month prior to the date of the performance, the manager observes the ticket sales for the show ("Period 1 Sales"), and updates her period 0 beliefs concerning the latent attractiveness of the show and the amount she plans to spend on advertising.

Period 2: Based on her updated beliefs concerning show attractiveness and period 1 sales, the manager decides how much to spend advertising the show, and purchases advertising in print and on the radio at the beginning of period 2. Advertising expenditures may also depend on the amount of budgetary funds remaining at the

[3] Expected advertising expenditures depend primarily on number of shows for the performance and the venue, as well as the manager's past experience.

beginning of the period. Ticket sales are then recorded up until the date and time of the performance.

Data and Summary Statistics

We obtained data from the Center regarding show characteristics, prices, and Period 1 and 2 ticket sales for each of the 146 shows during the years 1997-1999. Of key importance for this study, the Marketing manager provided us with her period 0 expectations regarding ticket sales for each show in the data set, as well as her period 0 planned advertising expenditures.[4]

Figure 1 shows that there is a strong positive relationship between the manager's expectations regarding ticket sales and actual ticket sales. The correlation between expected and actual tickets sold is high (0.85), with higher projected ticket sales generally associated with higher actual sales. However, the figure also indicates that the manager tends to under-predict ticket sales. In fact, the manager projects more than actual ticket sales for only 26% of shows. She also appears to be substantially more optimistic regarding avant-garde shows. In this case, the Manager's projections overstate actual tickets sold in 59% of cases.

Table 1 indicates that projected advertising expenditures per performance are roughly equal to actual expenditures. However, though statistically insignificant, actual advertising expenditures are higher than expected expenditures for avant-garde shows, and *vice versa* for other genres. Comparison of the actual and expected advertising expenditures by genre may suggest the consequences of the manager's beliefs concerning the latent attractiveness of avant-garde shows -- perhaps in response to slow first period sales, the manager substantially increase the actual advertising for avant-garde shows

[4] The manager's updated beliefs regarding ticket sales after Period 1 are not reported in the data.

than what the manager expected. The third row of the table shows that the average ticket price of each show is about $30, and there is no significant difference in pricing for avant-garde shows vs. other genres.

Some OLS Estimates of Ticket Sales

The difficulty faced by the econometrician in evaluating the impact of advertising, pricing, and product characteristics on tickets sold is illustrated by the results in Table 2. Columns (1) and (2) of the table present some OLS regression estimates of the price elasticity of demand for tickets sold for each performance, as well as the elasticity of advertising response and the impact of other product characteristics such as show genre and series membership. Taken at face value, the price coefficient implies that the demand curve for shows is upward sloping, since a 10 percent increase in price is associated with 3.2-3.4 percent increase in the number of tickets sold. On the other hand, advertising appears to reduce demand, since from column (1) a 10 percent increase in advertising expenditure is associated with a 1.2 percent decline in tickets sold! Even when the advertising effect is allowed to vary by show genre (avant-garde shows vs. not avant-garde shows), the OLS results in column (2) continue to show a negative effect of advertising.

The results in Table 2 suggest not surprisingly that pricing and advertising strategies are endogenous. If the manager believes a show is likely to be popular to potential customers, she will likely charge a higher price. Conversely, she might have an incentive to advertise more for less attractive shows, generating the negative relationship observed in the table. Other product characteristics are also likely to be endogenous. For example, believing that the “Vietnamese Water Puppets” is appealing to the audience the manager might move the show to a large venue. The potential endogeneity of many of the product attributes shown in Table 2 makes standard econometric approaches for generating unbiased estimates of the impacts of prices, advertising, and attributes particularly problematic. One is unlikely to find reasonable instruments for all the endogenous variables. In fact, even if product attributes are assumed to be exogenous, acceptable instruments may not be available for both pricing and advertising. Consequently, an alternative approach is required.

III. MANAGERIAL OBJECTIVES AND THE USE OF EXPECTATIONS DATA

In this section, we develop an econometric model using the managerial expectations data to recover some of the interested parameters in the market demand function, and to test whether the manager at the Center is simply choosing advertising levels to maximize revenues, or whether she also attempts to follow the Center's mission of bringing avant-garde art to the general community, without imposing some restrictive assumptions on the managerial behavior and information set. We begin by specifying a general parametric demand function for product *j* which specification is known to both researchers and the manager, up to a parameter set Θ which represents the effects of product attributes and managerial policies on the demand function, as

$$y_j = y(X_j, z_j, \omega_j; \Theta), \tag{1}$$

where X_j is a vector of the product attributes (which may include variables describing the competitive environment), and z_j consists of managerial decision variables (e.g., advertising expenditures, prices etc.). The stochastic component ω_j denotes the true product quality or attribute which is unobserved by researchers; it is in general correlated with z_j and X_j. This brings us to the classical endogeneity problem: a simple linear or non-linear regression on (1) will produce biased estimates for Θ, as we illustrated in the previous section. Empirical researchers usually assume that ω_j is only correlated with z_j but not with X_j. However, in some cases and most certainly for our data, such an assumption is invalid.

The manager may only partially observe the true ω_j. In a general case, let Ω_j be the managers' information set for product *j*, her belief of the true ω_j is a conditional distribution function $F^0(\omega|\Omega_j)$. This belief may be very uncertain if the information set is very limited, and may be even biased. Further, let Θ^0 be a parameter set representing how the manager perceives as the effects of product attributes and managerial policies on the demand function and let y_j^0 be the manager's *expected* market demand. We can write down

$$y_j^0 \equiv E[y_j|\Omega_j] = \int y(X_j, z_j, \omega_j; \Theta^0) dF^0(\omega|\Omega_j), \qquad (2)$$

We allow for the case that Θ^0 is different from the true parameters Θ in the model, *i.e.*, the manager may make *systematic errors* in forming expectations for the market demand; hence, her expected market demand y_j^0 in (2) may be systematically different from the true market demand y_j in (1). In other words, our model does not impose rational expectations assumption to decision making of managers.

We further assume a general parametric managerial expected objective function when making decisions for z_j as

$$V(z_j; X_j, W_j; \Theta^0, \Omega_j; \Psi^0) = \int u(X_j, W_j, y(X_j, z_j, \omega_j; \Theta^0); \Psi^0) dF^0(\omega|\Omega_j) \qquad (3)$$

where W_j is a set of variables excluded from the market demand function (e.g., cost variables). What is different from the traditional profit maximization assumption in most of the empirical literature is that *V(.)* allows for other managerial objectives, such as maximizing total market share or valuing certain products more than the others, which is implied by the set of parameters Ψ^0. Under this specification, the manager chooses the optimal level of z_j to maximize her objective function. That is

$$z_j^* = \arg\max_{z \in Z} V(z_j; X_j, W_j; \Theta^0, \Omega_j; \Psi^0) \equiv h(X_j, W_j; \Theta^0, \Omega_j; \Psi^0). \tag{4}$$

It is common to write the observed z_j as

$$z_j = z_j^* + \varepsilon_j = h(X_j, W_j; \Theta^0, \Omega_j; \Psi^0) + \varepsilon_j, \tag{5}$$

where the stochastic variable ε_j is assumed to be independent with X_j and W_j. However, since the unobserved ω_j enters $h(\cdot)$ through the manager's expectation, it may be correlated with X_j. Another difficult issue is that if $\Theta^0 \neq \Theta$ and the manager's information set Ω_j is not perfect, we cannot separately identify Ψ^0 from her beliefs. For example, observing the manager advertising more for avant-garde shows from data we are unable to infer whether this is because avant-garde shows have more weight in the manager's objective function, or because the manger wrongly believes that advertising is more effective in generating revenue for avant-garde shows.

