Information reliability and welfare: A theory of coarse credit ratings

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Abstract

An enduring puzzle is why credit rating agencies (CRAs) use a few categories to describe credit qualities lying in a continuum, even when ratings coarseness reduces welfare. We model a cheap-talk game in which a CRA assigns positive weights to the divergent goals of issuing firms and investors. The CRA wishes to inflate ratings, but prefers an unbiased rating to one whose inflation exceeds a threshold. Ratings coarseness arises in equilibrium to preclude excessive rating inflation. We show that competition among CRAs can increase ratings coarseness. We also examine the welfare implications of regulatory initiatives.

Keywords: credit ratings, coarseness, cheap talk, credit quality
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"Junk Bonds prove there's nothing magical in a Aaa rating" – Merton Miller

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1. Introduction

It is well known that credit ratings consist of a relatively small number of ratings categories, whereas the default risks of the debt instruments being rated lie in a continuum. Why is there such a mismatch? Note that there is no ”technological” impediment to having continuous ratings, nor is there any legal barrier. Precise forecasts of future outcomes are not uncommon in financial markets, so coarse ratings are by no means a “hard-wired” phenomenon. While the benefit of rating coarseness is elusive, the potential costs are easy to conjecture. For example, since a credit rating provides valuable information to investors, coarseness reduces the precision and value of the information being communicated by ratings. If this information is used for real decisions, welfare may be reduced by coarseness. Moreover, to the extent that the fees of rating agencies are increasing in the value of the rating to issuers and investors, coarseness can diminish both the fees of rating agencies and the value generated for market participants. Thus, it remains a puzzle why credit ratings are coarse.

One might propose a simple explanation like the difficulty for the rating agency in providing point estimates of default probabilities or credit qualities. After all, is it not easier to provide a range within which a default likelihood lies than to be more precise? If you pick a point estimate, it is easier to be wrong, to be “nit picked”, and then you might even be sued for being wrong.

This simple explanation has too many holes, unfortunately. First, there is no reason why investors should use the same “standard” for judging whether the rating agency is right or wrong when ratings lie in a continuum as they do when ratings lie in coarse categories. That is, the judgment standard should *adapt* to the degree of coarseness of the ratings, so that the legal/reputational liability of the rating agency does not depend on the degree of coarseness. To see this, suppose there is a rating from a coarse grid that implies a default probability in the (0.001,0.01) range and there is a reputational/legal risk associated with the ex post inferred default probability being outside the range. Then the reputational/legal risk of being “wrong” should be the same if ratings lie in a continuum instead of the coarse grid and the rating agency assigns a rating from within this range that implies a default probability of say 0.009. In other words, as long as the ex post inferred default probability is within (0.001,0.01), the rating agency should face no legal/reputational risk in the second regime if it did not do so in the first. Second, rating agencies did not face legal liability for providing ratings – viewed as “forward-looking information" – until the recent passage of the Dodd-Frank Act. Third, there are many instances of point estimates being drawn from a continuum in other financial market contexts, such as earnings forecasts, IPO prices set by investment bankers, valuations provided by equity research analysts, etc.

In this paper, we provide a theoretical explanation for ratings coarseness. We develop a model in which there is a rating agency whose objective in setting ratings is to balance the divergent goals of the issuing firm and the investors purchasing the issuing securities. An issuer wants a high rating to minimize the cost of external financing. Investors, by contrast, want as accurate a rating as possible. The rating agency's objective is a weighted average of these two goals. We model the ratings determination process as a cheap-talk game (Crawford and Sobel, 1982), and show that, in equilibrium, the divergence of interests between issuers and investors leads to the endogenous determination of coarse ratings.

In this model, ratings indicate project/credit quality to both the firm issuing securities to finance a project and the investors purchasing these securities. The issuer's level of investment depends on its assessment of project quality. More precise information about project quality permits more efficient investment, which is valuable to both the issuer and the investors. The rating agency's incentive to inflate ratings stems from the issuer's preference for higher ratings because these are associated with lower costs of debt financing. This incentive prevents the CRA from credibly communicating its information about project quality, which leads to a breakdown in the market for credit ratings that lie in a continuum. The market for ratings is resurrected by the rating agency's incentive to report a rating whose inflation lies below an upper bound that is acceptable to the rating agency. Sufficient coarseness in credit ratings forces the rating agency to choose between an accurate (not inflated) rating, and one that is inflated beyond its acceptable upper bound, and the scheme is designed to tilt the choice in favor of reporting an uninflated, accurate rating. The ratings coarseness arising in our model does not result in any ratings bias such as ratings inflation. However, this coarseness of credit ratings has a cost because the imprecise quality inferences generated by coarse ratings lead to investment inefficiencies and thus reduce welfare.

Our model predicts that a *ceteris paribus* reduction in the coarseness of credit ratings will improve the informativeness of ratings and increase the sensitivity of the investments of borrowers to their credit ratings. Empirical evidence in support of this prediction is provided by Tang (2009). He examines how Moody's 1982 credit rating refinement affected firms' investment policies. Starting April 26, 1982, Moody's reduced the coarseness of its ratings by increasing the number of credit rating categories from 9 to 19. Consistent with the prediction of our model, firms that were upgraded due to the change exhibited higher capital investments and faster asset growth than downgraded firms.

Competition among rating agencies is no panacea when it comes to reducing ratings coarseness. We show that going from one rating agency to two can actually increase ratings coarseness. Nonetheless, holding the credit rating agency's objective function fixed, welfare increases due to the additional information provided by the second rating. When competition is allowed to alter the credit rating agency's objective function, greater competition is likely to increase welfare when the number of rating agencies is small, but decrease welfare when the number of competing rating agencies is large.

Our analysis predicts that initiatives that increase the weight rating agencies attach to the concerns of investors and/or reduce the weight they attach to the concerns of issuers will reduce the coarseness of credit ratings. This implies, for example, that if *all* issuers of a particular security were required to obtain ratings and disclose all ratings obtained — so that rating agencies would attach smaller weight to the desires of issuers — then coarseness is likely to diminish.

This paper is related to the emerging literature on credit ratings. The early work of Allen (1990), Millon and Thakor (1985), and Ramakrishnan and Thakor (1984) provided the theoretical foundations for thinking about rating agencies as diversified information producers and sellers. More recently, Boot, Milbourn and Schmeits (2006) have proposed that a credit rating agency (CRA) can arise to resolve a specific kind of coordination problem in financial markets (see also Manso, 2013). In particular, they show that two institutional features – "credit watch" and the reliance on ratings by investors – can allow credit ratings to serve as the focal point and provide incentives for firms to expend the necessary "recovery effort" to improve their creditworthiness. Bongaerts, Cremers and Goetzmann (2012) provide evidence about why issuers choose *multiple* credit rating agencies. They show that their evidence is most consistent with the need for certification with respect to regulatory and rule-based constraints. Goel and Thakor (2010) argue that the change in pleading standards for rating agencies under Dodd-Frank – a change that created a harsher legal requirement for rating agencies – can have a perverse effect.

There is also an emerging literature on failures in the credit rating process. Bolton, Freixas and Shapiro (2012), and Sangiorgi, Sokobin and Spatt (2009) examine competition among rating agencies and consequences of this, including the incentives of rating agencies to manipulate ratings. They model "ratings shopping," something that occurs because issuers can choose which credit ratings to purchase *after* having had a glimpse of those ratings, thereby creating incentives to publish only the most favorable ratings. As Spatt (2009) points out, ratings shopping can occur only if the security issuer gets to determine which credit ratings to choose and publish, a flexibility that is limited in the U.S. because Moody's and S&P rate *all* taxable public corporate bonds, even if issuers do not pay for those ratings. Sangiorgi and Spatt (2013) show that opacity about the contacts between the issuer and the rating agencies provides issuer a valuable option to cherry-pick which ratings to announce and enables ratings agencies to extract some of the surplus associated with this option value. Opp, Opp and Harris (2013) focus on the feedback effect of mechanical rules based on ratings on the incentives of the CRA to acquire and disclose information. Becker and Milbourn (2011) empirically examine the effect of an increase in competition among CRAs on their reputational incentives. Their evidence shows that increased competition caused an increase in ratings levels, a decline in the correlation between ratings and market-implied yields, and a deterioration in the ability of ratings to predict default.

