Optimal Pricing of Seasonal Products in the Presence of Forward-Looking Consumers

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Abstract

We study the optimal pricing of fashion-like seasonal goods, in the presence of forward-looking (strategic) customers, characterized by heterogeneous valuations that decline over the course of the season. We distinguish between two classes of pricing strategies: Inventory-contingent discounting strategies, and announced fixed-discount strategies. For the first class, we find a subgame-perfect Nash equilibrium for the game between the seller and the customers. For the second class, we develop an optimization problem for the seller, taking into account the consumers’ response to any feasible pre-committed price path. When inventory is limited, strategic consumers need to consider not only future prices, but the likelihood of stockouts, which depends on other customers’ behavior. Under both classes of pricing strategies, we show that it is optimal for the consumers to purchase according to individual thresholds that depend on personal base valuations and arrival times to the store. We conducted a numerical study to explore the way by which strategic consumer behavior impacts pricing policies and expected revenue performance, and to examine the way by which it interferes with the drivers of the benefits of price segmentation. We discuss the way by which equilibrium in the contingent pricing case is affected by various key factors. We also examine the performance of announced fixed-discount strategies, and argued that pre-commitment can bring an advantage to the seller, of up to 8.32% increase in expected revenues. Unlike the case of myopic customers, under strategic consumer behavior, inventory has a significant impact on the announced depth of discounts, particularly when the rate of decline in valuations is low-to-modest. Finally, we considered the case in which the seller incorrectly assumes that strategic customers are myopic in their purchasing decisions. This misperception can be quite costly, reaching a loss of 20% in expected revenues.

Key Words: Dynamic Pricing, Game Theory Applications, Marketing-Operations Interface, Revenue Management.
1 Introduction

Dynamic pricing and revenue management has gained an increasing popularity in retail settings, and has engendered a growing body of academic research in recent years (for a recent survey, see Bitran and Caldentey, 2003). When applied to the sales of fashion-like goods, sellers need to account for key characteristics of the sales environment, including the scarcity of goods, demand uncertainty, and consumer behavior. In this work, we develop a stylized model to study the optimal pricing of a finite inventory, in the presence of forward-looking customers that time their purchases in anticipation of future discounts. Consumers’ valuations for seasonal goods tend to peak at the beginning of the selling period, and decline with time. Consumers who arrive early in the season usually have higher valuations and lower price sensitivity than those arriving at a later stage. A possible explanation for this property is that early arriving customers are commonly the most eager for the good, while subsequent arrivals exhibit lower need for it (Desiraju and Shugan 1999).

Limited inventory is often a result of the supply chain structure (supplier’s location, production lead times, capacity constraints, etc.; see Abernathy et al. 1999), but it could originate from deliberate marketing strategies. For example, Zara, a large Spanish producer and retailer of fashion goods, limits the number of clothing items in each store in order to create a sense of urgency among consumers (Ghemawat and Nueno 2003). When supply is limited, forward looking consumers are forced to consider not only future prices, but the likelihood of stockouts as well. Since the level of remaining inventory depends on the individual purchase decisions, each customer has to take into account the behavior of other (past and future) customers. Furthermore, the uncertainty about the arrival times of consumers to the store adds significant complexity to the consumers purchasing decisions and the seller’s pricing strategy.

A seller faced by heterogeneous and declining consumer valuations, uncertain arrival times, and constrained to a finite number of units of a product, can ideally maximize the expected revenues from sales by continuously changing the price over the course of the season. In other words, discounts are used as a segmentation mechanism (also known as price skimming): A strategy via which a seller takes advantage of differences in customer valuations in order to increase expected revenues. To reflect settings in which prices cannot be changed (e.g., due to cost and process delays;
We conducted a numerical study to explore the way by which strategic consumer behavior impacts pricing policies and expected revenue performance, and to examine the way by which it interferes with the drivers of the benefits of price segmentation. Our results provide a sharper understanding of the consumers' and seller's strategies in equilibrium. We discuss the way by which the equilibrium is affected by key factors, such as the level of heterogeneity in consumers' valuations, the rate of decline in valuations over the course of the sales horizon, the time of discount, and the initial level of inventory.

Surprisingly, we found that announced fixed-discount strategies perform essentially the same as contingent pricing policies in the case of myopic consumers. Under strategic consumer behavior, we found that pre-commitment to discounts can bring an advantage to the seller, of up to 8.32% increase in expected revenues. We specify the conditions under which it is appropriate to announce discount, but we also argue that under those circumstances, it may be optimal for the seller to restrict himself to a single price policy upfront. We also argued that the sensitivity of the depth
of discount with respect to the rate of decline in valuations appears to me more significant when customers are strategic, compared to the case of myopic customers. Unlike the case of myopic customers, under strategic consumer behavior, inventory has a significant impact on the announced depth of discounts, particularly when the rate of decline in valuations is low-to-modest. Finally, we explore the case in which the seller incorrectly assumes that strategic customers are myopic in their purchasing decisions. This misperception can be quite costly, reaching a loss of 20% in expected revenues. A similar result was demonstrated for other revenue management settings; see, e.g., Besanko and Winston (1990) and Levin et al. (2005).

The paper is organized as follows: In §2 we begin with a review of the relevant literature. In §3 we present the main features of our model. In §4 and §5, we describe the analysis of the two classes of pricing strategies described above, contingent discounting, and announce fixed-discounts, respectively. We then continue in §6 with a presentation of our numerical studies and report on various insights that can be drawn from the study. Section 7 concludes our paper.

2 Literature Review

The relevant literature on dynamic pricing of finite inventory has mostly ignored the effect of forward-looking consumer behavior. Gallego and van Ryzin (1994) and Bitran and Mondschein (1997) analyze a continuous and discrete time pricing schemes, in environments where the distribution of consumer valuations does not change over time. Lazear (1986) and Aviv and Pazgal (2005) discuss dynamic pricing of fashion goods where some characteristics of the demand are unknown. In these papers, prices serve both as a tool for revenue generation as well as a learning mechanism. Desiraju and Shugan (1999) analyze the strategic pricing of capacity-constrained services. They find that yield-management pricing systems work best when price insensitive consumers prefer to buy later than price sensitive consumers. As an alternative to dynamic pricing, Vulcano et al. (2002) utilize an auction framework to characterize the optimal division of a finite inventory over a sequence of a finite number of auctions. For further information on the above body of research, we refer the reader to Elmaghraby and Keskinocak (2003) and Chan et al. (2001).

Research on intertemporal price discrimination that considers forward-looking consumers, such as Stokey (1979) and Lansberger and Meilijson (1985), assume that a monopolist pre-commits to
a price strategy for the entire selling horizon in order to maximize expected revenue. Heterogeneous consumers are present throughout the entire season and optimally select the timing of their purchases to maximize individual surplus. Besanko and Winston (1990) extends the above models by proposing a contingent subgame perfect pricing strategy for the monopolist. They consider a seller facing a finite pool of rational consumers that aim to maximize their individual utilities by optimally timing their purchases. They show that in equilibrium, price skimming strategies should be used by the monopolist. Similarly to Besanko and Winston (1990), we consider the existence of rational, forward-looking customers. Additionally, as in Gallego and van Ryzin (1994) and Bitran and Mondschein (1997), we take into account scarcity of goods (finite inventory levels) as well as uncertain demand processes.

The interdisciplinary research we pursue in this paper joins a set of parallel research projects in the management science field. As is typically the case, different researchers examine the topic at hand from a variety of angles, confining themselves to specific assumptions about relevant key factors; see, e.g., a detailed discussion in the introduction of Liu and van Ryzin (2005). Elmaghraby et al. (2004) consider a monopolist who wishes to sell a finite number of units of a product. The seller determines and commits to a multi-period discounts path (a markdown mechanism). Customers are strategic in nature and may request more than a single unit of the product. The authors compare the expected seller’s profit that can be gained via a markdown mechanism to the expected profits under an optimal single-price policy. Levin et al. (2005) propose a dynamic pricing model for a monopolist selling to a finite population of strategic customers. In their model, pricing decisions are made after each sale of a unit of the product. To model strategic behavior, the authors introduce a per-period discount factor for the customers. When the discount factor is large, customers behave myopically, whereas for a low discount of future utilities the customers are more inclined to wait if they anticipate lower prices in the future; in other words, they behave strategically. The authors prove the existence of a unique subgame-perfect equilibrium in the game between the seller and the customers. Unlike our model, inventory is completely observable by customers. Su (2005) studies a model in which customers belong to four classes, different from each other in two dimensions: High versus low valuations and myopic versus strategic behavior. The demand is modeled as continuous (infinitesimally small), following a deterministic flow at a
constant rate. The class of pricing policies used by the seller is one in which the seller commits to a price path along with a rationing function that specifies the fraction of demand that is fulfilled at any given time. After the specific pricing policy is announced, strategic consumers can weigh the benefits of waiting for a discount (if any is offered). This paper demonstrates that markups are optimal when high-valuation customers are proportionately more strategic, whereas markdowns are optimal if the high-valuation customers are proportionately more myopic. Liu and van Ryzin (2005) develop a stylized model to study conditions under which it is optimal for a seller to create shortages (by understocking products) in order to introduce rationing risk that will discourage strategic consumer behavior. They consider a seller that pre-announces a price path, including a premium for the first part of the sales horizon, and a discount price for the second part of the horizon. All customers are present when the sales begin, and they wait until purchase or until the end of the horizon. The authors analyze the cases in which customers are either risk-neutral or risk-averse. They find that when there is a sufficiently large segment of high-valuation customers that are highly risk-averse, capacity rationing is desirable. The authors also discuss the effectiveness of rationing in an oligopoly context.

