Outsourcing, Information Leakage and Consulting Firms

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Web Appendix

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1 Proofs of Section 3

Proof of Proposition 1 Let $\underline{c}_j$ be the best technology learned by the monopolistic contractor, and $n-\gamma$ be the measure of firms that did not invest in technology. To show the first part of the claim we need to show that $c_j^* = \underline{c}_j$. The demand for technology is given by

$$\phi(c_j, \beta) = A(c_j, \beta) \left\{ c_j^{-\frac{\alpha}{1-\alpha}} - \left[\frac{\sigma}{n-\gamma-\beta} \underline{c}_j^{-\frac{\alpha}{1-\alpha}} + \frac{n-\gamma-\beta-\sigma}{n-\gamma-\beta} \right] \right\}$$

where$\underline{c}_j$ is the technology spilling from the monopolistic contractor to a measure σ of firms and

$$A(c_j, \beta) = \frac{W(1-\alpha)}{(\gamma+\sigma)\underline{c}_j^{\frac{-\alpha}{1-\alpha}} + \beta c_j^{\frac{-\alpha}{1-\alpha}} + (n-\gamma-\sigma-\beta)}$$

Notice that the monopolist of information, contractor j, has to solve the problem (**??**). For a given $\beta \in [0, n-\gamma-\sigma]$, observe that the problem becomes

$$\max_{c_j \geq \underline{c}_j} \frac{\left\{ c_j^{\frac{-\alpha}{1-\alpha}} - \left[\frac{\sigma}{n-\gamma-\beta} \underline{c}_j^{\frac{-\alpha}{1-\alpha}} + \frac{n-\gamma-\beta-\sigma}{n-\gamma-\beta} \right] \right\}}{(\gamma+\sigma)\underline{c}_j^{\frac{-\alpha}{1-\alpha}} + \beta c_j^{\frac{-\alpha}{1-\alpha}} + (n-\gamma-\sigma-\beta)}$$

The derivative of the objective function is

$$\frac{\left(\frac{-\alpha}{1-\alpha}\right) c_j^{\frac{-1}{1-\alpha}} \left[\left(\gamma+\sigma+\frac{\sigma\beta}{n-\gamma-\beta}\right) \underline{c}_j^{\frac{-\alpha}{1-\alpha}} + \frac{(n-\gamma)(n-\gamma-\sigma-\beta)}{n-\gamma-\beta} \right]}{\left[(\gamma+\sigma)\underline{c}_j^{\frac{-\alpha}{1-\alpha}} + \beta c_j^{\frac{-\alpha}{1-\alpha}} + (n-\gamma-\sigma-\beta) \right]^2} < 0$$

As the derivative is always negative, we have $c_j^* = \underline{c}_j$.

To prove the second part of the claim, observe that, if $c_j^* = \underline{c}_j$ for any given β, we have

$$\phi(\beta) = A(\underline{c}_j, \beta) \frac{n-\gamma-\beta-\sigma}{n-\gamma-\beta} \left\{ \underline{c}_j^{\frac{-\alpha}{1-\alpha}} - 1 \right\}$$

where

$$A(\underline{c}_j, \beta) = \frac{W(1-\alpha)}{(\gamma+\sigma+\beta)\underline{c}_j^{\frac{-\alpha}{1-\alpha}} + (n-\gamma-\sigma-\beta)}$$

However, since $\left[\underline{c}_j^{\frac{-\alpha}{1-\alpha}}-1\right]$ is a constant at the time the monopolist solves the profit maximization problem, problem (3) (in the paper) is equivalent to

$$\max_{\beta\in[0,n-\gamma-s]} \frac{\beta^{\frac{n-\gamma-\beta-s}{n-\gamma-\beta}}}{(\gamma+s+\beta)\left[\underline{c}_j^{\frac{-\alpha}{1-\alpha}}-1\right]+n} \tag{1}$$

It is easy to check that the second order condition of problem (1) is satisfied. Let us define $\Phi(\beta)\equiv\phi'(\beta)\beta+\phi(\beta)$. As the second order conditions of (1) is satisfied, $\Phi'(\beta)<0$. Then, the monopolist chooses β^* such that $\Phi(\beta^*)=0$. As from the definition of $\phi(\beta)$ it is easy to check that $\phi(n-\gamma-s)=0$, we have that $\Phi(n-\gamma-s)<0$, thus $\beta^*<n-\gamma-s$ ■