Suppose that we observe data on managerial expectations of outcomes, y_j^0. Making use of this unique data, we propose a methodology to "invert" the unobservables from the observed and expected demand functions. To do so, we introduce a two-step estimation strategy in model estimation. The first step is to model the manager's updated belief regarding the latent product attribute ω_j after new information comes in period 1 as we discussed above the sequence of decisions making, by combining the observed and expectations data. This allows us to "invert" the unobservables and hence specify the pure demand shock unexpected to the manager in both periods 1 and 2, which is an important identification condition in the model estimation. By doing so we obtain consistent estimates for (Θ-Θ^0), as well as a subset of the true Θ and the perceived Θ^0 that relates to the effectiveness of decision variables z_j.[5] Conditional on these estimates, the next step is to recover the parameter set Ψ^0 in the managerial objective function *V(.)*.

In summary, the expectations data allow us to make the following contributions:

1. We allow for a general case where the unobservable stochastic component ω_j is correlated with X_j as well as z_j. That is, we do not need to impose the restriction that X_j is exogenous, as is assumed in most of the empirical literature.
2. We allow for a general case where managers may make systematic errors in decision making, *i.e.,* $\Theta^0 \neq \Theta$, and may have partial or even biased information about the unobservables in the demand function. We leave for the data to decide whether or not the "rational expectations" assumption made in most empirical literature is valid.
3. Because of (1) and (2), we are able to estimate the managerial objective function without imposing potentially restrictive assumptions. For example, one can recover managerial objectives that are potentially inconsistent with the static profit maximization assumption.[6]

[5] This will be explained clearly in the model estimation section.
[6] Such a relaxation is important in the non-profit environments. It can also be important for private firms. For example, a firm's long-term profit maximization objective may lead to decisions inconsistent with the static profit maximization objective. Further, a manager's decisions may be inconsistent with the firm's profit maximization objective if there exist agency problems.

III.A. Using the Expectations Data to Infer Advertising Effectiveness

We assume a Cobb-Douglas ticket sales function for show *j* as the following:

$$\ln(Y_j) \equiv y_j = \alpha + X_j\beta + Ad_j \cdot \gamma + \omega_j \tag{6}$$

where Y_j is the total ticket sales of *j*, α is a constant, X_j the show attributes that include genre, series, day of week, show time, venue, period dummies, year dummies, and (ln of) prices. The variable Ad_j is the (ln of) advertising expenditures for the show. A key difference between X_j and Ad_j is that the former is determined before the start of any ticket sales (period 0) and cannot be adjusted during the season, while the latter is determined only about one month before the performance date (period 2), at which time some tickets have already been sold. Finally, ω_j is the show attractiveness unobserved by researchers and may be partially observed by the manager.

Recall that advertising for any performance comes only in period 2. We use another Cobb-Douglas function to specify the demand in period 1:

$$\ln(Y_{1,j}) \equiv y_{1,j} = \alpha_1 + X_j\beta_1 + \omega_{1,j} \tag{7}$$

where $Y_{1,j}$ is the total amount of ticket sold in period 1, and definitions of other variables are similar as above. Note that X_j in (7) are the same as in (6) since they cannot be changed after the season starts; Ad_j does not enter (7) because there is no advertising in period 1. The variable $\omega_{1,j}$ is the period 1 unobserved show attractiveness that will be explained in detail later.

From (6) and (7) we can write the ticket sales function in period 2 as:

$$\begin{aligned} Y_{2,j} = Y_j - Y_{1,j} &= e^{\alpha + X_j\beta + Ad_j \cdot \gamma + \omega_j} - e^{\alpha_1 + X_j\beta_1 + \omega_{1,j}} \\ &= Y_{1,j} \cdot (e^{\alpha_2 + X_j\beta_2 + Ad_j \cdot \gamma + \omega_{2,j}} - 1) \end{aligned} \tag{8}$$

where $\alpha_2 = \alpha - \alpha_1,\ \beta_2 = \beta - \beta_1,\ \omega_{2,j} = \omega_j - \omega_{1,j}$. Such a flexible specification is desirable since some show attributes may have different impacts on ticket sales in different periods.[7]

Managerial Expectations of Ticket Sales

We specify the manger's perceived total ticket sales function as:

$$Y_j^0 = e^{\alpha^0 + X_j \beta^0 + Ad_j \cdot \gamma^0 + \omega_j}$$

where the subscript "*0*" represents the manager's beliefs. The above specification is equivalent to (6) except that the manager's perceptions concerning the effects of show attributes may be different from the actual effects, *i.e.*, $\alpha \neq \alpha^0$, $\beta \neq \beta^0$, and $\gamma \neq \gamma^0$. Consequently, we allow the potential case that manager would make systematic mistakes in forecasting policy implications, perhaps due to her limited information or the lack of experience. The manager may be uncertain about the true appeal of a show to the public and the final advertising expenditures; hence, ω_j and Ad_j are stochastic in the above specification. Ad_j is uncertain if the manager expects demand shocks and hence the necessity to adjust advertising policies. The manager uses her own information to form *expected* show attractiveness ω_j^0 and advertising spending Ad_j^0. We use a standard way to specify her beliefs as $\omega_j \sim normal(\omega_j^0, \sigma_0^2),$ and $Ad_j \sim normal(Ad_j^0, \sigma_{Ad^0}^2),$ where σ_0^2 and $\sigma_{Ad^0}^2$ are the variances in the prior beliefs. These specifications imply that the manager believes in general she does not have biased expectations regarding the true

[7] For example, series package, in which shows that belong to same genre are bundled together and sold before the season starts, may affect ticket sales positively in period 1 but negatively in period 2. In which case β_1 for series package is positive and β_2 is negative.

show attractiveness and advertising expenditures; however, it may be incorrect in the model.

Let $\Omega_{j,0}$ be the information set for performance *j* in period 0, which includes show attributes X_j and the manager's expected advertising spending and show attractiveness. Based on the above assumptions, the manager's (*ln* of) expected total ticket sales is:

$$\ln(E[Y_j^0 \mid \Omega_{j,0}]) \equiv y_j^0 = a^0 + X_j\beta^0 + Ad_j^0 \cdot \gamma^0 + \omega_j^0 \tag{9}$$

where $a^0 = \alpha^0 + \frac{(\gamma^0)^2 \cdot \sigma_{Ad^0}^2}{2} + \frac{\sigma_0^2}{2}$.

Further, we assume that the manger's perceived period 1 ticket sales function as the following:

$$Y_{1,j}^0 = e^{\alpha_1^0 + X_j\beta_1^0 + \omega_{1,j}}.$$

Given that we only have total expected ticket sales in the data, it is necessary to make assumptions regarding the manager's expected show attractiveness in period 1. We assume that the manager expects the show attractiveness has equal effect on the ticket sales in periods 1 and 2. Hence, we can similarly specify the manager's prior belief of the show attractiveness in period 1 as $\omega_{1,j} \sim normal(\omega_j^0, \sigma_1^2)$, where σ_1^2 is the variance which may be different from σ_0^2. Therefore, the manager's (*ln* of) expected ticket sales in period 1 is:

$$\ln(E[Y_{1,j}^0 \mid \Omega_{j,0}]) \equiv y_{1,j}^0 = a_1^0 + X_j\beta_1^0 + \omega_j^0 \tag{10}$$

where $a_1^0 = \alpha_1^0 + \frac{\sigma_1^2}{2}$. Note that ω_j^0 in (9) and (10) are the same.

