A Field Study on Matching with Network Externalities*

Mariagiovanna Baccara† Ayse Imrohoroglu‡ Alistair J. Wilson§
Leeat Yariv¶

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Abstract

We study the effects of network externalities on a unique matching protocol for faculty in a large U.S. professional school to offices in a new building. We collected institutional, web, and survey data on faculty's attributes and choices. We first identify the different layers of the social network: institutional affiliations, coauthorships, and friendships. We demonstrate and quantify the effects of network externalities on choices and outcomes. Furthermore, we disentangle the different layers of the social network and estimate their relative impact. Finally, we assess the matching protocol from a welfare perspective. Our study suggests the importance and feasibility of accounting for network externalities in general assignment problems and evaluates a set of techniques that can be employed to this end.

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†Olin School of Business, Washington University in St.Louis, mbaccara@wustl.edu.

‡Department of Finance and Business Economics, Marshall School of Business, University of Southern California, aimrohor@marshall.usc.edu.

§Department of Economics, New York University, alistair@nyu.edu.

¶Division of Humanities and Social Sciences, California Institute of Technology, lyariv@hss.caltech.edu.

1 Introduction

1.1 Overview

Externalities are commonplace within assignment processes: In the housing market, the value of a property depends on the demographics of neighboring homeowners. In an oligopolistic market, the returns from joining one firm depend on the composition of rivals. In universities, the desirability of a specific dorm room may depend on the peers in nearby rooms. In politics, the benefit from joining a particular party or coalition varies with the other political alliances formed. In team sports, the value in joining one team depends on the quality of other teams' players. And so on and so forth. Despite the wide range of applications featuring externalities, the matching literature, both positive and prescriptive, has largely ignored their presence.[1]

One of the significant challenges in assessing the role of externalities is that the underlying networks generating them are often unobservable or difficult to pin down. In particular, while attributes such as income, professional qualification, and education are frequently available, other important measurements of social connection—friendship, a shared professional history, etc.—are more difficult to obtain. Beyond the scarcity of data, the matching literature lacks a definitive framework that accounts for externalities, while still enabling empirical evaluations.

The current paper uses unique field data from *a centralized assignment process in which connections between individuals were mapped at both the professional and social level.* Specifically, our data originate from a matching process that assigned faculty members from a U.S. professional school to newly renovated offices. Using web and survey sources, we identify the institutional, coauthorship, and friendship networks of associations between the faculty involved.

Our study has three goals: First, to provide an empirical account of the effects of externalities resulting from agents' connections in assignment processes on behavior and outcomes; Second, to assess the differing networks' relative effects;[2] Third, to evaluate the efficiency

[1] For several exceptions, see the literature review below.

[2] Most existing work estimating peer effects elicits one layer of interactions (be it social, professional, or geographical). In our study, we allow the data to speak as to which of the network layers matters and to

of matching protocols in terms of welfare, accounting for the identified externalities. As a by-product, our analysis suggests and appraises several econometric and computational techniques for estimating network externalities.

Our data describe the matching process and final assignment of 73 faculty into offices. The offices vary in their physical attributes—in particular, elevation, geographical exposure, and size, as well as their position and spatial relation to other offices. School officials designed a serial-dictatorship matching protocol in which faculty members were coarsely ranked into four tiers according to career seniority, those with the greatest seniority choosing first, where the order of choice within each seniority level was determined by a random draw. Based on the resulting order, each faculty member was called upon in sequence to select an office from those remaining, having observed all preceding selections. After the selection process was completed, faculty were free to trade offices, and, additionally, were permitted to use transfers from their research budgets to facilitate trades.

In this environment, the externalities across agents can be easily mapped and separated into three layers of a social network. The first is institutional: the faculty members are divided into *departments*. The second social network is mapped using the past and current *coauthorship* links between faculty members. This network provides an alternative map of professional proximity, in which links between individuals were not incumbent on institutional affiliation or choice of research interest, but allowed to arise spontaneously through a bilateral choice. Finally, making use of a survey, we map a third social network, the *social interactions and friendships* between faculty members.

Our analysis follows several stages. As a preliminary step, we estimate an array of discrete-choice models in which, at each decision node, a faculty member chooses from a menu of offices, and decides based on each office's physical attributes, as well as the network characteristics at the time of choice (for example, the number of coauthors located nearby). If network effects play no role in choice, the corresponding network elements in our model should have

what extent. For some related work, see Conley and Udry (2010) and Kremer and Miguel (2007) discussed in Section 2.1.

no weight. However, all of our specifications generate significant network effects—in fact, the estimates suggest that network effects have a comparable impact to those of physical attributes. Nonetheless, this approach, while standard, is tantamount to assuming that faculty are myopic, ignoring the implications of their choices on their peers' subsequent selections. In that respect, while we reject the hypothesis that networks have no impact on *choice*, the magnitude of these effects should be interpreted carefully. This leads us to more closely inspect the dynamic and strategic aspects of the matching process.

In order to quantify the magnitudes of network sorting effects on *outcomes*, while accounting for the strategic aspects present during the matching process, we compare the observed assignment to a counterfactual in which faculty choose offices based only on physical attributes. Using the sequence in which faculty made decisions, we examine the outcomes that would result were faculty to locate solely on the basis of offices' physical characteristics—for instance, preferring offices on higher floors to lower ones, larger offices to smaller ones, etc. Where faculty face a choice from a subset of offices with the same physical characteristics, we assume one office is chosen randomly from the subset. This allows us to simulate the resulting network 'clustering' (for several preference specifications over offices' physical attributes) and compare it to that observed in the data.[3] The results from this comparison suggest that office proximity among linked individuals (both at the floor and office-neighbor level) occurs significantly more frequently in the observed assignment than in the simulated ones. Specifically, in the simulated assignments, members of the same department, coauthors, and friends are on the same floor at least 8%, 36%, and 30% less often than in our data, respectively. Similarly, proximity of office neighbors from every network layer were lower by 21%–59%. From a general perspective, these results are illustrative, in both significance and magnitude, of the potential importance of network externalities on assignment outcomes.

Next, we disentangle the relative importance of each of the three network layers. In particular, we are interested in separating the effects of the institutional network, generated

[3]This is sometimes referred to as a dartboard approach in the context of spatial econometrics, see Ellison and Glaeser (1997), Guimarães, Figueiredo, and Woodward (2009), and Head and Reis (2005).

by department affiliation, from the idiosyncratic choice networks, described by coauthorships and friendships.

As mentioned before, following the sequential choice process, faculty were allowed to exchange their allocated offices using transfers between their research budgets. This allows us to define a simple notion of stability pertaining to the final assignment (after all swaps were carried out). We say that an assignment is *pairwise stable with transfers* if there is no trade in office assignments between two faculty members that results, with a transfer, in an improvement for both faculty, keeping all other office assignments fixed. We show that pairwise-stable assignments exist when utilities are such that: (i) the effects of offices' physical attributes are common across faculty and separable from network effects; and (ii) network effects are symmetric across linked individuals and separably additive (for example, utilities depend linearly on the number of peers that are within the relevant neighborhood).[4]

Pairwise stability (with transfers) entails a sequence of constraints corresponding to all faculty pairs in our data. Using techniques developed recently for matching games without externalities (Bajari and Fox, 2010 and Fox, 2010), we estimate utility parameters for each of our network layers. We find that the coauthorship network has a greater impact than both the institutional and the friendship networks, where we find the latter to have a negligible effect. Nonetheless, the interaction between coauthorship and friendship has a sizable effect on preferences. Beyond the relevance to the matching process per se, this observation highlights the importance of studying the appropriate network of connections when examining peer effects. From a methodological perspective, these estimations underscore the importance of accounting for strategic behavior in dynamic matching markets. Indeed, the relative magnitudes of our estimates are different than those we achieve using standard discrete-choice models, which, as stressed above, omit the forward-looking strategic aspects.

Given the significance of externalities in individuals' utilities, it is interesting to contem-

[4]This is interesting from a theoretical perspective. Indeed, the literature on matching with externalities has mostly concentrated on existence of stable outcomes. As discussed in the literature review below, the difficulty arises due to the freedom one has in specifying beliefs over other players' reactions upon deviation. Our notion essentially entails myopic beliefs about the swaps that ensue. This assures existence.

plate the design of efficient assignments. In principle, designing the most efficient assignment is a complex problem due to the vast number of possible assignments ($73! > 10^{105}$ in our data set).[5] As it turns out, designing the most efficient assignment for a class of preferences allowing for network externalities (that encompasses those we estimate) is a special case of the *quadratic assignment problem* (see Koopmans and Beckman, 1957). While generally difficult computationally, and subject of an active line of investigation in operations research, we show how new techniques, still unexploited in the economics literature, can be used to approximate an optimal assignment.

Under our assumptions that the utilities from offices' physical attributes are shared across faculty and are separable from network characteristics, utilitarian efficiency is influenced only through the network effects present in our population. In fact, given our utility specification and the estimation results, any assignment that would increase the proximity of members from the different network layers would increase efficiency. Using our estimates for the relative preference weights of the different network variables, we can evaluate the efficiency of the matching protocol at hand. Namely, we identify an assignment that achieves a 183% improvement in network utility relative to the implemented assignment.

Finally, having identified individuals' preferences and an efficiency benchmark, we study some properties of assignment mechanisms. First, we compare the assignment implemented by the school with the best-found assignment in terms of fairness across seniority levels and departmental affiliations. We find that the best-found assignment implies more egalitarian outcomes across seniority, but somewhat less egalitarian outcomes across departments. Second, we consider several commonly used versions of the serial-dictatorship mechanism (varying the order in which the individuals choose and banning ex-post swaps), and evaluate their efficiency performance. In the presence of externalities, outcomes appear consistently lower than the socially optimal benchmark. From a general institutional-design point of view, this analysis suggests the importance of recognizing and accounting for the underlying networks of

[5] Furthermore, the presence of network externalities makes the problem significantly more intricate than those pertaining to well-known problems of assigning goods exhibiting complementarities (e.g., spectrum rights' auctions).

relevant connections when constructing assignment mechanisms, and illustrates computational techniques for doing so in practice.

1.2 Related Literature

The idea that externalities may play a crucial role in group formation appears in some of the recent theoretical work on cooperative games. The general setup of games that are often referred to as "partition function games" (Lucas and Thrall, 1963 and Myerson, 1977) or "global games" (Gilboa and Lehrer, 1991) presumes that players' payoffs depend on the *partition* of the population. There are two general approaches that the literature takes. One strand focuses on core-like or Shapley value notions in which a particular belief structure (pertaining to the entire population's reaction to a coalitional deviation) is imposed (for example, Gilboa and Lehrer, 1991, De Clippel and Serrano, 2008, and Hafalir, 2008). The goal of this literature is to provide conditions under which the relevant solution concept exists. The other line of work is more explicitly dynamic in that it proposes a particular "bargaining protocol" by which coalitions are formed and analyzes the resulting set of equilibria in terms of efficiency and the pattern of emerging coalitions (see Bloch, 1996, Maskin, 2003, Ray and Vohra, 1999, and Yi, 1997).

In the context of matching, Sasaki and Toda (1996) illustrate the large freedom in beliefs upon deviations assuring the existence of stable matches *for any* prevailing preferences.[6]

Without externalities, there is a large body of theoretical work that studies housing matching environments similar to ours (starting from Shapley and Scarf, 1974 and more recently explored in, for example, Che and Gale, 2009, Ehlers, 2002, Pycia and Ünver, 2007, and references therein).[7]

[6] The matching literature has also considered different types of externalities in many-to-one matching environments in which agents (say, workers in a firm, or students in a school) care about the peers who are assigned with them (but *not the entire population assignment*). That literature focuses mostly on conditions under which particular notions of stability generate non-empty predictions. See Echenique and Yenmez (2007) and Pycia (2009) for details.

[7] In the no externalities world, there is also a budding literature studying decentralized dynamic matching games in which, similarly to our setting, agents may consider other agents' actions when deciding to commit to an irreversible match, see Niederle and Yariv (2009).

Empirically, while we are not aware of any studies quantifying the effects of network externalities in cooperative setups (matching environments in particular), the idea that peers may affect behavior and outcomes has been explored in many realms (see, for instance, Jackson, 2008 and Wasserman and Faust, 1994 for references).[8] In fact, recent field data suggest differential effects of multi-layered networks on outcomes (see Conley and Udry, 2010 and Kremer and Miguel, 2007). Another related strand of empirical work considers field performance of assignment mechanisms, without accounting for externalities (see Abdulkadiroğlu, Pathak, Roth, and Sönmez, 2006, and references therein).[9]

Methodologically, the dartboard approach used in Section 4 to estimate the impact of network externalities on the observed bunching of connected faculty has been used in other empirical studies on spatial clustering (for instance, Ellison and Glaeser, 1997, who use a similar approach to estimate geographic concentration of U.S. manufacturing industries). The estimations we perform in order to assess the relative magnitudes of the effects of the different layers of the underlying networks utilize identification techniques developed by Bajari and Fox (2010) and Fox (2010).

Finally, our welfare analysis involves finding the optimal solution for a quadratic assignment problem, which dates back to the specification of location assignments with externalities in Koopmans and Beckmann (1957). Solving this problem, which is NP-hard, is a continuing area of research within the operations-research and combinatorics literatures.[10]

Several recent papers contain welfare assessments of assignments via random serial dictatorship *without externalities*. Manea (2007) characterizes subgame-perfect equilibrium outcomes of serial-dictatorship procedures for multiple objects, and finds that outcomes are not generically efficient, in contrast with the single-object case. Budish and Cantillon (2011)

[8]Several recent papers have mapped friendship networks in order to test for their effects on behavior in experimental games (for example, Leider, Möbius, Rosenblat, and Do, 2009 and Goeree, McConnell, Mitchell, Tromp, and Yariv, 2009).

[9]For anecdotal evidence on how office locations impact faculty interactions, see Kraut, Egido, and Galegher (1988).

[10]Liola, Nair, de Abreu, Boaventura-Netto, Hahn, and Querido (2007) provide an extensive list of references in the operations research literature and Brandeau and Chiu (1989) provide a general taxonomy for a planner's location/assignment problem.

analyze data from a university's course-assignment process and find that the university's manipulable mechanism provides an ex-ante welfare improvement over the strategy-proof and ex-post efficient random serial dictatorship.