3 Model Assumptions and Terminology

We consider a seller that has \( Q \) units of an item, available for sale during a sales horizon of length \( H \). The sales season \([0, H]\) is split into two parts, \([0, T]\) and \([T, H]\), for a given fixed value \( T \). During the first part of the season, a “premium” price \( p_1 \) applies, and during the second phase of the season, a “discount” price \( p_2 \) is set (where \( p_2 \leq p_1 \)). The seller’s objective is to maximize the expected total revenues collected during the sales horizon. Customers arrive to the store according to a Poisson process with a rate of \( \lambda \) arrivals per time unit\(^1\). This arrival process is independent of the pricing policy and the available level of inventory in the store\(^2\). Customers’ valuations of the

\(^1\)Our models throughout this paper can accommodate situations in which the rate of the arrival process is not uniform throughout the season, but rather exhibits a time-dependent pattern \( \lambda(t) > 0 \). Handling this complication can be done via a simple time-transformation that brings the arrival process to a constant. The primary effect of such time transformation is that we lose the nice exponential decline pattern in consumers’ valuations; but again, this is a relatively minor technical concern.

\(^2\)This assumption is consistent with the finding that the arrival pattern of potential customers to retail stores is mainly influenced by their regular purchasing behavior, rather than a function of individual item valuations and prices (see, e.g., Bitran and Mondschein 1997).
product vary across the population, and decline over the course of the season. To reflect this, we use a multiplicative valuation function of the type:

$$V(t) = V \cdot e^{-\alpha t}$$

Specifically, each customer’s base valuation $V$ is drawn from a given continuous\(^3\) distribution form $F$. Then, depending on the particular time of purchase ($t$), the realized valuation is discounted appropriately by a known exponential decline factor $\alpha \geq 0$, fixed across the population. For instance, the case $\alpha = 0$ represents a situation in which consumers are not time sensitive in their valuations. The assumption that valuations decline over the course of the season seems to be prevalent in the sales of fashion and seasonal items (Desiraju and Shugan 1999). In summary, customers are heterogeneous in their base valuations as well as in their arrival times. The assumption that the decline in valuation is at a common rate for all consumers is done to enable technical tractability (see, e.g., Besanko and Winston 1990, and Levin et al. 2005). An interesting alternative approach to model the patterns of consumer valuations, is via time dependent Markovian intensities; see Feng and Gallego (2000) for details.

Our model is characterized by the set of parameters $\{\lambda, T, H, \alpha, Q, F\}$, assumed to be known to the seller and all consumers. Additionally, each consumer has private information about his own valuation and arrival time, while the seller monitors the level of inventory continuously. Consumers only know the initial inventory quantity $Q$.

Upon arrival to the store, each customer observes the price $p_1$ only, but he does not see other customers or the current level of inventory. Therefore, what distinguishes the behavior of one customer from another are just the individual arrival time and the personal base valuation. In our main models, customers exhibit strategic behavior, in that they may decide to postpone their purchases if they believe that a later purchase at a lower price may bring a higher expected surplus than what they can gain by an immediate purchase. Indeed, Holak et al. (1987) demonstrate through an experimental study, that there is a clear correlation between consumers’ expectations about product attributes (including pricing), and the time they buy the product. In our model, customers that arrive prior to time $T$ behave according to the following lines: A given customer $j$,

\(^3\)We confine ourselves to continuous distribution forms in order to simplify the mathematical exposition at some points in our analyses.
arriving at time $t$, will purchase immediately upon arrival (if there is inventory) if two conditions are satisfied about his current surplus $V_je^{-at} - p_1$: (i) it is non-negative; and (ii) it is larger or equal to the expected surplus he can gain from a purchase at time $T$ (when the price is reset to $p_2$). Of course, the latter expected surplus depends on the customer’s belief about $p_2$ as well as the likelihood that a unit will be available to the customer. If the customer purchases a unit, he leaves the store immediately. Otherwise, the customer stays until time $T$. At time $T$, all existing customers take a look at the new price $p_2$ and if they can gain a non-negative surplus, they request a unit of the remaining items (if any). In case there are less units than the number of customers who wish to buy, the allocation is made randomly. After time $T$ new customers buy according to whether or not they can immediately gain a non-negative surplus. Clearly, it does not make sense for a customer to wait in the store after time $T$, since prices will not drop.

The seller observes the purchases, or equivalently, his level of inventory. Hence, the discounted price $p_2$ can be contingent on the remaining inventory at time $T$, as we discuss in the next section.

4 A Contingent Pricing Model with Strategic Customers

In this section, we develop a subgame perfect equilibrium strategy for the game between the customers and the seller. The seller’s strategy is characterized by the initial premium price $p_1$ and the discounted price menu $\{p_2(q)\}_{q=1}^Q$. The customers’ strategy is one that prescribes purchasing decisions for every possible pair of individual arrival time $t$, and base valuation $V$.

4.1 An Optimal Class of Purchasing Strategies

We begin by characterizing the best response of the customers to a given seller’s pure strategy of the form $p_1, p_2(1), \ldots, p_2(Q)$. The response strategy clearly needs to be based on a competitive situation that exists among consumers, which arises due to the fact that an individual consumer’s decision impacts the product availability for others. Theorem 1 below shows the existence of a time-dependent threshold. Consumers purchase the good immediately upon arrival if and only if their valuations are higher than this threshold.

**Theorem 1** For any given pricing scheme $\{p_1, p_2(1), \ldots, p_2(Q)\}$, it is optimal for the customers to base their purchasing decisions on a threshold function $\theta(t)$. Specifically, a customer arriving at
time $t \in [0, H]$ will purchase an available unit immediately upon arrival if $V(t) \geq \theta(t)$. Otherwise, if $V(t) < \theta(t)$ and $t < T$, the customer will revisit the store at time $T$, and purchase an available unit if $V(T) \geq \theta(T)$. The threshold function $\theta(t)$ is given by

$$
\theta(t) = \begin{cases} 
\psi(t) & 0 \leq t < T \\
p_2 & T \leq t \leq H
\end{cases}
$$

(1)

where $\psi(t)$ is the unique solution to the implicit equation

$$
\psi - p_1 = E_{Q_T} \left[ \max \left\{ \psi e^{-\alpha(T-t)} - p_2(Q_T), 0 \right\} \cdot 1(A|Q_T) \right]
$$

(2)

The random variable $Q_T$ represents the remaining inventory at time $T$, and the event $A$ represents the allocation of a unit to the customer upon request.

The strength of Theorem 1 is that it demonstrates the optimality of a threshold-type policy for every individual customers, under any arbitrary purchasing strategies of the others. The left-hand-side of (2) represents the current surplus the customer can gain by purchasing a unit, whereas the right-hand-side of the equation represents the expected surplus that will be gained by the customer if he postpones his purchase to time $T$. The latter expected value takes into account two conditions. The first condition is that the discounted price needs to leave the customer with a non-negative surplus. This is simply given by the condition $\psi e^{-\alpha(T-t)} - p_2(Q_T) \geq 0$, where $Q_T$ is a random variable. The second condition is that in order to materialize a surplus, a unit needs to be available and be allocated to the customer. Provided a specific realization of $Q_T$, the allocation probability depends on the statistical distribution of the number of other customers that postpone their purchases to time $T$. At the moment, we skip the technical details of how to evaluate such allocation probabilities, but we shall get back to this shortly ($\S$4.3).

In our analyses below we will refer to the function $\psi$, defined in the range $[0, T)$, as the customer’s strategy. Following the proof of Theorem 1, one can easily see that

**Corollary 1** The threshold function $\psi(t) : [0, T) \to [p_1, \infty)$ is increasing in $t$.

This property is useful in our analyses below.
4.2 The Seller’s Contingent Pricing Scheme

We now study the seller’s best contingent pricing \( \{p_2(1), \ldots, p_2(Q)\} \) in response to a given purchasing strategy \( \psi \), and a given initial premium price \( p_1 \). We distinguish between five types of customers, as illustrated in Figure 1 below.