We can write down the relationship between the effect of show attractiveness in period 1 and the managerial expected show attractiveness as $\omega_{1,j} = \omega_j^0 + \xi_{1,j}$, and hence rewrite the ticket sales function in period 1 as the following:

$$\ln(Y_{1,j}) \equiv y_{1,j} = \alpha_1 + X_j\beta_1 + \omega_j^0 + \xi_{1,j}. \tag{11}$$

For the effect of show attractiveness on total ticket sales we can also write down $\omega_j = \omega_{1,j} + \xi_{2,j} = \omega_j^0 + \xi_{1,j} + \xi_{2,j}$, and hence rewrite the total demand function as:

$$\ln(Y_j) \equiv y_j = \alpha + X_j\beta + Ad_j \cdot \gamma + \omega_j^0 + \xi_{1,j} + \xi_{2,j}. \tag{12}$$

Note that $\xi_{1,j}$ is a pure demand shock to the manager in period 0 when X_j are determined, while $\xi_{2,j}$ is an additional shock occurring in period 2. As $y_{1,j}$ is known to the manager when she makes advertising decisions in period 2, $\xi_{1,j}$ may correlate with the actual advertising expenditures Ad_j. Following the standard assumption we assume that $E[\xi_{1,j} \mid \Omega_{j,0}] = 0,$ and $E[\xi_{2,j} \mid \Omega_{j,0}] = 0.$ These assumptions are very reasonable since in the model we allow the intercepts in the expected and true demand functions to be different, i.e., $\alpha \neq \alpha^0$ and $\alpha_1 \neq \alpha_1^0$. If the manager constantly over- or under-estimates the true show attractiveness ω_j, such a difference should be reflected in differences $\alpha - \alpha^0$ and $\alpha_1 - \alpha_1^0$.

Updating Beliefs

As it will become clear later the identification of our estimation model relies on the specification of the *perceived* demand shocks unexpected by the manager when she reports the expectations data and makes advertising decisions, respectively. In particular we have to specify the demand shock to the manager at the end of period 1. Given that we only have data on the manager's period 0 expectations of ticket sales and advertising expenditures and not the updated beliefs after period 1, we need to model how the manager updates her beliefs after observing ticket sales in period 1, and the updating framework should general enough to allow for alternative types of decision rules potentially used by the manager.

Note that the prior beliefs for the show attractiveness are $\omega_j \sim normal(\omega_j^0, \sigma_0^2)$, and $\omega_{1,j} \sim normal(\omega_j^0, \sigma_1^2)$. Let $\xi_{1,j}^0$ be the demand shock perceived by the manager after period 1. Since she may be wrong in her beliefs of the true impacts of show attributes, it can be expressed as

$$\xi_{1,j}^0 = (\alpha_1 - \alpha_1^0) + X_j(\beta_1 - \beta_1^0) + \xi_{1,j}.$$

Such a demand shock comes from two sources: the true shock, $\xi_{1,j}$; and the systematic error in predicting the true impact of show attributes of which the manager is unaware, $(\alpha_1 - \alpha_1^0) + X_j(\beta_1 - \beta_1^0)$. Therefore to researchers $E[\xi_{1,j}^0 \mid \Omega_{0,j}] = (\alpha_1 - \alpha_1^0) + X_j(\beta_1 - \beta_1^0)$, which may not be zero. If, as is generally assumed in the empirical literature, there are no systematic errors, then $\xi_{1,j} = \xi_{1,j}^0$ and $E[\xi_{1,j}^0 \mid \Omega_{0,j}] = 0$.

Let $\xi_{2,j}^0 = \omega_j - \omega_j^0 - \xi_{1,j}^0$ [8], and also let $\Omega_{j,1}$ be the information set for the manager after period 1, which now includes the ticket sales information in period 1. We assume that the manager use the following linear updating rule to update her belief regarding $\xi_{2,j}^0$:

$$E[\xi_{2,j}^0 \mid \Omega_{1,j}] = \theta \cdot \xi_{1,j}^0$$

and

$$\mathrm{var}[\xi_{2,j}^0 \mid \Omega_{1,j}] = \sigma_2^2$$

[8] Similar to $\xi_{1,j}^0$, we use $\xi_{2,j}^0$ to distinguish from another stochastic variable $\xi_{2,j} = \omega_j - \omega_j^0 - \xi_{1,j}$ when there are no systematic errors.

Under this assumption, the updated belief of the total ticket sales after period 1, conditional on advertising spending Ad_j, will be

$$E[\ln(Y_j^0) \mid \Omega_{j,1}, Ad_j] = \alpha^0 + \frac{\sigma_2^2}{2} + X_j\beta^0 + Ad_j \cdot \gamma^0 + \omega_j^0 + (1+\theta)\cdot \xi_{1,j}^0 . \qquad (13)$$

Define $\eta_j = \xi_{2,j} - \theta \cdot \xi_{1,j}^0$, which is the unexpected demand shock from the manager's perspective after period 1 when she makes the advertising decisions. Based on the above discussion about the potential biases in the manager's beliefs of the impacts of show attributes, $E[\eta_j \mid \Omega_{1,j}]$ will be equal to $-\theta \cdot [(\alpha_1 - \alpha_1^0) + X_j(\beta_1 - \beta_1^0)]$.

The above linear updating rule is general enough to include various types of updating rules. The following are some examples:

Example 1: Simple Adaptive Learning Let $\omega_{1,j}^0 = \omega_j^0 + \xi_{1,j}^0$ be the effect of show attractiveness on ticket sales in period 1 perceived by the manager. Assume the manager uses the simple adaptive learning as $E[\omega_j \mid \Omega_{1,j}] = \upsilon \cdot \omega_j^0 + (1-\upsilon)\cdot \omega_{1,j}^0$, where υ is between 0 and 1. In this case θ in the linear updating rule is equal to υ within the range of 0 and 1.

Example 2: Bayesian Updating Suppose that in period 0 the manager's prior belief of the period 2 demand shock is $\xi_{2,j}^0 \sim N(0, \tilde{\sigma}_2^2)$ and the manager uses the Bayesian rule to update her beliefs. Together with her belief that $\omega_j \sim normal(\omega_j^0, \sigma_0^2)$ we can derive that $E[\omega_j \mid \Omega_{1,j}] = \omega_j^0 + \upsilon \cdot \xi_{1,j}^0$, where $\upsilon = \dfrac{\sigma_0^2}{\sigma_0^2 + \tilde{\sigma}_2^2}$. In this case θ in the linear updating rule is equal to $\upsilon - 1$, which is in the range of -1 and 0, and $\sigma_2^2 = \dfrac{\sigma_0^2 \cdot \tilde{\sigma}_2^2}{\sigma_0^2 + \tilde{\sigma}_2^2}$.