Proof of Corollary 2 If $s=0$, since we still have that, by Proposition 1, $c^*=\underline{c}$, the demand for information becomes

$$\varphi(\beta)=\frac{W(1-\alpha)\left[c^{\frac{-\alpha}{1-\alpha}}-1\right]}{(\gamma+\beta)\left[\underline{c}^{\frac{-\alpha}{1-\alpha}}-1\right]+n}$$

The monopolist has to maximize the revenue $\varphi(\beta)\beta$. However, notice that $\varphi'(\beta)\beta+\varphi(\beta)>0$ for any β. In fact, $\varphi'(\beta)\beta+\varphi(\beta)>0$ if

$$-\frac{\left[\underline{c}^{\frac{-\alpha}{1-\alpha}}-1\right]\beta}{(\gamma+\beta)\left[\underline{c}^{\frac{-\alpha}{1-\alpha}}-1\right]+n}+1>0$$

which is equivalent to

$$\beta<\frac{(\gamma+\beta)\left[\underline{c}^{\frac{-\alpha}{1-\alpha}}-1\right]+n}{\left[\underline{c}^{\frac{-\alpha}{1-\alpha}}-1\right]}$$

which is always satisfied. This guarantees that $\beta^*=n-\gamma$ ■

Proof of Proposition 7 Let us show first that if $s=0$ it is impossible to have a monopolistic market for information in equilibrium. Let γ be the measure of investing firms and β be the measure of firms that buy the information from a contractor j. The willingness to pay of a non-investing firm for a technology c reachable with investment k is

$$\phi(c,\beta) = \frac{W(1-\alpha)}{(\gamma+\beta)\left[\underline{c}^{\frac{-\alpha}{1-\alpha}}-1\right]+n}\left[(1+k)^{\frac{\alpha\rho}{1-\alpha}}-1\right]$$

where $\underline{c}$ is the level of technology adopted by the investing firms and the other buying firms and reachable with the investment $\overline{k}$. In equilibrium any investing firms invests k such that

$$\frac{W\alpha\rho}{(\gamma+\beta)\left[\left(1+\overline{k}\right)^{\frac{\alpha\rho}{1-\alpha}}-1\right]+n}(1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} = 1 \tag{2}$$

Notice that in equilibrium all the investing firms adopt the same technology. This implies that $k = \overline{k}$. Keeping $\overline{k}$ constant, it is possible to integrate the LHS of (2) with respect to k between 0 and $\overline{k}$. We have

$$\begin{aligned}
\varphi\left(\beta,\overline{k}\right) &= \frac{W(1-\alpha)\left[\left(1+\overline{k}\right)^{\frac{\alpha\rho}{1-\alpha}}-1\right]}{(\gamma+\beta)\left[\left(1+\overline{k}\right)^{\frac{\alpha\rho}{1-\alpha}}-1\right]+n} \\
&= \int_0^{\overline{k}} \frac{E\alpha\rho}{(\gamma+\beta)\left[\left(1+\overline{k}\right)^{\frac{\alpha\rho}{1-\alpha}}-1\right]+n}(1+v)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\,dv \\
&> \overline{k}
\end{aligned}$$

where the last inequality is guaranteed by identity (2), by $k = \overline{k}$, and by the fact that the LHS of (2) is decreasing in k (as guaranteed by Assumption 1). This guarantees that if $s = 0$ investing in technology always strictly dominates buying a technology from a monopolistic contractor.■

2 Proofs of Section 4

Proof of Lemma 8 From the profit definition (1) in the paper, denoting by $\pi(i)$ the *economic* profit of a firm, we have

$$\pi(i) = \frac{W(1-\alpha)}{\int_{\mathcal{N}} c(j)^{\frac{-\alpha}{1-\alpha}} dj} c(i)^{\frac{-\alpha}{1-\alpha}}$$
$$= \frac{W(1-\alpha)}{\int_{\mathcal{N}} (1+k(j))^{\frac{\alpha\rho}{1-\alpha}} dj} (1+k(i))^{\frac{\alpha\rho}{1-\alpha}}$$

From now on, let

$$A \equiv \frac{W(1-\alpha)}{\int_{\mathcal{N}} (1+k(j))^{\frac{\alpha\rho}{1-\alpha}} dj}$$