Another assignment mechanism extensively studied in the literature is auctions. Bajari and Fox (2010) analyze the welfare loss in the sale of FCC spectrum licenses via auctions after constructing estimates of license complementarities. Again, externalities across different bidders' license assignments are assumed not to be present. Sönmez and Ünver (2009) discuss the welfare losses caused through auction mechanisms with endowment of fiat currency, demonstrating the failure of these markets over straightforward statements of ordinal preferences. Krishna and Ünver (2008) empirically analyze the results from course assignments with bidding, finding auctions inferior to a standard Gale-Shapley mechanism.

2 The Allocation Process

In this section, we describe the details of the matching protocol that was utilized in the field experiment, as well as the components of our data set.

2.1 The Matching Protocol

In 2006, plans to renovate one building of a large U.S. professional school were revealed to the faculty. The renovation would result in 74 vacant offices. Dean-level negotiations produced an initial list of 74 faculty members from 4 departments to occupy the new building.[11]

The assignment process used was a random serial-dictatorship procedure. As a first step, the school officials produced a coarse ranking of the 74 faculty members according to career seniority: priority was given first to chaired professors and department chairs, then full professors, followed by associate professors and, finally, assistant professors. The ordering of faculty within each group was determined by a *random draw* administered by the dean's office, under the supervision of department representatives.

[11]Before the renovation, three different buildings housed the offices of the school's faculty members, with departments assigned to different floors within these buildings.

Once the ranking was determined, the faculty members bound for the new building received a memo providing the complete sequence, as well as instructions on how the process would evolve. These instructions indicated that all the office choices were to be conducted in one day. Each faculty member was able to see all the choices made up to the time of his/her own choice. Faculty members who could not be present on the day of the draft were asked to fill out a proxy form detailing their preferences and give it to a faculty who would be present.[12]

Conversations and discussions among the faculty took place prior to the selections. Faculty members were encouraged to make pre- and post-draft exchanges (prior to the draft, exchanges of rank numbers, and after the draft, exchanges of offices). Furthermore, *faculty were allowed to use funds from their research accounts to facilitate both types of exchange.* Indeed, while no ex-ante draft-number trades took place, ex-post trades involving 7 offices occurred immediately following the draft. Specifically, there were three office changes observed after the initial assignment: (i) A bilateral swap; (ii) A swap triggered by one faculty member leaving the building, followed by a second faculty member taking his office, a third faculty member taking the office of the second, and a fourth faculty member taking the office of the third; (iii) A move of one faculty to a vacant office. Both swaps (i) and (ii) involved research money transfers, while of course (iii) did not. We have detailed data regarding 73 of these faculty, which are the subject of our study.

2.2 The Assignment Data

We collected three types of data: data on office characteristics, population characteristics, and the matching process, which we now describe in turn.

2.2.1 Office Characteristics

The building had housed one of the departments for many years prior to the renovation. Therefore, faculty members from that department had detailed information on the desirability

[12] Also, three offices were reserved for potential new hires, who were given a number in the ranking similarly to the other faculty members. Department representatives were in charge of selecting those offices.

Table 1: Summary Statistics of Office and Population Characteristics

Variable	Mean	Std. Dev.	Min	Max	N
Office Variables:					
Large, Corner Office	0.10	0.30	0	1	73
Western Exposure	0.38	0.49	0	1	
Floor	6.40	1.24	4	8	
Faculty Variables:					
Years since PhD	13.64	11.09	0	37	72
Years since Joining School	9.51	8.78	0	35	
Female	0.21	0.41	0	1	73
Coauthors	1.56	1.63	0	6	
Lunches†	2.5	2.34	0	9	54
Social Events‡	0.94	1.22	0	4	
Friends	2.86	2.47	0	9	
Coauthors and not-Friends	0.81	1.01	0	3	
Coauthors and Friends	0.97	1.32	0	4	
Department Links	18.6	5.21	12	25	73
Research Cluster Links	5.32	2.31	0	8	
Institutional Variables:					
Departments	18.25	5.73	13	26	4
Research Clusters	4.87	2.90	1	9	15

Note: (†) Refers to colleagues with whom survey respondents report having regular lunches in Question 6 of the survey; (‡) Refers to colleagues with whom survey respondents report meeting socially outside school in Question 8 of the survey.

of different offices. Moreover, before the office selections were made, the dean's office provided detailed descriptions regarding office attributes to all faculty.

The top panel of Table 1 summarizes the characteristics of the available offices, which vary in floor, view, and size. The offices are located on the top five floors of an eight-floor building. Half of floor 4 and floors 5, 6, 7, and 8 had undergone renovation.[13] Each floor has offices that face east, west, and south.

In terms of size, there are three office types. The majority of offices are identically sized (at about 213 square feet). These are the 56 offices facing either west or east, aligned on the two sides of a main corridor, on floors 5, 6, 7, and 8 (accounting for 76% of all offices). Then

[13]The lower floors and the other half of the 4th floor contain classrooms (and were not modified).

there are 8 large offices in the corners of the south sides of floors 4 through 8 (corresponding to 10% of all offices). These have an area of approximately 261 square feet and include an additional 20 square feet of closets Finally, there are 10 smaller offices in the south sides of floors 4 through 8 that have an area of approximately 200 square feet. Since the offices are very similar in terms of size, the view was considered an important distinguishing characteristic. Faculty were told that the preferred views tend to be on the higher floors, and on the sides facing west and south (in high floors, the west- and south-exposed offices have open city views, while the east ones look onto a high-traffic artery).[14] See Figure 2 for a graphical illustration of the offices' spatial arrangement. In what follows, we will consider two offices to be in the same *neighborhood* if the distance between the office doors is less than *30 feet*.[15]

2.2.2 Population Characteristics

The bottom two panels of Table 1 contain the summary statistics of our population, and Section 3 provides a detailed description of its characteristics. Faculty members' attributes were collected using two sources:

1. Web-harvested individual data on department, research cluster, arrival at the school, Ph.D. cohort, coauthorship, education background, and gender.

2. Survey results. Faculty members were surveyed after the draft and the ex-post trades took place. There were 36 completed survey respondents (50% of the total number of faculty members). The survey elicited information on the faculty's social network as well as their preferences over the offices' physical (floor, view, and size) and non-physical (colleagues' proximity) attributes. The respondents were also asked to assess the importance of each office attribute keeping the other attributes constant. The complete list of questions asked in the survey and the aggregate responses are available in Appendix A.

[14] Faculty members were encouraged to tour the building with its current tenants and examine the offices.

[15] The 30 feet figure is chosen to include offices that are *less than two doors away* on either side of the main corridor. Since there are no offices directly in front of them, offices facing south have neighbors only on their own side of the corridor.

2.2.3 The Matching Process Data

Our data contain the complete results from the matching process. In particular, besides the final assignment of offices after swaps, for every choice made we know the set of faculty who had already chosen an office, how the partial assignment looked at the time of choice, and the remaining faculty who still had to make a choice.

3 The Underlying Networks

Individuals interact in different spheres. Since the location of an office may affect one's quality of work-life on both purely social and purely intellectual levels, we elicited peer connections on three dimensions: institutional links, determined according to the department or research cluster each faculty belongs to, coauthorship links, and friendship links. Below, we describe each of these layers of the network of connections and the correlations between them.

3.1 Institutional Network

The first network we consider addresses the research interests of faculty members, dividing them according to their specific research fields. The 73 faculty members are divided into 4 departments according to main research fields. Each department is further divided into research clusters according to sub-fields, resulting in a total of 15 clusters. The average department size is 18.25 individuals (ranging from 13 to 26 members). The average cluster size is approximately 5 individuals (ranging from 1 to 9 members).[16] The research network appears in Figure 1. In the figure, each node's shape corresponds to a particular research department, where each of the four departments is located in a different quadrant. Nodes are encircled in a shaded circle if they belong to the same research cluster. In both the *Department* and *Research Cluster* networks all individuals are all connected to one another,

[16]In our analysis, cluster affiliation did not play an important role and our results are therefore presented using department affiliation for the institutional network. We provide details of the school's research clusters for the sake of completeness.

so each component in these networks is complete (in particular, the average distance[17] within a connected component is 1).[18]

3.2 Coauthorship Network

The second network encapsulates professional interactions among faculty, as captured by the existence of coauthored work. This network has been constructed by combining web-harvested and survey data. In particular, this network layer considers two faculty connected if they coauthored at least one paper together in the past or are currently collaborating on a project (the latter element declared in survey responses). The coauthorship network is described by the solid lines between nodes (both bold and faint) in Figure 1.

At the time of choice, the average number of coauthors each faculty member had among other faculty members was 1.6, ranging from 0 to 6. The average distance between connected individuals in this network is 3.28, ranging from 1 to 10.

3.3 Friendship Network

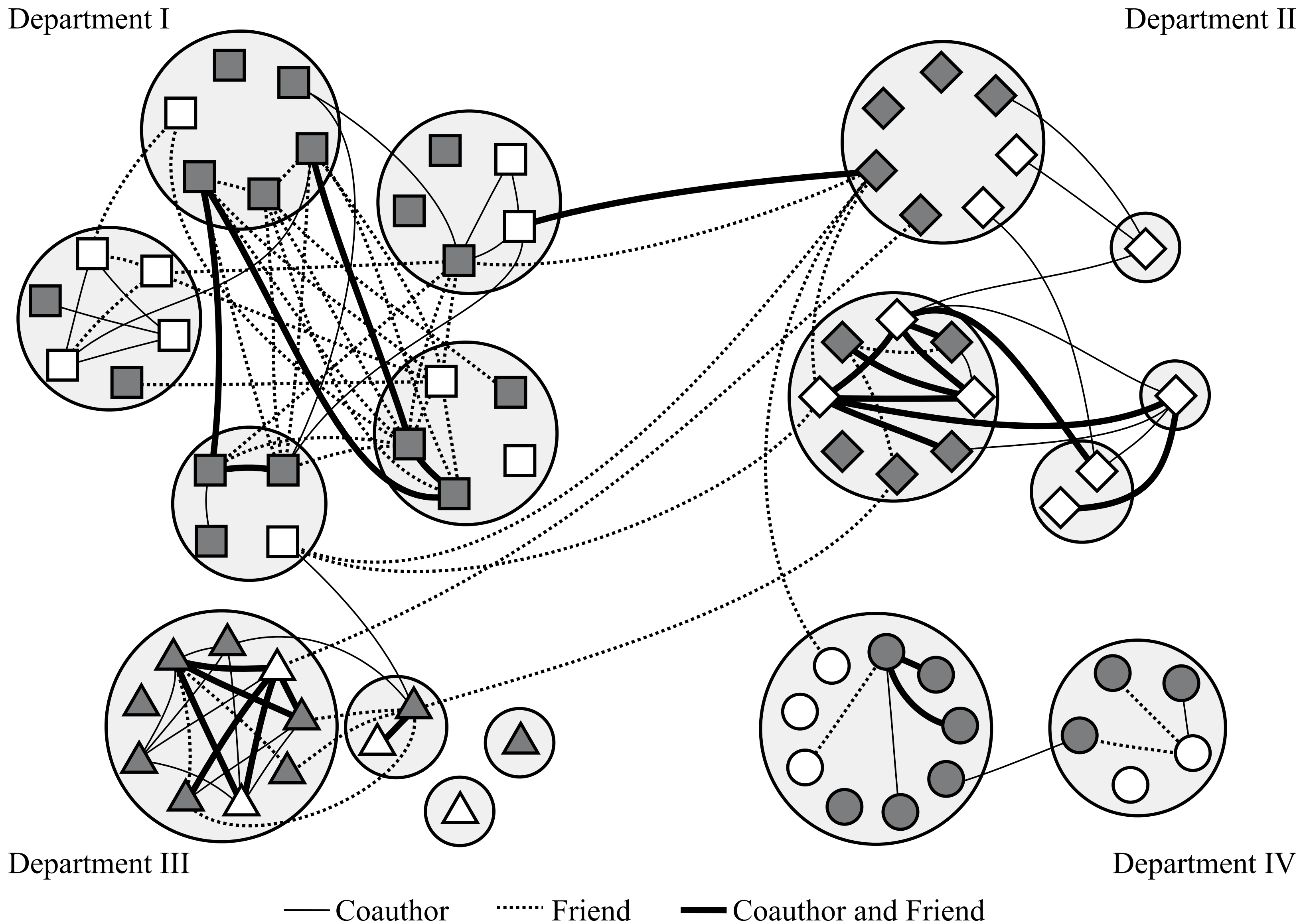
The friendship network captures the interactions among the 36 faculty members who completed the survey, as well as the individuals socially connected to them (that is, individuals who were not survey respondents themselves, but were declared as a friend by at least one survey respondent).

In detail, Question 6 in the survey asked participants to name up to 5 fellow faculty members with whom they had lunch on a regular basis. Question 8 asked participants to name up to 5 personal friends (people with whom the participant interacts socially with outside school at least once a month) from within the school (see Appendix A). To build a friendship network, we combined the answers to these two questions. Two faculty are connected if they were mentioned one by the other in responses to either Question 6 or 8. In particular, not all

[17] The distance between two nodes is defined as the number of links on the shortest path in the network connecting the two nodes.

[18] Moreover, we distinguish the faculty seniority levels in Figure 1 using the shading of nodes (white for senior faculty and gray for junior faculty).

Figure 1: Network Diagram



Note: Differing shapes represent department affiliations; Circles research clusters; Node shading represents seniority level, with white for seniors, and gray for juniors.

faculty included in this network are necessarily survey respondents—they are either survey respondents or individuals connected to a survey respondent. There were 21 faculty who specified at least one individual on either one of these questions, leading to 54 *extended survey participants* who compose the social network. We stress that survey respondents who did not specify colleagues' names answering Questions 6 and 8 could either (i) not have any social interactions with other members of the faculty, or (ii) have social interactions they prefer not to disclose in the survey.

This generates the network given by the dotted and bold line connections in Figure 1.[19] We assume links are bi-directional. Indeed, Questions 6 and 8 were phrased so that individuals were specifically asked to report the frequency of interactions (lunches or social events outside the school), that are inherently symmetric.[20]

As reported in Table 1, the survey respondents are reported to have an average of 0.9 individuals (ranging from 0 to 4) whom they interact with socially outside the school, and 2.5 colleagues with whom they regularly have lunch (ranging from 0 to 9). The average degree in the friendship network is 2.9, ranging from 0 to 9, and the average distance between individuals is 4.91, ranging from 1 to 12. Moreover, simple regression analysis reveals that the faculty that filled the survey do not exhibit significantly different observable attributes from those who did not.[21]

3.4 Overlap of Networks

Figure 1 demonstrates the complexity of the social network under examination, and the difference between the coauthor and friendship relations. Table 2 provides the exact correlations between the different layers of the social network. The correlations above the main diagonal

[19] Dotted lines correspond to the pure friendship network–links are between agents who are friends but not coauthors; Bold lines represent the intersection between the coauthorship and friendship networks–links are between individuals who are both friends and coauthors.