Example 3: Linear Least Square Updating Suppose the manager has the prior beliefs $\begin{pmatrix} \xi_{1,j}^0 \\ \xi_{2,j}^0 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tilde{\sigma}_1^2 & \tilde{\sigma}_{12} \\ \tilde{\sigma}_{12} & \tilde{\sigma}_2^2 \end{pmatrix}\right)$. The linear least square updating rule for the manager will be

$E[\xi_{2,j}^0 \mid \Omega_{1,j}] = \frac{\tilde{\sigma}_{12}}{\tilde{\sigma}_1^2} \cdot \xi_{1,j}^0$, and $\text{var}[\xi_{2,j}^0 \mid \Omega_{1,j}] = \tilde{\sigma}_2^2 \cdot (1 - \frac{\tilde{\sigma}_{12}^2}{\tilde{\sigma}_1^2 \cdot \tilde{\sigma}_2^2})$. In this case, $\theta = \frac{\tilde{\sigma}_{12}}{\tilde{\sigma}_1^2}$.

It is important to note that the true data generating process for ($\xi_{1,j}, \xi_{2,j}$) need not be specified, except that their expectations conditional on $\Omega_{j,0}$ are zero. Although we parametrically model the prior beliefs ($\xi_{1,j}^0, \xi_{2,j}^0$) for the manager, these assumptions are not more restrictive than what is assumed in most structural models in the literature, and are consistent with various updating rules. If there are no systematic errors in the manager's beliefs, ($\xi_{1,j}, \xi_{2,j}$) are equivalent to ($\xi_{1,j}^0, \xi_{2,j}^0$).

In summary, the manager's expected show attractiveness, ω_j^0, may be correlated with performance attributes X_j as well as expected and actual advertising expenditures Ad_j^0 and Ad_j, respectively. Conditional on the information set Ω_0, the stochastic variable $\xi_{1,j}$ is unexpected to the manager hence is uncorrelated with X_j and Ad_j^0. However, it does affect Ad_j since it may be used by the manager to update her belief about the true show attractiveness ω_j at the end of period 1. Conditional on information set Ω_1, the stochastic variable η_j is unexpected to the manager hence is uncorrelated with X_j, Ad_j^0, and Ad_j. These are important identification conditions for our model estimation.

Estimating Advertising Effectiveness

As the first step in model estimation, we can subtract the ticket sales function in period 1 in (11) from the manager's expected total ticket sales function (10) to have:

$$y_j^0 - y_{1,j} = (a^0 - \alpha_1) + X_j(\beta^0 - \beta_1) + Ad_j^0 \cdot \gamma^0 - \xi_{1,j} \tag{14}$$

Let $Z_{1j} = \{1, X_j, Ad_j^0\}$. We have the following moment condition

$$E[\xi_{1,j} \mid Z_{1j}] = 0$$

which can be used to obtain consistent estimates for the parameters $\{(a^0 - \alpha_1), (\beta^0 - \beta_1), \gamma^0\}$ in (14).

As $\omega_j = \omega_j^0 + \xi_{1,j}^0 + E[\xi_{2,j}^0 \mid \Omega_{1,j}] + \eta_j$, we can substitute the expressions $\xi_{1,j}^0 = (\alpha_1 - \alpha_1^0) + X_j(\beta_1 - \beta_1^0) + \xi_{1,j}$ and $E[\xi_{2,j}^0 \mid \Omega_{1,j}] = \theta \cdot \xi_{1,j}^0$ into the total ticket sales function in (6) to have

$$y_j = \alpha + X_j\beta + Ad_j \cdot \gamma + \omega_j^0 + \xi_{1,j} + \theta \cdot [(\alpha_1 - \alpha_1^0) + X_j(\beta_1 - \beta_1^0) + \xi_{1,j}] + \eta_j .$$

Further substitute equations (7) and (10) into the above expression we have

$$\begin{aligned} y_j = {} & (1+\theta) \cdot y_{1,j} - \theta \cdot y_j^0 + \mu + X_j\chi + Ad_j \cdot \gamma + Ad_j^0 \cdot (\theta \cdot \gamma^0) \\ & + \eta_j + \theta \cdot [(\alpha_1 - \alpha_1^0) + X_j(\beta_1 - \beta_1^0)] \end{aligned} \tag{15}$$

where $\mu = \alpha - (1+\theta) \cdot \alpha_1 + \theta \cdot a^0$, and $\chi = \beta - (1+\theta) \cdot \beta_1 + \theta \cdot \beta^0$. Let $\tilde{\eta}_j = \eta_j + \theta \cdot [(\alpha_1 - \alpha_1^0) + X_j(\beta_1 - \beta_1^0)]$, and $Z_{2j} = \{1, X_j, Ad_j, Ad_j^0\}$. We have the following moment condition:

$$E[\tilde{\eta}_j \mid Z_{2j}] = 0$$

that can be used to obtain consistent estimates of $\{\mu, \chi, \gamma, \theta \cdot \gamma^0\}$. The above two moment conditions can be used to recover consistent estimates of $\{(a^0 - \alpha_1), (\beta^0 - \beta_1)', \gamma^0;$ $(\alpha - a^0), (\beta - \beta^0)', \gamma, \theta\}$.

From the above models we can estimate the true and expected advertising effects γ and γ^0, respectively, but we can only recover the differences $(\beta - \beta_1)$ and $(\beta^0 - \beta_1)$. This is because advertising expenditures can be adjusted after $\xi_{1,j}$ is realized, while other performance attributes cannot. In a more general application in which we observe other time-varying choices, such as prices that are dynamically adjusted in response changing market conditions, we could recover their effects with expectations data.[9] However, for decisions that are fixed after a product is introduced into the market (e.g., product designs) only the difference between expected and actual effects can be estimated. However, suppose we followed standard assumptions in the previous literature that product attributes are exogenous and hence uncorrelated with the unobserved demand shocks. We could have directly estimated the impacts of product attributes on demand from equation (6), after obtaining the estimate for the advertising effect.

Our model is similar in many respects to the strategy used in Olley and Pakes (1996) and Levinsohn and Petrin (2003) that estimates the production function. These papers decompose the unobserved state variables in the production function into two components: first ω_t is the firm's unobserved productivity shocks in period *t*. Its expected value conditional on the past, $E[\omega_t|\omega_{t-1}]$, will affect capital input decisions k_t (analogous to ω^0 on performance attributes in our model), while its realized value will

[9] In our data of art performances, prices are fixed in the pre-season that cannot be changed. Therefore price coefficients cannot be recovered by using the expectation data.

affect the labor input decisions l_t (analogous to the impact of ($\omega^0 + \theta \cdot \xi_1^0$) on advertising decisions in our model). Another state variable η_t (similar to our η) has no effect on both k_t and l_t. Their estimation strategy, as the first step, is to use other inputs such as investment i_t or intermediate input ι_t in their data as an instrument, and invert ω_t as a non-parametric function of inputs except l_t. Conditional on the estimated labor coefficient obtained from the first step, their second step is to assume that ω_t follows a first-order Markov process, i.e., $\omega_t = E[\omega_t \mid \omega_{t-1}] + \xi_t$, where ξ_t is an unexpected productivity shock uncorrelated with k_t. Instead of instrumental variables, we use the manager's expectations data. We model how the manager updates her belief of the unobserved show attractiveness ω, after observing the first period ticket sales. This is similar to the second step in Olley and Pakes and Levinsohn and Petrin, except that we use the linear updating rule $E[\omega \mid \Omega_1] = \omega^0 + \theta \cdot \xi_1^0$. Though our method may be more restrictive than the non-parametric specification,[10] it does not require other potentially restrictive conditions in the model, e.g., the *monotonicity* condition that relates to the objective function of decision makers.[11] Our model also allows that the manager may not have perfect information and may make systematic errors.