Consider first the case in which the set of outsourcing firms, H, includes at least two firms, and the set of non-outsourcing firms, $\mathcal{N}\backslash H$ is non-empty. Notice that in the set H there can be at most one firm investing in technology. Indeed, a second firm would have a profitable deviation in not investing and learning the technology through the spill. Now, is it possible to have an equilibrium with exactly one investing and outsourcing firm? Suppose it is, and let $k_H > 0$ be the investment of such firm, say firm i. Notice that that each not-investing firm in H must be at least as well off as each firm in $\mathcal{N}\backslash H$, since if this is not the case, that firm would be better off following the strategy of the firm in $\mathcal{N}\backslash H$ (notice that these firms' behavior does not have any significant influence on A). Notice also that each firm in $\mathcal{N}\backslash H$ must be at least as well off as any non-investing firm in H, since if the opposite is true, it would have a profitable deviation in outsourcing and not investing. If only one firm i in H invests $k_H > 0$, i has to be worse off than the others firms in H, which we just claimed are at least as well off as the not outsourcing firms. Firm i is then worse off than the ones not outsourcing, so it would have a profitable deviation by following their strategy. Indeed, if we denote by $k_{\mathcal{N}\backslash H}$ the optimal investment of the not-outsourcing firms, we have

$$\begin{aligned} A_L (1+k_H)^{\frac{\alpha\rho}{1-\alpha}} - k_H &< A_L (1+k_H)^{\frac{\alpha\rho}{1-\alpha}} \\ &= A_L \left(1+k_{\mathcal{N}\backslash H}\right)^{\frac{\alpha\rho}{1-\alpha}} - k_{\mathcal{N}\backslash H} - t \\ &< A_{NL} \left(1+k_{\mathcal{N}\backslash H}\right)^{\frac{\alpha\rho}{1-\alpha}} - k_{\mathcal{N}\backslash H} - t \end{aligned}$$

where γ is the measure of the set of investing firms, A_L is defined as

$$A_L \equiv \frac{W(1-\alpha)}{(n-\gamma)(1+k_H)^{\frac{\alpha\rho}{1-\alpha}} + \gamma\left(1+k_{\mathcal{N}\backslash H}\right)^{\frac{\alpha\rho}{1-\alpha}}}$$

and A_{NL} as

$$A_{NL} \equiv \frac{W(1-\alpha)}{n + \gamma\left(1+k_{\mathcal{N}\backslash H}\right)^{\frac{\alpha\rho}{1-\alpha}}} > A_L$$

By avoiding the leakage, and then increasing the demand for all firms (higher A), firm i would improve everybody's profits in the set $\mathcal{N}\backslash H$, and a fortiori, it would be better off than by outsourcing and investing k_H.

In the case in which there is only one outsourcing firm, if this firm invests, the investment has to be at the same level of all other non-outsourcing firms (as they solve the same maximization problem). Then, one non-outsourcing (and investing) firm has a profitable deviation in outsourcing and not investing. Finally, if all firms outsource, at most one firm can invest in equilibrium k_H (as all the others would free-ride on that investment). However, this implies

$$\frac{W(1-\alpha)}{n(1+k_H)^{\frac{\alpha\rho}{1-\alpha}}}(1+k_H)^{\frac{\alpha\rho}{1-\alpha}} - k_H < \frac{E(1-\alpha)}{n}$$

i.e., the investing firm is completely expropriated from its investment (i.e., its economic profit is the same it would get without investment). Thus, the firm has a profitable deviation in not investing ■

Proof of Proposition 9 (i) The profit of a not-outsourcing firm is

$$\pi(i) = A(1+k(i))^{\frac{\alpha\rho}{1-\alpha}} - k(i) - t$$

The first order condition of the optimal investment problem is

$$A\frac{\alpha\rho}{1-\alpha}(1+k(i))^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} - 1 = 0 \tag{3}$$

which is such that $\frac{dk(i)}{dA} > 0$. If everybody else outsource, A is the maximum possible, i.e. $\overline{A} = \frac{W(1-\alpha)}{n}$, which implies that $\overline{k} \equiv \left(\frac{n}{W\alpha\rho}\right)^{\frac{1-\alpha}{\alpha\rho-1+\alpha}} - 1$ is the maximum possible investment. If

$$\frac{W}{n}(1-\alpha)\left(1+\overline{k}\right)^{\frac{\alpha\rho}{1-\alpha}}-\overline{k}-t\leq\frac{W}{n}(1-\alpha)$$

or

$$t\geq\left(\frac{n}{W\alpha\rho}\right)^{\frac{1-\alpha}{\alpha\rho-1+\alpha}}\left(\frac{1-\alpha}{\alpha\rho}-1\right)-\frac{W}{n}(1-\alpha)+1\equiv\overline{T}$$

then the firms strictly prefers outsourcing and not investing rather than not outsourcing and investing. Since the profit of a not outsourcing firm is increasing in A, if the maximum possible A does not refrain a firm to contract, any lower A, corresponding to different strategies chosen by the other firms do not refrain such firm to contract either, so that the equilibrium is unique.