[20] In our data, if we restrict attention to survey respondents alone, the probability that f considers f' a friend, conditional on f' considering f a friend, is 52%.

[21] This is the result of a set of regressions including variables such as gender, department affiliation, order of choice in the implemented serial-dictatorship mechanism, years since PhD, years since joining the school, and degree in the coauthorship network.

Table 2: Network Correlations

	Department	**Cluster**	**Coauthor**	**Friend**
Department	1	0.478	0.240	0.235
Cluster	0.413	1	0.318	0.231
Coauthor	0.254	0.343	1	0.374
Friend	0.387	0.302	0.452	1

Note: Correlations for dummy variable indicating a link between all faculty pairs (f, f'). Numbers above the diagonal correspond to the full sample, $N = 2628$, numbers below the diagonal correspond to the survey sample, $N = 630$.

are computed with observations associated with all faculty members. The correlations below the main diagonal are computed restricting the data to survey respondents only. As clusters are subsets of departments, the two networks are highly correlated. While friendship ties seem fairly uncorrelated with department and cluster links, they are correlated with coauthorship ties. We account for this correlation throughout our analysis.[22]

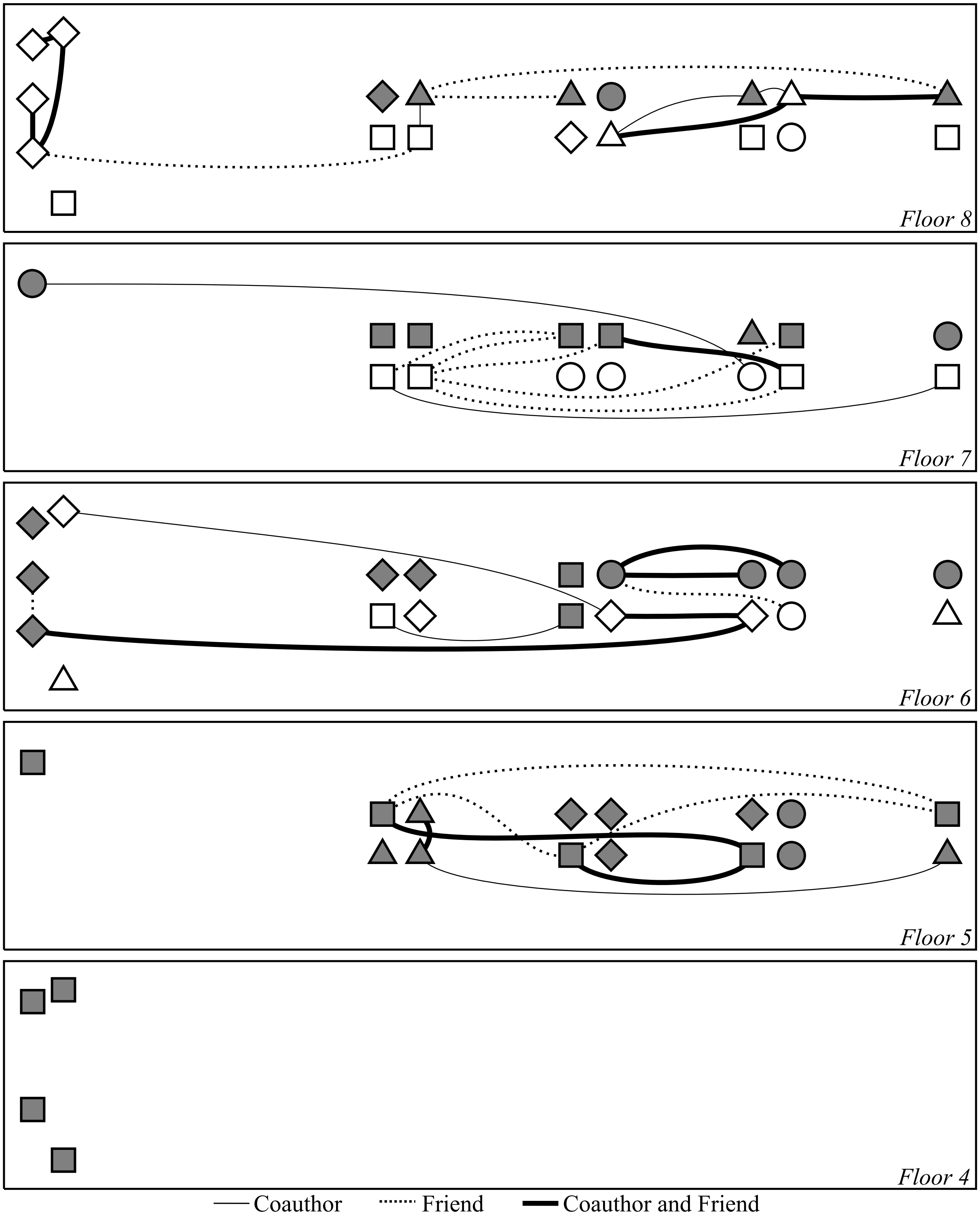
3.5 Existence of Network Effects in Office Selection

The choice made by each faculty member during the matching process could be influenced both by the office's physical attributes (floor, exposure, size), as well as the choices made (or expected to be made) by others. Figure 2 describes the outcome of the matching process (after ex-post trades took place), with dotted lines representing the friendship links within a floor, faint solid lines the coauthorship links, and bold solid lines the intersection of both network links. In particular, the figure represents the final spatial assignment by floor, with nodes schematically placed at the doorway of the chosen offices.

We start our investigation by assessing the *null hypothesis that network externalities are not taken into consideration during office selection.* As a first take, we consider the discrete choice each faculty is facing. Each observation in our sample corresponds to a pair (f, o), where f is a faculty member and o is an office *available to this faculty member at her time of choice.* We specify an array of models in which choices are explained by variables corresponding to

[22]We note that the preferences over physical office characteristics stated in the survey are not significantly correlated within any of the networks (e.g., department affiliation does not help to predict these preferences).

Figure 2: The Observed Assignment



Note: Differing shapes represent department affiliations; Circles research clusters; Node shading represents seniority level, with white for seniors, and gray for juniors.

Table 3: Conditional Logit Regressions

Variable	$CL(i)$	$CL(ii)$	$CL(iii)$	$CL(iv)$
Large Corner Office	0.155	0.284***	0.237***	0.199***
	(0.097)	(0.078)	(0.088)	(0.113)
Western Exposure	0.312***	0.394***	0.377***	0.352***
	(0.045)	(0.033)	(0.036)	(0.048)
Highest Available	0.368***	0.221***	0.291***	0.235***
	(0.034)	(0.071)	(0.053)	(0.076)
Department Neighbor		0.174***	0.229***	0.233***
		(0.043)	(0.038)	(0.049)
Department on Floor		0.082***		
		(0.027)		
Coauthor Neighbor			0.186*	
			(0.103)	
Coauthor and not-Friend Neighbor				-0.040
				(0.247)
not-Coauthor and Friend Neighbor				0.095
				(0.171)
Coauthor and Friend Neighbor				0.319**
				(0.156)
N	73	73	73	54

Note: Coefficients represent marginal effects between two otherwise identical offices; Standard deviations given in parentheses. (*) indicates significance with 90% confidence, (**) with 95% confidence, and (***) with 99% confidence.

both physical and network characteristics. Such an approach is inherently non-strategic in that we do not take into account forward-looking strategic aspects that are potentially present if network effects are at play (in particular, the approach does not allow us to quantify the effects of externalities). However, note that if the null hypothesis that network externalities are irrelevant to choice holds, any coefficient pertaining to network variables should not be significantly different from 0.

Table 3 contains the results of four conditional-logit regression specifications.[23] The vari-

[23] Note that the set of available offices decreases by one unit after each choice is made. Thus, a faculty at position $k = 1, .., N$ in the ranking has $N - k + 1$ possible offices to choose from. Each observation in our data corresponds to a faculty and their menu of offices (excluding the last faculty who was left with no choice). Thus, for our 73 faculty, we have a total of $\frac{73\times 74}{2} - 1 = 2700$ possible choices.

ables associated with offices' physical attributes are *Large Corner Office*, *Western Exposure*, and *Highest Available* (which are the corresponding dummy variables for the respective characteristics). The rest of the variables are associated with the department, coauthorship, and friendship networks. Specifically, consider an observation pertaining to a particular faculty-office pair (f, o). *Department Neighbor* is a count of the number of offices neighboring o that are already taken by another member of f's department before his/her turn in the sequence;[24] *Department Floor* indicates the number of individuals in f's department that were present on the floor corresponding to o at the time of choice. Similarly, *Coauthor Neighbor* is an integer variable representing the number of neighboring offices close to o that have been taken by faculty members with coauthor ties to f at the time of choice. Similarly *Coauthor and not-Friend Neighbor*, *not-Coauthor and Friend Neighbor*, and *Coauthor and Friend Neighbor* represent the counts of faculty members that are coauthors but not friends, friends but not coauthors, and both friends and coauthors, respectively, in offices neighboring o at the time of choice.

Throughout all the specifications of Table 3, the coefficients measure the marginal increase in the probability of an office being selected as a result of a unit increase in the variable under consideration. For example, in the first specification, denoted $CL(i)$, we include only the physical attributes of offices. Given two offices that differ only in their exposure, the office with the western exposure is 31.2% more likely to be selected. In the subsequent specifications, denoted $CL(ii)$–$CL(iv)$, we introduce the variables associated with network externalities.[25] Network variables have significant explanatory power. For instance, each additional coauthor located in a neighboring office raises the probability of an office being selected by 19%.

The results of these specifications provide two main insights. First, network variables' coefficients are positive and, at the micro-neighborhood level, significantly different from zero

[24] Recall that in Section 2.2.1 we defined two offices as *neighbors* if the distance between office doors is less than 30 feet.

[25] Specifications CL(*ii*–*iv*) were chosen to correspond to our ensuing specifications in Section 5. The last specification, $CL(iv)$, is restricted to faculty who have links in our friendship network and as such pertains to fewer observations.

at any reasonable confidence level.[26] In particular, *we reject the null hypothesis that network externalities (at all three levels) did not influence faculty's office choices.* Second, the regressions suggest the importance of accounting for network effects when estimating such matching processes. Indeed, the coefficients corresponding to offices' physical attributes change significantly when we include network variables. Note that these coefficients respond in different ways to the omission of network variables: the effects of large corner offices are underestimated in $CL(i)$ relative to $CL(iii)$ since faculty choose offices close to colleagues even when large offices are available; the effects of highest floor are overestimated in $CL(i)$ relative to $CL(iii)$, suggesting that faculty may be choosing higher floors to be in proximity to particular colleagues, rather than out of a preference for higher floors per se.[27]

We stress that given the significant network effects, the magnitudes of the coefficients we estimate need to be interpreted cautiously. Indeed, when network effects are present, individuals making choices within the serial-dictatorship protocol may consider future choices of others, making the conditional logit specification problematic. With that caveat, the results reported in Table 3 are useful in two respects. First, as we have discussed, they illustrate the existence of network effects in *choices*. Second, they provide a methodological baseline for our investigation since conditional logit analysis is common in studies such as ours. In what follows, we estimate the extent to which network effects affected *final outcomes*. When these effects turn out to be important, we introduce new techniques for estimating preferences accounting for strategic motives present in the mechanism under consideration.

4 Dartboard Approach: Network Effects and Outcomes

The exploratory regressions discussed in Section 3 suggest the existence of non-trivial effects of network externalities on match outcomes. In order to assess the magnitude of these effects,

[26]The joint hypothesis that the network variables are all zero in specifications CL(ii–iv) is rejected at the 99.9% level.

[27]In principle, there could be instances in which physical characteristics of an office are correlated with network attributes (for instance, corner offices are by geography more isolated than others). When controlling for offices' spatial arrangement by including offices' number of neighbors and a corner office dummy, we find no significant effects associated with these variables.

Table 4: Conterfactual Sorting in Simulated Office Assignments

Variable	**Proximity**	**Observed**	**Counterfactual**		
			$CF(i)$	$CF(ii)$	$CF(iii)$
Department	Neighbor	80	63.0**	61.2***	63.2**
			(6.5)	(5.2)	(6.6)
	Floor	164	151.1*	147.0***	151.1
			(9.4)	(0.0)	(9.4)
Coauthor	Neighbor	14	5.3***	3.3***	5.7***
			(2.2)	(1.3)	(2.3)
	Floor	22	12.7***	13.0***	13.8**
			(3.1)	(0.0)	(3.4)
Friend	Neighbor	15	6.2***	6.0***	7.0***
			(2.4)	(1.7)	(2.5)
	Floor	24	14.9**	14.0***	16.7**
			(3.4)	(0.0)	(3.5)

Note: Standard deviations given in parentheses. (*) indicates significance with 90% confidence, (**) with 95% confidence, and (***) with 99% confidence. Simulation specifications: $CF(i)$ Random preferences; $CF(ii)$ Lexicographic preferences (Size$\succ$View$\succ$Floor); $CF(iii)$ is based on results of $CL(i)$ in Table 3. The number of simulations is 10^5.

we start by considering counterfactual assignment procedures that do not account for any observable externalities and compare the degree to which such procedures generate network proximity relative to that observed in our data. Put differently, we assess to what extent a random assignment based on purely physical office attributes can explain the observed patterns of social connection, accounting for the mechanism in place (namely, the order in which faculty chose offices). We consider three benchmarks that differ in the prevailing faculty preferences for offices.