III.B. Managerial Objectives and Advertising Choices

One of our main interests in this study is to understand the advertising decision making within the institution. To specify the objective function for advertising decision-making, we have to consider several general issues: First, as discussed before, the

[10] We could have used a non-parametric updating rule such as $E[\omega \mid \Omega_1] = g(\omega^0, \xi_1^0)$. We choose a more restrictive linear rule because of data limitation: we do not have as many data points as in their data; hence, estimator efficiency is critical.
[11] As a non-profit organization, the Center is very likely to have objectives other than profit maximization. A lot of research has been done related to the comparison of the objective functions between for- and non-profit organizations.

manager may make systematic errors due to limited information or experience. Second, the Center is a non-profit organization; hence, its objective may be different from pure profit-maximization. As stated in its mission statement, the Center "promotes an aesthetic of fusion and diversity" and non-mainstream or avant-garde artists. Therefore it is possible that the manager puts more emphasis on promoting avant-garde performances. Third, the fact that the manager is an agent implies that her objectives may not be entirely consistent with the Center. For example, she may have an incentive to see more stable ticket sales across shows. Finally, the manager has a marketing budget, hence she may want to avoid over- or under-spending on advertising in each year.

Our goal is to model the advertising decision making under a framework that is general enough to incorporate the above factors. We choose a reduced-form specification to approximate the manager's objective function: Suppose at time *t* the manager has to decide how to allocate the advertising budget for all shows scheduled after *t* within the season, based on her information set at *t*, Ω_t. We write down her objective function as the following:

$$\begin{aligned}\max_{\{Ad_j,\,j>t\}} V_t &\equiv \sum_{j>t} E[U_j \mid \Omega_t] \\ &= \sum_{j>t} \Big\{ p_j \cdot E[y_j \mid \Omega_t;\Theta^0] - Ad_j \\ &\quad + \psi_A \cdot Ad_j \\ &\quad + \psi_F \cdot E[y_j \mid \Omega_t;\Theta^0] \cdot \{F_j\} \\ &\quad + \psi_H \cdot E[y_j \mid \Omega_t;\Theta^0] \cdot \{H_j\} + \psi_L \cdot E[y_j \mid \Omega_t;\Theta^0] \cdot \{L_j\} \Big\} \\ &\quad + \psi_{B,t} \cdot (\sum_{j>t} Ad_j - B_t)\end{aligned} \tag{16}$$

where $j>t$ denotes all performances after time t, p_j is the price for performance j, $\Theta^0 = \{\alpha^0, \beta^0{}', \gamma^0, \theta, \sigma_2^2\}$ is the set of parameters related to the manager's beliefs,

$E[y_j | \Omega_t; \Theta^0]$ is her expected total ticket sales conditional on the information set Ω_t and Θ^0, B_t is the advertising budget remained in the season at time t, and $\{x\}$ is an indicator function which equals to 1 if x is true and 0 otherwise. The second line in (16) captures the profit maximization purpose. The coefficient ψ_A in the third line may capture other long term benefits due to advertising. For example, advertising may be an effective way of establishing brand name for the Center that helps to generate benefits other than ticket sales.[12] $\{F_j\}$ in the fourth line equals 1 if j is an avant-garde performance. If the manager puts more emphasis on selling tickets for avant-garde performances, we will have $\psi_F > 0$. $\{H_j\}$ and $\{L_j\}$ in the fifth line are indicator functions referring whether the first period ticket sales $y_{1,j}$ is much higher or lower than the expected ticket sales y_j^0, respectively. If the manager prefers more stable ticket sales, we will have $\psi_H < 0$ and $\psi_L > 0$. Finally, the last line in (16) captures the impact of the budget constraint on advertising decisions. We know that the Center allows, if necessary, that realized total advertising expenditures to be higher than the planned budget,[13] but an excessive over-spending over the remaining budget B_t at the end of season will put on some pressure on the manager that she may want to avoid, which is captured by the coefficient $\psi_{B,t}$. In model estimation we parameterize this coefficient as a function of how much has been over-spent for the previous shows within the season: let OB_t be the amount of total over-spending before t, we assume that $\psi_{B,t} = \psi_{0,B} + \psi_{1,B} \cdot OB_t$, where $\psi_{0,B}$ and $\psi_{1,B}$ are parameters to be estimated.

[12] For example, many art-performance theaters rely on donation as a major source of revenue. Advertising for various art performances is useful to impress prospective donors.
[13] Therefore the manager's problem is not the one with fixed advertising budget.

Let t be the time when the manager has to make advertising choice for a particular show j, when she has the information of the first period ticket sales, *i.e.*, $\Omega_t = \Omega_{1,j}$. Also let the updated expectation for the show attractiveness be $E[\omega_j \mid \Omega_{1,j}] \equiv \omega_j^1$, with variance $\mathrm{var}[\omega_j \mid \Omega_{1,j}] = \sigma_2^2$. We can write $E[y_j \mid \Omega_{1,j}; \Theta^0] = e^{\alpha^0 + \frac{\sigma_2^2}{2} + X_j\beta^0 + Ad_j\gamma^0 + \omega_j^1}$, then derive the first-order condition for the optimal level of advertising spending for j, Ad_j^*, in (16) as the following:[14]

$$
\begin{aligned}
&(p_j + \psi_F \cdot \{F_j\} + \psi_H \cdot \{H_j\} + \psi_L \cdot \{L_j\}) \cdot e^{\alpha^0 + \frac{\sigma_2^2}{2} + X_j\beta^0 + Ad_j^* \cdot (\gamma^0 - 1) + \omega_j^1} \cdot \gamma^0 - 1 + \psi_A + \psi_{0,B} + \psi_{1,B} \cdot OB_j = 0 \\
&\Rightarrow (\gamma^0 - 1) \cdot Ad_j^* = \ln(1 - \psi_A - \psi_{0,B} - \psi_{1,B} \cdot OB_j) - \ln(p_j + \psi_F \cdot \{F_j\} + \psi_H \cdot \{H_j\} + \psi_L \cdot \{L_j\}) - \ln \gamma^0 \\
&\qquad\qquad - (\alpha^0 + \frac{\sigma_2^2}{2} + X_j\beta^0 + \omega_j^1)
\end{aligned}
$$

We assume that the observed advertising spending is $Ad_j = Ad_j^* + \tilde{\varepsilon}_j$, where the stochastic component $\tilde{\varepsilon}_j$ is uncorrelated with show attributes X_j.[15] Then we have the following condition:

$$
\begin{aligned}
(\gamma^0 - 1) \cdot Ad_j &= \ln(1 - \psi_A - \psi_{0,B} - \psi_{1,B} \cdot OB_j) - \ln(p_j + \psi_F \cdot \{F_j\} + \psi_H \cdot \{H_j\} + \psi_L \cdot \{L_j\}) - \ln \gamma^0 \\
&\quad - (\alpha^0 + \frac{\sigma_2^2}{2} + X_j\beta^0 + \omega_j^1) + \varepsilon_j
\end{aligned} \tag{17}
$$

where $\varepsilon_j = (\gamma^0 - 1)\tilde{\varepsilon}_j$. Since ω_j^1 may be correlated with X_j and Ad_j, a simple regression of (17) will produce biased estimates.