(ii) Let $t<\overline{T}$. *First step*: Let us first show that there cannot be an equilibrium in which two firms invests two strictly positive but different amounts $k'\neq k''$. To see this, suppose that there exists an equilibrium in which this is the case. Let $A'\equiv\frac{W(1-\alpha)}{\int_{\mathcal{N}}c(k(j))^{-\frac{\alpha}{1-\alpha}}dj}$ in this equilibrium. Observe that, but Lemma 1, it must be the case that the two firms are both not outsourcing. But then, at least one would have a profitable deviation in investing k^* uniquely defined as

$$k^*\equiv\arg\max_{k\in R_+}A'(1+k)^{\frac{\alpha\rho}{1-\alpha}}-k$$

Second step: Recall from (i) that in this case, if all (or all but a zero-measured set of) the firms outsource, there is a profitable deviation in not outsourcing and investing $\overline{k}$. Then, in equilibrium it must be the case that a positive measured set of firms invest and do not outsource (i.e., $\gamma<n$). On the other hand, notice that for any A there is a unique level of $k(i)$ which satisfies (3), and since all the not-outsourcing firms face the same A, they must invest the same in R&D. This implies that in equilibrium a positive measured set of firms outsources ($\gamma>0$). In fact, if $\gamma=0$, they would all invest the same k in R&D. In this case, one non-outsourcing firm would be better off outsourcing because it would gain t and the leakage cannot lower its profit (not having any impact on A). This implies that in the only possible equilibrium left there must be some positive measure $\gamma\in(0,n)$ of firms not outsourcing and a positive measure $n-\gamma$ of outsourcing firms. By Lemma 1, we have that there is no $i\in H$ such that $c(i)<\overline{c}=1$, so in equilibrium it must be the case that $c(i) = \overline{c} = 1$ for all $i \in H$.

The considerations made so far allow us to write A as a function of the measure of investing firms, γ, i.e.,

$$\begin{aligned} A(\gamma) &\equiv \frac{W(1-\alpha)}{(n-\gamma) + \int_{\mathcal{N}/H} c\left(k^*(j)\right)^{-\frac{\alpha}{1-\alpha}} dj} \\ &= \frac{W(1-\alpha)}{(n-\gamma) + \gamma\left(1 + k_{-i}\right)^{\frac{\alpha\rho}{1-\alpha}}} \end{aligned}$$

where the fact that, by symmetry, all the not-outsourcing firm invest the same amount in R&D guarantees the second equality. Then, for a given γ to find the equilibrium $k(i)$ for $i \in \mathcal{N}/H$, we need to find the solution $k_{-i} = k(i)$ of

$$\frac{W(1-\alpha)}{(n-\gamma) + \gamma\left(1 + k_{-i}\right)^{\frac{\alpha\rho}{1-\alpha}}}\left(1 + k(i)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\left(\frac{\alpha\rho}{1-\alpha}\right) - 1 = 0$$

or

$$\frac{W\alpha\rho}{(n-\gamma) + \gamma\left(1 + k_{-i}\right)^{\frac{\alpha\rho}{1-\alpha}}}\left(1 + k(i)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} - 1 = 0$$

Such solution $k_{-i} = k(i) = k^*(\gamma)$ is unique for each γ since

$$\frac{dk(i)}{dk_{-i}} = -\frac{\frac{\partial A}{\partial k(\gamma)} c\left(k(i)\right)^{-\frac{1}{1-\alpha}}\left(-\frac{\alpha}{1-\alpha}\right) c'\left(k(i)\right)}{\frac{d^2\pi(i)}{dk(i)^2}} < 0$$