Our marginal contribution relative to this literature is that we focus on the endogenous determination of rating categories to explain why equilibrium ratings are coarse indicators of credit quality, despite the adverse impact of coarseness on welfare. This takes us a step closer to understanding how the credit ratings market works, how the incentives of different groups interact, and how market and regulatory forces impinge on ratings. Note that ratings coarseness is a puzzle only if the additional information conveyed by finer ratings would improve welfare in the economy, as is the case in our model. This distinguishes our paper in a significant way from models with binary investment choices in which the only relevant information is whether the project should be financed or not. For example, in Lizzeri (1999), the intermediary only certifies that quality is greater than or equal to zero, so more information is completely superfluous in that setting.[1] By contrast, we assume

[1]Another feature of Lizzeri (1999) model is that the information intermediary can

that information has a continuous effect on welfare via the optimal level of investment. Only in such a circumstance is it worthwhile explaining ratings coarseness. Kartasheva and Yilmaz (2013) extend the model in Lizzeri (1999) to show that ratings become more precise if gains from trade are increasing in issuer quality. They do not discuss endogenous ratings coarseness because the underlying information examined in the model is assumed to be coarse to begin with.[2]

The rest of the paper is organized as follows. Section 2 contains the model and the analysis that shows how ratings coarseness arises endogenously. Section 3 discusses the implications of competition among CRAs on the ratings process. Section 4 discusses welfare and regulatory implications. Section 5 concludes. Appendix A provides a model motivating CRA's objective. All formal proofs are in Appendix B.

2. Model

Consider a firm that has an investment project available to it. The payoff $\Pi(I, q)$ from the project is risky and depends on the investment, I, in the project and the quality, q, of the project. The project quality is unknown but it is common knowledge that q is drawn from a continuous probability distribution with support $K \equiv (Q_l, Q_h)$.

The firm lacks internal funds and must raise the entire investment amount I from outside investors. The amount repaid to these outside investors is a function of the payoff from the project that is determined based on perceived project quality ($\hat{q}$) and the investment amount raised: $D(\Pi, I, \hat{q})$. The firm and the investors are risk neutral and the discount rate is zero. The firm acts to maximize the wealth of its current shareholders. The market for capital is competitive so that investors' expected return in equilibrium is zero.

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commit to a disclosure rule and can extract all the surplus in the benchmark scenario.

[2] Kovbasyuk (2013) also uses a cheap-talk model to show that ratings coarseness may arise if rating agencies are given private ratings-contingent payments and that optimal ratings are uninformative in this setting. In contrast, we do not assume ratings-contingent payments and show that ratings, while coarse, continue to be informative and enhance social welfare. Nonetheless, making ratings less coarse can improve welfare further. We also discuss the impact of competition on ratings coarseness. Benabou and Laroque (1992) and Morgan and Stocken (2003) consider the incentives of informational financial intermediaries to manipulate information.

2.1. Equilibrium in the absence of credit ratings

We assume that the firm raises external financing for the project. The firm determines the investment level in the project after taking the cost of external financing into account. However, since there is no asymmetric information between the firm and the investors, and the market for external financing is competitive, debt investors break even and the net present value (NPV) of raising external financing equals zero for the firm. Thus, investment and financing decisions are separable and the firm chooses an investment level I to maximize the NPV of the project:

$$V(I,q) = E[\Pi(I,q) - I]. \tag{1}$$

The following assumptions about the project payoff highlight the social value of precise information about project quality:

Assumption 1. *The NPV of the project is concave in investment and is maximized at the optimal investment level of* $I^*(E[q])$.[3]

Assumption 2. *The project payoff is increasing in project quality. Specifically,* $\Pi(I,q_2)$ *strictly first-order-stochastically-dominates* $\Pi(I,q_1)$ *if* $q_2 > q_1$.

It follows from Assumption 1 and Assumption 2 that $I^*(E[q])$ is increasing in $E[q]$ and that $E[V(I^*(E[q]),q)]$ is increasing in $E[q]$ and decreasing in variance of q. Thus, the value-maximizing investment level is an increasing function of q and a more precise estimate of project quality enables a more efficient investment so there is a social cost of uncertainty about the project quality.

The repayment terms are determined so that outside investors' expected payoff equals the investment amount:

$$E\left[D(\Pi(I,\hat{q}), I, \hat{q})\right] = I. \tag{2}$$

However, if perceived project quality differs from the true project quality, there is a net transfer of wealth between current shareholders and new investors.

[3] The assumption that the optimal investment level depends only on the expected value of the project quality rather than the entire distribution is without loss of generality. This is because the project quality can be redefined using a monotonic transformation to ensure that this assumption is valid.

Assumption 3. *The expected wealth transfer from new investors to current shareholders under the value-maximizing investment policy is increasing and concave in perceived project quality and decreasing in true project quality. That is,* $E[D(\Pi(q, I), I, \hat{q})]$ *is increasing and concave in* $\hat{q}$ *and decreasing in* q.

Thus, a higher project quality results in greater expected repayment to new investors, but a higher perception of project quality results in greater investment and more advantageous terms of financing leading to greater transfer of wealth from new investors to current shareholders. Thus, information about project quality q not only enhances welfare by increasing investment efficiency (Assumption 1), it also has a wealth distribution effect through its impact on the sharing of the proceeds from project between original shareholders and new investors (Assumption 3).

2.2. The credit rating agency

There is a credit rating agency (CRA) that can determine project quality and issue a credit rating, r, for the firm. The credit rating represents the CRA's report about the quality of the project. The credit rating is used by the firm to determine the investment level in the project and by investors to determine the terms of the financing raised by the firm.

The dual role served by the credit rating in determining the optimal investment level –which has social value implications – and in determining the terms of debt financing –which matter to the firm – creates a conflict of interest between the social value of the ratings and the value of the ratings to the firm. Both the firm and the new investors prefer a more accurate credit rating to a less accurate credit rating because the NPV of investment is decreasing in the uncertainty about project quality, implying that a more accurate rating would also be preferred by a social planner. However, the credit rating also determines the terms at which the firm can raise external financing. For a given investment level, a better rating generates a higher perceived project quality and leads the firm to raise external financing at more advantageous terms, resulting in a greater transfer of wealth from new investors to existing shareholders.[4] The firm's concern for maximizing the

[4] Graham and Harvey (2001) find that credit ratings are the second highest concern for CFOs when determining their capital structure. Kisgen (2006) finds empirical evidence

wealth of its original shareholders causes it to prefer a higher credit rating to a lower credit rating, whereas its desire to make an NPV-maximizing investment level choice generates a preference for credit rating accuracy. Since the social value of a credit rating depends only on the accuracy of the rating in helping the firm makes its investment-level choice, there is a divergence between the social value of a rating and its value to the firm.

We first examine the impact of the perception about project quality on the social value of the rating, defined as the NPV of investment. Suppose the true project quality is q but the firm and the investors believe, based possibly on the credit rating r, that the expected value of project quality is $\hat{q}(r)$. Then the firm will raise and invest $I = I^*(\hat{q}(r))$. The social value of the rating is the NPV of the investment at this investment level:

$$\text{Social value of the rating, SV} = E[V(I^*(\hat{q}(r)), q)]. \tag{3}$$

By the definition of optimal investment I^*, the social value of the rating is concave in $\hat{q}(r)$ and maximized at $\hat{q}(r) = q$. Next we examine the impact of the perception about project quality on the wealth of the firm's existing investors. The value of the stake (the wealth) of existing shareholders in the firm equals the NPV of the project minus the expected net transfer of wealth to new investors:

$$\begin{aligned}&\text{Value of the rating to the firm, FV} =\\ &\quad E\left[V(I^*(\hat{q}(r)), q) + I^*(\hat{q}(r)) - D(\Pi(I^*(\hat{q}(r)), q), I^*(\hat{q}(r)), \hat{q}(r))\right]. \end{aligned}\tag{4}$$

This expression in Eq. (4) is the ex post value of the firm to existing shareholders and it depends on the true project quality, q, in addition to the investor's inference $\hat{q}(r)$. The first term on the right side of the above equation is the NPV of the investment, which is also the social value of the rating, and it is maximized at $\hat{q(r)} = q$. The next two terms represent the expected transfer of wealth to original shareholders from the new investors. This net transfer equals zero if investors' inference of project quality is unbiased (see Eq. (2)). However, if the investment amount and repayment terms are based on project quality $\hat{q(r)}$ but the project quality is $q < \hat{q(r)}$, then it follows from Assumption 3 that the expected repayment to outside investors will fall

that is consistent with managers viewing ratings as signals of firm quality and being concerned with ratings-triggered costs or benefits.

short of the investment amount they financed and there is thus a positive expected wealth transfer from outsider investors to the original shareholders. So the value of the firm is concave in the inferred project quality $\hat{q}$, and is maximized at a rating that leads to an inflated inference of project quality. Further, the firm's marginal value of a higher inferred project quality is increasing in the true project quality: $\arg\max_{\hat{q}(r)} FV(\hat{q}(r), q) > q, FV_{11} < 0$, and $FV_{12} > 0$, where subscripts indicate partial derivatives.