![Figure 1: For a given realized price path \((p_1, p_2)\) and a customer threshold purchasing policy \( \psi(t) \), the customer’s space (arrival times and valuations) is split into five areas: (i) 'I' = Immediate buy at premium price; (ii) 'S' = Strategic wait and buy at discounted price; (iii) 'W' = Non-strategic wait and buy at discounted price; (iv) 'L' = Immediate purchasers at discounted price; and (v) 'N' = No buyers. In this context “buy” means a desire to buy.](image)

The first group is of customers that arrive during \([0, T]\) and purchase\(^4\) immediately at price \( p_1 \) (denoted by 'I'). It is easy to see that the expected number of customers of this type is

\[
\Lambda_I(\psi) = \lambda \int_{t=0}^{T} \bar{F}(\psi(t) e^{\alpha t}) \, dt
\]

We define the remaining four types of customers with respect to a realized value of \( p_2 \) at time \( T \). The second group of customers (denoted by 'S') consists of those who could get a non-negative surplus upon their arrival during \([0, T]\) but decided to wait “strategically” (i.e., in anticipation of a better expected surplus at time \( T \)), and indeed want to purchase a unit at the realized price

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\(^4\)We use the term “purchase” to reflect a desire to purchase a unit. Clearly, when inventory is depleted, we assume that the store closes and no further purchases are feasible.
\( p_2 \) (note that it may happen that the actual surplus realized is lower than the original surplus a customer could gain if purchased a unit immediately upon arrival). The number of customers that fall in this category has a Poisson distribution with a mean

\[
\Lambda_S (\psi, p_1, p_2) = \lambda \int_{t=0}^{T} \left[ \tilde{F} \left( \min \{ \max \{ p_1 e^{\alpha t}, p_2 e^{\alpha T} \} , \psi(t) e^{\alpha t} \} \right) - \tilde{F} \left( \psi(t) e^{\alpha t} \right) \right] dt
\]

The third group (denoted by ‘W’) includes those customers who waited for time \( T \) because their valuations upon arrival were below the premium price \( p_1 \), and want to purchase a unit at the realized discount price \( p_2 \). The number of such customers is Poisson distributed with a mean

\[
\Lambda_W (p_1, p_2) = \lambda \int_{t=0}^{T} \left[ \tilde{F} \left( \min \{ p_1 e^{\alpha t}, p_2 e^{\alpha T} \} \right) - \tilde{F} \left( p_1 e^{\alpha t} \right) \right] dt
\]

The fourth group of customers (denoted by ‘L’ for “late”) includes those who arrive at or after time \( T \) with a valuation higher than the posted discounted price \( p_2 \). Clearly, the number of customers in this group is Poisson with a mean of

\[
\Lambda_L (p_2) = \lambda \int_{t=T}^{H} \tilde{F} \left( p_2 e^{\alpha t} \right) dt
\]

Finally, a fifth group (denoted by ‘N’ in Figure 1) includes those customers who do not wish to purchase a unit of the product at any point of time. Note that in this group, we include customers who decided (strategically) to wait time \( T \) in anticipation of a better surplus, but then find out that the realized discounted price \( p_2 \) is higher than their individual valuations.

It is instructive to note that the values of \( \Lambda_S \), \( \Lambda_W \), and \( \Lambda_L \) depend on the value of \( p_2 \), which is generally unknown prior to time \( T \). Therefore, such uncertainty needs to be taken into account by customers who contemplate between an immediate purchase and a strategic wait (see next section). For the seller, the above values play a key role in setting the contingent pricing menu. Let \( Q_T \in \{0, \ldots, Q\} \) be the current inventory at time \( T \). Clearly, if \( Q_T = 0 \), then pricing is irrelevant. When \( Q_T \) is positive, the optimal discounted price \( p_2 (Q_T) \) is chosen so as to maximizes the revenues collected from customers of types ‘S’, ‘W’, and ‘L’. Specifically,

\[
p_2 (Q_T) \in \arg \max_{z \leq p_1} \{ z \cdot N (Q_T, \Lambda_S (\psi, p_1, z) + \Lambda_W (p_1, z) + \Lambda_L (z)) \}, \quad Q_T \in \{1, \ldots, Q\}, \quad (3)
\]

where \( N (q, \Lambda) = \sum_{x=0}^{\infty} \min (x, q) \cdot P (x|\Lambda) \), is the expected value of a Poisson random variable (with mean \( \Lambda \)), truncated at \( q \).
4.3 A Nash Equilibrium

Consider the subgame that begins after the premium price $p_1$ is set. A contingent pricing scheme \{\(p_2(q)\)\} and a purchase strategy $\psi$, followed by all consumers, form a Nash equilibrium in the subgame, if the following conditions are satisfied. First, each price $p_2(q)$ needs to satisfy (3); i.e., it is a best response of the seller if all customers follow the equilibrium strategy $\psi$. Second, the strategy $\psi$ needs to satisfy the conditions of Theorem 1; i.e., it is a best response to the contingent pricing scheme $\{p_2(q)\}$.

In our numerical study, we employed an iterative algorithm to find an equilibrium to the subgame. Each iteration consists of two main steps. In the first step of the $n$-th iteration we find the best contingent pricing policy, $\{p_{n,2}(q)\}$, conditional on the consumers’ threshold strategy $\psi_{n-1}$, calculated in the previous iteration\(^5\); see (3). The expression within the maximum operand in (3) is not necessarily unimodal over the range $[0,p_1]$. We therefore conducted an exhaustive search to identify the optimal price $p_2(q)$ for each one of the $Q$ inventory contingencies. While one could possibly find ways to conduct a more efficient search for the solution to the contingent pricing problem, our objective was not speed.

In the second step of iteration $n$, we used the new values $\{p_{n,2}(q)\}$ to update the threshold function. For each arrival time $t \in [0,T]$, we found the unique value $\psi_n(t)$ that solves (2) in Theorem 1. Specifically, $\psi_n(t)$ is the solution to

$$\psi - p_1 = \sum_{x=0}^{Q-1} P(y|\Lambda_{I}(\psi_{n-1})) \cdot \max\left\{\psi e^{-\alpha(T-t)} - p_{n,2}(Q-x), 0\right\}$$

$$\cdot A(Q-x|\Lambda_S(\psi_{n-1},p_1,p_{n,2}(Q-x))) + A_W(p_1,p_{n,2}(Q-x))$$

where $A(q|\Lambda) = \sum_{y=0}^{\infty} \max\{1+y,q\} P(y|\Lambda) = (1 - \frac{q}{\lambda}) \sum_{y=0}^{q-1} P(y|\Lambda) + \frac{q}{\lambda} [1 - P(q|\Lambda)]$, is the allocation probability of $q$ units. In fact, it is possible to write an exact expression (in a non-implicit form) for $\psi_n$ as a function of $\psi_{n-1}$ and $\{p_{n,2}(q)\}$. Yet, to avoid additional notation and tedious proofs, we decided not to include this result in the paper. We found the exact expressions to be useful in our computational algorithm, since convergence of the sequence $\{\psi_n\}$ can be examined in the space of a finite number of parameters, rather than in the space of functions.

\(^5\)We initialized our algorithm, by setting $\psi_0(\cdot) = p_1$. 

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In search of maximizing the expected total revenue over the sales horizon, the seller needs to pick the best premium price \( p_1 \). This is done with the anticipation that a choice of \( p_1 \) will be followed by the subgame we described above. Given\(^6\) a Nash equilibrium \( \psi^* (p_1) \) and \( \{p^*_2 (q|p_1)\}_q \), the seller is faced with an optimization problem of the following type. Let \( J(q|p_1) \) be the expected revenues collected during the second part of the season ([\( T, H \)]) given the subgame Nash equilibrium strategies:

\[
J(q|p_1) = p^*_2 (q|p_1) \cdot N(q, \Lambda_S (\psi^* (p_1), p_1, p^*_2 (q|p_1))) + \Lambda_W (p_1, p^*_2 (q|p_1)) + \Lambda_L (p^*_2 (q|p_1))
\]

Then, the seller’s task is to maximize the expression

\[
\pi^*_C \ni \max_{p_1} \left\{ p_1 \cdot Q \cdot \left( 1 - \sum_{x=0}^{Q-1} P(x|\Lambda_I (\psi^*(p_1))) \right) \right. \\
+ \left. \sum_{x=0}^{Q-1} (p_1 \cdot x + J(Q-x|p_1)) \cdot P(x|\Lambda_I (\psi^*(p_1))) \right\}
\]

The subscript ‘C/S’ denotes the case of contingent pricing policies with strategic customers.

### 4.4 A Reference Model: The Case of Non-Strategic Consumers

In the case of non-strategic customers, we assume that the trade-off between current and expected future surplus is never considered. Specifically, a consumer arriving at time \( t \in [0, T] \) with valuation \( V(t) \) will purchase a unit (given one is available) if \( V(t) \geq p_1 \). If an immediate purchase is not valuable, the customer will remain in the store until time \( T \) (or practically, come back at time \( T \)), and purchase a unit if \( V(t) \geq p_2 \). Similarly, customers arriving at or after time \( T \) will base their decisions on whether they can gain a non-negative surplus from an immediate purchase at the discounted price \( p_2 \).

It is easy to see that the seller’s optimal strategy in this case follows the model we outlined above (for strategic customers), with \( \psi(p_1) = p_1 \). Computationally, we search for an optimal value of \( p_1 \) that attains the maximum expected revenue

\[
\pi^*_C \ni \max_{p_1} \left\{ p_1 \cdot Q \cdot \left( 1 - \sum_{x=0}^{Q-1} P(x|\Lambda_I (p_1)) \right) \right. \\
+ \left. \sum_{x=0}^{Q-1} (p_1 \cdot x + R(Q-x|p_1)) \cdot P(x|\Lambda_I (p_1)) \right\}
\]

\(^6\)For each and every parameter combination we tested in our study, the above algorithm converged to a fixed point constituting a Nash equilibrium.
where \( R(q|p_1) = \max_{z \leq p_1} \{ z \cdot N(q, \Lambda_W(p_1, z) + \Lambda_L(z)) \} \). The subscript ‘C/N’ denotes the case of contingent pricing policies with non-strategic customers.