Now, the equilibrium condition that outsourcing and not outsourcing firms must have the same payoff allow us to determine the equilibrium measure γ. Recall that if we denote by $\pi_H(\gamma)$ the profit of an outsourcing firm, we have

$$\pi_H(\gamma) = A(\gamma) = \frac{W(1-\alpha)}{(n-\gamma) + \gamma\left(1 + k^*(\gamma)\right)^{\frac{\alpha\rho}{1-\alpha}}}$$

On the other hand, if $\pi_{\mathcal{N}\setminus H}(\gamma)$ denotes the profit of a not-outsourcing firm, we have

$$\begin{aligned} \pi_{\mathcal{N}\setminus H}(\gamma) &= A(\gamma)\left(1 + k^*(\gamma)\right)^{\frac{\alpha\rho}{1-\alpha}} - k^*(\gamma) - t \\ &= \frac{W(1-\alpha)}{(n-\gamma) + \gamma\left(1 + k^*(\gamma)\right)^{\frac{\alpha\rho}{1-\alpha}}}\left(1 + k^*(\gamma)\right)^{\frac{\alpha\rho}{1-\alpha}} - k^*(\gamma) - t \end{aligned}$$

The equilibrium γ is a solution of the equation $\pi_H(\gamma) = \pi_{\mathcal{N}\backslash H}(\gamma)$. For $\gamma = 0$, we have that, as $t \leq \overline{T}$, $\pi_H(0) < \pi_{\mathcal{N}\backslash H}(0)$. On the other hand, notice that for $\gamma = n$, we have $k^*(\gamma) = \frac{W\alpha\rho}{n} - 1$, and $A(n) = (1-\alpha)\left(\frac{W}{n}\right)^{\frac{-\alpha\rho+1-\alpha}{1-\alpha}} (\alpha\rho)^{-\frac{\alpha\rho}{1-\alpha}}$. This implies

$$\pi_H(n) = (1-\alpha)\left(\frac{W}{n}\right)^{\frac{-\alpha\rho+1-\alpha}{1-\alpha}} (\alpha\rho)^{-\frac{\alpha\rho}{1-\alpha}}$$

and

$$\pi_{\mathcal{N}\backslash H}(n) = (1-\alpha)\left(\frac{W}{n}\right)^{\frac{-\alpha\rho+1-\alpha}{1-\alpha}} (\alpha\rho)^{-\frac{\alpha\rho}{1-\alpha}} \left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha}} - \frac{W\alpha\rho}{n} + 1 - t$$

This implies that $\pi_H(n) \geq \pi_{\mathcal{N}\backslash H}(n)$ if and only if

$$t \geq \frac{W}{n}\left[1-\alpha-\alpha\rho-\left(\frac{W\alpha\rho}{n}\right)^{-\frac{\alpha\rho}{1-\alpha}}(1-\alpha)\right] + 1 \equiv \underline{T}$$

Let us now show that there is a unique γ^* satisfying the condition $\pi_H(\gamma) = \pi_{\mathcal{N}\backslash H}(\gamma)$, and thus there is a unique equilibrium for $t \in \left[\underline{T}, \overline{T}\right]$. The proof of uniqueness consists of two steps: (a) I first show that $\frac{\partial A(\gamma)}{\partial\gamma} < 0$ for all γ, (b) then I show that $\frac{\partial A(\gamma)}{\partial\gamma} < 0$ for all γ implies uniqueness.

(a) To show that $\frac{\partial A(\gamma)}{\partial\gamma} < 0$ for all γ, let us first compute the derivative $\frac{\partial k^*(\gamma)}{\partial\gamma}$. Recall that from (3), we have

$$\frac{W\alpha\rho}{(n-\gamma)+\gamma(1+k^*)^{\frac{\alpha\rho}{1-\alpha}}}(1+k^*)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} - 1 = 0$$

which implies

$$\frac{\partial k^*(\gamma)}{\partial\gamma} = \frac{\left[(1+k^*(\gamma))^{\frac{\alpha\rho}{1-\alpha}} - 1\right]}{\frac{\alpha\rho-1+\alpha}{(1-\alpha)(1+k^*(\gamma))}\left\{n+\gamma\left[(1+k^*(\gamma))^{\frac{\alpha\rho}{1-\alpha}}-1\right]\right\} - \frac{\alpha\rho\gamma}{1-\alpha}(1+k^*(\gamma))^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}} \tag{4}$$