Reports in the media and research both indicate that a firm's choice of the CRA it purchases its ratings from seems to depend on the willingness of the CRA to assign the firm a sufficiently high rating (e.g., see Bolton et al., 2012, Opp et al., 2013, and Sangiorgi et al., 2009). This ratings-shopping practice implicitly conditions the payoff of the CRA on the rating it assigns to the issuer. In line with the dual role of credit ratings described earlier, we assume that the CRA's choice of credit rating is influenced by two considerations: the social value of the rating and the objective of the firm. Its concern with the social value of the rating causes the CRA to exhibit a preference for an efficient investment level that maximizes project value, whereas its concern with the objective of the firm causes it to prefer a higher assessment of project quality to enable the firm to raise debt financing at a lower cost and increase the wealth of its existing shareholders.

There are economic microfoundations for these two considerations. The CRA's incentive to maximize the efficiency of investment with an accurate credit rating can arise from reputational concerns. If there is uncertainty about the CRA's ability to judge project quality accurately, a credit rating that results in higher investment efficiency enhances the CRA's reputation by signaling higher ability, thereby elevating the fees the CRA can charge for its future credit ratings. The CRA's concern for maximizing the wealth of the issuing firm's existing shareholders may be driven by the expectation that doing this will increase the likelihood that the firm will reward the CRA with future credit rating requests or other business opportunities. This is often viewed as an outcome of the practice of the issuer paying the CRA for credit ratings, referred to as the "issuer pays" model. Even if the firm does not exert direct influence on the CRA, such a *perception* can influence the CRA. Additionally, the CRA may itself prefer a higher credit rating that induces higher investment and thereby makes future investments and credit rating requests more likely.

In Appendix A, we present a model that provides a microfoundation for the CRA's objective function to be a weighted average of the social value of the rating (given by Eq. (3)) and the value of the rating to the firm (given by Eq. (4)):

$$Z(\hat{q}(r), q) = \alpha SV(\hat{q}(r), q) + \beta FV(\hat{q}(r), q), \tag{5}$$

where α and β are positive constants, the social value of the rating is given by Eq. (3) and the value of the rating to the firm is given by Eq. (4). The CRA reports the rating that maximizes Z. The social value of the rating is maximized at $\hat{q}(r) = q$. The value of the rating to the firm, consisting of the social value, which again is maximized at $\hat{q}(r) = q$, and the expected wealth transfer from outside investors to original shareholders, which is increasing in the inferred project quality, is maximized at an inflated inference of project quality. The CRA's objective, a weighted average of social value and firm value, is increasing and concave in inferred project quality $\hat{q}(r)$ (see Assumption 1 and Assumption 4) and is maximized at a rating which leads to an inflated inference of project quality:

$$h(q) \equiv \arg\max_{\hat{q}(r)} Z(\hat{q}(r), q) \geq q + \eta, Z_{11} < 0, Z_{12} > 0, \tag{6}$$

where $\eta > 0$ is the minimum value of the bias in the rating that maximizes the CRA's objective.

The CRA's reporting of a credit rating is an information-transmission mechanism that is an example of a "cheap talk" game.[5] The reason is that the CRA's payoff in Eq. (5) is not directly affected by the credit rating r it reports. The payoff is only indirectly affected by the effect of the credit rating on the firm's investment level and the terms of the financing raised, both of which depend on investors' inference about project quality $\hat{q}(r)$ rather than the actual content of the credit rating r. In particular, a change in the language, scale, or presentation of the credit rating will have no impact on the payoffs of the game as long as investors are aware of the change and can extract the same information from the credit rating. This would change if regulators were fixated on the actual rating, rather than the information conveyed by the rating. In this case, regulations like capital requirements may be based on actual ratings, so that the scale of credit ratings would matter.

[5] See Farrell and Gibbons (1989) and Krishna and Morgan (2007) for surveys of this literature.

2.3. Equilibrium with credit rating

An equilibrium consists of the CRA's rule for credit rating $\rho(r|q)$ such that

1. ρ is a probability distribution: $\int \rho(r|q)\mathrm{d}r = 1$.
2. The credit rating rule $\rho(r|q)$ maximizes CRA's objective in Eq. (5), given the project quality q and investors' perceived expected project quality $\hat{q}(r)$.
3. Investors update their beliefs about project quality q using Bayes' rule. If $\rho(r|q) > 0$ for some q, then investors' posterior probability distribution is

$$g(q|r) = \frac{\rho(r|q)g(q)}{\int_K \rho(r|\chi)g(\chi)\mathrm{d}\chi}. \tag{7}$$

Equilibrium condition 3 requires that the investors' inference about expected project quality be rational:

$$\hat{q}(r) = E[q\,|r]. \tag{8}$$

Equilibrium condition 2 – that the CRA's equilibrium rating choice maximize its objective in Eq. (5) – requires that[6]

$$Z(\hat{q}(r), q) \geq Z(\hat{q}(r'), q) \quad \forall q, r, r', q' \quad \text{if} \quad \rho(r|q) > 0, \rho(r'|q') > 0. \tag{9}$$

2.4. Coarse credit ratings

The purpose of this section is to demonstrate that credit ratings will be inherently coarse in equilibrium. This result is an application of Crawford and Sobel's (1982) result that when the sender and the receiver of the information

[6] Confining alternative ratings to the set of equilibrium ratings is without loss of generality. This is equivalent to an assumption that if the CRA reports an out-of-equilibrium rating, investors choose an investment level corresponding to one of the equilibrium ratings ($\forall r' \, \exists r, q \ni \hat{q}(r') = \hat{q}(r), \rho(r \mid q) > 0$) or such an extreme investment level that the CRA will always prefer an equilibrium investment level to that investment level ($\forall r', q' \, \exists r, q \ni Z(\hat{q}(r), q') \geq Z(\hat{q}(r'), q'), \rho(r \mid q) > 0$). If these conditions are not specified, the CRA's equilibrium rating strategy is not incentive compatible and the equilibrium does not exist.

in a cheap-talk game have divergent interests, information communication is unavoidably imprecise. Crawford and Sobel (1982) derive their results with exogenously assumed objectives of the sender of the information and the receiver of the information. In the context of credit ratings, there are two receivers of information — the firm and the investors. We specify how agency conflicts among stakeholders in the firm can lead to a divergence in the objectives of the firm and the investors, and show how these differences lead to an endogenous conflict of interest between the investors and a CRA which maximizes a weighted average of the objectives of the firm and the investors. This conflict of interest is measured by the weight β that the CRA places on the value of the rating to the firm.[7]

Definition 1. A credit rating is "coarse" if there exists $\epsilon > 0$ such that $|\hat{q}(r) - \hat{q}(r')| \geq \epsilon$ for all r and $r' \neq r$ such that $\rho(r|q) > 0$ and $\rho(r'|q') > 0$ for some q and q'.

Thus, the credit rating in a period is coarse if the actions induced by credit ratings are discrete – there exists $\epsilon > 0$ such that any two actions that can be induced in equilibrium must differ by at least ϵ. The action induced by the credit rating is the inference investors draw about the expected project quality based on the rating, which in turn, determines both the investment level and the terms of debt financing for the rated firm. Notice that investors' objective is a continuous function of inferred project quality $\hat{q}$, *so the optimal investment level with full information about project quality is a continuous function of the project quality.* This means that investors cannot achieve the full-information outcome with coarse credit ratings, so ratings coarseness is a source of welfare losses. As we indicated in the Introduction, this is an essential feature of a model that explains the puzzle of ratings coarseness.

Proposition 1. *The credit rating is coarse in equilibrium. Specifically, if r and r' are two credit ratings reported by the CRA, then $|\hat{q}(r) - \hat{q}(r')| > \eta > 0$.*

[7] The weight α assigned by the CRA to the welfare of investors can arise from the CRA's reputational concerns and provides a counterweight to the CRA's concern with maximizing the wealth of the firm. If the CRA does not face this tension in its objective function (that is, $\beta = 0$), reputational concerns are not needed (α can be 0) for perfect information revelation by the CRA in our model. Ottaviani and Sorensen (2006) show if reputational concerns are present despite no conflict of interest or tension in the objective function of the sort we model, strategic behavior by the sender to signal a higher ability can actually limit information revelation by the sender under some specific information structures.

This proposition shows that if the interests of the CRA and the investors are not aligned, the CRA will issue discrete credit ratings, and the coarseness in credit ratings will increase as the gap between the interests of the investors and the CRA (measured by η) increases. The intuition is as follows. There does not exist an equilibrium in which investors infer the CRA's information precisely based on continuous ratings *and*, given investors' expectations about the CRA's ratings reporting strategy, the CRA actually reports ratings in a manner consistent with those expectations. This is due to the CRA's incentive to manipulate ratings in order to exploit investors' expectations – if investors draw a precise inference about project quality based on the rating, the CRA, with an objective that diverges from the objective of the investors, has an incentive to manipulate the reported rating. To see this, suppose the CRA observes the credit quality as a number in a continuum and reports credit quality as another number in a continuum, with a higher credit quality represented by a bigger number. If investors believed that the CRA reported credit quality truthfully, they would infer that the credit quality equals the reported credit rating. However, given these beliefs, the CRA would report an inflated credit quality as a number larger than the true credit quality, so that investors' inference of credit quality would exceed the true credit quality by the CRA's preferred inflation.