5 Announced Fixed Discount Strategies

In this section, we consider a two-period pricing problem in which the seller commits \textit{upfront} to a \textit{fixed} price path \((p_1, p_2)\). Specifically, under this policy class, which we name “announced discount strategies,” the discounted price \( p_2 \) is not contingent upon the remaining inventory at time \( T \). Interestingly, sellers such as Filene’s Basement (Bell and Starr 1994) and Syms use this method for pricing some of their products. Nonetheless, our treatment of this case is not driven by anecdotal evidence. Rather, it is motivated by the following economic puzzle. Consider for a moment the case of myopic customers. Clearly, in such settings, a fixed discount is generally not optimal. The seller could obviously increase his expected revenues by waiting until time \( T \) and then determine the best price discount according to the remaining amount of inventory on hand. The same logic does not necessarily apply to the case of strategic customers. Game theory provides us with variety of examples in which limiting one’s choices of future actions (“burning the bridges”) may put one in a better position at equilibrium. But the effect of limiting strategic choices may also work the other way around and be more acute in a gaming environment than in a non-strategic setting.

In our revenue management context, we believe that an announced pricing path would not be as bad (i.e., inferior to a contingent pricing policy) as it would be in the case of non-strategic customers. In plain words, our intuition works as follows: If many customers make a strategic choice to wait, the seller would be left with a (statistically) larger number of units at time \( T \). This, in turn, may lead the seller to sharply drop the price of the product at that time. In contrast, after a price path is fixed and announced upfront, customers’ decisions will not have any impact on the price \( p_2 \). Clearly, the intuitive explanation is incomplete, since there are many intervening factors that need to be taken into account. For example, in the case of contingent pricing policies, the seller may not react to large inventory level by sharply dropping the price at time \( T \), if he believes that many customers are waiting to the discounted price. Additionally, the premium price \( p_1 \) is typically not the same for a contingent pricing policy as it is for an announced discount strategy. How could these considerations be taken into account? How would the initial level of inventory
(Q) affect the answer to the above question? For instance, when the level of inventory is very high, there is practically no difference between contingent and non-contingent policies. In §6, we report on results from a numerical study that we performed to study this important pricing question.

We now turn to the technical details, beginning with the specification of the customers’ purchasing policy in response to an announced price path \( (p_1, p_2) \).

**Theorem 2** Consider any given (and credible) announced pricing path \( \{p_1, p_2\} \). Then, the threshold-type policy defined by the function \( \theta \) below constitutes a Nash equilibrium in the customers’ purchasing strategies. Let

\[
\theta(t) = \begin{cases} 
\psi_A(t) = \max \left\{ p_1, \frac{p_1 - wp_2}{1 - we^{-\alpha(T-t)}} \right\} & 0 \leq t < T \\
p_2 & T \leq t \leq H 
\end{cases}
\]

where \( w \) is a solution to the equation:

\[
w = \sum_{x=0}^{Q-1} P(x|\Lambda_I(\psi_A)) \cdot A(Q-x|\Lambda_S(\psi_A,p_1,p_2) + \Lambda_W(p_1,p_2))
\]

(Note that \( \psi_A \) is dependent on \( w \), as given in (5).) Specifically, a customer arriving at time \( t \in [0, H] \) will purchase an available unit immediately upon arrival if \( V(t) \geq \theta(t) \). Otherwise, if \( V(t) < \theta(t) \) and \( t < T \), the customer will revisit the store at time \( T \), and purchase an available unit if \( V(T) > \theta(T) = p_2 \).

Like in the case of contingent pricing policies, in equilibrium, every customer needs to take into account the behavior of the other customers. This is reflected by the parameter \( w \) which represents the likelihood of receiving a unit of the product at time \( T \). Clearly, if \( Q \) is relatively large, the interaction between customers is negligible, and their optimal purchasing policy is the same as given in (5), with \( w \approx 1 \).

It is easily seen that the game between the seller and the customers has a Stackelberg form, with the seller being the leader in his announcement of the strategy \( (p_1, p_2) \) and the customers following by selecting their strategy \( w \). Given the seller’s knowledge of the customers’ response to any particular pair of prices, his task is to find the optimal solution to maximizes the expected
We use the notation $\pi^{*}_{A/S} = \max_{p_1, p_2 \leq p_1} \{ \pi_{A/S} (p_1, p_2) \}$, with the subscript ‘A/S’ denoting the case of announced pricing path, with strategic customers.

5.1 A Reference Model: The Case of Non-Strategic Consumers

The case of non-strategic customers can be described as an optimization problem in which the seller looks for a solution $(p_1, p_2)$ to the maximization problem

$$\pi^{*}_{A/N} = \max_{p_1, p_2 \leq p_1} \left\{ p_1 \cdot Q \cdot \left( 1 - \sum_{x=0}^{Q-1} P(x|\Lambda_I (\psi_A)) \right) \right.$$

$$+ \sum_{x=0}^{Q-1} P(x|\Lambda_I (\psi_A)) \times$$

$$\left[ p_1 \cdot x + p_2 \cdot N (Q - x, \Lambda_S (\psi_A, p_1, p_2) + \Lambda_W (p_1, p_2) + \Lambda_L (p_2)) \right]\right\}$$

(The subscript ‘A/N’ stands for announced pricing path, with non-strategic customers.)

6 Numerical Studies: Results and Insights

We conducted a numerical study to explore the way by which strategic consumer behavior impacts pricing policies and expected revenue performance. To this end, we constructed a group of 1,800 parameter-combinations, to which we will refer as “instances.” To represent the level of heterogeneity in the customers’ base valuations, we use the Gamma distribution form:

$$f(x|\mu, c) = \frac{1}{\mu c} \left( \frac{1}{\mu c} x \right)^{\frac{1}{c} - 1} e^{-\frac{1}{\mu c} x}, \quad x \geq 0$$

where $\mu$ is the mean valuation across the population, and $c$ is the coefficient of variation (=standard deviation/mean). Additionally, in a matter of personal taste, we chose to specify the value $\rho = e^{-\alpha}$, instead of $\alpha$. The value $\rho$ has a clearer interpretation, being the fraction of the customers’ valuations at the end of the season relative to their valuations at time zero (base valuations). In total, our models include the seven parameters $\{\lambda, T, H, \rho, Q, \mu, c\}$. Without loss of generality, we set $H = 1$ (change of the time scale), and so time units should be interpreted as fractions of the sales season.
Similarly, the value of $\lambda$ represents the expected number of customer arrivals during the whole season. We also set the average base valuation in the population to $\mu = 1$ (change of currency); the values of $p_1$ and $p_2$ should be interpreted accordingly. The instances we examined reflect the full range of combinations of the five parameters $\{\lambda, T, \rho, Q, c\}$, spanned by:

$$\lambda \in \{1, 2, 5, 10\}, T \in \{0.5, 0.75, 0.85, 0.95, 0.99\}, \rho \in \{0.05, 0.25, 0.5, 0.75, 0.95\}$$

$$Q \in \{1, 2, 3, 4, 5, 10\}, c \in \{0.1, 0.5, 1\}$$

For each instance, we used the models outlined in §§4-5 to calculate various metrics related to the four cases: ‘C/S’, ‘C/N’, ‘A/S’, and ‘A/N’ (‘C’ = contingent; ‘A’ = announced; ‘S’ = strategic; ‘N’ = non-strategic/myopic).

Price segmentation is a strategy via which a seller takes advantage of differences in customer valuations in order to increase expected revenues. In our model, differences in consumer valuations arise due to the heterogeneity in base valuations, and due to the decline in individual valuations over time. As explained above, the two sources of differences in valuations are reflected in our study by the parameters $c$ and $\rho$, respectively. We measure the effectiveness of price segmentation by increase in expected revenues resulting from moving from an optimal single-price (fixed-price) strategy, to an optimal two-price strategy (either contingent discounting, or announced discounting). Note that under a fixed-price policy, the optimal expected revenue is equal to

$$\pi^*_F = \max_p \left\{ p \cdot N \left( Q, \lambda \int_0^H \tilde{F} (p \cdot e^{\alpha t}) \, dt \right) \right\}.$$  \hspace{1cm} (7)

Clearly, when differences in valuations are minimal (i.e., $c \to 0, \rho \to 1$), no advantage can be gained via discounting. In fact, the optimal price to set for the entire horizon is $\mu$ – the exact static valuation of all customers.

### 6.1 The Effectiveness of Segmentation under Strategic Customer Behavior

One of our main objectives in the analysis below is to examine the way by which strategic consumer behavior interferes with the drivers of the benefits of price-segmentation that can be achieved via contingent pricing policies. Table 1 below describes the percentage-wise benefits of ‘C/N’ and ‘C/S’, compared to fixed-price policies. The benefits are defined by $\left( \pi^*_{C/N} - \pi^*_F \right) / \pi^*_F$ and $\left( \pi^*_{C/S} - \pi^*_F \right) / \pi^*_F$, respectively.
Consider first the case of myopic customers. Segmentation offers high benefits when the level of heterogeneity in consumer valuations \( (c) \) is high. The initial price is used by the seller to bet on high valuations, and the discount price is used a “clearance” mechanism to extract revenues from customers with lower valuations. Similarly, we anticipate high benefits of segmentation in cases with high rates of decline in consumer valuations (low \( \rho \) values). Here, the seller can comfortably use a high premium price to extract revenues from early arriving customers, and then reduce the price when consumer valuations decline. The interaction between the values of \( c \) and \( \rho \) appears to play an important role. For example, if the value of \( c \) is large, but the value of \( \rho \) is small, the seller would be concerned about betting on high valuations with the premium price, because the valuations of waiting\(^7\) customers will decline. In plain words, the seller pays a penalty for dragging customers to the discount time. As we explained above, under large values of \( \rho \) and small values of \( c \), the benefits of segmentation are insignificant.