As we have discussed,

$$\omega_j^1 = \omega_j^0 + (1+\theta) \cdot \xi_{1,j}^0 = \omega_j^0 + (1+\theta) \cdot [(\alpha_1 - \alpha_1^0) + X_j(\beta_1 - \beta_1^0) + \xi_{1,j}].$$

---

[14] We can write down the first-order condition for each $j>t$ in (16) separately since we assume no dynamic linkage between shows except from the budget constraint. If, for example, there is a spill-over advertising effect from one show to another, advertising for j will have an impact on future performances and our problem will become one of true dynamic optimization.

[15] This assumption is reasonable since the above expression for Ad_j^* has included the expected show attractiveness.

We can substitute these into (17) to obtain the following expression:

$$(\gamma^0-1)\cdot Ad_j=\ln(1-\psi_A-\psi_{0,B}-\psi_{1,B}\cdot OB_j)-\ln(p_j+\psi_F\cdot\{F_j\}+\psi_H\cdot\{H_j\}+\psi_L\cdot\{L_j\})-\ln\gamma^0$$
$$-(\alpha^0+\frac{\sigma_2^2}{2}+X_j\beta^0)-\omega_j^0-(1+\theta)\cdot[(\alpha_1-\alpha_1^0)+X_j(\beta_1-\beta_1^0)+\xi_{1,j}]+\varepsilon_j$$

We can invert ω_j^0 by making use of the expectations data. We can substitute $\omega_j^0=y_j^0-a^0-X_j\beta^0-\gamma^0\cdot Ad_j^0$ into the above equation and, after some algebraic manipulation, we have the following:

$$y_j^0+\gamma^0\cdot(Ad_j-Ad_j^0)-Ad_j+\ln\gamma^0=\ln(\tilde{\psi}-\psi_{1,B}\cdot OB_j)-\ln(p_j+\psi_F\cdot\{F_j\}+\psi_H\cdot\{H_j\}+\psi_L\cdot\{L_j\})$$
$$+\varphi-(1+\theta)\cdot X_j(\beta_1-\beta_1^0)+e_{1,j} \tag{18}$$

where $\tilde{\psi}=1-\psi_A-\psi_{0,B}$, $\varphi=(\frac{\sigma_0^2}{2}-\frac{\sigma_2^2}{2})+\frac{(\gamma^0)^2\cdot\sigma_{Ad^0}^2}{2}-(1+\theta)\cdot(\alpha_1-\alpha_1^0)$, and $e_{1,j}=-(1+\theta)\cdot\xi_{1j}+\varepsilon_j$, all variables same as previously defined. By assumption $e_{1,j}$ is uncorrelated with the variables on the right-hand side of (18).

We can also invert ω_j^0 from the true market demand function. We can substitute $\omega_j^0=y_j-\alpha-X_j\beta-Ad_j\cdot\gamma-\xi_{1,j}-\xi_{2,j}$ into the equation before (18) and, after some algebraic manipulation, we will have another regression equation:

$$y_j-(\gamma-\gamma^0)\cdot Ad_j-Ad_j+\ln\gamma^0-(\alpha-a^0)-X_j(\beta-\beta^0)$$
$$=\ln(\tilde{\psi}-\psi_{1,B}\cdot OB_j)-\ln(p_j+\psi_F\cdot\{F_j\}+\psi_H\cdot\{H_j\}+\psi_L\cdot\{L_j\})+\varphi-(1+\theta)\cdot X_j(\beta_1-\beta_1^0)+e_{2,j} \tag{19}$$

where φ is defined above, and $e_{2,j}=-\theta\cdot\xi_{1j}+\xi_{2j}+\varepsilon_j$. Again based on model assumptions $e_{2,j}$ is uncorrelated with the variables on the right-hand side of (19).

We plug in the estimates of {γ^0, $(\alpha-a^0)$, $(\beta-\beta^0)'$, γ, θ} from the first stage, and then estimate the non-linear simultaneous equation system (18) and (19). The set of parameters to be estimated is $\{\tilde{\psi}, \psi_{1,B}, \psi_F, \psi_H, \psi_L, \varphi, (\beta_1 - \beta_1^0)'\}$, which is related to the managerial advertising decision-making. We emphasize that without this structural objective function parameters γ, γ^0, and the errors in decision makers' prediction $(\beta - \beta^0)$ in the ticket sales function can still be consistently estimated.

IV. ESTIMATION RESULTS

We use the data from the 146 shows staged by the Center between 1997-1999 to estimate the economic model of managerial behavior described in Section III. Table 3 presents the results from Step 1 of the analysis in which we examine the relationship between managerial expectations and advertising effectiveness. As discussed above, we are unable to consistently estimate the impact of time-invariant show attributes including price on demand. However, we are able to estimate the extent to which the manager's beliefs concerning the effect of these attributes on demand deviates from their actual value. The first row of Panel A suggests that while the manager slightly underestimates the price elasticity of demand for Center performances, the difference is not statistically significant. Her period 0 expectations concerning the demand curve appear to be borne out by the actual data.

In contrast to the findings concerning price, the positive and significant coefficient for Avant-Garde (AG) suggests that the manager is over-optimistic concerning the appeal of this type of show in the Center's market. The magnitude of this effect is substantial; the manager initially believes that, holding prices and other attributes constant, AG shows will generate 25.9% more ticket sold than they actually do. The implication of this over-optimism is that the manager may eventually increase advertising expenditures for these shows after finding that ticket sales in period 1 are lower than her expectations. This may explain the large positive difference between actual and planned advertising expenditures in Table 1 for AG shows. Table 1 also shows that the ticket prices for AG and non-AG performances were approximately equal. If the manager had not been over-optimistic, she may have set lower ticket prices for AG shows in order to generate additional demand, assuming that attendance is price sensitive.

We are able to recover from our model both the manager's beliefs concerning the impact of advertising on demand, as well as an unbiased estimate of the true advertising elasticity. Panel B of Table 3 indicates that the manager believes that advertising generates additional demand. Moreover, the results suggest that the manager believes that the marginal effect of advertising is roughly the same for AG and non-AG shows, since the difference in the elasticities across show types is not statistically significant. These beliefs imply that the manager would be indifferent between spending an additional dollar advertising an AG vs. non-AG show if her objective was purely revenue maximization.

Panel C of Table 3 demonstrates the value of our approach in generating plausible estimates of the impact of advertising. While the simple OLS estimates in Table 2 indicated that advertising had a significant and negative impact on demand, we now find that a 10% increase in advertising expenditure generates 0.7-0.8% more tickets sold. The manager's expectations concerning advertising effectiveness is also consistent with actual effectiveness in two ways. First, the marginal effect of actual advertising effectiveness does not differ significantly across performance type. Second, although the manager appears to be slightly optimistic concerning the advertising effectiveness, the differences in the corresponding coefficients in Panels B and C are not statistically significant.[16] Consequently, while the manager is initially too optimistic concerning the appeal of AG shows relative to other genres, the expectations she has when choosing advertising levels for each performance at the end of Period 1 appear to be appropriate. The results also suggest that for the purpose of maximizing revenue for each show, the optimal level of advertising expenditures should be about 7-8% of ticket revenues, or 8-11% under the perceived advertising effectiveness of the manager. This is much lower than the actual advertising expenditures from the data, which is about 30% on average. Such difference perhaps implies that the benefits from advertising other than ticket sales such as attracting future donation are much higher than the impact of advertising on revenue for each particular show.

