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The Valuation of Assets under Moral Hazard

RAM T. S. RAMAKRISHNAN and ANJAN V. THAKOR*

ABSTRACT

The design of managerial incentive contracts is examined in a setting in which economic agents are risk averse, and the actions of managers can affect asset returns which contain both systematic and idiosyncratic risks. It is shown that in the absence of moral hazard, owners of assets will insure managers against idiosyncratic risks, but with moral hazard, contracts will depend on both systematic and idiosyncratic risks. The traditional recommendation of asset pricing models, namely, to focus only on systematic risks, is thus proved to be valid only when there is no moral hazard. The major empirically testable predictions of the model are (1) managerial incentive contracts will generally depend on systematic as well as idiosyncratic risks, (2) idiosyncratic risks will generally be important in investment decisions, (3) the managers of firms with relatively high levels of idiosyncratic risks will have compensations that are less dependent on their firms' excess returns, and (4) the compensations of managers of larger firms will be relatively more dependent on the excess returns of their firms.

EXTANT ASSET VALUATION MODELS assume that the probability distribution of asset returns is exogenous to the valuation process itself and beyond the influence of those involved in the ownership and management of the asset. Since the plausibility of such an assumption is suspect for an actively managed asset, our main objective is to relax this assumption and characterize Pareto optimal managerial incentive contracts in a setting in which managers can affect asset returns through ex post unobservable actions.

The economy has two types of economic agents—principals and agents. Principals, who may be risk averse, are endowed with capital, but they lack the skills to efficiently manage the assets they own. Thus, they must hire managers who have mean-variance utility functions and are in elastic supply.\footnote{This means a competitive equilibrium will result.} Managers are assumed to possess no initial capital and, consequently, they must sell their services to satisfy their consumption needs. The incentive contracts by which managers are compensated are linear. For simplicity, all managers have identical preferences and skills.

Allowing principals to be risk averse is important if the asset's profitability is not a diversifiable risk. The assumption of mean-variance utility is common in portfolio theory. The linearity constraint on incentive contracts is admittedly

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strong. However, apart from the analytical convenience they afford, such contracts are appealing because they are ubiquitous in practice.\footnote{For other analyses that use linear contracts, see Ross [8, 11].}

In Section I the model is described and optimal incentive contracts are obtained. Section II analyzes how principals would select the magnitude of systematic risk, if they could. Section III concludes.

I. Asset Valuation and Incentive Contracts

The issues addressed in this section are similar to our companion piece [7] and Diamond and Verrechia [1]. The major extensions in this paper relative to [7] are that principals are assumed to be risk averse as opposed to risk neutral, a linear returns generating process is used, and the analysis is cast within the framework of a well-developed asset pricing theory.\footnote{In the subsequent discussion, we use the Arbitrage Pricing Theory (APT) which requires few restrictions on preferences. All that is required is that utility functions be bounded from below or be uniformly integrable. See Ross [9, Appendix 2]. Also see Huberman [4].} Diamond and Verrechia’s paper [1] also differs from the model developed here in many ways. Diamond and Verrechia assume that the manager has a square root utility function over wealth and that principals are risk neutral.\footnote{The fact that we assume principals are risk averse and maximize market values, while Diamond and Verrechia [1] assume principals are risk neutral and maximize expected payoffs is an important distinction between the two models. For example, the risk neutrality assumption leads them to (erroneously) conclude that incentive contracts will not depend on systematic risks. We find that when shareholders have general concave utility functions, managerial contracts will contain nondiversifiable risks.} Moreover, because they base their analysis on an illustration from Holmstrom [3], their distributional assumptions, unlike ours, are very different from the commonly accepted (and empirically tested) assumptions of portfolio theory.\footnote{For example, the “market return” \( m \) in Diamond and Verrechia [1] is assumed to be uniformly distributed over \([-1, 1]\). And, the conditional distribution of the unsystematic component of returns is exponential.} The most substantial similarity between Diamond and Verrechia’s work and ours is the conclusion that with moral hazard idiosyncratic risks are not inconsequential to asset valuation.