We now turn to the case of strategic customers. Clearly, the benefits of segmentation cannot be better than the case of myopic customers. Our results suggest that price segmentation can be quite effective when both \( \rho \) and \( c \) are small, and \( T \) is relatively large. The rationale behind this is that under these conditions, there is typically a little difference in price valuations at time \( T \), so the price \( p_2 \) is generally set in a way that does not offer a substantial surplus to consumers. For

\(^7\) Waiting happens here simply due to the fact that current valuations are lower than the premium price, and not due to strategic considerations.
this reason, it is not beneficial for strategic customers to wait. Price segmentation in this case is used as a mechanism to discriminate between early high-valuation arrivals and later arrivals with lower valuations.

Unlike the case of myopic customers, strategic consumer behavior suppresses the benefits of segmentation, under medium-to-high values of $c$ and $\rho$. In fact, in some cases, the performance of contingent pricing schemes in equilibrium may be worse than fixed price strategies (reflected, e.g., by the negative numbers in Table 1). The reason for this is explained in more details in the next section. But the general idea is that when $\rho$ is relatively high, customers are patient in waiting for the discount time. Not only that, but unlike the case of fixed-price policies, the customers rationally expect discounts to take place.

### 6.2 The Adverse Impact of Strategic Customer Behavior

In this section we discuss in greater detail the counter-productive effect of strategic consumer behavior. We distinguish between the cases of large vs. small level of initial inventory ($Q$). In order to provide a clearer focus, we consider the case of large $c$ values (high level of heterogeneity) and large $\rho$ values (low rate of decline in consumer valuations).

When $Q$ is relatively large, customers need not be concerned about product availability. Anticipating high-likelihood of product availability at discount time, customers may be inclined to wait and purchase at time $T$. Nevertheless, in equilibrium, the seller anticipates such behavior\(^8\) and responds with a lower discount at time $T$. Additionally, the seller may choose to lower the premium price in order to reduce the incentive of customers to wait. Consider for instance the case \(\{\lambda = 1, T = 0.95, \rho = 0.95, Q = 10, c = 1\}\). When customers are myopic, we get $\pi^*_{C/N} = 0.504$ and $p_1 = 1.567$. The distribution of expected revenues by source is as follows: Immediate purchasers (at premium price) – 59.3%; non-strategic waiting – 37.5%, and late buyers (customers arriving after time $T$) – 3.2%. With strategic customers, we get $\pi^*_{C/S} = 0.352$ and $p_1 = 1.068$. Indeed, in this case the seller sets the premium price at a dramatically lower rate than in the case of myopic customers. Nevertheless, the distribution of the expected revenues by the four sources is now: Immediate buy – 3%, strategic waiting – 81%, non-strategic waiting – 11%, and late buy –

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\(^8\)This is reflected by the term $\Lambda_S (\psi, p_1, z)$ in (3); the larger the value of $\Lambda_S$ is, the larger would be the discounted price charged by the seller.
5%. As can be seen, despite the use of low initial prices, most of the revenues are collected from (strategically) returning customers, at a late part of the season. This raises an interesting point – Why wouldn’t the seller drop the premium price even further so as to motivate more immediate purchases during the earlier part of the season? Possibly, this can be due to the seller’s wish to bet on “extremely-high-valuation” customers. However, the fact that only 3% of the revenues are generate through immediate buyers suggests that this may not be the primary motivation. We believe that the following reason provides a better explanation. Recall that while high premium prices increase the incentive to wait, they also permit high levels of discounted prices at time $T$ (because of the constraint $p_2(q) \leq p_1$ for all $q$). Consequently, strategic customers feel threatened by high premium prices. In summary, the seller cannot effectively prevent the negative impact of strategic consumer behavior.

When the value of $Q$ is small, the seller expects customers to be more concerned about product availability at discount time, and that they would act in a way that is practically similar to myopic customers. However, if customers act myopically, the seller could possibly benefit from high-price experimentation. In other words, the seller would set a high price for the initial period of time in order to bet on collecting large revenues, speculating that if a small number of units will be left on shelf by time $T$, they could be sold relatively easy at discount. For example, consider the previous illustration but with $Q = 1$. Using a fixed price policy, we get $\pi^*_F = 0.304$ ($p^* = 1.142$).

A contingent pricing policy in the case of myopic customers yields $\pi^*_C/N = 0.421$ ($p_1 = 1.780$). But high-price experimentations in the ‘C/S’ model would drive customers to wait even if the availability probability is not high. Indeed, for the illustration above, we get $\pi^*_C/S = 0.317$ ($p_1 = 1.139$). The distribution of expected revenues by source for this case is: Immediate buy – 38.8%, strategic waiting – 36.4%, non-strategic waiting – 21.2%, and late buy – 3.6%. As can be seen, the seller avoids a high-price betting, and so customers are more inclined to purchase immediately than wait and take the risk of a stockout. We conclude, again, that the seller cannot effectively avoid the adverse impact of strategic behavior even under low levels of initial inventory.

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9If we set $\lambda = 10$, instead of 1 (i.e., $Q/\lambda = 0.1$) we still observe the same sort of phenomenon: $\pi^*_F = 1.447$ ($p^* = 1.993$), $\pi^*_C/N = 1.855$ ($p_1 = 2.848$), and $\pi^*_C/S = 1.699$ ($p_1 = 2.35$).
6.3 Timing of Discounts

We now extend our discussion to explore the impact of the time of discount $T$ on expected revenues. Our treatment in this section is confined to pre-announced (fixed) discount times. We begin with a description of a few special cases which we find useful in interpreting the results of our numerical study. First, note that the special case $T = 0$ is equivalent to a fixed-price strategy; in other words, $\pi^*_C(Q|T=0) = \pi^*_C(Q|T=0) = \pi^*_F(Q)$. Clearly, since there is no gaming in the C/N case, we expect $\pi^*_C(Q|T) \geq \pi^*_F(Q)$ for all values of $T$. The best time of discount in the ‘C/N’ model depends on two factors. On one hand, the seller may want to push the time of discount to a later part of the season, in order to permit segmentation across as many customers as possible. On the other hand, if customers’ valuations decline over the course of the horizon, the seller would pay a penalty for dragging customers until discount time – hence, the pressure to set $T$ to an earlier part of the season. Consider the case of high $c$ and $\rho$ values.

Proposition 1 Suppose that $c = 1$, $\rho = 1$, and $Q/\lambda \rightarrow \infty$. In this case, we have:

$$\pi^*_C = (\mu \lambda H e^{-1}) e^{\frac{T}{\mu \lambda}} \geq \mu \lambda H e^{-1} = \pi^*_S$$

The case in the proposition indicates that when customers are myopic, $Q$ is large, and $\rho$ is very close to 1, it is useful to postpone the discount time as much as possible in order to facilitate price segmentation. But when customers are strategic, the seller cannot segment customers, and hence he effectively uses the constant price $\mu$ throughout the season. In plain words, the seller practically uses an optimal fixed-price policy. As explained in the previous section, large $c$ and $\rho$ values are indeed among the cases in which strategic consumer behavior has a significant adverse impact on the seller’s ability to segment customers. Under the conditions of the proposition we get: $(\pi^*_C - \pi^*_F) / \pi^*_F = \exp(1) - 1 = 44.47\%$ (under the choice of $T = H$) and $(\pi^*_C - \pi^*_F) / \pi^*_F = 0$. When $\rho$ and $c$ are large, and the initial level of inventory is small, we argue that postponing $T$ to the latest part of the season may be the best action by the seller both under strategic and non-strategic consumers. When consumers are strategic, setting $T$ to a large value increases the stockout risk that they face, and hence the seller is able to better utilize segmentation.

Another case of interest is one which the values of $\rho$ and $c$ are very small. As we argued previously, we anticipate the effectiveness of contingent pricing schemes to be high, both under
Proposition 2 Consider the case $c = 0$ and $Q/\lambda \to \infty$. Suppose that the seller's strategy is confined to an upfront commitment to three values: $(p_1, p_2, T)$. Then, the optimal prices (say, $p^*_1$ and $p^*_2$) are equal to the solution to the problem

$$\max_{p_1 \in [\mu_\rho, p_1], p_2 \in [\mu_\rho, p_1]} \left\{ (p_1 - p_2) \cdot \lambda H \frac{\ln (\frac{p_1}{\mu})}{\ln (\rho)} + p_2 \cdot \lambda H \frac{\ln (\frac{p_2}{\mu})}{\ln (\rho)} \right\}$$

In the case of myopic customers (C/N), the optimal value of $T$ is any point within the time-range $[H \cdot \ln (\frac{p_1^2}{\mu}) / \ln (\rho), H \cdot \ln (\frac{p_2^2}{\mu}) / \ln (\rho)]$. In the case of strategic customers (C/S), the optimal discount time is $H \cdot \ln (\frac{p_2^2}{\mu}) / \ln (\rho)$. The expected revenues under ‘C/N’ and ‘C/S’ are the same.