In our first specification, denoted $CF(i)$, we consider all offices equivalent, so that at each stage, faculty choose one of the available offices at random. In specification $CF(ii)$, we assume that each faculty has a lexicographic preference in which large corner offices are valued most, followed by western-exposure offices, followed by higher offices.[28] Again, within each class

[28]The preference for higher offices over lower ones is consistent with survey results. Indeed, in Question 11 in the survey, 86% of respondents declared the top floors, floors $6-8$, as their most preferred, and 83%

of offices, faculty randomly select an office. In the third specification, denoted $CF(iii)$, we suppose faculty have preferences as in our discrete-choice model $CL(i)$ in Table 3, where the probability of choosing any office is determined by the set of available offices, and the physical characteristics of those offices.[29]

For each of the specifications, we used the order in which faculty chose to simulate the random-matching procedure 10^5 times. For every set of simulations, we considered the three network layers: institutional affiliation (captured through department), coauthorship, and friendship (encapsulating the connections determined through social interaction or lunch companionship as described in Section 2). We calculated the resulting average volume of faculty from each network in a participating faculty's macro-neighborhood (the floor of their office), and their micro-neighborhood (the set of close office neighbors), which are reported in the counterfactual columns in Table 4. For each simulation we run a one-sided test of whether the observed outcome is in the right-tail of the simulated distribution.

Table 4 illustrates how the simulated faculty placements exhibit network links that are consistently lower than those observed in the data, for each of the three layers. For example, the number of faculty members who share a department affiliation and locate on the same floor is 164 in our data, but at most 151 in each of the counterfactuals, an 8% difference. Floor-level proximity is also significantly lower by at least 36% for coauthors and 30% for friends.

At the micro-neighborhood level, our results are most striking, with proximity of office neighbors under the coauthorship and friendship layers lower than the observed number by at least 59% and 53% respectively (significant at any conventional confidence level). Department-level sorting was significantly lower as well, by approximately 21%.

In our survey, looking at responses to questions asking about the *importance* of office floor, exposure, and size (on a 1–10 scale), we find no significant differences in responses, though the

declared the bottom floors, floors $4-5$, as their least preferred. The assumption that faculty prefer larger offices to smaller ones and offices with city views to large road views seem natural first steps.

[29]That is, the simulated choice probability for office i within choice set C is given by $\frac{e^{x_i\beta}}{\sum_{j\in C} e^{x_j\beta}}$, where the values for β are obtained from the $CL(i)$ in Table 3.

distribution of responses of office floor did stochastically dominate those corresponding to size and exposure. As a robustness check, we implemented an additional set of simulations. We used individual responses of survey participants to run an auxiliary conditional logit specification that includes expressed preferences (effectively interacted with survey participation) as additional variables. We then simulated the dartboard assignment as in $CF(iii)$, and obtained virtually identical results.

The conclusion from this array of counterfactual simulations is that the order in which faculty chose their offices does not seem, in itself, to explain the spatial concentration of those connected in these networks. Furthermore, in terms of outcomes, the differences between procedures that disregard network attributes and those observed in the data is substantial.

In comparison with the discrete choice models assessed in Table 3, the results in this section illustrate the potential importance of network externalities in determining outcomes. However, they do not allow us to back out the preferences of participants. Most importantly, the dartboard technique does not allow us to disentangle the differential effects from each of the different networks on agents' choices. This is so for two reasons: First, since the different layers of social networks are correlated, cross-layer comparisons of the dartboard approach results cannot be directly associated to agents' utility. Second, because of the strategic nature of the matching process, significant differences in how agents perceive the different layers of the social networks may result in small differences in the counterfactual estimates based on outcomes, and vice-versa.

5 Estimating the Relative Effects of Different Networks

The previous two sections motivated the importance of networks to the office selection mechanism. In particular, they make the case that externalities matter. In this section, we seek to disentangle the relative importance of each of the three layers on office choice: the department affiliation, the coauthorship network, and the friendship network. In particular, we are interested in separating the effects of the institutional network generated by membership in a department from those of the spontaneous networks based on coauthorship and friendship.

In principle, the mechanism used for allocating offices defines an extensive-form game. Assume that agents' preferences take some functional form allowing for the weight placed on each network and each physical office characteristic to be parametrized. Then, for any parameter value, there would be a corresponding set of equilibria of the assignment mechanism. In principle, this approach would allow us to select the parameters that best match our data. Note, however, that strategies in this game are contingent plans that specify, for each agent, an office selection that depends on the entire history of choices made by all predecessors. Since our data set contains 72 faculty who have a non-trivial choice, the set of strategies is vast and finding parametric equilibria profiles is not computationally feasible.[30]

In order to overcome this computational difficulty and still exploit the strategic elements inherent in the matching process, we focus on natural restrictions on the final assignment, using the fact that faculty members were allowed to swap offices after the draft was completed, and that monetary transfers across research accounts were allowed to facilitate such swaps. In what follows, we assume that the transfers were not subject to budget constraints. Indeed, all faculty were allotted identical budget allocations and were allowed to borrow against future years' provisions. Therefore, once the assignment has been determined (after all ex-post office swaps had been carried out), we can assume that there are no remaining beneficial swaps, that is, the assignment is *stable*. In the spirit of Bajari and Fox (2010), and Fox (2010), we require that no two faculty would benefit from exchanging offices (accounting for network effects derived from such an exchange) regardless of the monetary transfers between them. This requirement provides us with a manageable set of restrictions that allows for preference estimation.

[30]The computation of the set of Nash equilibria requires a factorial time algorithm. In fact, for N faculty members, the number of edges of the extensive-form game is

$$N + N(N-1) + \ldots + \frac{N!}{2!} + \frac{N!}{1!} = N! \sum_{j=1}^{N-1} \frac{1}{j!} \approx N!(e-1) - 1$$

5.1 Stability with Externalities

Consider a finite set of faculty $\mathcal{F} = \{1, \dots, N\}$ and a finite set of offices $\mathcal{O} = \{1, \dots, N\}$. We ultimately observe an assignment $\mu : \mathcal{F} \to \mathcal{O}$, a bijection assigning each faculty member to a particular office. The utility of faculty member f can be generically represented by the utility function $u_f(\mu)$.[31] For any assignment μ, we denote by $\mu_f^{f'}$ the assignment derived from μ by exchanging the office assignments of f and f':

$$\mu_f^{f'}(x) := \begin{cases} \mu(f') & \text{if } x = f \\ \mu(f) & \text{if } x = f' \\ \mu(x) & \text{otherwise} \end{cases}$$

The notion of stability (with transfers) we use requires that for any faculty pair (f, f') there does not exist a transfer t from f to f' such that the bilateral exchange of offices specified by μ improves both their outcomes. That is, there does not exist a transfer t such that $u_f(\mu_f^{f'}) - t \geq u_f(\mu)$ and $u_{f'}(\mu_f^{f'}) + t \geq u_{f'}(\mu)$, with at least one of these inequalities being strict. Or, equivalently:

Definition 1 (Pairwise Stability)**.** *An assignment μ is* pairwise stable *if for every pair $(f, f') \in \mathcal{F} \times \mathcal{F}$*

$$u_f(\mu) + u_{f'}(\mu) \geq u_f(\mu_f^{f'}) + u_{f'}(\mu_f^{f'}).$$

We remain agnostic as to the exact nature of any bargaining or distribution of any pairwise surplus from a switch, but maintain the condition that should a pairwise reassignment be improving, that it be carried out. It is useful to contrast the notion of stability we use, exchange of an assigned object between two faculty f and f', and the blocking-pair notion of stability in two-sided matching, where a faculty-office pair (f, o) would block an assignment. Due to a lack of agency or preferences on the side of the offices, the blocking coalition is of the same size, two agents, but on just one side of the market. A similar comparison could be made with the stability notion used in the networks literature (see, e.g., Jackson, 2004).

[31] Unlike matching settings without externalities, in which an agent's utility depends solely on their own match, externalities imply that utilities may depend on the entire assignment.

We note two important observations regarding the assumptions underlying this definition. First, for technical tractability, our stability notion essentially assumes that faculty have myopic (or boundedly rational) beliefs over the process that ensues following a deviation. Indeed, in the presence of externalities, a switch by any pair of faculty affects others uninvolved in the swap. In general, one could contemplate beliefs specifying the reactions of all participants to such a deviation (in which case even existence can be problematic to obtain, see Sasaki and Toda, 1996 and Hafalir, 2008). Second, our notion considers only bilateral swaps, rather than exchanges among larger groups. We choose to focus on pairwise stability for simplicity and to match the behaviorally founded idea that it would be harder for larger coalitions to optimize collectively.[32]

Pairwise stability generates $\frac{(N-1)\times N}{2}$ necessary inequalities. In our data, one faculty moved to a different building after the initial assignment. Therefore, with 72 faculty left in the building after the ex-post swaps, we generate $2,556$ inequalities. We will assume that preferences take the following form:

$$u_f(\mu) := P_{\mu(f)} + \beta R(f;\mu), \tag{1}$$

where P_o represents the physical desirability of office o (its view, exposure, and size) and $R(f;\mu)$ is a vector of network effects specific to f induced by the assignment μ (proximity to coauthors, friends, departmental colleagues, etc.). In fact, throughout our analysis, we will assume that $R(f;\mu)$ depends (linearly) on the number of faculty from each network under consideration that end up on their floor or in their immediate neighborhood. That is, for any faculty f, let $k(f,\mu,l)$ be the number of faculty from network layer l, $l = 1,...,L$ (research, coauthorship, friendship, and so on) that are in f's neighborhood (say, floor) under the assignment μ. Then,

$$u_f(\mu) := P_{\mu(f)} + \sum_{l=1}^{L} \beta_l k(f,\mu,l).$$

This formulation allows for the volume of peers in close proximity to an assigned office to affect

[32] A similar analysis can be completed with three-way swaps, which we also carry out as a robustness check. We return to this point at the end of this section.

the occupants' well-being. For simplicity, we assume that the volume of faculty members not directly connected to the individual has no effect on well-being. This formulation is general in that: (i) Networks could be thought of as bilateral, with each pair of agents constituting a particular layer l, so it is additive separability, not linearity, that places the main constraint on utilities' functional form. (ii) The coefficients $\{\beta_l\}$ are not restricted in sign so that peer effects can be either positive of negative.[33]

Proposition 1, whose proof is given in Appendix B, shows that the market structure we impose allows for the existence of pairwise-stable assignments.[34]

Proposition 1 (Existence)**.** *There exists a pairwise-stable assignment.*

We now add a stochastic term to represent an idiosyncratic component for faculty f's preferences for a match μ so that preferences are represented by:

$$U(f, \mu) := P_{\mu(f)} + \sum_{l=1}^{L} \beta_l k(f, \mu, l) + \varepsilon^f_{\mu(f)},$$

where ε is the match-specific unobserved idiosyncratic error.[35] Given this specification, consider the pairwise-stability condition corresponding to the two faculty members. The physical attractiveness of the office essentially serves as a fixed effect when contemplating a swap, which can be directly compensated for with a transfer. Consequently, pairwise-stability constraints put restrictions on the network components of faculty's utility. Formally, pairwise stability of a match μ translates into the following: For any two faculty f, f', noting that $\mu(f) = \mu^{f'}_f(f')$ and $\mu(f') = \mu^{f'}_f(f)$,

$$\beta \cdot (R_f(\mu) + R_{f'}(\mu)) + \varepsilon^f_{\mu(f)} + \varepsilon^{f'}_{\mu(f')} \geq \beta \cdot \left(R_f(\mu^{f'}_f) + R_{f'}(\mu^{f'}_f)\right) + \varepsilon^f_{\mu(f')} + \varepsilon^{f'}_{\mu(f)}. \qquad (2)$$

[33]However, a layer must be symmetric: should faculty member f value the proximity of f' at x utiles, it must be the case that f' values the proximity of f at x too. This symmetry rules out cycles, where one faculty member desires close proximity to another who desires distance, and is key to the existence result in Proposition 1.

[34]We note that existence of stable assignments in the presence of externalities has been a major hurdle in the theoretical literature on the topic. Our existence result suggests that in environments such as those we study, stability is a managable concept.

[35]Strictly speaking, our existence result does not pertain to these modified utility functions. However, if one assumes that after the error terms are realized, they become common knowledge among the participants, existence follows in much the same way as in our original result.

The inequalities captured in (2) allow us to estimate the underlying parameter vector β.

5.2 Implementation

The set of inequalities defined by (2) serve as the basis for maximizing a score function (see Manski, 1975) defined as:

$$Q(\beta) := \sum_{f \neq f'} \mathbf{1} \left\{ \beta \cdot \left[R_f(\mu) + R_{f'}(\mu) - R_f(\mu_f^{f'}) - R_{f'}(\mu_f^{f'}) \right] > 0 \right\} \tag{3}$$

Three remarks about this objective function are in order. First, note that each term in (3) is defined in terms of a strong inequality. While inconsequential for the estimated parameters themselves, this allows us to get slightly more meaningful optimal score values. For example, in many cases our network measures are sparse–that is, two faculty are not likely to be connected across a particular measure. When individuals are not connected, the corresponding summand in (3) would always be satisfied if the inequality were weak. In particular, the values of the score would be shifted up by the number of faculty pairs who are not connected in any of the network layers relevant for the specification.[36] In our results we report the score with strong inequalities as above, but also with a modified indicator function that assigns value $\frac{1}{2}$ to pairs satisfying the condition with equality, and value 1 to those associated with a strong inequality.

Second, instead of maximizing the score $Q(\beta)$, one can consider a smoothed version of the score, a-là Horowitz (1992), which would be continuous and amenable to differential methods of optimization and would produce a point estimate. However, the point identification assumptions required for smoothed scores are similar to those required in the Manski score setup, and rely on data covariates being continuously distributed. Therefore, although the procedure would produce a point estimate, results would depend heavily on the smoothing function used and would not be asymptotically consistent for an arbitrary smoothing function.

The advantages of the objective given in (3) are threefold: First, the objective is computa-

[36]We stress that, since our scores are lower than the ones obtained with a weak inequality version of (3), there is a difference in magnitude between these scores and the ones found in the literature that employs maximum scores with weak inequalities (see, for instance, Bajari and Fox, 2010).

tionally simple. Second, although we do not meet the conditions required for asymptotic point identification, the boundaries of the estimated intervals are identified. Third, the score specification is robust to heteroskedasticity of the match-specific errors, which seems important in this setting [37]

Finally, the value of the score is invariant to scaling of the parameter β (for any $a > 0$, $Q(a\beta) = Q(\beta)$)–the scale is never identified, and estimation requires a normalization for one of the coefficients that must have a non-zero contribution to preferences. As previously demonstrated through our discrete-choice estimations (see Table 3) and illustrated in Figure 2, locating near department colleagues plays an important role in location choice. In addition, since the average degree corresponding to the department network is high (relative to the other network layers we consider), many of the inequalities in (3) have non-trivial elements pertaining to departmental network effects. We therefore normalize the coefficient for the proximity of a departmental neighbor to 1, denominating the remaining variables in terms of foregone departmental neighbors. In order to further justify this normalization, Table 5 provides the score Q when accounting for only one layer of the network. Since the magnitudes of the relevant coefficient β cannot be calibrated, we look at the scores for $\beta = 1$ and $\beta = -1$. Table 5 reports the score Q for both the entire data set and the subset of observations corresponding to participants of the extended survey. The *Department Neighbor* variable is the one generating the highest score levels over the full sample, and the second highest over the extended-survey sample.[38]

[37]Estimations were performed using *Mathematica*'s differential evolution algorithm, which has good properties when used to find global extrema of optimization problems (see Fox, 2010 and Santiago and Fox, 2008).