This divergence between the CRA's rating strategy and investors' expectation of the rating strategy leads to a breakdown of a ratings-based mechanism to credibly communicate the CRA's information about project quality precisely. Sufficiently coarse ratings can overcome this breakdown and be credible. To see how, suppose there are two coarse ratings and investors believe that the CRA's rating strategy is to report the higher rating if the true credit quality exceeds a threshold and the lower credit rating otherwise. When the CRA reports one of these ratings, investors interpret the expected credit quality to be the midpoint of the range of credit qualities represented by that credit rating. The CRA prefers to communicate a credit quality that exceeds the true credit quality by an amount equal to its preferred inflation. However, it is restricted to reporting one of the two coarse ratings that result in two different inferences of credit quality. The CRA consequently chooses the rating that results in an inferred credit quality that has the smallest deviation from the credit quality that the CRA prefers to communicate. When the true credit quality is less than the threshold, the CRA may report the lower credit rating, despite its incentive to inflate the reported rating. Specifically, the lower rating will be chosen if the credit quality inference corresponding to the higher rating exceeds the CRA's preferred inference by an amount greater than that by which the CRA's preferred credit quality inference exceeds the inference corresponding to the lower rating.[8]

We now show that there exist multiple equilibria and that, in each of these equilibria, the credit rating partitions the range of project qualities into discrete categories.

Proposition 2. *There exist equilibria with n distinct credit ratings r_1 to r_n for all $n \leq N$ where N is defined below. In an equilibrium with n credit ratings, the following statements are true:*

- *The CRA reports credit rating r_i if the project quality lies in a range (a_{i-1}, a_i), where the n ranges are uniquely defined by*

$$
\begin{aligned}
a_0 &= Q_l, && \text{(10a)}\\
Z(E[q\,|\,a_{i-1} \leq q \leq a_i], a_i) &= Z(E[q\,|\,a_i \leq q \leq a_{i+1}], a_i), 0 < i < n && \text{(10b)}\\
a_n &= Q_h. && \text{(10c)}
\end{aligned}
$$

- *When the CRA reports credit rating r_i, the firm invests $I = I^*(\hat{q}(r_i))$ and the outside investors are repaid $D(\Pi, I, \hat{q}(r_i))$, where $\hat{q}(r_i) = E[q \mid a_{i-1} \leq q \leq a_i]$.*

The maximum number of credit ratings, N, is nonincreasing in η and is the largest value of n such that there is a solution to

$$
\begin{aligned}
a_0 = a_1 &= Q_l, && \text{(11a)}\\
Z(E[q\,|\,a_{i-1} \leq q \leq a_i], a_i) &= Z(E[q\,|\,a_i \leq q \leq a_{i+1}], a_i), 0 < i < n && \text{(11b)}\\
a_n &\leq Q_h. && \text{(11c)}
\end{aligned}
$$

[8] In a standard signaling model (or a Revelation Principle game), perfect separation with truthful reporting/signaling is achieved by having the sender's objective function depend both on the sender's (privately known) true type and a payoff that is correlated with the sender's signal. In a cheap talk game, such as the one we study, the sender's (the CRA's) payoff does not depend on the signal. This makes it impossible to satisfy the incentive compatibility constraints associated with perfect separation using the standard specification of a marginal signaling cost that is lower for higher quality-types. Assuming that the sender's objective is concave in the signal with a unique maximum helps to achieve incentive compatibility, but its ability to do so is limited, and it takes coarse ratings to ensure global incentive compatibility. The reason is that, with coarse ratings, misrepresentation requires moving across a relatively wide rating category, which creates a sufficiently large misrepresentation cost, given the sender's objective function, to deter misrepresentation.

Any other equilibrium is equivalent to one of the above equilibria in the sense that the two equilibria will result in the same level of investment and the same terms of repayment to outside investors for the same value of project quality with probability 1.

The above proposition shows that there are multiple equilibria that differ in the number of discrete credit ratings reported by the CRA. An equilibrium partitions the range of project qualities into n intervals, and the credit rating reveals the interval in which the project quality lies. The credit rating does not reveal the exact project quality in this interval. The firm and the investors update beliefs about project quality rationally based on the assigned credit rating. These updated beliefs serve two purposes — they enable the firm to optimally choose investment level and they also help to determine the terms of external financing. While the credit rating allows the firm to invest more efficiently than it would in the absence of the credit rating, the residual uncertainty about project quality prevents elimination of the investment inefficiency. Note that since investors draw rational inferences from ratings, the coarseness in ratings does not result in any bias in investors' inference about project quality. That is, a point often not emphasized in discussions of ratings inflation is that if investors have rational expectations, then such inflation should *not* systematically bias the credit-quality inferences investors extract from observed ratings.

2.5. An example

To quantify the impact of credit ratings on investment efficiency, in what follows we assume a specific functional form for the investment payoff and also that outside investors provide debt financing.[9] In particular, we consider payoffs that are quadratic or linear in investment and project quality. We also assume that the probability distribution g of the project quality is uniform over (Q_l, Q_h). These assumptions result in quadratic objectives of the CRA and the investors, and facilitate the use of a cheap-talk approach to obtain closed-form expressions for the CRA's equilibrium rating policy.

[9]Equity financing or optimal security design may mitigate the conflict of interest between original shareholders and new investors and also influence the incentives of the CRA. We abstract from consideration of capital structure here by assuming the existence of an exogenous benefit to debt financing.

The payoff from the project equals

$$\Pi = \begin{cases} \Pi_h \equiv (a+1)I - b(I-q)^2 & \text{with probability } p \in (0,1) \\ \Pi_l \equiv I + cq - d & \text{with probability } 1-p. \end{cases} \tag{12}$$

where a, b, c, and d are constants, and $\Pi_h > \Pi_l > 0$. The payoff thus equals a high value, Π_h, with probability p, and a low value, Π_l, with probability $1-p$. This payoff specification captures two features. First, the high payoff is a quadratic function of project quality q and investment level I such that the marginal return on investment is increasing in q. As a result, the value-maximizing investment level is an increasing function of q, and a more precise estimate of project quality enables a more efficient investment. Second, the low payoff results in a loss of $d - cq$ relative to the amount invested, and this loss is decreasing in project quality. Thus, a higher-quality project has lower downside risk of a loss, so debt issued to finance the project will be less risky.

The firm chooses an investment level I to maximize the NPV of the project:

$$V(I,q) = E[\Pi - I] = E[p\{aI - b(I-q)^2\} + (1-p)(cq-d)]. \tag{13}$$

The first-order condition for maximizing the above NPV yields the optimal investment level:

$$I^*(q) \equiv E[q] + a/2b. \tag{14}$$

If project quality is unknown and the firm invests optimally according to Eq. (14), the NPV is:

$$E[V(I^*(E[q]),q)] = mE[q] - bp\text{Var}(q) + pa^2/4b - (1-p)d, \tag{15}$$

where $m = pa + (1-p)c$ and $\text{Var}(q)$ is the variance of project quality. We make the following assumption to model risky debt.

Assumption 4. *$a(Q_l + a/4b) > b(Q_h - Q_l)^2$ and $d > cQ_h$.*

The first condition in the assumption ensures that the high project payoff Π_h exceeds the investment level, and the second condition ensures that the low project payoff Π_l is less than the investment level.[10] The face value, F,

[10] The payoff Π_h exceeds investment if $aI > b(I-q)^2$. Substituting Eq. (14), this requires that $a(E[q] + a/4b) > b(E[q] - q)^2$. The left-hand-side is at least $a(Q_l + a/4b)$ while the right-hand-side is at most $b(Q_h - Q_l)^2$ so $a(Q_l + a/4b) > b(Q_h - Q_l)^2$ is a sufficient condition. The payoff Π_l is less than investment if $d > cq$. Since the right-hand-side is at most cQ_h, a sufficient condition is $d > cQ_h$.

of debt is determined so that the bondholders' expected payoff equals the investment amount:

$$pF + (1-p)(I + cE[q] - d) = I. \tag{16}$$

Suppose the true project quality is q, but the firm and the investors believe that the expected value of project quality is $\hat{q}(r)$ based on the credit rating r. The firm will raise and invest $I = I^*(\hat{q}(r)) \equiv \hat{q}(r) + a/2b$ (see Eq. (14)) and the NPV, given by Eq. (13), reduces to $mq - pb(\hat{q}(r) - q)^2 + pa^2/4b - (1-p)d$, which is a quadratic in $\hat{q}(r)$ that is maximized at $\hat{q}(r) = q$.