Interestingly, under the conditions of the proposition, the seller can completely prevent the strategic returns phenomenon through an appropriate choice of the time of discount. The discount price is offered at a time that leaves no surplus (i.e., $V(T) = \mu e^{-\alpha T}$ is equal or slightly higher than $p_2$), and hence strategic wait is not beneficial. Apparently, the ability to offer the discount at a time in which $\mu e^{-\alpha T} = p_2$, depends on the property that $p_2$ is known in advance. This property is satisfied under large $Q/\lambda$ values, or when $Q = 1$. When the value of $Q$ is at an intermediate level, the seller cannot eliminate strategic customer behavior. Nonetheless, our results suggest that the best revenues that can be collected in the ‘C/S’ setting are not substantially different than in the ‘C/N’ setting.

6.4 Announced Non-Contingent Pricing Strategies

In this section we explore the performance of announced, non-contingent (inventory independent) pricing strategies (below, we shall use the shorter title “announced discounts” to save space). Under pricing strategies of this type, the seller commits upfront to a price path; say $(p_1, p_2, T)$. The premium price $p_1$ is applied until time $T$, and then the price is reduced to the discounted price $p_2$, regardless of the prevailing level of inventory. There are a few reasons for why a seller may adopt an announced pricing strategy. The first reason has to do with simplicity. A non-contingent pricing policies relieves the seller from the burden of inventory counting and postponing the pricing decisions to the time of discount. The second reason can be due to lack of sophistication/capability
in handling contingent pricing policies. Surprisingly, we found that across all of our instances, announced pricing perform essentially the same as contingent pricing policies in the case of myopic consumers. In other words, the value of $\pi^*_A/N$ is virtually the same as $\pi^*_C/N$ for each one of our instances (on average, $\pi^*_C/N$ is only 0.1% better than $\pi^*_A/N$). One should be cautious in interpreting this result. First, recall that in the case of announced discounts, we consider the optimal choice of the price $p_2$. If a seller picks an arbitrary discount level, the sub-performance with respect to contingent pricing can be very high; see, e.g., results in this spirit in Bitran and Mondschein (1997; §4.2) and Mantrala and Rao (2001). Second, a great disadvantage of announced discounts is that it prevents the seller from acting upon learning about demand. If the seller can collect information about demand during the season, contingent pricing policies can be highly valuable; see, e.g., Aviv and Pazgal (2005) and Mantrala and Rao (2001).

When customers are strategic, and a contingent pricing strategy is used, there are cases in which customers anticipate that the seller would optimally set low prices at the time of discount. Therefore, a large number of customers may decide to wait. This observation hints at another possible reason for using announced discount strategies. If the seller credibly restricts himself to a pre-announced price path that does not offer a large discount, customers may forego the possibility of waiting, and purchase a unit immediately upon arrival. Indeed, we found that announced pricing policies can bring advantage to the seller, compared to contingent pricing schemes. Our results indicate that the gap $\left(\frac{\pi^*_A/S - \pi^*_C/S}{\pi^*_C/S}\right)$ is about 1.40% on average, with a minimum value of −0.43%, and a maximum value of 8.32%.

Consider the case in which $\rho$ and $c$ are large, in which strategic behavior has a large negative impact on the seller’s ability to segment customers. We found that pre-commitment to a price path cannot offer much value in the case of sufficiently large initial level of inventory (typically $Q \geq 1.5\lambda$)\(^{10}\). The reason for this is that customers are very patient in waiting ($\rho = 1$, and inventory availability is guaranteed), and hence they will always wait for the discounted price, unless $p_1 = p_2$.

We also found that announced pricing schemes are advantageous compared to contingent pricing

\(^{10}\)Similarly to Proposition 1 above, one can easily show that when $c = 1$, $\rho = 1$, and $Q/\lambda \to \infty$, $\pi^*_A/S = \pi^*_C/S = \frac{\mu\lambda}{\lambda e^{-1}}$. 

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schemes under the following conditions: (i) The number of units are sufficiently high (typically $Q \geq 1.5\lambda$); (ii) the level of heterogeneity in base valuations is high; (iii) the discounts are offered at a late part of the season; and (iv) the value of $\rho$ is at a medium level. For example, among the cases with \{\{Q \geq 1.5\lambda, T/H = 0.99, c = 1, \rho = 0.5\}\}, we found that announced pricing schemes perform similarly to a fixed price policies, whereas a contingent pricing scheme exhibits a poorer performance, by about 7.5%. Table 2 presents a set of results pertaining to an illustrative case with \{\{Q = 10, \lambda = 5, T/H = 0.99, c = 1, \rho = 0.5\}\). As can be seen in the table, under contingent pricing schemes, the customers expect the seller to drop prices quite deeply at the end of the season (discount in the range of $31\% - 56\%$), and hence the majority of them would wait. The seller could theoretically drop the premium price, but this proves to be counter-productive (note that 47.3\% of the expected revenues are generated via immediate buy at $p_1 = 0.826$). The announced pricing policy, with a pre-determined discount of 16\% only, discourages strategic consumer behavior. Nonetheless, the seller mitigates the adverse impact of strategic wait only to the extent that he can gain a similar performance to a fixed-price policy. The expected revenues are still very far from what can be achieved if customers were non-strategic ($\pi^*_{C/N} = 1.686$, higher by 30\% than $\pi^*_{A/S}$). In fact, our results suggest that in most cases where announced discounts perform significantly better than contingent pricing, they provide only a minimal advantage in comparison to fixed pricing policies.

Announced pricing policies offer another possible advantage in mitigating strategic behavior.

Table 2: The performance of various pricing policies for the case \{\{Q = 10, \lambda = 5, T/H = 0.99, c = 1, \rho = 0.5\}\), under strategic customer behavior. The distribution of expected revenues by source, corresponds with immediate purchases, strategic wait, non-strategic wait, and late buy, respectively. In the C/S setting, the ten $p_2$ prices within brackets pertain to the ten possible levels of inventory, \{1, \ldots, 10\}, respectively.

<table>
<thead>
<tr>
<th>Policy</th>
<th>Seller’s strategy</th>
<th>Customers’ strategy</th>
<th>Expected revenues</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>$p^* = 0.722$</td>
<td>Buy if $V(t) \geq p$</td>
<td>$\pi^*_F = 1.301$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(100%, 0%, 0%, 0%)</td>
</tr>
<tr>
<td>C/S</td>
<td>$p^*_1 = 0.826$</td>
<td>(omitted)</td>
<td>$\pi^*_{C/S} = 1.200$</td>
</tr>
<tr>
<td></td>
<td>$p_2 = {0.570, 0.456, 0.408, 0.382,}$</td>
<td></td>
<td>(47.3%, 26.1%, 25.8%, 0.8%)</td>
</tr>
<tr>
<td></td>
<td>$0.369, 0.364, 0.362, 0.362,$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.362, 0.362$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A/S</td>
<td>$p^<em>_1 = 0.757, p^</em>_2 = 0.636$</td>
<td>$w = 1$</td>
<td>$\pi^*_{A/S} = 1.302$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(87.8%, 9.5%, 1.9%, 0.8%)</td>
</tr>
</tbody>
</table>
Reconsider the conditions of Proposition 2 – the case of homogeneous base valuations.

**Proposition 3** Consider the case $c = 0$. Suppose that the seller’s strategy is confined to an upfront commitment to three values: $(p_1, p_2, T)$. Then, the optimal prices (say, $p_1^*$ and $p_2^*$) are equal to the solution to the problem

$$\max_{p_1 \in [\mu \rho, \mu]} \left\{ (p_1 - p_2) \cdot N \left( Q, \lambda H \frac{\ln \left( \frac{p_1}{\mu} \right)}{\ln (\rho)} \right) + p_2 \cdot N \left( Q, \lambda H \frac{\ln \left( \frac{p_2}{\mu} \right)}{\ln (\rho)} \right) \right\}$$

In the case of myopic customers (A/N), the optimal value of $T$ is any point within the time-range $\left[ H \cdot \ln \left( \frac{p_1^*}{\mu} \right) / \ln (\rho), H \cdot \ln \left( \frac{p_2^*}{\mu} \right) / \ln (\rho) \right]$. In the case of strategic customers (A/S), the optimal discount time is $H \cdot \ln \left( \frac{p_2^*}{\mu} \right) / \ln (\rho)$. The expected revenues in both cases, A/N and A/S, are the same.

The proof of this proposition is very similar to the proof of Proposition 2, and hence we avoid the details. The strength of Proposition 3 is in demonstrating that with homogeneous consumer base, the adverse impact of strategic waiting can be completely prevented via an appropriate choice of the discount time $T$, under any value of $Q$.

### 6.5 The Impact of Inventory on Announced Discount Levels

In this section, we explore the depth of discounts, under the announced non-contingent price policies (‘A/N’ and ‘A/S’). We focus our analysis on settings in which $T = 0.99$ (i.e., discounts are scheduled for a very late part of the sales horizon). For each instance, we calculated the depth of the announced discount, given by $(1 - p_2^*/p_1^*) \cdot 100\%$.