[38]Note that, as stressed above, we would have obtained higher scores using a weak inequality version of (3). In particular, the only inequalities that we would not have been able to predict are the ones explained by a negative coefficient ($\beta = -1$) when inequalities are strict. For instance, *Department Neighbor* would have predicted 2219 inequalities with a positive normalization. We also mention that considering interaction terms maintains the *Department Neighbor* variable as dominant in terms of number of inequalities explained.

Table 5: Single-Variable Explanatory Power

Normalization	$\boldsymbol{\beta = 1}$		$\boldsymbol{\beta = -1}$	
	Full	**Ext. Survey**	**Full**	**Ext. Survey**
Department Neighbor	51.8%	53.7%	13.2%	11.0%
Department Floor	41.6%	42.3%	16.4%	14.8%
Coauthor Neighbor	42.2%	51.0%	7.6%	6.5%
Coauthor Floor	40.0%	43.3%	11.3%	10.9%
Friend Neighbor	-	58.7%	-	9.9%
Friend Floor	-	50.6%	-	10.8%
Number of Inequalities	2556	1431	2556	1431

Note: The column headed $\beta = 1$ ($\beta = -1$) corresponds to a positive (negative) normalization for variables. The scores correspond to the percentage of inequalities predicted with objective (3).

5.3 Results

Holding constant the *Department Neighbor* normalization discussed above, we now estimate the intensities of each network layer relative to this variable. Our results are given in Table 6. We report estimates derived from the maximization of (3). The coefficients are reported as an identified interval $\left(\underline{\beta}^i, \overline{\beta}^i\right)$ for the specific variable i, where $\underline{\beta}^i$ is the minimal coefficient that maximizes the objective (3), and $\overline{\beta}^i$ is the maximal one.[39] That is:

$$\begin{aligned}
\underline{\beta}^i &= \arg\min \left\{ \tilde{\beta}^i \mid \tilde{\beta} \in \arg\max_{\beta \in \mathbb{R}^L} Q(\beta) \right\} \text{ and} \\
\overline{\beta}^i &= \arg\max \left\{ \tilde{\beta}^i \mid \tilde{\beta} \in \arg\max_{\beta \in \mathbb{R}^L} Q(\beta) \right\}.
\end{aligned}$$

This approach is required by the lack of point identification for β, which we discuss below. Roughly speaking, the objective defines a set of linear inequalities that generically would have multiple maximizing solutions. The inherent multiplicity of solutions is caused by two factors: (i) a discontinuous objective; and (ii) integer measures for our network layers.[40]

[39]Similarly, confidence intervals reported in Table 6 are constructed using the minimal coefficients for the lower limits and the maximal coefficients for the upper limit, a particularly conservative approach to finding the 95% confidence interval.

[40]Note that when the specification entails more than one coefficient in addition to the normalized one, $\underline{\beta}$ and $\overline{\beta}$ need not maximize the objective. In particular, focusing on a set of coefficients with minimal absolute values provides a conservative assessment of the effect of the corresponding variables.

Table 6: Pairwise-Stability Estimates

Variable	$PS(i)$	$PS(ii)$	$PS(iii)$
Department Neighbor	1	1	1
Department Floor	(-0.07, 0.08) [-0.33,0.39]		
Coauthor Neighbor		(4.00,5.00) [2.15,∞)	
Coauthor and not-Friend Neighbor			(3.00,4.47) [0.35,7.64]
not-Coauthor and Friend Neighbor			(0.00,0.50) [-0.84,1.90]
Coauthor and Friend Neighbor			(6.07,9.41) [2.95,∞)
Score$^{\blacktriangle}$	84.1% 69.3%	84.9% 75.2%	87.1% 82.2%
Number of Inequalities	$2,556$	$2,556$	$1,431$

Note: 95% confidence score derived from $2,000$ subsamples comprised of inequalities for 250 randomly selected faculty pairs. The confidence regions below the estimates represent the projection of the 95% confidence set projected onto the real line. *Department Neighbor* is normalized to 1 for scale identification. ($^{\blacktriangle}$) The top percentage denotes the fraction of all inequalities satisfied and the bottom percentage is the fraction of inequalities satisfied with weak inequalities receiving a weight of 1/2.

The first column in Table 6, titled $PS(i)$, gauges the relative importance of the micro- and macro-neighborhoods for department members. *Department Floor* is a count of the number of department colleagues currently located on the same floor for the particular faculty-office pair under consideration. The specification produces no significant effect for floor organization. However, even though the floor coefficient is small in size, the negative sign still explains approximately 500 inequalities over the single variable *Department Neighbor*. The negative sign could be interpreted as a decreasing-returns effect to *Department Neighbor* with which it is (unsurprisingly) correlated. Given the small estimated size of the coefficient, and the insignificance of the effect, we conclude that local neighborhood proximity is much more important than floor proximity.

In $PS(ii)$, we evaluate the importance of the coauthorship network relative to the department network. The first point to note from $PS(ii)$ is the large and significantly positive effect coming from *Coauthor Neighbor*. In particular, looking at the assessed interval we find that having a single coauthor nearby is enough, ceteris paribus, to compensate for four to five department colleagues in neighboring offices. The constructed inference region for this coefficient is large, and should be interpreted as inferring that with 95% confidence, the coefficient exceeds 2.15. The presence of ∞ in the estimates implies a lack of identification for the upper bound. When appearing in the confidence interval, it implies that in 5% or more of subsamples there is a lack of identification.

Finally, we introduce data from the friendship network in $PS(iii)$. In this specification we only include those inequalities corresponding to swaps between the 54 members of the extended-survey network.[41] Consequently, the sample size decreases by approximately 45%. In this specification, to avoid multicollinearity problems, we decompose the coauthorship and friendship networks and obtain the orthogonal variables *Coauthor and not-Friend Neighbor*, *not-Coauthor and Friend Neighbor*, and *Coauthor and Friend Neighbor*.

In our previous empirical approaches, in Sections 3 and 4, the friendship and coauthorship networks appear to have a comparable impact on the final outcome. Specifically, the coefficients in Table 3 represent similar effects from each network on the probability of choosing a particular office, and Table 4 points to a similar level of additional sorting in each network. In contrast, we find that the friendship network does not have a strong effect on office choice with respect to the other network layers. This result is due to an important difference between this approach and the previous techniques: indeed, the stability estimation incorporates the information derived from the lack of ex-post swaps that could have produced greater proximity among friends. However, the coefficients associated with *Coauthor and Friend Neighbor* in $PS(iii)$ illustrate that friendship and coauthorship are important together, exceeding a coefficient of 2.95 with 95% confidence.[42]

[41] However, the vector of characteristics for each of the remaining faculty are calculated using data on the entire population. For example, a faculty member outside of the extended-survey sample will still be counted as a coauthor/department neighbor when considering the observed assignment or a prospective swap.

[42] Specifications including floor-level variables for both coauthors and friendships found results similar to

The reported confidence intervals corresponding to the maximal score estimators utilize an i.i.d. subsampling methodology (for additional details on subsampling see Romano and Wolf, 1999; as it applies to maximum score estimators, see Delgado, Rodriguez-Poo, and Wolf, 2001) to estimate the 95% confidence interval of the maximum score estimator.[43,44]

Regarding the econometric techniques we employ and the estimated errors, Manski and Thompson (1986) present simulation evidence suggesting that the bootstrap method allows a good approximation for the estimator's root-mean-squared error. There is little additional evidence on the performance of the bootstrap method in this specific application before Delgado, Rodriguez-Poo, and Wolf (2001), who provide a theoretical justification for subsampling and present simulation evidence suggesting inconsistency of the bootstrap method.

We would like to point to some issues pertaining to the asymptotic identification assumptions required for these estimation methods. The assumptions in Bajari and Fox (2010) require that, given enough data, for any parameter vector different from the true one, more inequalities would be violated. In a setting in which all of the underlying explanatory variables are related to social-network degrees, one may expect problems since: (i) network degrees (and, in fact, many other network characteristics) are discrete by nature; and (ii) it is unlikely that individuals have unboundedly many social connections, even as we consider arbitrarily large samples. Point identification is therefore inherently problematic. In that respect, we find the intervals that maximize our objective function appealing in that they are pessimistic in terms of coefficients' significance.

We performed a series of robustness checks on our results. First, one might be concerned

$PS(iii)$. While the inclusion increases the score slightly, the magnitude and significance of the coefficients remain similar. This leads us to believe that floor-level variables do not have powerful explanatory value. Similarly, the inclusion of variables related to research clusters do not add explanatory power to the specifications $PS(ii) - PS(iii)$.

[43] For additional econometric results on partial identification see Imbens and Manski (2004), Chernozhukov, Hong, and Tamer (2007), Stoye (2009), and Romano and Shaikh (2010), as well as references therein.

[44] We also estimated the confidence regions using the Romano and Shaikh (2010) methodology. Results are qualitatively similar. Nonetheless, the subsampling results that we report provide tighter confidence intervals than those produced using the Romano and Shaikh technique, which, in our setting, are insensitive to the confidence level.

that senior and junior faculty behave differently in the assignment process.[45] Namely, they could have access to different resources to carry out trades, or they may have differing preferences over office attributes. To address the first concern, we run a set of estimations in which faculty are allowed to trade only within their tenure status (i.e., seniors with seniors, and juniors with juniors). As for the second concern, we also run estimations separately for senior and junior faculty. Both sets of estimations yield results similar to the ones we report.[46] Furthermore, we considered an alternative stability notion allowing three-way swaps, which yields virtually identical estimates to those reported. Finally, we obtained auxiliary data regarding the location of faculty prior to the move and generated a fourth network of connections based on the proximity of the old offices. We then replicated the stability estimations including additional variables reflecting this fourth network layer. In these estimations, the variables associated with previous proximity turn out not to be significant, while the other variables exhibit similar magnitudes to the above.

6 Welfare

Having established the importance of network externalities in individual preferences, a natural next step is identifying the socially optimal assignment, and evaluating the mechanism implemented by the school, as well as alternative variations of the serial-dictatorship mechanism. In this section, we illustrate techniques for doing so.

The analysis in the previous section allowed us to determine the network layers and the proximity notion that impact individuals' utilities the most. In this section we first identify a pairwise-stable assignment that, under the estimated preferences, would increase overall efficiency relative to the one implemented. We therefore provide a lower bound on the welfare loss generated by the observed assignment. We also inspect the trade-off between efficiency

[45] In fact, regression analysis using survey responses reveals that the only significant source of heterogeneity in preferences is associated with seniority levels with respect to office size. Specifically, senior faculty exhibits a stronger preference for large corner offices, which represent only 10% of the offices under consideration.

[46] However, the results do indicate a slightly stronger response to coauthors' proximity among seniors relative to juniors.

and 'fairness' of outcomes across different classes of faculty. Finally, under some simplifying assumptions, we estimate the welfare generated by commonly used variations of the serial dictatorship mechanism in the presence of network externalities.

6.1 The First-Best Assignment

Given the utility specification introduced in the previous section, *an efficient assignment* μ^* must satisfy (using our previous notation)

$$\begin{aligned}\mu^* \in \arg\max_{\mu\in\Theta} \textstyle\sum_{f\in\mathcal{F}} u_f(\mu) &= \arg\max_{\mu\in\Theta} \textstyle\sum_{f\in\mathcal{F}} \left[P_{\mu(f)} + \sum_{l=1}^{L} \beta_l k(f,\mu,l)\right] \\ &= \arg\max_{\mu\in\Theta} \textstyle\sum_{f\in\mathcal{F}} \sum_{l=1}^{L} \beta_l k(f,\mu,l),\end{aligned} \tag{4}$$

where Θ is the set of all possible assignments, and the second equality follows from the fact that the utility derived from offices' physical characteristics is homogeneous across agents.

In light of the preference-estimation results in Section 5, we now assume that the only links that matter to agents are those between departmental colleagues and coauthors in local neighborhoods. Even with this simplifying assumption, the problem of finding the most efficient assignment is still not trivial: there are $|\Theta| = 72! > 10^{103}$ possible assignments and the problem is inherently combinatorial, so differential techniques cannot be employed.[47] As it turns out, the problem fits into the class of *Quadratic Assignments Problems* (QAP), first described in Koopmans and Beckmann (1957) in the context of locating industrial plants with spillovers and transportation costs. Specifically, let $b_{ff'}$ (a generic entry in a symmetric $(N \times N)$-matrix B) be the overall intensity of network externalities between any two agents f and f' (obtained by summing up the links present in the different networks, weighted by their estimated relative importance β).[48] Also, let $h_{oo'}$ (a generic entry in a symmetric $(N \times N)$-matrix H) be a variable that takes value 1 if an office o neighbors an office o', and 0 otherwise. Finally, an *assignment matrix* X is an $(N \times N)$-matrix, where each row and column are made

[47] We note that the subtlety of the network architectures makes this a more intricate problem than others pertaining to efficient design in the presence of complementarities, such as, say, FCC spectrum allocations.

[48] Note that this formulation takes bilateral links as the de-facto networks. The values appearing in the matrix B essentially identify the coefficients $\{\beta_l\}$ in our utility specification, the utility flows between members in the (bilateral) network.

up of $N-1$ entries of 0 and a single entry of 1, with element x_{fo} corresponding to whether office o is assigned to agent f; the row and column restrictions guarantee a single assignment for each faculty member, and one faculty member to each office. Let Π be the set of all assignment matrices.