Suppose the return \( R_i \) on the \( i \)th asset in the economy is generated by a “bounded-variance” linear process of the form

\[
R_i = g_i(\alpha) + \theta_i = g_i(\alpha) + \beta_i m + \epsilon_i, \tag{1}
\]

where \( \alpha \) is the manager’s action, \( g(\alpha) \) is a positive, strictly increasing, concave function, \( \beta_i \) is a real valued scalar, \( m \) is the return on the market, \( \epsilon_i \) is a firm-specific return, \( E(m) = \bar{m}, E(\epsilon_i) = 0, \text{cov}(\epsilon_i, m) = 0 \forall i, \text{var}(m) = \sigma_m^2 < \infty, \text{var}(\epsilon_i) = \sigma_i^2 < \infty, E(R_i) = \bar{R}, \text{and} \text{cov}(\epsilon_i, \epsilon_j) = 0 \forall i, j, \text{where} E(\cdot), \text{var}(\cdot), \text{and} \text{cov}(\cdot, \cdot) \text{are the expectation, variance, and covariance operators, respectively. Throughout, bars are used to denote expected values.}

These specifications imply that

\[
E(\theta_i) = \beta_i \bar{m}, \quad \text{cov}(\theta_i, m) = \beta_i \sigma_m^2, \quad \text{and} \quad \text{var}(\theta_i) = \beta_i^2 \sigma_m^2 + \sigma_i^2.
\]

Although we refer to \( m \) as the return on the market, we note that any "well-
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diversified” portfolio (which satisfies that APT “zero residual risk” requirement and could be a subset of the market portfolio) will do.

Assume every manager’s utility function can be written as

\[
U(\alpha, \phi) = E(\phi) - \tau \sigma^2(\phi) - V(\alpha)(1 + r),
\]

where \(\phi\) is the manager’s state contingent compensation, \(E(\phi)\) and \(\sigma^2(\phi)\) are the expected value and variance of \(\phi\), respectively, \(\tau\) is a strictly positive risk aversion parameter, \(r\) is the riskless rate of interest, and \(V(\alpha)\), the effort disutility function, is increasing and strictly convex. For notational convenience, we drop the subscript \(i\) henceforth. Assuming that the principal can observe \(R\) and \(m\) but not \(\alpha\) or \(\epsilon\), the general linear contract can be expressed as

\[
\phi(R, m) = \eta_0 + \eta_1(R - \beta m) - \eta_2 m,
\]

where \(\eta_0, \eta_1, \) and \(\eta_2\) are scalars. It is assumed that the principal is aware of the \(\beta\) associated with the asset. As is self-evident, the managerial incentive contract here consists of three components: (i) a constant, (ii) a part depending on the excess return \((R - \beta m)\), and (iii) a part depending on the market return. The principal must optimize by searching for the appropriate weights to assign to these three components. Without loss of generality, the manager’s reservation expected utility (which is exogenous to the model) will be assumed to be zero henceforth.

Suppose the market values of all assets are determined in accordance with the single factor APT, and that the principal’s objective is to maximize market value. Thus, if the investment \(I\) in the asset is exogeneously fixed, the current price of the asset is

\[
P_0 = [1 + r]^{-1}[E(P_1) - \{\hat{m} - r\}\{\text{cov}(P_1, P_m)\}{\sigma_m \sigma(P_m)}^{-1}],
\]

where

\[
P_1 = \text{the net terminal wealth generated by the asset,}
\]

\[
= IR - \eta_0 - \eta_1[g(\alpha) + \epsilon] + \eta_2 m,
\]

\(P_m\) = the terminal value of the market portfolio,

\(\sigma(P_m)\) = standard deviation of \(P_m\).