$$\text{Social value of the rating SV} = -(\hat{q}(r) - q)^2. \tag{17}$$

The value of the stake (the wealth) of existing shareholders in the firm is given by $p(\Pi_h - F)$. Substituting the payoff Π_h from Eq. (12) and the face value of debt from Eq. (16), this simplifies to:

$$\begin{aligned}&\text{Ex post wealth of existing shareholders}\\ &\quad = p\left\{aI - b(I-q)^2\right\} - (1-p)\left\{d - c\hat{q}(r)\right\}.\end{aligned} \tag{18}$$

Substituting the investment level $I = \hat{q}(r) + a/2b$, this wealth simplifies to a quadratic expression in $\hat{q}(r)$ that is maximized at $\hat{q} = q + c(1-p)/2bp$:

$$\text{Value of the rating to the firm FV} = -\left(\hat{q}(r) - q - \frac{c(1-p)}{2bp}\right)^2. \tag{19}$$

The CRA's objective function, a weighted average of the social value of the rating (given by Eq. (17)) and the value of the rating to the firm (given by Eq. (19)), is

$$Z(\hat{q}(r), q) = -\alpha(\hat{q}(r) - q)^2 - \beta\left(\hat{q}(r) - q - \frac{c(1-p)}{2bp}\right)^2. \tag{20}$$

This objective is quadratic in $\hat{q}(r)$ and is maximized at $\hat{q}(r) = q + \delta$, where $\delta = \{\beta(1-p)c\}/\{2pb(\alpha+\beta)\}$. Thus, δ represents the bias in rating that maximizes the CRA's objective.

Equilibrium condition 2 – that the CRA's equilibrium rating choice maximize its objective in Eq. (5) – reduces to

$$(\hat{q}(r) - q - \delta)^2 \le (\hat{q}(r') - q - \delta)^2 \quad \forall q, r, r', q' \quad \text{if } \rho(r|q) > 0, \rho(r'|q') > 0. \tag{21}$$

With the specific functional forms assumed for the project payoff, the probability distribution of project quality, and debt financing, we get the following corollary from Proposition 2.

Corollary 1. *Suppose the probability distribution g is uniform over (Q_l, Q_h) and project is financed with debt. Then, there exist equilibria with n distinct credit ratings r_1 to r_n for all $n \leq N$, where N is the largest integer not exceeding $[(1 + 2(Q_h - Q_l)/\delta)^{1/2} + 1]/2$. In an equilibrium with n credit ratings, the following statements are true:*

a. *The CRA reports credit rating r_i if the project quality lies in range (a_{i-1}, a_i) where $a_i = Q_l + (Q_h - Q_l)i/n - 2i(n-i)\delta$.*

b. *When the CRA reports credit rating r_i, the firm invests $I = I^*(\hat{q}(r_i))$ and the face value of debt is $F = I + (d - c\hat{q}(r_i))(1-p)/p$ where $\hat{q}(r_i) = (a_{i-1} + a_i)/2$.*

Any other equilibrium is equivalent to one of the above equilibria in the sense that the two equilibria will result in the same level of investment and the same terms of debt financing for the same value of project quality with probability 1.

There are multiple equilibria that differ in the number of credit rating categories. Crawford and Sobel (1982) argue that the equilibrium with the most refined information communication Pareto dominates others. In our context, this is the equilibrium with the most credit ratings. We shall henceforth assume that given any set of parameter values, the equilibrium with the most credit ratings is implemented.

We now examine how the CRA affects social welfare through its impact on investment efficiency. With universal risk neutrality, social welfare is measured by the NPV of investment (see Eq. (15)) which equals $mE[q] - bp\text{Var}(q) + pa^2/4b - (1-p)d$. Thus, the welfare cost of imprecision about project quality is represented by a reduction of $bp\text{Var}(q)$ in the NPV of investment. From Corollary 1, we see that in an equilibrium with n credit ratings, the CRA reports credit rating r_i with probability $1/n + 2(2i - n - 1)\delta/(Q_h - Q_l)$ and the welfare cost equals $bp\text{Var}(q \mid r_i) = bp\{(Q_h - Q_l)/n + 2(2i-n-1)\delta\}^2/12$. Computing expectation across all credit ratings, the expected welfare cost of inefficient investment equals $bp((Q_h - Q_l)^2/12)\sum_{i=1}^{n}[1/n + 2(2i-n-1)\delta/(Q_h - Q_l)]^3$. This simplifies to $bp[(Q_h - Q_l)^2/12n^2 + \delta^2(n^2-1)/3]$. This welfare cost is less than the welfare cost of inefficient investment in an equilibrium with no credit ratings (or equivalently an equilibrium with $n = 1$ credit rating category) of $bp(Q_h - Q_l)^2/12)$. However, if the CRA could communicate project quality perfectly, the investment will always be efficient and the welfare cost will be zero. Thus, perfect information about project quality can improve welfare by $pb(Q_h - Q_l)^2/12$. However, the coarseness of credit ratings precludes this efficient outcome. An increase in the number of credit ratings, for a given bias δ in the CRA's objective, causes ratings to become more refined and leads to more efficient investments. Nonetheless, Proposition 1 shows that there is a limit to how precisely ratings can communicate project quality.

Fig. 1 illustrates how the bias δ in the CRA's objective affects the coarseness of ratings and thereby impacts the welfare cost of inefficient investment. If there is no bias in the CRA's objective ($\delta = 0$), the credit rating can be continuous, with infinitely many credit ratings. The firm invests optimally in this case and there is no welfare cost of inefficient investment. If the bias δ rises from zero to 0.1% of the standard deviation of project quality ($(Q_h - Q_l)/\sqrt{12}$), the maximum number of credit rating categories drops to 42 and the welfare cost of inefficient investment becomes 0.12% of the corresponding cost in the absence of credit ratings. As the bias in the CRA's objective increases to 1%, 5%, and 10% of the standard deviation of project quality, the maximum number of credit ratings declines to 13, 6, and 4, respectively. The corresponding welfare cost of inefficient investment rises to 1.2%, 5.7%, and 11.3%, respectively of the corresponding cost in the absence of credit ratings. We now discuss the economic determinants of the coarseness of credit ratings.

Proposition 3. *The number of credit ratings in the equilibrium with the most credit ratings is increasing in the weight the CRA places on maximizing the social value of the credit rating (α) and the marginal cost of uncertainty in project quality to the firm (pb), and decreasing in the weight that the CRA places on maximizing the wealth of the existing shareholders in the firm (β) and in the marginal value of project quality to debtholders ($(1-p)c$).*

Since the maximum number of credit ratings is a decreasing function of the divergence between the CRA's objective and the goal of maximizing the social value of the credit rating, credit ratings become more refined (the number of ratings increases) as the CRA increases the weight it places on the social value of ratings, and less refined as the CRA increases the weight it places on maximizing the wealth of the existing shareholders of the issuing firm. Moreover, the marginal social value is directly proportional to the sensitivity (pb) of the NPV of the investment to the variance of project quality (see Eq. (15)), so the number of credit ratings increases as this sensitivity increases. Finally, the divergence between the objectives of maximizing

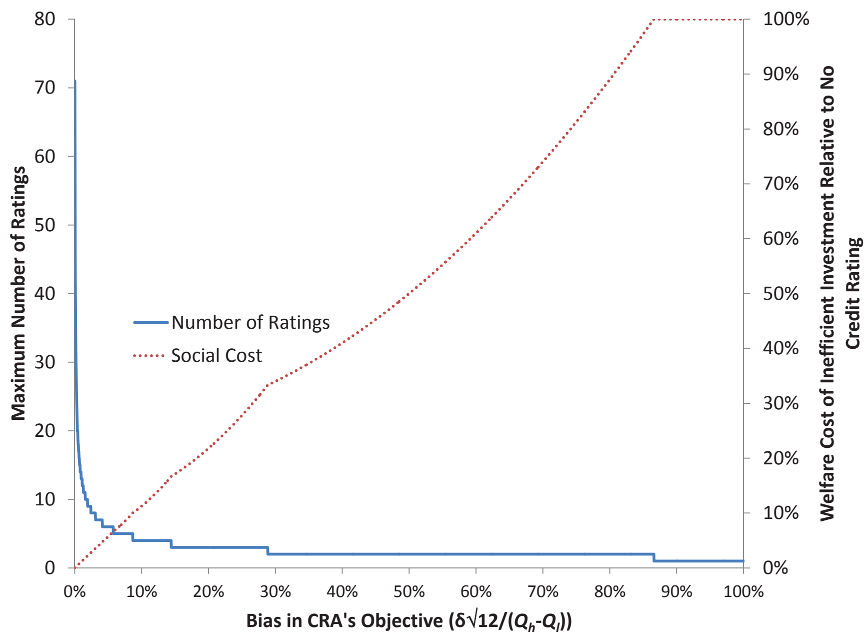

Fig. 1. The horizontal axis is the bias in the CRA's objective δ as a fraction of the standard deviation of project quality $((Q_h - Q_l)/\sqrt{12})$. The number of credit ratings is represented using the scale on the left vertical axis. The social welfare cost of inefficient investment as a fraction of the social welfare cost with no credit rating is represented using the scale on the right vertical axis. As the bias in the CRA's objective increases, the number of credit ratings decline and the welfare cost of inefficient investment increases.

the social value of credit ratings and maximizing the wealth of the current shareholders of the issuing firm arises from the possibility of a transfer of wealth between the current shareholders and the bondholders who provide the financing for the investment. Since the bondholders' expected payoff is $pF + (1-p)(I + cq - d)$ (see Eq. (16)), $(1-p)c$ represents the sensitivity of the wealth transfer between the existing shareholders and the bondholders to the project quality revealed by the credit rating. A higher value of this sensitivity leads to a stronger incentive for the CRA to inflate credit ratings, which in turn increases the coarseness of credit ratings.