In the benchmark case of myopic customers, discounts appear to increase in $c$, and decrease in $\rho$. The dependency on the level of inventory does not seem to be significant. For example, in the case of $\rho \to 0$, one would expect the levels of discounts to be close to 100% for any level of $Q$, since customers’ valuations decline to the vicinity of zero. Even in the other extreme situation, where the value of $\rho$ is very close to 1, the optimal level of discount can be very high. Recall that the reason for this is segmentation across the base-valuations. For instance, with large level of inventory, optimal discounts vary from 0% in the case $c = 0$, to $1 - (1 - \frac{1}{e}) / (2 - \frac{1}{e}) \approx 61\%$ in the case $c = 1$ and $T = H$; see Proposition 1.
In the case of strategic customers, we also observed that discounts appear to increase in $c$ and decrease in $\rho$. However, unlike the case of myopic customers, inventory ($Q$) has a significant impact on the optimal announced discount level, particularly when $\rho$ is large. With a small number of units on hand, strategic consumers are more concerned about product availability, and hence the seller can more effectively segment customers via a high level of discount. In contrast, when the initial level of inventory is high, customers are willing to wait for discounts, and hence it is best for the seller to limit the discount depth. In other words, small discounts are offered so as to mitigate the impact of strategic consumer behavior. Figure 2 below describes the optimal announced discount levels for a subset of our instances with $(\lambda = 2, T/H = 0.99, c = 1)$. This figure suggests, indeed,

![Figure 2: Optimal announced discount levels as a function of $\rho$. The four curves reflect either A/S or A/N settings (solid and dashed lines, resp.), and the number of units (Q=1 or Q=10; see chart legend). The results are based on a subset of our instances with $(\lambda = 2, T/H = 0.99, c = 1)$.

that there exists a range of modest-to-high levels of $\rho$ and $Q$, under which it is optimal for a seller facing strategic consumers to announce a relatively low-discount.

### 6.6 Policies Ignoring Strategic Customer Behavior

When the seller incorrectly assumes that customers are myopic in their purchasing decisions, he determines a pricing scheme that optimizes either the expected revenue expression described in §4.4 (the case of contingent pricing) or in §5.1 (the case of an announced fixed discount). In this
section, we examine ways by which various parameters impact the extent to which the seller’s expected revenues are affected by the misperception of consumer behavior. Besanko and Winston (1990) provide an illustration in which a monopolist’s profit can drop by more than 50% as a result of treating rational consumers (i.e., strategic) as myopic. We chose to focus our attention on the case of announced fixed discounts, in order to keep the technical details, and clarity of exposition at a reasonable level. For each one of the 1,800 combinations in our numerical study, we used the fixed pricing path of our “A/N” model (denoted by $\tilde{p}_1$ and $\tilde{p}_2$), and used the expected revenue function for the strategic customers case (i.e., $\pi_{A/S}(\tilde{p}_1,\tilde{p}_2)$) to evaluate the outcome. We then compared $\pi_{A/S}(\tilde{p}_1,\tilde{p}_2)$ to $\pi^*_{A/S}$, in order to assess the level of sub-optimality that results due to the above type of misperception. Similarly to our analyses in the previous sections, we consider the way by which the three parameters $(\rho,c,Q)$ help explain the question at hand. Table 3 below presents our results for the subset of instances with $T/H = 0.99$, and $\lambda = 2$. We use this table as an illustrative reference for our discussion\footnote{Results for other $T/H$ and $\lambda$ values lead to similar types of observations.}.

Table 3: The percentage sub-optimality gap resulting when strategic consumer behavior is ignored. Results are based on all instances with $\lambda = 2$, $T/H = 0.99$, and $Q \in \{1,10\}$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q = 1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.60%</td>
</tr>
<tr>
<td>0.25</td>
<td>0.00%</td>
<td>0.93%</td>
<td>2.99%</td>
</tr>
<tr>
<td>0.5</td>
<td>0.38%</td>
<td>4.22%</td>
<td>8.37%</td>
</tr>
<tr>
<td>0.75</td>
<td>1.35%</td>
<td>7.84%</td>
<td>13.68%</td>
</tr>
<tr>
<td>0.95</td>
<td>4.05%</td>
<td>10.54%</td>
<td>13.95%</td>
</tr>
<tr>
<td>$Q = 10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.00%</td>
<td>0.84%</td>
<td>1.01%</td>
</tr>
<tr>
<td>0.25</td>
<td>0.45%</td>
<td>7.58%</td>
<td>10.40%</td>
</tr>
<tr>
<td>0.5</td>
<td>1.75%</td>
<td>16.72%</td>
<td>20.54%</td>
</tr>
<tr>
<td>0.75</td>
<td>9.26%</td>
<td>19.67%</td>
<td>20.69%</td>
</tr>
<tr>
<td>0.95</td>
<td>4.37%</td>
<td>9.82%</td>
<td>11.70%</td>
</tr>
</tbody>
</table>

Mis-classification of customers can be very costly to the seller, reaching up to 21% loss of potential revenues in some cases. Levin et al. (2005) report on similar magnitudes of loss. As can be seen in Table 3, the sub-optimality gap increases in $c$. As we explained in our previous discussions, when $c$ is large, the seller sets the prices $p_1$ and $p_2$ apart from each other in order to segment consumers on the basis of differences in their base-valuations. Now, if valuations do not
decline significantly during the horizon, strategic customers would most likely wait.

The dependency of the sub-optimality gap on \( \rho \) works in two different ways. On one hand, when \( \rho \) is large, customers are typically more inclined to wait. But when they wait, their valuations do not decline significantly. When \( \rho \) is small, less customers will decide to wait. However, these customers’ valuations decline rapidly before time \( T \), hence hurting the seller’s ability to extract high revenues. Therefore, mis-classification may lead to higher losses for medium values of \( \rho \).

When the values of \( \rho \) and \( c \) are small, strategic customers do not have a substantial incentive to wait because their valuations are close to each other at time \( T \), and most likely, the price \( p_2 \) would be set to a level that does not leave significant surplus. Hence, mis-classification of customers is not expected to lead to substantial sub-performance; see Table 3.

7 Summary

We propose a stylized model of dynamic pricing for a retailer that sells a finite quantity of units of a seasonal item, to forward-looking (strategic) customers. We distinguish between two classes of pricing strategies: contingent (inventory-dependent), and announced discounts (inventory-independent). For the first case, we identify a subgame-perfect Nash equilibrium for the game between the seller (pricing strategy) and the customers (purchasing strategies). For the second case, we present an optimization problem for the seller, taking into account the consumers’ response to any feasible announced price path. For both cases, we develop a benchmark model in which customers are non-strategic (myopic). We conducted a numerical study to explore the way by which strategic consumer behavior impacts pricing policies and expected revenue performance.

Strategic customer behavior interferes with the drivers of the benefits of price segmentation. When the level of heterogeneity is small, the rate of decline is medium-to-high, and discounts are offered at a late part of the season, segmentation can be used quite effectively. The rationale behind this is that under these conditions, there is typically a little difference in price valuations at the time of discount. Hence, the discount price is generally set in a way that does not offer a substantial surplus to consumers. Unlike the case of myopic customers, strategic consumer behavior suppresses the benefits of segmentation, under medium-to-high values of heterogeneity and modest rates of decline in valuations. An underlying reason for this is that when the rate of decline in
valuation is small, customers are patient in waiting for the discount time. Not only that, but unlike case of fixed-price strategies, customers rationally expect discounts to take place. We provided a qualitative discussion in which we explained the impact of inventory on the equilibrium in the game between customers and the seller. While the reasoning is different for low vs. large levels of inventory, we observed that in general the seller cannot effectively manipulate prices in order to avoid the adverse impact of strategic waiting.

We explored the impact of the time of discount on expected revenues. For example, we demonstrated that when customers are myopic, the initial level of inventory is relatively large, and the decline in consumer valuations is insignificant, it is useful to postpone the discount time as much as possible in order to facilitate price segmentation. But when customers are strategic, the seller cannot segment customers, and hence he effectively uses a constant price throughout the season. The adverse impact of strategic behavior on the seller’s ability to segment customers can be as high as 44.47% reduction in expected revenues. Another case we examined is one in which consumer valuations are homogeneous, but the decline rate is relatively high. We showed that under such conditions, the seller can completely prevent the strategic returns phenomenon through an appropriate choice of the time of discount. We argue that in general, when valuations are homogeneous and the rate of decline in valuations is relatively large, price segmentation via contingent pricing policies offers substantial benefits regardless of the consumer behavior (strategic vs. myopic). The main difference between the settings, is that the time of discount will typically be pushed later into the season in the case of strategic customers.

Surprisingly, we found that announced fixed-discount strategies perform essentially the same as contingent pricing policies in the case of myopic consumers. But we suggest caution in interpreting this result. First, we consider the optimal announced discount. If a seller picks an arbitrary discount level, the sub-performance with respect to contingent pricing can be very high; see also Bitran and Mondschein (1997) and Mantrala and Rao (2001). Second, announced discounts prevent the seller from acting upon learning about demand; see, e.g., Aviv and Pazgal (2005) and Mantrala and Rao (2001). Under strategic consumer behavior, we found that announced pricing policies can bring an advantage to the seller (up to 8.32% increase in expected revenues), compared to contingent pricing schemes. Particularly, we observed that announced pricing schemes are advantageous compared to
contingent pricing schemes under the following conditions: (i) The number of units are sufficiently high; (ii) the level of heterogeneity in base valuations is high; (iii) the discounts are offered at a late part of the season; and (iv) the rate of decline in valuations is at a medium level. The underlying reason for the better performance of announced discount strategies, is that a credible pre-commitment to a fixed discount level removes the rational expectation of customers that at discount time the seller will optimally offer large discounts. Interestingly, we found that in those cases that announced discount strategies offer a significant advantage compared to contingent pricing policies, they appear to offer only a minimal advantage in comparison to fixed pricing policies.