Now, we have that

$$\frac{\partial A(\gamma)}{\partial\gamma} = -W(1-\alpha)\frac{\left[(1+k^*(\gamma))^{\frac{\alpha\rho}{1-\alpha}}-1\right] + \frac{\gamma\alpha\rho}{1-\alpha}(1+k^*(\gamma))^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\frac{\partial k^*(\gamma)}{\partial\gamma}}{\left[n+\gamma\left[(1+k^*(\gamma))^{\frac{\alpha\rho}{1-\alpha}}-1\right]\right]^2}$$

which implies that $\frac{\partial A(\gamma)}{\partial\gamma} < 0$ if and only if

$$\left[(1+k^*(\gamma))^{\frac{\alpha\rho}{1-\alpha}}-1\right]+\frac{\gamma\alpha\rho}{1-\alpha}\left(1+k^*(\gamma)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\frac{\partial k^*(\gamma)}{\partial\gamma}>0 \tag{5}$$

By plugging (4) into (5) and after some manipulations, one can see that condition (5) reduces to

$$\begin{aligned}\frac{\gamma\alpha\rho}{1-\alpha}\left(1+k^*(\gamma)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} &< \frac{\gamma\alpha\rho}{1-\alpha}\left(1+k^*(\gamma)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} \\ &\quad+\frac{1-\alpha-\alpha\rho}{1-\alpha}\left[n+\gamma\left[(1+k^*(\gamma))^{\frac{\alpha\rho}{1-\alpha}}-1\right]\right]\end{aligned}$$

which is satisfied because, by Assumption 1, we have $\frac{1-\alpha-\alpha\rho}{1-\alpha}>0$.

(b) Since $\frac{\partial A(\gamma)}{\partial\gamma}<0$, and $t\in\left[\underline{T},\overline{T}\right]$, there is a unique SP equilibrium in which $n-\gamma$ firms contract and do not invest and γ firms do not contract and invest. To see this, notice that for any γ it must be

$$\begin{aligned}\frac{\partial\pi_{\mathcal{N}\setminus H}(\gamma)}{\partial\gamma} &= \frac{\partial A(\gamma)}{\partial\gamma}\left(1+k(\gamma)\right)^{\frac{\alpha\rho}{1-\alpha}}+ \\ &\quad+\left[A\left(1+k(\gamma)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\frac{\alpha\rho}{1-\alpha}-1\right]\frac{\partial k(\gamma)}{\partial\gamma} \\ &= \frac{\partial A(\gamma)}{\partial\gamma}\left(1+k(\gamma)\right)^{\frac{\alpha\rho}{1-\alpha}} \\ &< \frac{\partial\pi_H(\gamma)}{\partial\gamma}<0\end{aligned}$$

Since $\pi_H(n)<\pi_{\mathcal{N}\setminus H}(n)$, there must be a unique $\gamma\in[0,n]$ such that $\pi_H(\gamma)=\pi_{\mathcal{N}\setminus H}(\gamma)$.

The last thing left to show is that $\underline{T}<\overline{T}$. To see that, recall that $A(n)<A(0)$ and notice that

$$\frac{\alpha\rho}{1-\alpha}A(n)\left(1+k\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}-k<\frac{\alpha\rho}{1-\alpha}A(0)\left(1+k\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}-k \tag{6}$$

for any $k\in R_+$. Equation (6), since $k(n)<k(0)$, implies

$$
\begin{aligned}
\underline{T} &= A(n)(1+k(n))^{\frac{\alpha\rho}{1-\alpha}} - k(n) - A(n) \\
&= \int_0^{k(n)} \left(\frac{\alpha\rho}{1-\alpha} A(n)(1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} - k \right) dk \\
&< \int_0^{k(0)} \left(\frac{\alpha\rho}{1-\alpha} A(0)(1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} - k \right) dk \\
&= A(0)(1+k(0))^{\frac{\alpha\rho}{1-\alpha}} - k(0) - A(0) = \overline{T}
\end{aligned}
$$

From the analysis carried out so far, it follow also that, if $t < \underline{T}$, we have no pure strategies equilibria ■

3 Proofs of Section 5

Proof of Proposition 10 The fact that k_0 is the minimal investment level comes from the fact that when the measure of investing firms is maximal, i.e. $\gamma = n$, the marginal return on the investment is the minimum possible, and so is the investment level. To show that $k_n > k_{mi}$, let us first show that $\gamma_n < \gamma_{mi} + \beta_{mi} + \sigma$. To show it, recall that the condition defining γ_n is

$$
A(\gamma_n)(1+k^*(\gamma_n))^{\frac{\alpha\rho}{1-\alpha}} - k^*(\gamma_n) - t = A(\gamma_n) \tag{7}
$$