The principal’s problem is to

\[
\text{maximize } P_0 \quad \quad \alpha \in A, \ (\eta_0, \eta_1, \eta_2) \in \mathbb{R}^3
\]

subject to \(\alpha \in \text{argmax}_{\alpha \in A} U(\alpha, \phi)\)

\[
U(\alpha, \phi) = 0
\]

\[\text{We use the APT rather than the Capital Asset Pricing Model (CAPM) because with the APT, systematic and unsystematic risks are well defined. In the case of the CAPM, that portion of an asset’s variance which cannot be explained by covariance with the chosen market portfolio is not the best measure of those risks which are unobservable and relevant for incentive purposes. In the discussion that follows, incentive problems, related to the task of suitably motivating the manager, arise from the presence of unobservable residual risks.}\]
where $A$ is the manager’s feasible action space and $\mathbb{R}$ is the real line. We can now explore the properties of the optimal incentive contracts when managerial actions (i) can be observed without error ex post and (ii) are unobservable. We assume managers are not allowed to trade in the market. Notationally, subscripts will denote derivatives.

**Case 1: $R$, $m$, and $\alpha$ Observable Ex Post**

**Theorem 1.** When managerial actions can be freely observed ex post, the optimal incentive contract completely shields the manager from the asset’s idiosyncratic risk and is equivalent to a contract of the form

$$
\phi(\alpha, m) = \begin{cases} 
\eta_0 + \eta_2 m & \text{if } \alpha = \alpha^* \\
0 & \text{otherwise}
\end{cases}
$$

(7)

where $\alpha^*$ is the (optimal) action desired by the principal and $\eta_0$ and $\eta_2$ are positive constants. The equilibrium price of the asset is

$$
P_0 = (1 + r)^{-1}[g(\alpha^*) + \beta r] - V(\alpha^*)(1 + r) + (\tilde{m} - r)^2(4\sigma_m^2)^{-1}.
$$

(8)

**Proof:** With $\alpha$ observable, the optimal contract pays $\eta_0 + \eta_1[R - \beta m] - \eta_2 m$ if the manager takes the desired action $\alpha^*$, and nothing otherwise. Thus, we just need to prove that $\eta_1 = 0$ and $\eta_2 < 0$ in the optimal contract. In equilibrium the manager will choose $\alpha = \alpha^*$, and since his reservation utility is zero, we have

$$
\eta_0 = -\eta_1 g(\alpha^*) + \eta_2 \tilde{m} + \tau[\eta_1^2 \sigma_1^2 + \eta_2^2 \sigma_m^2] + V(\alpha^*)(1 + r).
$$

(9)

Using (9) and the definition of $P_1$, we can obtain $E(P_1)$ and $\text{cov}(P_1, m)$ and substitute these in (4) to get

$$
P_0 = (1 + r)^{-1}[IR - \tau[\eta_1^2 \sigma_1^2 + \eta_2^2 \sigma_m^2] - V(\alpha^*)(1 + r) - (\tilde{m} - r)(I\beta + \eta_2)].
$$

(10)

Thus, for any $\alpha^*$ the principal chooses, $P_0$ is maximized at $\eta_1 = 0$. Since $\eta_1$ has no impact on $\alpha$, the issue is one of pure risk sharing as in Wilson [14]. Further, by setting $\partial P_0/\partial \eta_2 = 0$, we see that the optimal $\eta_2$ is a constant given by $-(\tilde{m} - r)(2\eta_2 \sigma_m^2)^{-1}$. Also, since $\tilde{m} > r$, we have $\eta_2 < 0$. The desired result (7) now follows by defining $\eta_2' = -\eta_2$, and (8) can be obtained by substituting for $\eta_0$ and $\eta_2$ in (10). Q.E.D.

Since the manager is risk averse, he prefers to bear as little risk as possible. The risk averse principals, therefore, insure him completely against diversifiable idiosyncratic risk, but not against nondiversifiable systematic risk which they share with him. The amount of market risk optimally borne by the manager depends on his personal risk aversion parameter, $\tau$; the more risk averse the manager the lower is the market risk imposed on him.

**Case 2: Only $R$ and $m$ Observable Ex Post**

**Theorem 2.** When managerial actions are not observable ex post, the Pareto optimal incentive contract will depend on the realized idiosyncratic return of the
asset as well as the market return. Further, this dependence is independent of the
correlation between the total asset return and the market return. The equilibrium
price of the asset in this case is lower than it is when managerial actions are freely
observable ex post.