The problem of finding the most efficient assignment μ^* can therefore be specified in the Koopmans-Beckmann formulation as:[49]

$$\max_{X\in\Pi}\sum_{f=1}^{N}\sum_{f'=1}^{N}\sum_{o=1}^{N}\sum_{o'=1}^{N} b_{ff'}h_{oo'}x_{fo}x_{f'o'},$$

Note that the matrix $BXHX^T$ has generic element $\sum_{f'=1}^{N}\sum_{o'=1}^{N}\sum_{o=1}^{N} b_{ff'}x_{f'o'}h_{o'o}x_{f'o}$. Thus, using the symmetry of H and changing summation order, the trace of $BXHX^T$ is equal to the objective function above. Therefore, we have the following proposition:

Proposition 2 (Quadratic Assignment Problem). *Finding an efficient assignment μ^* is a Quadratic Assignment Problem–that is, it is equivalent to identifying a matrix $X\in\Pi$ that maximizes* $\operatorname{tr}\left\{BXHX^T\right\}$*, where* $\operatorname{tr}\{\cdot\}$ *is the trace operator.*

The QAP has been shown to be NP-hard; in fact, even the problem of finding an ε-approximation is computationally complex.[50] Full solutions to this class of problems are still considered numerically intractable for $N>30$. We therefore assess a lower bound for the potential efficiency gain by identifying an alternative pairwise-stable assignment using an ant-colony algorithm (see Dorigo, 1992 and Dorigo, Di Caro, and Gambardella, 1999), which has been demonstrated to be effective in finding optima of the QAP.[51,52] This algorithm is

[49]The generalized formulation of QAP allows for an arbitrary term c_{ijlm} in place of $b_{ij}h_{lm}$. For details see Çela (1998).

[50]The proof that the problem is NP-hard can be seen by reinterpreting the locations as a time sequence of visits to differing cities, with b_{ij} representing the distance between a city pair, and h_{lm} assuming value 1 if l and m are sequential time-periods. This reinterpretation gives the fairly well-known NP-hard Traveling Salesman's Problem. The complexity of an approximation of the QAP is demonstrated in Sahni and Gonzalez (1976). They show that if an ε-approximation can be found in polynomial time, then $P=NP$.

[51]The efficiency gain we identify represents a lower bound on the potential efficiency gain only because, as is common in this literature, the most efficient assignment we identify represents a local maximum, but we are unable to tell whether it is also global.

[52]We can also identify a slack upper bound for the problem using the properties of the trace and the

discussed in more detail in Appendix C.

Using specification $PS(iii)$ in Table 6 from Section 5, we consider three combinations of coefficients corresponding to the variables *Coauthor Neighbor* and *Coauthor and Friend Neighbor* relative to *Department Neighbor*. The first corresponds to the lower bounds of the estimated intervals: 3 and 6, respectively; The second corresponds to the lower bounds of the confidence intervals: 0.35 and 2.95; The third approximates the midpoint of the estimated intervals: 3.75 and 8.

Figure 3 illustrates the most efficient assignment found by the algorithm under the first combination of coefficients, 3 and 6.[53] The comparison with the observed assignment in Figure 2 suggests the potential for more network clustering in the matching process. In fact, for this combination of coefficients, *the algorithm identifies a lower bound on the potential network efficiency gain of* 183%.[54] An identical welfare increase is generated when the coefficients are the midpoints of the estimated intervals. When the coefficients correspond to the lower bounds of the confidence intervals, the generated network-efficiency increase is 158%.

Given these results, we conclude that the assignment selected by the serial-dictatorship mechanism in place appears suboptimal. However, we stress that the limitations of our data set force us to assume homogeneous preferences across faculty. The efficiency of the observed assignment could, in principle, improve with respect to our estimates if individuals put different weights on network externalities. For example, if faculty members that care less about externalities tend to be more senior, they may sort themselves toward better offices at the cost

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eigenvalues of B and H. Given the ordered eigenvalues of H and B—($\upsilon_1 \leq \cdots \leq \upsilon_N$) and ($\rho_1 \leq \cdots \leq \rho_N$), respectively—we can give a simple upper bound on welfare since we have

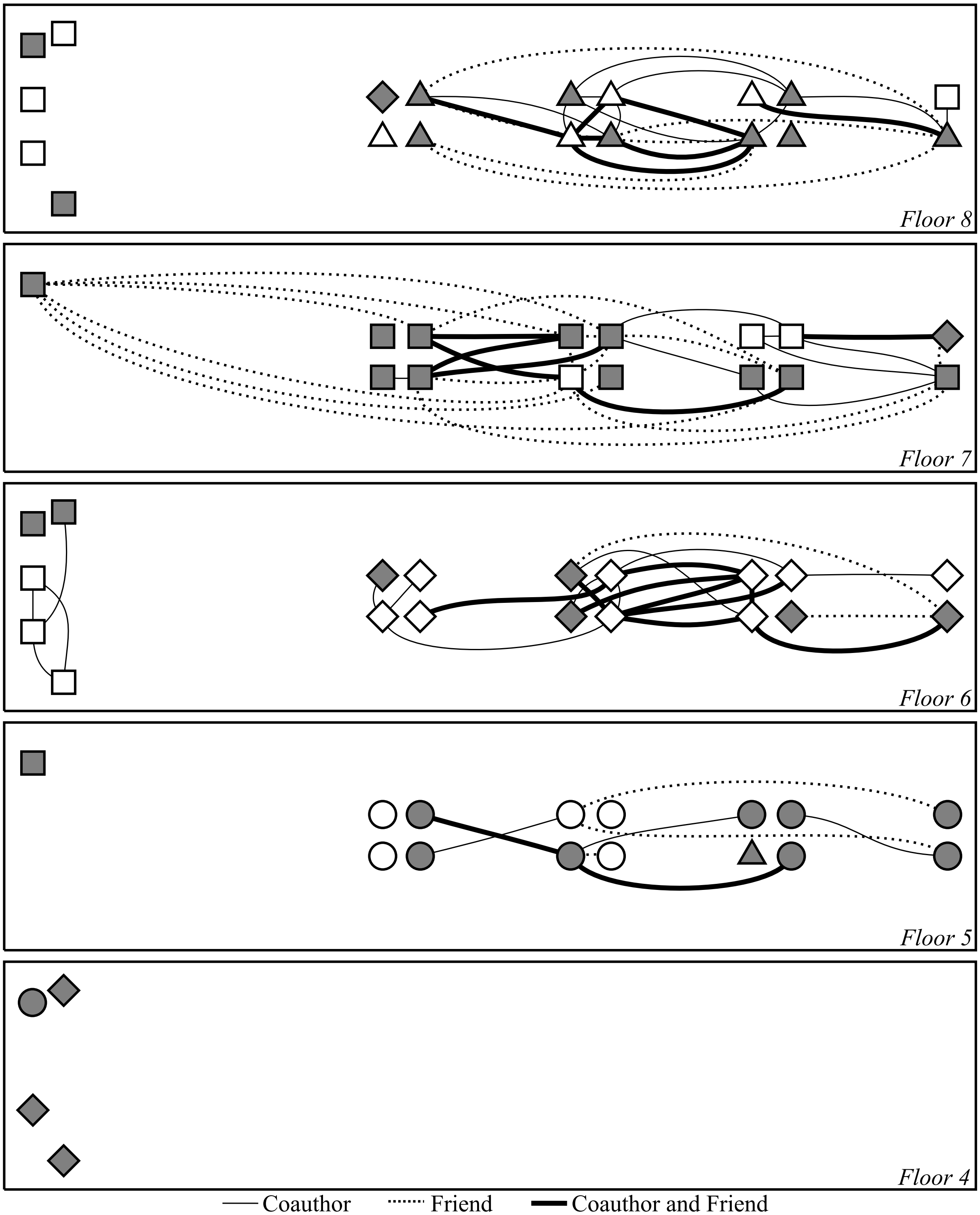
$$\operatorname{tr}\left\{BXHX^T\right\} \leq \sum_{i=1}^{N} \upsilon_i \rho_i$$

for any $X \in \Pi$. This points to an upper bound on the network gain of 270% to 337% across our three specifications for β.

[53]The most efficient assignment is not unique. Indeed, notice that floors 5 and 7 are interchangeable as are floors 6 and 8.

[54]We note that despite the fact that friendship enters the utility specification only through *Coauthor and Friend Neighbor*, the number of friendship links in local neighborhoods increases from 18 in the observed assignment to 32 in the best-found assignment.

Figure 3: The Best-Found Assignment



Note: Differing shapes represent department affiliations; Circles research clusters; Node shading represents seniority level, with white for seniors, and gray for juniors.

of less network links, allowing others to generate more high-efficiency links among themselves (however, as discussed in Section 5, allowing junior and senior faculty to put different weights on network externalities, yields results similar to the homogeneous preferences case).

6.2 Fairness Properties

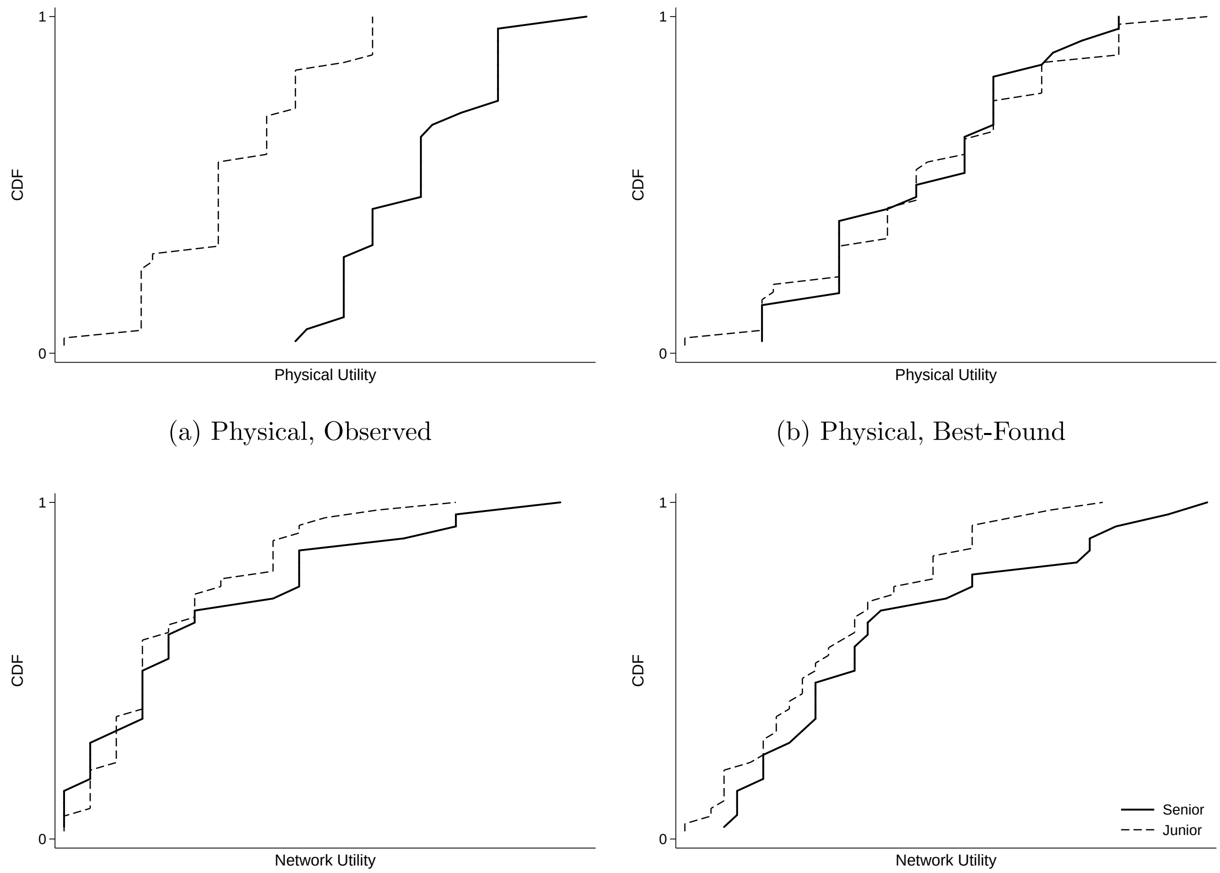
One aspect traditionally addressed by the matching literature has to do with fairness (see, for instance, Chapter 7 in Moulin, 2003 for an overview). Namely, one might desire a rather homogeneous division of welfare across different classes of individuals. Therefore, a natural question that arises is whether efficiency in our context comes at the expense of fairness.

In our application, there are two obvious dimensions according to which faculty can be classified: seniority and department affiliation. It is interesting to compare faculty outcomes at different seniority levels and across different departments under the observed assignment and the best-found one. Distributions over the assessed outcomes under each assignment are presented in Figures 4 and 5.

Panels (a) and (b) in Figure 4 illustrate the comparison between seniors' and juniors' physical outcomes, evaluated according to the weights estimated in $CL(i)$ for the physical office attributes.[55] As can be seen, the wedge between outcomes experienced by different seniority levels is substantially greater under the observed assignment relative to the best-found one. Panel (a) shows that in the observed assignment the physical utility distribution of senior faculty first-order stochastically dominates that for the junior faculty. This is intuitive: senior faculty chose first, and got better selections. Note that in Figure 2, white nodes, corresponding to senior faculty, are located predominantly on one side of the building, the more desirable western size. In contrast, in the best-found assignment, represented in Panel (b), the distributions for juniors and seniors are much closer.

In terms of network utility, junior and senior faculty experience similar outcomes under both assignments, as can be seen in panels (c) and (d). Ultimately, in terms of seniority,

[55] Note that efficiency levels remain the same if individuals on floors 5 and 7 or floors 6 and 8 are exchanged. Such a switch could, however, affect fairness levels (if juniors and seniors are not uniformly distributed across floors). Hence, exchanges of floors could, in principle, raise the similarity of outcomes in Panel (b).