Let us define a function $\Psi : [0, n] \to R$ as follows

$$
\Psi(x) \equiv A(x)(1+k^*(x))^{\frac{\alpha\rho}{1-\alpha}} - k^*(x) - A(x) \left[\frac{s}{n-x+s} (1+k^*(x))^{\frac{\alpha\rho}{1-\alpha}} + \frac{n-x}{n-x+s} \right]
$$

Observe that from condition (7) we have $\Psi(\gamma_{mi} + \beta_{mi} + s) = 0$. Since $\Psi(0) > 0$ and $\Psi(n) < 0$, if $\Psi(\gamma_n) > 0$, then $\gamma_{mi} + \beta_{mi} + s > \gamma_n$. However, if s is small enough, by (7) we have

$$
\begin{aligned}
\Psi\left(\gamma_n\right) &= A\left(\gamma_n\right)\left(1+k^*\left(\gamma_n\right)\right)^{\frac{\alpha\rho}{1-\alpha}} - k^*\left(\gamma_n\right) \\
&\quad - A\left(\gamma_n\right)\left[\tfrac{s}{n-\gamma_n+s}\left(1+k^*\left(\gamma_n\right)\right)^{\frac{\alpha\rho}{1-\alpha}} + \tfrac{n-\gamma_n}{n-x+s}\right] \\
&> A\left(\gamma_n\right)\left(1+k^*\left(\gamma_n\right)\right)^{\frac{\alpha\rho}{1-\alpha}} - k^*\left(\gamma_n\right) - t \\
&\quad - A\left(\gamma_n\right)\left[\tfrac{s}{n-\gamma_n+s}\left(1+k^*\left(\gamma_n\right)\right)^{\frac{\alpha\rho}{1-\alpha}} + \tfrac{n-\gamma_n}{n-x+s}\right] \\
&= A\left(\gamma_n\right) - A\left(\gamma_n\right)\left[\tfrac{s}{n-\gamma_n+s}\left(1+k^*\left(\gamma_n\right)\right)^{\frac{\alpha\rho}{1-\alpha}} + \tfrac{n-\gamma_n}{n-x+s}\right] \\
&\to 0
\end{aligned}
$$

which guarantees $\Psi\left(\gamma_n\right) > 0$. Since $\gamma_n < \gamma_{mi} + \beta_{mi} + \sigma$, the claim $k_n > k_{mi}$ follows from the fact that in equilibrium, if x is the measure of firms adopting a technology reached by k^*, we have $\frac{dk^*(x)}{dx} < 0$■

Proof of Proposition 11 (i) To show that γ_n decreases as n increases, first observe that, if we start with an equilibrium γ^* measure of investing firms such that $\pi_{\mathcal{N}\backslash H}\left(\gamma^*\right) = \pi_H\left(\gamma^*\right)$,and we denote by $k\left(n\right)$ the equilibrium investment level at γ^*, we have

$$
\begin{aligned}
\frac{\partial \pi_{\mathcal{N}\backslash H}\left(n\right)}{\partial n} &= \frac{\partial A\left(n\right)}{\partial n}\left(1+k\left(n\right)\right)^{\frac{\alpha\rho}{1-\alpha}} + \\
&\quad + \left[A\left(1+k\left(n\right)\right)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}\frac{\alpha\rho}{1-\alpha} - 1\right]\frac{\partial k\left(n\right)}{\partial n} \\
&= \frac{\partial A\left(n\right)}{\partial n}\left(1+k\left(n\right)\right)^{\frac{\alpha\rho}{1-\alpha}} \\
&< \frac{\partial A\left(n\right)}{\partial n} = \frac{\partial \pi_H\left(n\right)}{\partial n} < 0
\end{aligned}
$$

This implies that if n increases, $\pi_{\mathcal{N}\backslash H}\left(n\right)$ decreases faster than $\pi_H\left(n\right)$. So, we have that, if we increase n, the new payoff functions computed at $\gamma^*$are such that $\pi_{\mathcal{N}\backslash H}\left(\gamma^*\right) < \pi_H\left(\gamma^*\right)$,which implies that the intersection between the two curves occurs at a lower γ. The proof of (ii) is similar to (i)■