Proof: The manager's expected utility is

$$U(\alpha, \phi) = \eta_0 + \eta_1 g(\alpha) - \eta_2 \bar{m} - \tau[\eta_1^2 \sigma_r^2 + \eta_2^2 \sigma_m^2] - V(\alpha)(1 + r).$$  \hspace{1cm} (11)

The optimal managerial choice of action, \(\tilde{\alpha}\), is obtained from \(\partial U / \partial \alpha = 0\), and is given by

$$\eta_1(\tilde{\alpha}) = \left[ V_\alpha(\alpha)(1 + r) \right] [g_\alpha(\tilde{\alpha})]^{-1}. \hspace{1cm} (12)$$

Replacing \(\alpha^*\) by \(\tilde{\alpha}\) in (9) gives us \(\eta_0\). Note that

$$E(P_1) = IR - \tau[\eta_1^2 \sigma_r^2 + \eta_2^2 \sigma_m^2] - V(\tilde{\alpha})(1 + r), \hspace{1cm} (13)$$

$$\text{cov}(P_1, m) = [I\beta + \eta_2] \sigma_m^2. \hspace{1cm} (14)$$

Substituting \(\eta_0\) in (13) and then using (4), (13), and (14) yields

$$P_0 = (1 + r)^{-1}[IR - \tau[\eta_1^2 \sigma_r^2 + \eta_2^2 \sigma_m^2] - V(\tilde{\alpha})(1 + r)] - [I\beta + \eta_2][\bar{m} - r]. \hspace{1cm} (15)$$

The first-order condition, \(\partial P_0 / \partial \eta_2 = 0\), now yields

$$\eta_2 = -(\bar{m} - r)(2\tau \sigma_m^2)^{-1}. \hspace{1cm} (16)$$

Since \(\bar{m} > r\), (16) implies \(\eta_2 < 0\). Substituting (16) in (15) gives

$$P_0 = (1 + r)^{-1}[IR(\tilde{\alpha}) + \beta r] - \tau\eta_1^2 \sigma_r^2 - V(\tilde{\alpha})(1 + r) + [\bar{m} - r][4\tau \sigma_m^2]^{-1}. \hspace{1cm} (17)$$

The optimal \(\eta_1\) (and thus \(\tilde{\alpha}\)) can be obtained by setting \(\partial P_0 / \partial \eta_1 = 0\). Now using (16) the Pareto optimal contract can be written as

$$\phi(r, m) = \eta_0 + \eta_1[R - \beta m] + [\bar{m} - r][2\tau \sigma_m^2]^{-1} m. \hspace{1cm} (18)$$

From (1) we know that the idiosyncratic return \([R - \beta m]\) is independent of \(m\). From (18) it is clear that the manager's effective consumption must depend on the market return as well as the idiosyncratic return observed ex post. Further, from (12) and (16) it is apparent that both \(\eta_1\) and \(\eta_2\) are independent of \(\beta\). Finally, the decline in the equilibrium price of the asset due to the ex post unobservability of managerial actions is evident from a comparison of (15) with (8). Q.E.D.

This theorem is appealing because of its obvious implications for the design of incentive contracts in practice. The market return in the optimal incentive contract here facilitates efficient risk sharing. From Theorem 1 we know that idiosyncratic risk is inconsequential for risk sharing purposes. Thus, its presence in (18) is simply to motivate the manager to take the desired action. Systematic risk, on the other hand, is important for risk sharing purposes because the principal is concerned about the market risk he bears. The inclusion of a nonzero \(\eta_2\) in the optimal contract achieves an optimal sharing of risk between the two parties.
Since the asset’s total return contains both risks, a contract contingent only on the asset’s total return will obviously accomplish some risk sharing and have some motivational effect. However, such a contract implicitly assigns equal weights to the systematic and idiosyncratic risks. Because there is no a priori reason to expect \( \eta_1 \) and \( \eta_2 \) to be equal in absolute magnitude in the optimal contract, such an arrangement will generally be suboptimal.\(^7\)

The decline in the asset’s equilibrium price due to the ex post unobservability of managerial actions sheds light on the nature of agency costs in investment decisions. The decline in the asset price due to the agency relationship here is \( \tau \eta_1 \sigma^2 \). This cost is proportional to \( \tau \) (the manager’s risk aversion), \( \eta_1 \), which is a function of \( \bar{\alpha} \) (the principal’s desired effort level), and \( \sigma^2 \), the idiosyncratic variance.