Panel D of Table 3 reports the estimates of the coefficient in the linear updating rule $E[\xi_{2,j}^0 \mid \Omega_{1,j}] = \theta \cdot \xi_{1,j}^0$. From equation (13), the result implies that, if the first period ticket sales are 10% higher than expectation, the manager would update her expectations of the total ticket sales by about 8% ($1+\theta$). We can also use this result to check with the various updating rules we discussed in section III. The negative estimate is inconsistent with the simple adaptive learning since under such rule θ has to be in the range of 0 and 1. Bayesian updating rule implies that θ is negative but larger than -1, which is consistent with our finding. Using the linear updating rule, our result implies that $\tilde{\sigma}_{12}$ is negative and is much smaller in magnitude compared with $\tilde{\sigma}_1^2$ (see our discussion above).

Table 4 presents the results from the structural estimation of the manager's objective function in advertising decision making. Suppose we assume that the effect of

[16] For example, the difference between expected and actual advertising effectiveness for non-AG shows reported in Panels B and C is -0.041 with a standard error of 0.030.

over-budgeting $\psi_{0,B}$ is insignificant from zero (this may be reasonable considering that other coefficients $\psi_{1D,B}$ and $\psi_{1S,B}$ are insignificantly different from zero and very small in magnitudes. Further, the Center adopts a soft budget rule.[17] The pressure of over-spending may be small from the manager's perspective.) The small estimate for Constant in the table suggests that the marginal benefits from advertising other than increasing ticket revenue is almost equal to the advertising cost itself, *i.e.*, ψ_A is almost equal to 1. Again this is consistent with our previous discussion about the importance of benefits other than ticket revenue for art performance theaters.

The estimate for the coefficient ψ_F, the additional benefit of increasing demand for avant-garde shows to the manager, is 11.5. Given that the average ticket price for a show is about $30. This shows that an additional ticket sold for avant-garde show has a marginal value to the manager at $30 + $11.5 = $41.5, or about 30% higher than an additional ticket sold for non avant-garde shows. The manager's objective function seems to be consistent with the Center's mission of promoting avant-garde or non-mainstream art performances. Note that the structural model is estimated conditional on the manager's expectations. As we discussed above, the manager is over-optimistic concerning the appeal of avant-garde shows hence she will over-spend on advertising avant-garde shows for the purpose of revenue maximization. This additional benefit of $11.5 will further increase the advertising. These two results together explain why actual advertising expenditures for avant-garde shows are on average much higher than that for non avant-garde shows (see Table 1).

[17] From personal communication with the Center staff.

In order to understand the magnitude of impacts on advertising decisions of the manager's over-optimism concerning the appeal of avant-garde shows and additional preference weight for increasing demand for avant-garde shows, we conduct a counterfactual policy experiment. We fixed the total advertising expenditures for each season from 97-99 at the same level as in the data. Then we compute the optimal advertising expenditure for each show based on the estimates of the managerial objective function. Next we assume that the manager corrects her biased belief concerning the appeal of avant-garde shows, i.e., the coefficient for 'Avant-Garde" in Table 3 is equal to 0, and compute the optimal advertising expenditure for each show again. Finally we assume that the manager also does not have special preference for increasing demand for avant-garde shows, i.e., the coefficient ψ_F in Table 4 is equal to 0, and compute the optimal advertising expenditures. We find that advertising expenditure for avant-garde shows will be cut by 15% had the manager corrected her biased belief concerning the appeal of avant-garde shows, while advertising expenditure for other types of shows will be increased by 11%. Had the manager also does not have special preference for avant-garde shows in her objective function, advertising expenditure for avant-garde will be further cut by 17%, while that for other types of shows further increased by 13%. These results show that biased beliefs and special preference for avant-garde shows have a significant impact on the manager's advertising decision making.

Finally, the estimate for ψ_L, the additional benefit of increasing ticket sales for the manager if ticket sales in period 1 are much lower than expectations, suggests that the manager does not prefer to see extreme low audience for each show and will use advertising to stimulate demand if period 1 sales are too low. Given that the average ticket price for a show is about $30, an additional ticket sold for shows which period 1 ticket sales lag behind expectations brings a marginal value to the manager at $30 + $18.7 = $48.7.

The two non-linear equations (18) and (19) are based on different data: equation (18) has the expected total demand y_j^0 while equation (19) has the actual total demand y_j on the left side. This suggests that we can perform some sort of over-identification tests to test our model specifications. The F-test statistic 0.36 shows that we cannot reject the null hypothesis that our model is correctly specified.

V. CONCLUSION

In this paper we develop an econometric framework that combines observed market data with self-reported, subjective expectations data to jointly estimate demand and objective functions to assess managerial choices. Our methodology combining objective and subjective data addresses four critical issues of great concern in the application of many structural econometric models: First, endogeneity issues that arise when almost *all* product attributes and managerial choices are correlated with unobserved (to the researcher) product quality. Second, the decision-maker may be uncertain of true product quality, and may even be biased in her beliefs regarding the appeal of certain product attributes. Third, she may also have biased beliefs concerning the outcomes associated with her actions. Finally, the manager may have objectives other than pure static profit maximization that guide her choices. As a result, we are able to relax some strong behavioral assumptions, such as rational expectations, that are typically embedded in the structural modelling approach.

We apply our methodology to the analysis of the advertising decisions of the marketing manager of a large university performing arts center. Our findings highlight the value of our approach. OLS estimates of the impact of advertising (and prices) on demand yield nonsensical results: increased advertising lowers tickets sales. Moreover, the manager spends substantially more advertising “avant-garde” shows despite low demand for them. The self-reported data on the manager’s expectations for ticket sales of each show help us to recover her prior beliefs regarding the appeal of each performance. We incorporate these beliefs into a learning model that allows us to obtain unbiased estimates of the true impact of advertising on demand, and to test whether the manager’s beliefs regarding this relationship coincide with actual outcomes. While we find that the manager’s beliefs concerning advertising effectiveness are unbiased, it appears that the manager has biased beliefs for particular product attributes, namely, her belief that avant-garde shows are significantly more appealing to the public than they actually are. Incorporating these beliefs into the estimation of the manager’s objective function, we find that the manager departs from pure static profit maximization by exhibiting special preference for promoting avant-garde shows. Additional attendance at an avant-garde show generates 30% more utility than at a “traditional” performance. This finding coincides with the stated mission of the arts center for bringing avant-garde performance to the local community.

Although the advertising decisions at a performing arts center may not be of particular interest, we emphasize that the approach taken in this paper may be used in a wide range of applications. For example, empirical researchers may access to the consumer expectations of future income, health, or education through surveys, or to firm expectations of future sales, market share growth, and profitability through company financial reports. Use of such subjective data in the context of a well developed empirical model of behaviour may allow researchers to address some of the shortcomings that have limited the application and impact of structural econometric models.