These comparative statics indicate how ratings coarseness can vary across different kinds of debt instruments. For example, consider ratings of structured products such as mortgage-backed securities and credit default obligations (CDOs). These ratings are primarily used for portfolio allocation by investors and have lesser relevance for real investment decisions. There are two reasons for this. First, ratings of structured securities typically lag real investments financed through the underlying securities. Moreover, the anticipation of a rating does not influence investment in our model because there is no information asymmetry between the issuer and the investors.[11] Second, there are fewer issuers of structured securities than say issuers of bonds or mortgages and the quality of a typical structured security depends on the quality of a portfolio of many underlying securities. This means that a rating for a structured security conveys relatively little information about the efficiency of the investment financed through an individual underlying security. Hence, we expect the parameter α, the weight placed by the CRA on the social value of the rating, to be lower for structured securities than for corporate bonds. Proposition 3 then suggests that there should be fewer ratings for structured products than for bonds.

While ratings provide a measure of the creditworthiness of firms, credit scores serve a similar purpose for consumers, but with the important difference that the revenue of the company assigning credit scores depends less on a consumer's decision than does the revenue of a rating agency on the decision of the issuing firm. There are two reasons for this. First, a consumer may pay for access to her credit score, but most individuals or businesses

[11] Nonetheless, the real investments expected to be financed in the future through similar securities can be impacted by the spillover effects of the ratings of structured securities and the anticipation of similar ratings for future structured securities.

interested in assessing the creditworthiness of the consumer obtain the credit score directly. Second, in contrast to firms, a single consumer is a minuscule fraction of the consumer population. Thus, we expect the conflict of interest (β) to be much smaller for credit scores than for credit ratings. Our theory then indicates that credit scores should be less coarse than credit ratings.

3. Competition

How might inter-agency competition affect ratings coarseness? If there are multiple credit rating agencies that compete, so the firm can choose the CRA from which to purchase a credit rating, would the ratings be more or less coarse? A plausible conjecture is that competition among CRAs will counteract the effects of conflicts of interest and lead to more informative credit ratings. An opposite view is that competition dilutes the reputational incentives of CRAs and causes ratings to be less informative; see Becker and Milbourn (2011) for empirical evidence.

We now assume that there are two credit rating agencies – CRA A and CRA B – that are ex ante identical. We abstract from the firm's considerations about which CRA's rating to procure by assuming that each CRA issues a credit rating about the quality of the firm's project. The credit ratings issued by the two CRAs can differ if the CRAs disagree about the project quality or if the CRAs report different credit rating categories despite having identical information about project quality. The CRAs can disagree because each CRA's credit rating is based on its privately observed *noisy* signal of the project quality. The signal s^i observed by CRA i, $i \in \{A, B\}$, has a probability distribution $\pi^i(q)$ over support K^i, conditional on project quality q. The expected project quality based on the CRA's updated beliefs is $q^i \equiv E[q \mid s^i]$ with support (Q^i_l, Q^i_h). We can consider q^i rather than s^i as CRA i's signal, without loss of generality.

We make the following assumption about the information structures of the CRAs:

Assumption 5.

1. ***Signals are conditionally independent.*** *Signals q^A and q^B are stochastically independent, conditional on a value of q.*
2. ***Signals are informative.*** *Signal $q^j, j \in \{A, B\}$ satisfies the monotone likelihood ratio property. That is, the ratio of $\left[\frac{\partial Pr(q^j<s|q)}{\partial s}\right]_{s=q_2}$ to*

$\left[\frac{\partial Pr(q^j<s|q)}{\partial s}\right]_{s=q_1}$ *is increasing in* q *if* $q_2 > q_1$.

3. ***Signals are substitutes.*** *There exist constants* β_l *and* β_h *such that* $0 < \beta_l \leq \frac{E[q|q^j=q_2,q^k=q_3]-E[q|q^j=q_1,q^k=q_3]}{E[q|q^j=q_2]-E[q|q^j=q_1]} \leq \beta_h < 1$ *for* $j,k \in \{A,B\}, q_2 > q_1$. *Moreover,* $\beta_h - \beta_l < \frac{2\delta(2-\beta_h-\beta_l)}{Q_h-Q_l}$.

The first condition, namely that signals are conditionally independent, means that the noise terms in the signals of the CRAs are uncorrelated and ensures that each CRA's information is marginally informative. The second condition ensures that a higher value of a CRA's signal connotes higher project quality, holding fixed the signal of the other CRA. The third condition states that the CRAs' signals are partial substitutes in the sense that the marginal informativeness of a CRA's signal decreases when the other CRA's signal is available. The assumption that parameters β_l and β_h are close ensures that the percentage reduction in the marginal informativeness of the CRA's signal, due to the availability of the other CRA's signal, does not vary much across the support of the CRA's signal.

The two CRAs first observe their private signals of project quality and then simultaneously announce their credit ratings. The CRAs can differ in the menu of credit ratings they assign. Let r^i be the credit rating assigned by CRA i. Since each CRA observes only its own signal, the credit rating it assigns is based on its expectation of the credit rating that the other CRA will announce and on its expectation of how the two ratings will be used by the firm and the investors to revise beliefs about project quality.

An equilibrium consists of the CRAs' rules for credit ratings, $\rho^A(r^A|s^A)$ and $\rho^B(r^B|s^B)$, such that:

1. ρ^j is a probability distribution: $\int \rho(r^j|s^j)\mathrm{d}r^j = 1$.
2. The credit rating rule $\rho^j(r^j|s^j)$ of CRA j maximizes

$$\begin{aligned} Z = f \quad &- \quad \alpha E\left[\left(\hat{q}(r^j,r^k) - E[q \mid s^j, r^k]\right)^2\right] \\ &- \quad \beta E\left[\left(\hat{q}(r^j,r^k) - E[q \mid s^j, r^k] - \frac{c(1-p)}{2bp}\right)^2\right], \end{aligned} \quad (22)$$

given its signal s^j, the credit rating rule $\rho^k(r^k|s^k)$ of the other CRA, and investors' inference $\hat{q}(r^j,r^k) = E[q]$ based on posterior distribution $g(q|r^A,r^B)$ of project quality.

3. Investors update their beliefs about project quality q using Bayes' rule. If $\rho(r^A|s^A) > 0$ for some s^A and $\rho(r^B|s^B) > 0$ for some s^B, then investors' posterior probability distribution is

$$g(q|r^A, r^B) = \frac{\iint\limits_{K^B,K^A} \rho(r^A|s^A)\rho(r^B|s^B)\pi^A(s^A|q)\pi^B(s^B|q)g(q)\mathrm{d}s^A\mathrm{d}s^B}{\iiint\limits_{K,K^B,K^A} \rho(r^A|s^A)\rho(r^B|s^B)\pi^A(s^A|\chi)\pi^B(s^B|\chi)g(\chi)\mathrm{d}s^A\mathrm{d}s^B\mathrm{d}\chi}. \tag{23}$$

The first equilibrium condition requires that each credit rating function is a probability distribution, the second condition requires that each CRA's equilibrium credit rating choice is incentive compatible, and the third condition requires that the investors' inference about expected project quality is rational along the path of play.