Before doing the comparative statics we will derive the optimal \( \eta_1 \). To do this, substitute (12) in (17) and use the first-order condition \( \partial P_0 / \partial \eta_1 = 0 \) to obtain

\[
\eta_1 = \frac{1}{\left(1 + r(2\pi \sigma^2)\right)^3} \left[I[g_a(\bar{\alpha})]^3 - (1 + r)V_a(\bar{\alpha})[g_a(\bar{\alpha})]^2[g_a(\bar{\alpha})V_{aa}(\bar{\alpha}) - V_a(\bar{\alpha})g_{aa}(\bar{\alpha})]^{-1}\right]. \tag{19}
\]

The four most interesting properties of \( \phi(R, m) \) are discussed below.

(i) An increase in \( \bar{m} \) increases the weight assigned to \( m \) (see (18)). From (19) we know that \( \eta_1 \) is unaffected, but by substituting (16) in \( \eta_0 \) it follows that \( \eta_0 \) goes down. Effectively, therefore, the larger the expected market return (for a fixed level of market risk) the greater should be the manager’s “investment” in the market portfolio. Further, the fixed component of the manager’s compensation is reduced appropriately to keep the manager’s expected utility at zero.

(ii) As \( \sigma^2 \) rises, the dependence of the optimal contract on \( m \) declines (see (18)). The term in the contract representing the weight of the market component is a decreasing function of \( \sigma^2 \), whereas \( \eta_1 \) does not depend on \( \sigma^2 \). With (16) substituted in \( \eta_0 \), it is evident that the reduction in the market based portion of the manager’s fee is partially offset by a concomitant increase in \( \eta_0 \).

(iii) A higher \( \sigma^2 \) reduces the weight, \( \eta_1 \), assigned to the idiosyncratic component of the asset’s return. However, unlike the previous case, \( \eta_0 \) does not necessarily adjust upward. This is because the reduction in \( \eta_1 \) is also accompanied by a curtailed input of effort by the manager. So the principal accepts a lower \( \bar{\alpha} \), because the tradeoff between output \( (g(\bar{\alpha})) \) and risk (\( \sigma^2 \)) has worsened, and it is more expensive to induce an additional unit

\(^7\) In practice, one routinely observes managerial contracts which consist of a fixed component and a “bonus” component which depends only on the managed asset’s (firm’s) total return. Examples of such “bonuses” are executive stock options and stock plans. While we do not propose to explain why such arrangements are so popular, our analysis should cause one to suspect their optimality. In fact, the Institutional Investor Study Report [13] recognizes the drawbacks of a contract that rewards the manager with some fraction of the asset’s total return, and recommends a contract in which managerial rewards depend entirely on ex post superior performance. The only source of risk in the suggested contract there, however, is idiosyncratic (see U.S. Congress [13, pp. 262–66]). Thus, it focuses on the motivational aspects of the optimal contract, but appears to ignore optimal risk sharing.
of effort. The smaller $\alpha$ increases the manager’s expected utility through the effort disutility function, $V(\alpha)$, and the lower $\eta_1$ augments the manager’s welfare due to its association with the risk aversion parameter, $\tau$. Thus, it may not be necessary to raise $\eta_0$ to satisfy the zero reservation utility constraint.

(iv) A higher initial investment $I$ increases $\eta_1$. This follows readily from (19), because $g(\cdot)$ is strictly increasing. But $\eta_0$ does not necessarily adjust downward. This is due to the incentive effect of $\eta_1$. From (12) we know that a higher $\eta_1$ will result in a higher $\alpha$. Thus, the managerial effort disutility, $V(\alpha)$, will also be greater. Consequently, $\eta_0$ may have to rise to offset this decline in the manager’s welfare. From an empirical standpoint, we should therefore expect the compensations of the managers of larger firms to be more dependent on the excess returns of their firms but not necessarily to contain larger fixed components in relative terms.