(a) Physical, Observed

(b) Physical, Best-Found

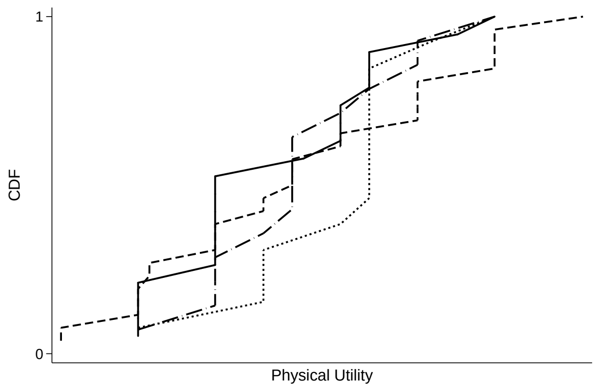
(c) Network, Observed

(d) Network, Best-Found

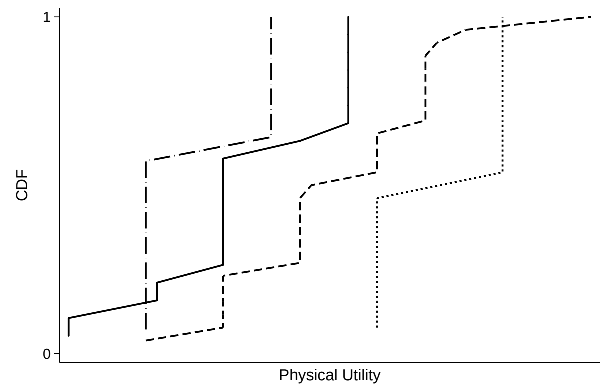
Figure 4: Utility Cumulative Distribution, by Seniority

there does not seem to be a trade-off between fairness and efficiency. In fact, the best-found assignment appears to generate more egalitarian outcomes in terms of physical office attributes.

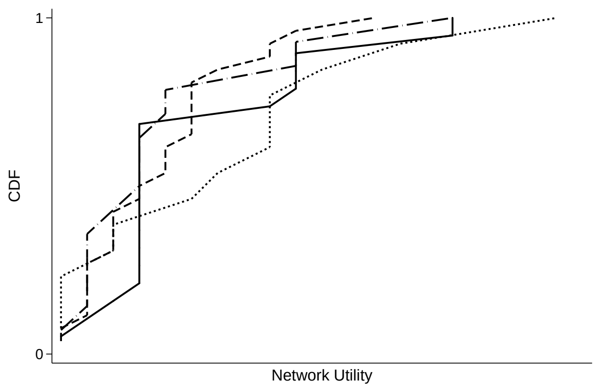
Figure 5 illustrates outcomes for faculty within different departments under the observed and best-found assignments. Panels (a) and (b) suggest that, with respect to department, there could be a tension between efficiency and fairness. Indeed, under the best-found assignment, the physical utility distribution of the departments are first-order stochastically ranked. The underlying reason for this is that, while efficiency pushes same-department faculty to be placed in proximity, this is likely to result in individual departments dominating



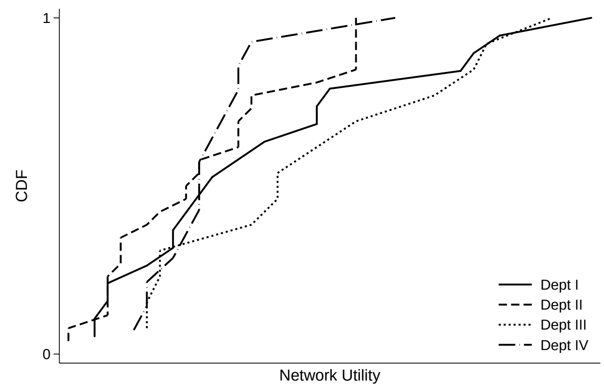
(a) Physical, Observed



(b) Physical, Best-Found



(c) Network, Observed



(d) Network, Best-Found

Figure 5: Utility Cumulative Distribution, by Department

different floors. Consequently, departments occupying higher floors will experience greater physical utility levels.[56] Panels (c) and (d) suggest that, even in terms of network utility, the best-found assignment entails greater variance in outcomes across departments.

6.3 Welfare Properties of the Serial-Dictatorship Mechanism

Our results thus far allow the evaluation of different mechanisms. Indeed, our estimates from Section 5 provide individual utilities, while the analysis of Section 6.1 provides a benchmark

[56]The fact that efficient outcomes imply more fairness across seniority levels but less fairness across departments is robust to aggregating physical and network utilities into an overall utility using the methodology presented in the next Section 6.3.

against which to compare any mechanism in welfare terms. In this section, we consider a class of variations of the mechanism implemented by the school. Namely, we consider several natural re-orderings of the faculty, and as is often the case in standard implementations of serial dictatorship, we do not allow for ex-post swaps.

As before, assessing the welfare properties of these mechanisms by calculating the corresponding Nash equilibria is computationally unfeasible. We therefore make simplifying assumptions on agents' strategic sophistication in predicting subsequent choices. In particular, we assume that each agent believes that all successors will select the office preferred according to its physical attributes.[57]

We simulate the following three versions of serial-dictatorship: *Random Ordering*, under which faculty are allocated a draft order at random; *Seniority-Random Ordering*, in which higher seniority levels are given priority, and draft order within each seniority level is determined at random (as in the implemented mechanism); and *Department-Random Ordering*, in which departments choose in sequence, and within departments, members are ordered randomly.[58]

To compute the overall utilities obtained by these mechanisms, we exploit the observed choices in our data as follows. We use the simplified beliefs described above to generate, for each individual's feasible office at the time of choice, a projected final assignment. Using the estimates of $PS(iii)$ to weight the relative importance of the network attributes, we can therefore simulate the likelihood function for each choice, and estimate the scale of the network components vis-à-vis the physical ones by maximizing this likelihood. The estimated marginal effects of the network component on utility are reported as *Network Utility Scale* in Table 7.

As in Section 6.1, Table 7 includes three specifications for the relative network weights (derived from the results of $PS(iii)$), using the lower bounds of the estimated intervals, the lower bounds of the confidence intervals, and the mid-points of the estimated intervals. We

[57]This notion is reminiscent of the level-1 behavior described in the cognitive hierarchy literature (see, e.g., Dahl and Wilson, 1994, 1995, Crawford, Costa-Gomez, and Broseta, 2001, and references therein).

[58]Among all possible department orderings, we report results for the one generating the highest welfare levels.

name these specifications $S(i)$, $S(ii)$, and $S(iii)$, respectively. In each column, we first report the network weights used. The *Lower Bound* is the portion of the utility derived from physical office attributes only. Since we assume that faculty value offices' physical characteristics identically, this value does not depend on the assignment chosen and represents a lower bound on the overall utility obtained by any assignment. The remaining percentages are all expressed relative to the relevant best-found assignment detailed in Section 6.1. Each mechanism's performance is measured in two ways: in terms of the network utility, and in terms of the overall utility (the latter reported in square brackets and calculated using the *Network Utility Scale*).

The best ordering we identify corresponds to individuals belonging to the same department choosing in sequence. In particular, the *Department-Random Ordering* obtains up to 54.5% of the maximal network utility and 84.6% of the maximal overall utility level (see $S(ii)$). However, the wedge between the maximal welfare generated by serial dictatorship and the first best assignment remains important. In fact, even in $S(ii)$, the best performance of the serial dictatorship mechanism generates utility levels that approximately mid-way between the lower bound on utility levels (given by 66.2%) and the best found assignment (100% by construction).

As a robustness check, we have also conducted similar analyses with more sophisticated beliefs. Specifically, we suppose that faculty members believe that subsequent individuals select offices under the assumption that *their followers* will base their decision only using offices' physical characteristics (thereby adding another 'level' to the cognitive process). The results are similar to the ones obtained with the described, more naive, belief specification.

7 Conclusion

We document an assignment protocol of faculty to offices in which locations (offices) varied in physical characteristics. We elicited three layers of network connections: institutional and choice-based (coauthorship and friendship). Our data allow us to study the role of network

Table 7: Welfare Analysis of Serial Dictatorship

	$S(i)$	$S(ii)$	$S(iii)$
Network Weights			
Department	1.00	1.00	1.00
Coauthor	3.00	0.35	3.75
Coauthor and Friend	6.00	2.95	8.00
Estimates			
Network Utility Scale (†)	7.3%	11.9%	5.9%
	(1.5)	(2.2)	(1.2)
Lower Bound	67.8%	66.2%	69.2%
Simulations			
Seniority-Random Ordering	36.2% [79.5%]	40.3% [79.5%]	35.9% [80.3%]
	(5.8)	(5.6)	(5.5)
Random Ordering	33.2% [78.5%]	37.8% [79.0%]	32.2% [79.2%]
	(5.2)	(5.6)	(5.5)
Department-Random Ordering	46.2% [82.7%]	54.5% [84.6%]	44.7% [83.0%]
	(6.4)	(6.8)	(6.5)

Note: Standard errors in parentheses under estimates of network effect; standard deviations in the simulations. (†) *Network Utility Scale* is measured as an increase in the offices' selection probability given an additional department link, as in Table 3.

externalities.

Three main insights stand out. First, network externalities have a crucial impact on behavior and final outcomes in the assignment process. Second, the different network layers have unequal impacts on outcomes. Third, from a normative perspective, identifying the relevant networks is important for the design of efficient assignments.

From a methodological point of view, our study suggests the usefulness of a modified notion of stability for the estimation of network externalities in assignment processes. The paper also contributes to the empirical literature on social networks. Namely, we show how to account for the relative impact of different layers of peer connections. We also point out techniques that can be employed to evaluate the welfare performance of assignments in the presence of externalities. Ultimately, this paper highlights the conceptual significance and empirical feasibility of considering network externalities in matching setups.

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Appendix A: The Faculty Survey

In this appendix we report the individual survey questions, the aggregate responses, and, in square brackets, the number of respondents to each question.

Hello and thank you for responding!

Your survey responses will be strictly confidential and data from this research will be reported only in the aggregate. Your information will be coded and will remain confidential. Please start with the survey now by clicking on the Continue button below.

1. Your name:

   Department:

2. How many days a week do usually come into your office? [38 and 37]

Teaching period	1(2.63%)	2(2.63%)	3(13.16%)	4(39.47%)	5(23.68%)	$>$ 5(18.42%)
Non-teaching period	1(2.70%)	2(18.92%)	3(10.81%)	4(32.43%)	5(27.03%)	$>$ 5(8.11%)

3. How many hours (on average) do you spend at the office? [38]

Teaching period	$<$ 2(0%)	$2-5$(5.26%)	$5-8$(39.47%)	$>$ 8(55.26%)
Non-teaching period	$<$ 2(0%)	$2-5$(2.63%)	$5-8$(60.53%)	$>$ 8(36.84%)

4. In a typical week, which days of the week to do you come into your office? [38]

Monday	(18.24%)
Tuesday	(19.50%)
Wednesday	(17.61%)
Thursday	(18.87%)
Friday	(22.01%)
Weekend	(3.77%)

5. Do you try to come to the office when your office neighbors are around? [38]

Yes, I try to coordinate	(31.58%)
I do not think about it	(68.42%)
No, I try to arrive when they are not there	(0%)

6. Please name up to 5 people you have lunch with on a regular basis and specify the number of times in a typical week that you have lunch with each of these. [27]

7. Please name up to 5 of your most recent coauthors within the business school and the year in which you have last worked together. [21]

8. Please name up to 5 personal friends (people with whom you interact socially with outside school at least once a month) from within the business school. [13]

9. Please name up to 5 colleagues that would be valuable for you to have on your floor. [28]

10. On a scale of 1-10, how important to you are the floor (4-8), exposure (east, west, or south), and size (corner office or standard office) for the quality of an office (where 1 is least important and 10 is most important)? [37]

Floor	1(10.81%)	2(5.41%)	3(8.11%)	4(5.41%)	5(10.81%)	6(5.41%)	7(13.51%)	8(8.11%)	9(10.81%)	10(21.62%)
Exposure	1(8.11%)	2(8.11%)	3(5.41%)	4(8.11%)	5(18.92%)	6(8.11%)	7(8.11%)	8(16.22%)	9(8.11%)	10(10.81%)
Size	1(10.81%)	2(5.41%)	3(2.70%)	4(8.11%)	5(18.92%)	6(10.81%)	7(5.41%)	8(13.51%)	9(10.81%)	10(13.51%)

11. For a particular exposure and size of office, please rank the floors from 1-5 (where 1 would be your most preferred floor and 5 would be your least preferred floor). [43]

Floor 4	1(14.29%)	2(0.00%)	3(5.71%)	4(0.00%)	5(80.56%)
Floor 5	1(0.00%)	2(14.29%)	3(11.43%)	4(71.43%)	5(2.78%)
Floor 6	1(8.57%)	2(11.43%)	3(74.29%)	4(5.71%)	5(0.00%)
Floor 7	1(8.57%)	2(68.57%)	3(2.86%)	4(17.14%)	5(2.78%)
Floor 8	1(68.57%)	2(5.71%)	3(5.71%)	4(5.71%)	5(13.89%)

12. On a scale of 1-10, what was the importance of your office neighbors to you prior to moving (where 1 is least important and 10 is most important)? [37]

1(10.81%) 2(0.00%) 3(5.41%) 4(8.11%) 5(2.70%) 6(10.81%) 7(5.41%) 8(8.11%) 9(24.32%) 10(24.32%)

13. If you are part of a particular research cluster within your department, please identify it. [26]

14. On a scale of 1-10, how important is it for you to be on the same floor with members of your own department and research cluster (where 1 is least important and 10 is most important)? [37 and 35]

Department	1(5.41%)	2(2.70%)	3(2.70%)	4(2.70%)	5(5.41%)	6(10.81%)	7(13.51%)	8(16.22%)	9(16.22%)	10(24.32%)
Research Cluster	1(8.57%)	2(0.00%)	3(5.71%)	4(0.00%)	5(2.86%)	6(5.71%)	7(8.57%)	8(8.57%)	9(14.29%)	10(45.71%)

15. On a scale of 1-10, how important is it for you to be a direct neighbor, that is, sit in an adjacent office to, or across the hallway from members of your own department and research cluster (where 1 is least important and 10 is most important)? [37 and 35]

Department	1(16.22%)	2(2.70%)	3(10.81%)	4(5.41%)	5(13.51%)	6(5.41%)	7(10.81%)	8(10.81%)	9(16.22%)	10(8.11%)
Research Cluster	1(8.57%)	2(2.86%)	3(14.29%)	4(2.86%)	5(8.57%)	6(2.86%)	7(8.57%)	8(17.14%)	9(17.14%)	10(17.14%)

16. At the time of your selection, how likely did you think you were to switch offices (where 1 corresponds to no switch and 10 corresponds to sure switch)? [36]

1(22.22%) 2(11.11%) 3(22.22%) 4(8.33%) 5(5.56%) 6(8.33%) 7(2.78%) 8(8.33%) 9(2.78%) 10(8.33%)

17. To what extent was your initial selection of a new office influenced by the possibility of ex-post trade, that is, by how desirable the office would be for others (where 1 corresponds to unimportant and 10 corresponds to very important)? [36]

1(36.11%) 2(5.56%) 3(13.89%) 4(11.11%) 5(5.56%) 6(11.11%) 7(2.78%) 8(5.56%) 9(5.56%) 10(2.78%)

18. Did you exhaust your research account this past year? [34]

Yes	(61.76%)
No	(38.24%)

19. Did you exchange offices with anyone using your research account? [35]

Yes	(11.43%)
No	(77.14%)
Tried but failed	(11.43%)

20. Did you exchange offices with anyone without using your research account? [33]

Yes	(6.06%)
No	(81.82%)
Tried but failed	(12.12%)

21. We would appreciate it greatly if you could describe to us how you would made your decision of office in the space below. [29]

22. Suppose that an additional office were made available and auctioned off in the business school. Please specify your 3 top choices for the location of that office (in terms of floor - 4 through 8 and exposure

- east, west, or south) and the maximal bid you would be willing to pay out of your research account in order to move from your current allocated office to the new available one. Thus, if you specify an amount X for any particular office, and all other bids fall below that, you would move to that office and pay X out of your research account. If any other bid surpasses X, you would stay in your current office. If several other colleagues would specify precisely the same X, we would randomly select one of you and exchange their office for X out of their research account.