Lemma 1.

a. *Each CRA's credit rating is coarse in equilibrium. Specifically, if r^i and r'^i are two credit ratings reported by CRA i, then $|\hat{q}(r^i) - \hat{q}(r'^i)| \geq 2\delta$ where $\hat{q}$ is the mean project quality based on the posterior beliefs of the firm and the investors.*

b. *There exist equilibria with n^A distinct credit ratings r_1^A to $r_{n^A}^A$ of CRA A and n^B distinct credit ratings r_1^B to $r_{n^B}^B$ of CRA B such that the following is true:*

   - *CRA j, $j \in \{A, B\}$, reports credit rating category r_i^j if its expectation of project quality, q^j, lies in range $\left[a_{i-1}^j, a_i^j\right]$, where the ranges are uniquely defined by*

$$a_0^j = Q_l^j \quad j \in \{A, B\}, \tag{24a}$$

$$\begin{aligned}
&\sum_{n=1}^{n^k} Pr(q^k \in [a_{n-1}^k, a_n^k] \mid q^j = a_i^j) \times \\
&\qquad \big(E\left[q \mid q^j \in [a_{i-1}^j, a_i^j], q^k \in [a_{n-1}^k, a_n^k]\right] - \\
&\qquad\quad E\left[q \mid q^j = a_i^j, q^k \in [a_{n-1}^k, a_n^k]\right] - \delta\big)^2 \\
&= \sum_{n=1}^{n^k} Pr(q^k \in [a_{n-1}^k, a_n^k] \mid q^j = a_i^j) \times \\
&\qquad \big(E\left[q \mid q^j \in [a_i^j, a_{i+1}^j], q^k \in [a_{n-1}^k, a_n^k]\right] - \\
&\qquad\quad E\left[q \mid q^j = a_i^j, q^k \in [a_{n-1}^k, a_n^k]\right] - \delta\big)^2, \\
&\quad j, k \in \{A, B\}, j \neq k, 0 < i < n^j,
\end{aligned} \tag{24b}$$

$$a_{n^j}^j = Q_h^j \quad j \in \{A, B\}. \tag{24c}$$

- *When CRA A and CRA B report credit ratings r_i^A and r_n^B, respectively, the firm invests $I = I^*(\hat{q}(r_i^A, r_n^B))$ and the face value of debt is $F = I + (d - c\hat{q}(r_i^A, r_n^B))(1-p)/p$ where $\hat{q}(r_i^A, r_n^B) = E[q \mid a_{i-1}^A \leq q^A \leq a_i^A, a_{n-1}^B \leq q^B \leq a_n^B]$.*

c. *Any other equilibrium is equivalent to one of the above equilibria in the sense that the two equilibria will result in the same level of investment and terms of debt financing for the same signals of the CRAs.*

Part a of this lemma shows that equilibrium credit ratings continue to be coarse when there are multiple competing CRAs. The reason is that the coarseness of a CRA's rating arises from the CRA's inability to credibly commit to truthfully report a continuous rating, given its incentive to inflate the rating. With multiple CRAs, the fact remains that a given CRA's credit rating still influences the posterior beliefs about project quality – and hence the wealth of the issuing firm's existing shareholders – so if the CRA's objective is increasing in the wealth of the existing shareholders of the issuing firm, it still has an incentive to manipulate ratings to benefit these shareholders. This tilt in the objective of the CRA toward maximizing the wealth of the issuing firm's shareholders causes ratings to be coarse and prevents the CRA's information from being fully revealed by the credit rating it assigns.

Parts b and c of the lemma characterize the equilibria with two CRAs. Based on their privately observed signals, the two CRAs simultaneously announce possibly different credit ratings. The credit rating assigned by CRA

j partitions the range of expected project qualities based on its signal (q^j). Conditions (24a)-(24c) are the incentive compatibility conditions for CRA j's credit rating strategy conditional on q^j and based on its beliefs about the other CRA's (k's) information q^k and CRA k's equilibrium rating strategy. Condition (24b) specifies that the boundary a_i^j between ratings corresponding to ranges $[a_{i-1}^j, a_i^j]$ and $[a_i^j, a_{i+1}^j]$ of q^j is such that CRA j is indifferent between assigning those two ratings if it expects project quality to be a_i^j based on its own signal. The two ratings will result in the same expected squared deviation between the project quality inferred by the investors and the biased project quality inference that maximizes the CRA's objective. The left-hand side of Eq. (24b) is the expected value of the squared deviation when CRA j assigns the rating corresponding to range $[a_{i-1}^j, a_i^j]$ and the right-hand side is the expected value of the squared deviation when CRA j assigns the rating corresponding to range $[a_i^j, a_{i+1}^j]$. The equilibrium also specifies that the firm's investment as well as terms of debt financing are based on beliefs about project quality that are rationally determined based on the assigned credit ratings and equilibrium strategies of the CRAs. We now examine how competition among CRAs affects ratings coarseness.

Proposition 4. *The maximum number of credit ratings reported by a CRA in an equilibrium with two CRAs is less than or equal to the maximum number of credit ratings reported by the CRA in an equilibrium when it is the only CRA. Despite this increase in coarseness, the welfare associated with the most informative equilibrium is higher when there are two CRAs than when there is only one CRA.*

The proposition shows that, rather than mitigating the coarseness of ratings, greater competition among CRAs can result in an equilibrium with *more* coarse credit ratings issued by each CRA. The economic intuition is as follows. The divergence between the CRA's and investors' objectives limits the precision of information that can be credibly communicated, leading to coarse ratings such that the project qualities inferred by investors are also coarse and differ by at least 2δ across ratings. With two CRAs, the inference about project quality drawn by investors depends on the credit ratings assigned by *both* CRAs, and a particular CRA's rating will cause a smaller shift in the investors' inference in this case compared to the case in which there is a credit rating from only one CRA. So to influence investors' inference by the same amount as with a single agency, each CRA must choose wider rating categories. This means that, when there are two CRAs with identical objectives, each CRA's most informative credit ratings will be coarser than the credit ratings that will arise in an equilibrium with only one CRA.

Nonetheless, when there are two ratings, *more precise information about credit qualities will be communicated in equilibrium* compared to the single rating case. If CRA A reports n_A ratings and CRA B reports n_B ratings, both n_A and n_B are less than the maximum number of ratings when there is only one CRA. Yet, there are effectively $n_A \times n_B$ rating buckets from the investors' perspective, and competition enhances welfare, despite coarser ratings.

The above result relies on the assumption that competition does not affect the objectives of the CRAs. However, competition can, in fact, change each CRA's objective by exerting an ex ante influence on the weights the CRA's objective puts on the interests of the issuer and the social value of credit ratings. It can also potentially affect the informativeness of ratings and hence their effect on real outcomes. The net effect of competition on the informativeness of credit ratings, and hence on social welfare, will depend on the relative impact of competition on the values of the parameters α and β in Eq. (5), an issue that we discuss below.

Suppose there is unobservable heterogeneity among CRAs with respect to the precision with which they discover the credit qualities of issuers, and CRAs are developing reputations for this precision. A more reputable CRA is associated with a greater responsiveness of bond yields to ratings as investors attach higher values to ratings issued by more reputable CRAs. This, in turn, induces issuing firms to prefer more reputable CRAs to those with lesser reputations *ceteris paribus*. The consequence is the generation of an economic incentive for the CRA to acquire a reputation for precise ratings in order to boost future investor demand for its ratings and thereby influence the issuer's purchase decision. To the extent that the value of boosting future investor demand for accurate ratings increases with inter-CRA competition – say because having a larger number of CRAs to choose from allows issuers to be more "picky " in selecting more reputable CRAs – an increase in competition exerts upward pressure on the ratio α/β.

Pitted against this reputational force to report precise ratings is the CRA's desire to inflate ratings due to the component of a CRA's objective that is based on the maximization of the wealth of the issuing firm's existing shareholders.[12] That is, as the inter-CRA market becomes more competitive, the likelihood of the CRA being able to capture an issuer's *current* business declines *ceteris paribus*, thereby strengthening incentives to cater to the interests of the issuer's current shareholders and inflate ratings, i.e., greater competition exerts a downward pressure on α/β. This incentive is exacerbated by the deleterious impact of higher competition on the CRA's survival probability, since this reduced survival probability diminishes the present value of future reputational rents to the CRA. It appears therefore that an increase in competition among CRAs can strengthen both the investor-demand-driven reputational incentive to issue more precise ratings and the issuer-catering-driven incentive to inflate ratings. To see which effect dominates requires more careful and formal reasoning.

To provide such reasoning, we capture the forces discussed above in a simple model in Appendix A. The model shows that when the number of CRAs is relatively small, an increase in competition is likely to increase α/β and thereby increase welfare through its effect on the CRA's objective function. However, when the number of competing CRAs is large, a further increase in competition is likely to reduce α/β, make ratings coarser, and reduce welfare. These conclusions are consistent with the findings in the literature on market structure and product quality about an inverted-U relationship between competition and product quality (see, for example, Aghion, Bloom, Blundell, Griffith and Howitt, 2005 and Dana Jr. and Fong, 2011).[13]

Our model assumes that multiple rating agencies simultaneously issue ratings. However, if rating agencies can observe other ratings when they issue or revise their rating, an interesting possibility to explore is the revelation of information generated by the aggregation of ratings issued by multiple rating agencies and comparisons of these ratings with actual default outcomes. With multiple CRAs, rating agencies that are revealed by comparison to be "wrong" less often will get higher future business as investors will value their ratings more and yields will be more responsive to their ratings. In other words, CRAs will be engaged in an implicit "reputational tournament."[14]

[12]The incentive to maximize shareholder wealth is an outcome of the ability of issuing firms to engage in ratings shopping and choose CRAs that provide higher ratings.