It may seem restrictive to assume that managers are precluded from investing in the market, but examining this case is an effective means of determining the extent to which managerial compensation should vary with firm-specific events and the extent to which it should vary with more general economic events. It is natural to ask what would happen if managerial trading were allowed. The key to understanding this point is a proper interpretation of the managerial reservation utility constraint. In our partial equilibrium framework there are two ways of looking at the reservation constraint. One is to assume that the manager must be guaranteed an expected utility of zero just on the basis of his compensation from the agency relationship and any additional welfare generated from other activities should not affect the zero utilities that the incentive contract must provide. Another interpretation is that the manager’s total welfare (within the agency relationship and outside) should be zero utilities.

We claim that the second approach is the correct way of interpreting the reservation constraint. Our reasoning is as follows. The manager can only work for one firm and he will clearly make his choice of employer on the basis of which firm offers him a contract that generates the highest expected utility in conjunction with all the other actions he can take, including the investment of his personal funds in the market. This implies that the zero reservation utility must be viewed as the expected utility after all permitted investments have been considered. It is easy to see that in this case equilibrium asset values will be identical whether managerial investment in the market is permitted or not. When the manager is not allowed to trade, we can view the owners as providing an indirect means for him to access the market by offering a contract that invests an amount $K^*$ in the market portfolio. This optimal risk sharing benefits the principal to the tune of $(\bar{m} - r)(4\sigma_m^2)^{-1}$. But if the manager has already traded up to his optimal investment level $K^*$, an expected utility of $(\bar{m} - r)(4\sigma_m^2)^{-1}$ is generated for the manager through this personal trading prior to his acceptance of the contract. Thus the incentive contract would only need to guarantee the manager an

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8 Given any currency wealth $W$, the manager’s expected utility is $W(1 + r) + K(\bar{m} - r) - rK^2\sigma_m^2$. If allowed to invest in the market, the manager’s optimal investment is $K^* = (\bar{m} - r)(2\sigma_m^2)^{-1}$ and this generates for him an expected utility of $(\bar{m} - r)(4\sigma_m^2)^{-1}$. 

expected utility of \(-\bar{m} - r)(4\tau \sigma_m^2)^{-1}\). This means that the value of the asset will be insensitive to managerial trading. The only restriction that is (obviously) required is that the manager cannot short sell his own firm’s shares.

II. Project Uncertainty Selection and Moral Hazard

Our objective now is to highlight the potential interrelationships between the diversifiability of risk, moral hazard, and equilibrium asset prices. The main reason for moral hazard is idiosyncratic risks. If the principal can reduce these risks, the manager can be motivated with less risk being imposed on him and hence the risk premium paid to him can be lower. Systematic risks do not create agency costs, and the manager bears such risks only to the extent that it is optimal for him to do so as a member of the economy. Thus, if the owners of assets had a choice, they would opt for less idiosyncratic risk under moral hazard than they would otherwise.

This point is illustrated through a specific model. Note, however, that the result itself transcends the model, and that the model is only a particular means of conveying it. Suppose there are at least some assets in the economy whose returns are described by the equation

\[ R = g(\alpha) + \beta m + (1 - \beta)\epsilon, \tag{20} \]

with \(E(m) = \bar{m}, \text{var}(m) = \sigma_m^2 < \infty, E(\epsilon) = \bar{\epsilon}, \text{var}(\epsilon) = \sigma_\epsilon^2 < \infty, \text{cov}(\epsilon, m) = 0,\) and \(\beta \in [0, 1].\) Also assume that \(\bar{\epsilon} > r\) and \(\bar{m} > r.\)

Note that the \(\beta\) here has a slightly different interpretation from that in the previous sections; if \(\beta = 1,\) the asset’s return is perfectly correlated with the market return. Assume that the function \(g(\alpha)\) varies cross-sectionally, and is the only component of the asset’s return that is exogenously fixed. Prospective owners of the asset can pick any \(\beta\) they want. In a competitive market for real assets, then the \(\beta\) chosen in equilibrium will be the one that maximizes the asset’s price.