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TABLE 1
Summary Statistics per Show, Overall and by Genre

Variable	Show Type		
	All	Not Avant-garde	Avant-garde
Advertising $ (actual)	$5,654 (2798)	$5127 (2557)	$6,495 (2971)
Advertising $ (expected)	$5,587 (1747)	$5619 (1575)	$5,536 (1999)
Price	$30.26 (8.07)	$30.49 (9.27)	29.89 (5.68)
# Performances	2.39 (2.15)	1.49 (0.99)	3.83 (2.67)
Series 1	0.05 (0.21)	0.05 (0.22)	0.04 (0.19)
Series 2	0.72 (0.45)	0.81 (0.39)	0.56 (0.50)
Genre - Traditional	0.44 (0.50)	0.68 (0.47)	0
Genre – Avant-Garde	0.39 (0.49)	0	1
Small Venue	0.33 (0.47)	0.34 (0.48)	0.31 (0.47)
Large Venue	0.60 (0.49)	0.62 (0.49)	0.57 (0.50)
Weekend	0.62 (0.49)	0.59 (0.49)	0.67 (0.47)
Daytime	0.04 (0.19)	0.04 (0.19)	0.04 (0.19)
Mid-Year	0.32 (0.47)	0.37 (0.48)	0.25 (0.44)
Late Year	0.18 (0.38)	0.18 (0.38)	0.18 (0.39)
Year 1998	0.31 (0.46)	0.36 (0.48)	0.23 (0.42)
Year 1999	0.21 (0.41)	0.23 (0.42)	0.17 (0.37)
N	146	112	34

Note: Standard deviations are in parentheses.

TABLE 2
OLS ESTIMATES OF DETERMINANTS OF TICKETS SOLD
(Dependent Variable is ln(Tickets Sold))

Variable	(1)	(2)
ln(Advertising $)	-0.122 (0.045)	
ln(Advertising $)*Avant-Garde		-0.064 (0.076)
ln(Advertising $)*not Avant-Garde		-0.146 (0.052)
ln(Price)	0.343 (0.182)	0.324 (0.183)
Avant-Garde	-0.101 (0.086)	-0.796 (0.731)
Traditional	-0.053 (0.091)	-0.057 (0.092)
Series 1	0.416 (0.175)	0.417 (0.175)
Series 2	0.200 (0.089)	0.193 (0.089)
Small Venue	-0.676 (0.151)	-0.664 (0.151)
Large Venue	0.403 (0.138)	0.403 (0.138)
Daytime	0.115 (0.169)	0.099 (0.170)
Weekend	0.026 (0.066)	0.025 (0.066)
Mid-Year	-0.017 (0.074)	-0.012 (0.074)
Late Year	0.057 (0.094)	0.049 (0.094)
Year 98	0.173 (0.078)	0.167 (0.078)
Year 99	0.088 (0.090)	0.086 (0.090)
R^2	0.585	0.584

Note: Standard errors in parentheses. Model also includes a constant.

TABLE 3 MANAGERIAL EXPECTATIONS AND ADVERTISING EFFECTS	
Variable	
Panel A: Deviation of Manger's Expectations from Actual Impact of Selected Show Characteristics on Demand	
ln(Price)	-0.020 (0.085)
Avant-Garde	0.259 (0.043)
Traditional	-0.039 (0.031)
Series 1	-0.167 (0.086)
Series 2	-0.078 (0.034)
Small Venue	-0.328 (0.028)
Large Venue	-0.158 (0.031)
Daytime	-0.139 (0.065)
Weekend	0.042 (0.035)
Mid-Year	-0.013 (0.022)
Late Year	-0.022 (0.024)
Year 98	0.056 (0.025)
Year 99	-0.0004 (0.023)
Panel B: Manager's Beliefs Concerning Advertising Effectiveness (Elasticity)	
Avant-Garde Shows	0.082 (0.037)
Non Avant-Garde Shows	0.108 (0.032)
Panel C: Actual Advertising Effectiveness (Elasticity)	
Avant-Garde Shows (elasticity)	0.076 (0.016)
Non Avant-Garde Shows (elasticity)	0.067 (0.014)
Panel D: Period 1 to Period 2 Updating Parameter	
θ	-0.156 (0.049)

Note: Difference between beliefs and actual ad effectiveness for non AG shows is -0.041 with SE = 0.030

TABLE 4 STRUCTURAL ESTIMATES OF MANAGERIAL UTILITY PARAMETERS	
Variable	
Constant (1 - ψ_A - $\psi_{0,B}$)	0.010 (0.006)
Avant-Garde Show (ψ_F)	11.464 (4.244)
ln(Budget Deficit) ($\psi_{1D,B}$)	-0.0001 (0.002)
ln(Budget Surplus) ($\psi_{1S,B}$)	0.0001 (0.0002)
High Period 1 Sales (ψ_H)	1.343 (3.105)
Low Period 1 Sales (ψ_L)	18.670 (4.801)
Criterion Function Value	569.5
Over-Identification F-Test	0.358

Note: Standard errors in parentheses.

FIGURE 1
TICKETS SOLD PER SHOW - ACTUAL vs. MANAGER'S EXPECTATION

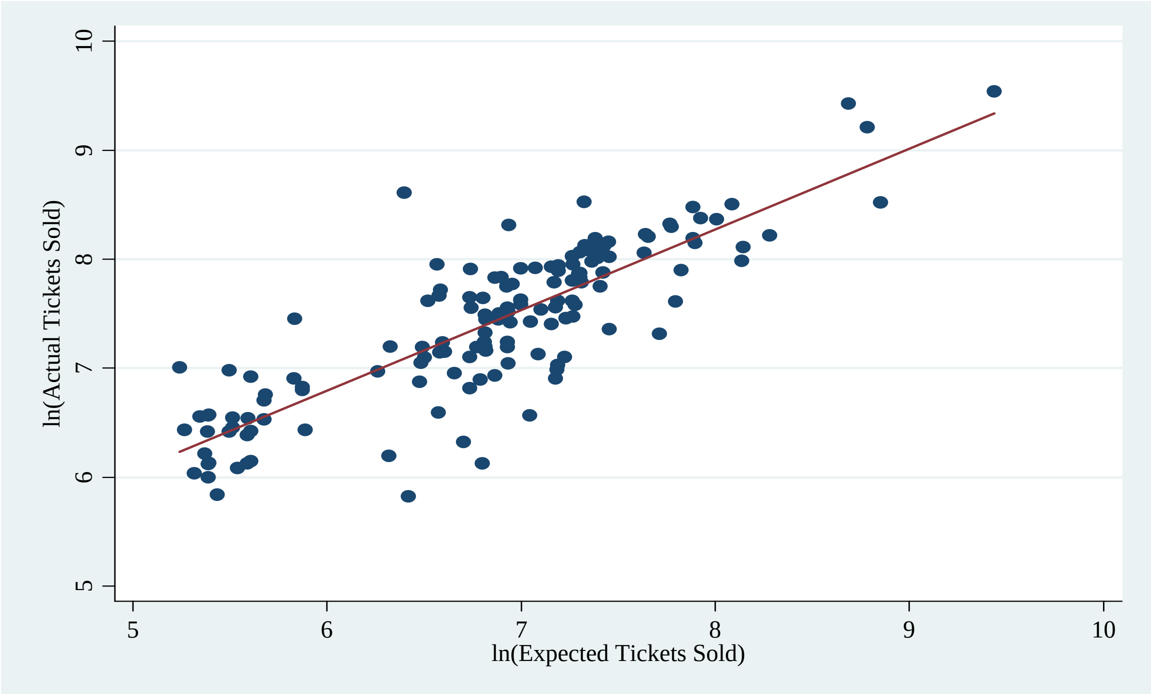

FIGURE 2
PERIOD 2 vs. PERIOD 1 TICKET SALES