Appendix B: Proof of Proposition 1

Since the number of possible assignments is finite, there exists a most efficient one. Let μ be the most utilitarian efficient assignment. We now show that μ is pairwise stable. Indeed, suppose that faculty f and f' form a blocking pair. That is, there exists some transfer t such that:

$$\begin{aligned} P_{\mu_f^{f'}(f)} + \sum_{l=1}^{L} \beta_l k(f, \mu_f^{f'}, l) + t &\geq P_{\mu(f)} + \sum_{l=1}^{L} \beta_l k(f, \mu, l); \\ P_{\mu_f^{f'}(f')} + \sum_{l=1}^{L} \beta_l k(f', \mu_f^{f'}, l) - t &\geq P_{\mu(f')} + \sum_{l=1}^{L} \beta_l k(f', \mu, l) \end{aligned}$$

and at least one of the inequalities is strict. Since $\mu_f^{f'}(f) = \mu(f')$ and $\mu_f^{f'}(f') = \mu(f)$, summing up these inequalities implies that:

$$\delta \equiv \sum_{l=1}^{L} \beta_l k(f, \mu_f^{f'}, l) + \sum_{l=1}^{L} \beta_l k(f', \mu_f^{f'}, l) - \sum_{l=1}^{L} \beta_l k(f, \mu, l) - \sum_{l=1}^{L} \beta_l k(f', \mu, l) > 0. \tag{5}$$

Suppose that under μ, in office $\mu(f)$, faculty f has $k_1, ...k_L$ connected faculty from layers $1, ..., L$ (including themselves), and, in office $\mu(f')$, faculty f has $r_1, ..., r_L$ faculty from layers $1, ..., L$. Then, after some manipulation, Condition (5) can be written as:

$$\delta = \sum_{l=1}^{L} \beta_l \left(r_l + r_l' - k_l - k_l' + 2 \right) > 0 \tag{6}$$

Consider now the effects on utilitarian efficiency from a shift from μ to $\mu_f^{f'}$. For each layer l, we need to take into account the changes (an addition or reduction of one peer for the shift of f and the shift of f') for all members of that network layer other than f and f'. Using (6), the overall efficiency gain is then

$$\Delta = \sum_{l=1}^{L} \beta_l \left(r_l + r_l' - (k_l - 1) - (k_l' - 1) \right) + \delta = 2\delta > 0$$

in contradiction to μ being the most efficient assignment. ■

Appendix C: The Ant-Colony Algorithm

Ant Colony Algorithms are probabilistic search methods for optimization in combinatorial problems, first introduced by Dorigo (1992). They are named after the natural process they emulate: in the task of finding and retrieving food, ants deposit a pheromone trail along the path between a new food source and their colony. Other ants are attracted by these trails and follow them to the food source, leaving their own trail as they go. Given some randomness in the ants' behavior, and the fact that old pheromone deposits decay over time, the shortest path ends up being chosen more frequently. The pheromones act as a method of communication between the individuals, helping the colony as a whole optimize. The algorithm utilizes a number of probabilistic agents, the 'ants,' that make successive random assignments within a graph. Assignments that are ranked highly by the objective are reinforced through a larger likelihood of occurring in the future, a process that Dorigo, Maniezzo, and Colorni (1996, DMC henceforth) term *autocatalytic*—a self-sustaining positive-feedback process.

In our application, each faculty in $\mathcal{F} = \{1, \ldots, N\}$ and each office in $\mathcal{O} = \{1, \ldots, N\}$ constitute nodes on a completely connected bipartite graph. That is, each 'faculty' node f is connected to each 'office' node o, and vice-versa. Therefore, each edge is indexed by a faculty-office pair (f, o), which represents the assignment of faculty f to office o. The algorithm describes a process in which a probability distribution over the edges of the graph is used to generate M sample assignments. The sample assignments are then assessed by the objective function, and the probability distribution is updated to increase the likelihood of better assignments.

The probability distribution is constructed from two matrices: a *pheromone matrix* $\Xi(t)$, which changes over the algorithm's run, and a fixed matrix Ω, termed *the heuristic*. Each has a specific function within the algorithm. The heuristic, Ω, provides an *ex ante* measure for the desirability of each edge (f, o), and guides the assignments in the early phase of the algorithm. The pheromone $\Xi(t)$ encodes the information learned during the algorithm's run. At the outset, we start with an initial distribution of the pheromone $\Xi(0) = \left\{\xi_{f,o}(0)\right\}$ that assigns a small equal level to all the edges in $\mathcal{F} \times \mathcal{O}$ (i.e., for each faculty-office pair (f, o), $\xi_{f,o}(0) = \varepsilon$). We follow DMC in constructing the heuristic as follows: Given the network connection matrix B, and the office proximity matrix H, we define $\Omega = B\iota(H\iota)'$.[59] That is, the heuristic for each pair (f, o) is given by $\omega_{f,o} = (\sum_{f' \in \mathcal{F}} b_{ff'}) \cdot (\sum_{o' \in \mathcal{O}} h_{oo'})$, the product of the faculty member f's total connection value and the office o's neighbors.[60] As such, higher values in Ω are given to faculty-office pairs in which the office allows for many neighbors and the faculty member has many connected colleagues.

We now outline the assignment algorithm in detail, given the heuristic Ω and a particular starting level of the pheromone matrix $\Xi(0)$. At each iteration $t \geq 0$, the algorithm carries out the following procedure.

(a) Ordering Randomization. Determine a random ordering of faculty, $\{f_1, \ldots f_N\}$.

(b) Faculty Assignment. For $n = 1, .., N$, assign each faculty f_n to a particular location o_n among the offices still unassigned, i.e., $o_n \in \mathcal{O}_n$, where $\mathcal{O}_1 \equiv \mathcal{O}$, and $\mathcal{O}_n \equiv \mathcal{O}_{n-1} \setminus \{o_{n-1}\}$. The probability of a particular location $o \in \mathcal{O}_n$ being chosen for faculty f is given by

$$g^t_{f_n,o} = \frac{\xi^{\rho}_{f,o}(t)\ \omega^{\sigma}_{f,o}}{\sum_{k \in \mathcal{O}_n} \xi^{\rho}_{f,k} \omega^{\sigma}_{f,k}},$$

where $\xi_{f,o}(t)$ is the generic element of the pheromone level matrix in period t, $\Xi(t)$, and ρ and σ are parameters that control the weight given to the heuristic and the pheromone levels, respectively, in determining the location probabilities.

(c) Ant iteration. At each iteration $t \geq 0$, steps (a) and (b) are repeated M times (where each run represents a particular 'ant'). Each repetition generates a candidate assignment $\widetilde{\mu}^t_m$.

(d) Pairwise stability. For all $n = 1, \ldots, N$, $m = 1, .., M$, we then use our notion of pairwise stability to improve the candidate assignment $\widetilde{\mu}^t_m$ through a local search.[61] Formally, the local search starts at $\widetilde{\mu}^t_m$, draws a random faculty pair (f, f') and checks if an office swap

[59] From Section 6.1, recall that B describes the overall intensity of network externalities between any two agents f and f' and H describes the proximity of any two offices o and o'.

[60] In our program, we actually used the normalized heuristic matrix $\widetilde{\Omega} = \frac{\Omega}{\overline{\omega}}$, where $\overline{\omega}$ was the average entry of Ω.

[61] Note that Proposition 1 guarantees, given our assumptions, that any bilateral improvement is also a global improvement.

(with transfers) would be mutually profitable. If it is, the pair swap is implemented and the pairwise-stability process starts over with the new assignment $\left(\widetilde{\mu}_m^t\right)_f^{f'}$. If no office swap is profitable, the process selects another faculty pair (f, f') from those remaining. The local-search algorithm ends when the assignment is pairwise stable.[62] The benefit of adding this step is that the local search algorithm is fairly quick, and can move quite far from the initial candidate assignment $\widetilde{\mu}_m^t$, taking the local network structure into account more explicitly. The initial placements are structured by the ant-colony probabilities, so assignments 'closer' to previous local maxima are more likely to occur, but the algorithm's "deviations" are now used in a more efficient way.[63] The denote the resulting assignment by μ_m^t.

(e) Welfare Computation. The value for the assignment resulting from step (d), μ_m^t, is calculated according to the aggregate welfare function, given by

$$W(\mu_m^t) = \sum_{f \in \mathcal{F}} u_f(\mu_m^t) = \sum_{f=1}^{N} \sum_{l=1}^{L} \beta_l k(f, \mu, l))$$

where, according to the notation used in Section 5, $k(f, \mu, l)$ is the number of faculty from network layer l, $l = 1, ..., L$ that are in f's neighborhood under the assignment μ, and β_l is the estimated coefficient associated with network layer l. Let μ^* be the current best-found assignment. If $W(\mu_m^t) > W(\mu^*)$, we set $\mu^* = \mu_m^t$.

(f) Updating the pheromone matrix $\Xi(t)$ between t and $t+1$. The algorithm changes the distribution from which assignments are drawn by making the assignments that generate high welfare levels more likely to occur. In the simplest formulation, this is achieved by adding the term $\gamma W(\mu_m^t)$ (where γ is a chosen scale parameter) to the pheromone level of all edges used in each assignment. More specifically, for every edge (f, o), we calculate the new pheromone level according to

[62] Because any profitable swap strictly increases total welfare, and the globally best assignment is pairwise stable, this process ends in a finite number of iterations at a pairwise-stable assignment.

[63] In particular, simulation evidence suggests that in lower-dimensional problems, the ant-colony algorithm with local search is better at finding global maxima than the ant-colony alone.

$$\xi_{f,o}(t+1) = \lambda\xi_{f,o}(t) + \gamma \sum_{i=1}^{M} W(\mu_m^t) 1\{\mu_m^t(f) = o\},$$

where λ is the decay parameter of pheromones. If we write the assignment function μ as an assignment matrix $X(\mu)$ (so that $X(\mu)_{f,o} = 1$ whenever $\mu(f) = o$, and 0 otherwise), this has the matrix form:

$$\Xi(t+1) = \lambda\Xi(t) + \gamma \sum_{i=1}^{M} W(\mu_m^t) X(\mu_m^t).$$

The process places higher weight on highly efficient assignments (i.e., assignments that generate more network connections).

Alternatively, it is possible to reward each edge via its contribution to total welfare—that is, we could instead deposit $\sum_{l=1}^{L} \beta_l k(f, \mu, l))$ pheromones on each utilized edge $(f, \mu(f))$. This process is called an *Ant-Quantity* algorithm, as opposed to the previous specification, which is known as an *Ant-Cycle* algorithm. DMC provide simulation results suggesting better performance from the Ant-Cycle specification in the Traveling-Salesman Problem, attributing the effect to the higher saliency for global placements in the latter stages of an algorithm's run. One final variation on the procedure is called an *Elitist Ant Colony*, in which we follow the above algorithm with an additional pheromone component derived from the best assignment μ^* found over the M repetitions. That is, the pheromone update process is given by

$$\Xi(t+1) = \lambda\Xi(t) + \gamma \left[m^* W(\mu_m^t) X(\mu^*) + \sum_{i=1}^{M-m^*} W(\mu_m^t) X(\mu_m^t) \right],$$

where m^* is a parameter representing the number of elitist ants who follow the best-found assignment.

(g) End Condition. Since for any two consecutive periods the objective $W(\mu^*)$ is likely to remain the same, a convergence condition cannot easily be used as an end condition. Consequently, the end-condition for the algorithm is either a certain number of iterations T, or a limit on the run-time. However, during the algorithm's run it is sometimes necessary to reset the process. This is because the pheromone matrix can converge in a way that does not leave enough variation to the stochastic assignment process. One way to avoid this is to reduce the parameter ρ, keeping in mind that, as we do that, the positive-feedback process becomes less important.[64]

In our specific application, a problem arising with the procedure is that location probabilities for faculty members are independent. Ideally, we would like to probabilistically re-sample the location for a defined group. However, the algorithm internalizes the social network structure only through the objective W and the heuristic matrix Ω, and samples deviations of individual members independently. While we added Step (d) to the DMC procedure to mitigate this limitation, more complicated structures might allow for correlated locations of small groups within the innermost loop.

The main advantages of the ant-colony algorithm are its fairly robust global-search properties and relatively simple implementation. In addition, the algorithm seems flexible and easy to customize to particular applications, for example by treating pheromone levels in alternative ways as illustrated above. The main drawbacks are the large number of parameters that need to be specified by the user.

[64] DMC recommends setting $\rho = 1$, and $\sigma = 5$ and $\lambda = 0.5$. However, in general, one may have to experiment with different parameters to uncover potential trade-offs. Decreasing λ leads to a greater ability to forget previous assignments, but decreasing it too much results in a large variance in the path of the algorithm. Increasing ρ leads to a greater focus on the pheromone process and less on the heuristic one, which has negative implications in the early iterations, where we want to use the heuristic to guide search. Similarly, increasing σ places greater weight on the heuristic, which is beneficial in early iterations, but reduces the global search properties in later iterations. In many ways, the algorithm provides an interesting starting point from which one can set up search procedures tailored for a specific problem.