**Theorem 3.** If the principal can choose \(\beta\) in the returns model (20), his optimal choice will be

(i) \(\beta = 0\) in the absence of moral hazard, and
(ii) \(\beta \neq 0\) in the presence of moral hazard.

Moreover, in the latter case the optimal \(\beta\) is an increasing function of the level of effort desired by the principal.

**Proof:** Suppose \(\alpha\) is observable ex post. Then from (8), the market value of the asset is

\[ P_0^\alpha = (1 + r)^{-1}[I[g(\alpha^*) + \beta r + (1 - \beta)\bar{\epsilon}] - V(\alpha^*)(1 + r) + (\bar{m} - r)^2(4\tau \sigma_m^2)^{-1}]. \tag{21} \]

Since \(\bar{\epsilon} > r,\) the optimal choice of \(\beta\) is clearly zero.

If \(\alpha\) is unobservable ex post, we can appeal to Theorem 2 to write

\[ P_0 = (1 + r)^{-1}[I[g(\bar{\alpha}) + \beta r + (1 - \beta)\bar{\epsilon}] - \tau \eta_1(1 - \beta)^2 \sigma_\epsilon^2 - V(\bar{\alpha})(1 + r) + (\bar{m} - r)^2(4\tau \sigma_m^2)^{-1}]. \]
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From the first-order condition, $\partial P_o / \partial \beta = 0$, we get the optimal $\beta$ as

$$1 - \beta^* = I(\bar{c} - r)(2\tau(1 - \alpha)^2\sigma_\alpha^2)^{-1}$$

(22)

Since the right-hand side of (22) need not be unity, $\beta^* = 0$ need not be the optimal choice. Finally, from (12) we have

$$d\eta_1 / d\alpha = \left[ g(a(\alpha))V_{\alpha\alpha}(\alpha) - V_\alpha(\alpha)g_{\alpha\alpha}(\alpha) \right](1 + r)(g_{\alpha}(\alpha))^{-2}.$$  

(23)

Because $g(\cdot)$ is increasing and concave and $V(\cdot)$ is increasing and convex, it follows that $d\eta_1 / d\alpha > 0$. From (22) we, therefore, conclude that $\beta^*$ increases with $\alpha$. Q.E.D.

A practical implication of this theorem is that, ceteris paribus, those firms (assets) which enjoy a higher level of "managerial-effort-induced" returns will also be characterized by a higher level of systematic or market risk. The CAPM and the APT imply the converse of this result, namely that the higher the systematic risk of a firm the greater its expected return.

The term $\tau\sigma_\alpha^2(1 - \beta)^2\sigma_\alpha^2$ represents the agency cost created by moral hazard. Thus, if the principal desires a higher $\alpha$, ceteris paribus, he faces a higher agency cost because $\eta_1$ increases with $\alpha$. To counteract this he increases the $\beta$ of the asset under management. The reason for doing this is that increases in systematic risk are not accompanied by increases in agency costs. In other words, the heightened moral hazard generated by a higher $\alpha$ alters the principal's risk-return trade-off such that it is optimal to lay off some idiosyncratic risk and take on additional systematic risk.

III. Concluding Remarks

Agency theory has shed light on the design of multilayered organizations (Mirrlees [6]), capital structure decisions (Jensen and Meckling [5]), and contract design to induce optimal financial signalling and activity choice decisions (Ross [12]). Our paper, however, is one of the first to address managerial contract design and asset pricing in a capital market setting. We hope that it serves as a precursor to more complete studies of asset pricing under moral hazard.9

9 As in other agency theoretic models (see Harris and Raviv [2]), managerial risk aversion plays an important role in our analysis. In practice, if the manager's compensation happens to be extremely small relative to his wealth, his risk attitudes will be relatively unimportant (see Ross [10]).

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