Under the class of announced fixed discount strategies, we explored the optimal depth of discounts when scheduled for a very late part of the sales horizon. Discounts are higher when the level of heterogeneity is large and when the rate of decline in valuations is larger. The sensitivity of the depth of discount with respect to the rate of decline in valuations appears to me more significant when customers are strategic, compared to the case of myopic customers. Unlike the case of myopic customers, under strategic consumer behavior, inventory has a significant impact on the announced depth of discounts, particularly when the rate of decline in valuations is low-to-modest. With a small number of units on hand, strategic consumers are more concerned about product availability, and hence the seller can more effectively segment customers via a high level of discount. In contrast, when the initial level of inventory is high, customers are willing to wait for discounts, and hence it is best for the seller to limit the discount depth.

When the seller incorrectly assumes that strategic customers are myopic in their purchasing decisions, it can be quite costly. We found that the loss of potential revenues can reach a level of about 20%. Besanko and Winston (1990) and Levin et al. (2005), too, reported high expected losses of more than 50% and about 20% (respectively), resulting from a mis-classification of this type. When the level of heterogeneity is large, misclassification results in offering high discounts. Now, if valuations do not decline significantly during the horizon, strategic customers would most likely wait. The dependency of the sub-optimality gap on rate of decline in valuations works in two different ways. On one hand, when it is small, customers are typically more inclined to wait, but their valuations do not decline significantly. When the rate of decline is high, less customers will decide to wait, but the rapid decline in these customers’ valuations would hurt the seller’s ability
to extract high revenues at time of discount. Finally, when customers’ valuations are homogeneous and the decline rate is large, strategic customers do not have a substantial incentive to wait, and so mis-classification is not expected to lead to substantial loss.

The interdisciplinary research we pursue in this paper joins a set of parallel research projects in the management science field. As is typically the case, different researchers examine the topic at hand from a variety of angles, confining themselves to specific assumptions about relevant key factors; see, e.g., a detailed discussion in the introduction of Liu and van Ryzin (2005). For example, Elmaghraby et al. (2004) introduce the possibility of multi-unit bids, and the level of information about the consumers’ valuation (complete knowledge vs. known common distribution). Levin et al. (2005) model the level of strategicity of customers through a common discount factor. Su (2005) considers the correlation between valuations and strategic behavior, under the assumptions of deterministic demand and two possible valuations. Liu and van Ryzin (2005) introduce risk-aversion and capacity rationing aspects. Our work brings stochasticity of the arrival process and customer valuations along with the property of unobservable inventory. We also discuss the value of credible commitment to discounts. We believe that collectively, the above scientific works contribute to a better understanding of strategic consumer behavior in revenue management systems.

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References


Appendix: Proofs

Proof of Theorem 1. Consider a customer arriving at time $t$. First, we can trivially argue that a threshold-type policy with $\theta(t) = p_2$ is optimal for customers arriving during $[T, H]$. Similarly, it is straightforward to show that for a customer that arrives during $[0, T)$ and decides to wait for the discounted price $p_2$, the optimal purchasing time (if at all) is at $t = T$. With this optimal behavior in mind, consider any arbitrary but known purchasing strategies for the other customers. The focal customer needs to compare the current surplus $\psi - p_1$, with the expected surplus at time $T$.

To prove the uniqueness of a solution $\psi \geq p_1$ to (2), observe that both sides of the equation increase in $\psi$. The left-hand-side $\psi - p_1$ is a linear function with a slope of $1$, and gets the value $-p_1$ at $\psi = 0$. The right-hand-side of the equation is non-negative, and its derivative is given by

$$
\frac{d}{d\psi} E_{QT} \left[ \max \left\{ \psi e^{-\alpha(T-t)} - p_2 (QT), 0 \right\} \cdot 1 \{ A \mid QT \} \right]
= E_{QT} \left[ e^{-\alpha(T-t)} \cdot 1 \left\{ \psi e^{-\alpha(T-t)} \geq p_2 (QT) \right\} \cdot 1 \{ A \mid QT \} \right]
= e^{-\alpha(T-t)} \cdot Pr \left\{ \psi e^{-\alpha(T-t)} \geq p_2 (QT) \mid A \right\} < 1
$$

Consequently, there exists a unique value of $\psi \in [p_1, \infty]$ for which (2) holds. Finally, note that for any $t \in [0, T)$, we compared the current surplus with the surplus at time $T$ only. Yet, it is straightforward to see that a purchase at any other time cannot be optimal for the customer. 

Proof of Theorem 2. Similarly to the proof of Theorem 1, we argue that a threshold-type policy with $\theta(t) = p_2$ is optimal for customers arriving during $[T, H]$. Additionally, any customer that arrives during $[0, T)$ and decides to wait for the discounted price $p_2$, will attempt to purchase unit at $t = T$, but not afterwards. The interesting part of the strategy is that pertaining to the consumer behavior during the time interval $[0, T)$. An important observation to keep in mind is that (6) implies that $w$ is equal to the probability that a unit would be allocated to a given customer, provided that all other customers follow the strategy $\theta$. We now consider a specific customer that arrives at time $t < T$, and distinguish between two cases: (i) $e^{-\alpha(T-t)} \leq \frac{p_2}{p_1} \leq 1$. In this case, it is clear that the customer would not purchase the product.
immediately if \( V(t) < p_1 \). However, if \( V(t) \geq p_1 \), then

\[
V(t) - p_1 \geq V(t) e^{-\alpha(T-t)} - p_1 e^{-\alpha(T-t)} \geq V(T) - p_2 \geq (V(T) - p_2) \cdot w
\]

The left-hand-side of the inequality, \( V(t) - p_1 \), is the surplus gained by an immediate purchase, whereas the right-hand-side is the expected surplus gained if the customer decides to wait for the discount. Therefore, \( p_1 \) is the threshold. (ii) \( e^{-\alpha(T-t)} > \frac{p_2}{p_1} \). Here, we argue that the threshold is given by \( \frac{p_1 - wp_2}{1 - we^{-\alpha(T-t)}} \geq p_1 \) (the proof of the “\( \geq p_1 \)” inequality is trivial). First note that if \( V(t) \) is larger or equal to \( \frac{p_1 - wp_2}{1 - we^{-\alpha(T-t)}} \), then the first condition for an immediate purchase is satisfied (i.e., \( V(t) \geq p_1 \)). Next, in order to satisfy the second condition for an immediate purchase, we need to have:

\[
V(t) - p_1 \geq w \left( V(t) e^{-\alpha(T-t)} - p_2 \right)
\]

where \( w \) is the likelihood that a unit of the product would be available for the specific customer at time \( T \). But this probability is precisely the one provided in (6). Using simple mathematical arguments, one can show that the above inequality is satisfied if and only if \( V(t) \geq (p_1 - wp_2) / (1 - we^{-\alpha(T-t)}) \). The theorem now follows. □

**Proof of Proposition 1.** In the C/S case, customers expect the discounted price to be \( \mu \), the price that maximizes the expected revenue gained per customer \( p \cdot \bar{F}(p) = pe^{-\frac{t}{\mu}} \). None of the customers would purchase the product prior to time \( T \), unless \( p_1 = \mu \). The expected revenues collected during the horizon is hence \( \pi^*_{C/S} = \lambda H \mu e^{-1} \). In the case of myopic customers, the seller needs to choose the prices by solving the following problem

\[
\pi^*_C = \max_{p_1, p_2 \leq p_1} \left\{ p_2 \cdot \lambda H e^{-\frac{p_2}{\mu}} + (p_1 - p_2) \cdot \lambda T e^{-\frac{p_1}{\mu}} \right\}
\]

The solution to the latter problem is \( p_1^* = \mu \left( 2 - \frac{T}{\mu} e \right) \geq \mu, \) \( p_2^* = p_1^* - \mu \leq \mu, \) and \( \pi^*_{C/N} = \lambda H e^{-1+\frac{T}{\mu}} \).

□

**Proof of Proposition 2.** The proof is straightforward for the case of non-strategic customers. It is done via a few observations: First, for any pair of prices \((p_1, p_2)\), it is never optimal (assuming \( \rho < 1 \)) to offer the discount prior to time \( \tau_1 \) if \( \alpha \cdot \ln \left( \frac{p_1}{\mu} \right) / \ln (\rho) \), since all customers arriving until this time are willing to pay the premium price \( p_1 \). Second, it is never optimal to offer the discount price after \( \tau_2 \), since no customer will be ready to pay the discounted price after that time. The third observation is that all customers arriving between \( \tau_1 \) and \( \tau_2 \) will agree to pay the discounted price, regardless of when it is offered during \([\tau_1, \tau_2]\). For the case of strategic customers, we argue that the same optimal revenue as in the case of C/N can be obtained; clearly, the seller cannot do better. But we need to show that equilibrium conditions in the game between the seller and the customers are satisfied. To this end, note that if the seller sets the discount to the time \( \tau_2 \), at which no surplus is offered to consumers, the customers will never want to postpone their purchases, hence making the case equivalent to the C/N setting. □