[13]Reputation can be valuable in oligopolistic environment, as in our analysis. However, Hörner (2002) points out the problems faced by models of reputation building in a competitive environment and provides a model that overcomes these problems.

[14]See Goel and Thakor (2008) for how an intrafirm reputational tournament among

This can generate "reputational herding" incentives for CRAs. In a world in which CRAs cannot directly collude and coordinate the ratings they assign, such herding, based on independently drawn signals, is made easier by ratings coarseness; for example, this is trivially true when there is only one rating. That is, ratings coarseness becomes *more* attractive as the number of CRAs increases. So, while multiple equilibria are likely in such an environment, it is plausible that one of these is an equilibrium in which greater inter-CRA competition leads to more ratings coarseness (see Proposition 4).

4. Welfare and regulatory implications

As we have discussed before, ratings coarseness reduces welfare by lowering the precision of the information available for investment decisions. Hence, regulatory actions should be focused on finding ways to induce CRAs to increase effective rating categories, according to our analysis. The focus of regulatory actions instead has been to take the number of rating categories as given and seek to ensure that ratings assigned to debt issues are "accurate" in the sense that a particular credit quality corresponds to the rating investors would expect it to be. This misses the point, however. As our analysis shows, if investors have rational expectations, then ratings inflation does not lead to biased inferences by investors, so that ratings will always be "accurate," given the rating categories deployed.[15] But accuracy, for a fixed number of ratings, does *not* connote precision, and welfare can be improved by increasing the number of rating categories and hence elevating the precision of ratings.

How can regulators induce CRAs to endogenously offer more refined ratings? A strong implication of the analysis is that this can be achieved by reducing the weight the CRA attaches to the wealth of the issuing firm's shareholders. One possible way to do this is to require all issuers to purchase credit ratings—as in the case of all taxable corporate bonds in the U.S.—so that the demand for ratings becomes independent of the extent to which a CRA caters to the issuer's interest. Of course, while this ensures that the

managers competing to be CEO can distort project choices, with reputational herding taking the form of all managers choosing excessively risky projects.

[15]If investors do not have rational expectations, then the focus of regulation ought to shift to addressing the problem of improving ratings-based inferences and perhaps requiring CRAs to more clearly explain how ratings map into default probabilities.

aggregate demand for ratings is independent of the extent to which CRAs cater to the wishes of issuers, it does *not* eliminate ratings shopping, which could cause competing CRAs to continue to attach considerable weight to the wishes of issuers, especially when investors cannot determine the extent of ratings shopping and issuers can benefit from "cherry picking." Sangiorgi and Spatt (2013) indicate that the problem of ratings bias/inflation is exacerbated by the opacity of the contracts between issuers and rating agencies; such opacity creates uncertainty for investors about whether the issuer obtained ratings that are not being disclosed. Joining that insight with the implication of our analysis suggests that a mandatory increase in *transparency* about *all* ratings acquired by an issuer would help to reduce coarseness since it would diminish the benefit of ratings shopping and thereby lower the weight the CRA attaches to the issuer's shareholder wealth.

This discussion also exposes the weakness of regulators mechanically tying regulatory benefits to categories so that firms with higher ratings reap higher benefits regardless of the inference investors draw from these ratings. Such a practice strengthens the issuing firm's preference for a higher rating and widens the divergence between the social value of ratings and the CRA's objective which is a weighted average of the social value and the issuing firm's objective. This widening of the divergence further limits the precision of ratings. So attaching regulatory benefits to rating labels lowers the upper bound on the precision of ratings.

5. Conclusion

In this paper we have provided a theory that explains why credit ratings are coarse indicators of credit quality. We model the credit-ratings determination process as a cheap-talk game and show that a rating agency that assigns positive weights in its objective function to the divergent goals of issuers and investors will come up with coarse credit ratings in equilibrium. The analysis also shows that the coarseness reduces welfare because it leads to investment inefficiencies relative to a system in which the CRA communicates its signal one-to-one to the public. Moreover, competition among rating agencies may cause ratings to become even more coarse. The reason is that the availability of ratings from competing CRAs lowers the marginal impact of a CRA's rating on investors' inference about credit quality, which then induces the CRA to increase ratings coarseness. Nonetheless, greater competition increases aggregate information about credit quality and raises social welfare when there is a small number of competing agencies.

Regulators can affect ratings coarseness in many ways. In particular, these regulations can target investors or issuing firms. On the investor front, if regulators decide to confer benefits on issuers that obtain higher ratings – say by imposing lower capital requirements on investors who invest in higher-rated bonds – so that issuers of higher-rated bonds enjoy lower yields regardless of the inference investors draw from these ratings, then ratings coarseness will increase and welfare will decline. On the issuing firms front, if regulators require all issuers to obtain ratings and also disclose all ratings that were obtained, then coarseness will decline and welfare will be enhanced because each CRA will attach a smaller weight to the issuer's shareholder wealth in its objective function.[16] Thus, the *nature* of regulatory intervention may matter a great deal in the ratings coarseness that arises in equilibrium.

An interesting issue that we have not addressed is why regulatory reliance on ratings is often more coarse than even the (coarse) underlying ratings. That is, why do regulators wish to distinguish only between investment grade and junk bonds or rely on only six risk categories (as the NAIC)? While we have not examined this issue, we can make a few observations. First, because the credit rating itself is a sufficient statistic for the regulator's classifications, there is no information loss associated with this regulatory practice. Second,

[16]For example, the recent regulation by the European Union (see Council of the European Union, 2013) introduced a mandatory rotation rule that requires issuers of structured finance products to switch to a different CRA every four years. The ostensible goal is to reduce the issuer-catering incentives created by the issuer-pays model. The regulation also has other clauses for improved disclosure transparency. Our analysis implies that the greater transparency and a lower β can enhance welfare.

By contrast, some other initiatives to reduce regulatory-mandated reliance on credit ratings may actually reduce welfare, according to our analysis. An example is the recent change in the manner in which capital requirements are computed for the insurance holdings of mortgage-backed-securities. The change replaced credit ratings with regulator-paid risk assessments by Pimco and Blackrock. Becker and Opp (2013) document that this led to significant reductions in aggregate capital requirements. To the extent that a rating issued by a CRA with reputational concerns in an oligopolistic industry becomes more precise when inter-CRA competition increases, this move to replace a rating with an assessment that has not been shaped by similar reputational concerns can reduce welfare not only due to a less precise risk assessment, but also because the consequently lower aggregate capital is inimical to financial stability (see Thakor, 2014, for more on the stability argument).

one reason why regulators may wish to rely on coarser categories than the ratings themselves is that, to the extent that there are costs or benefits associated with how ratings are used for regulatory purposes, incentives for ratings manipulation are generated, and these entail social costs that regulators may wish to minimize. If regulators rely on just a subset of rating categories, there would be little incentives for firms and CRAs to distort ratings *within* each subset. It would be interesting to formally model the tradeoffs engendered by regulation-dependent ratings manipulation incentives, and examine the theoretical soundness of this conjecture in future research.

Appendix A: A simple model of the CRA's objective - Endogenizing α and β

Suppose there two periods, n ex ante identical CRAs, and M ex ante identical firms that need ratings from the CRAs in each of the two periods. A higher value of n indicates higher competition among CRAs. There is turnover among CRAs so that an incumbent CRA in the first period can be replaced with a new CRA in the second period. Following the empirical literature on industry turnover, we assume that the probability that a first-period CRA survives for the second period, $\phi(n)$, is decreasing in n.[17]

Each CRA has an unknown ability that determines the accuracy of its ratings. The probability distribution of the ability in the first period is the same for all CRAs. Ex ante identical CRAs use identical equilibrium reporting strategies in the first period but since a higher ability CRA observes a more precise signal of project quality, its reported rating is expected to result in more efficient investment and greater social welfare. Hence, a CRA whose ratings result in greater social welfare in the first period develops a reputation for higher ability entering the second period.

Each firm independently chooses the CRA from which it buys a rating and pays the CRA a fee f^t for the rating in period t. Firms' choices of CRAs depend on two considerations – the accuracy of ratings and the impact of rating on shareholder wealth. Specifically, firms prefer more accurate ratings that lead to more efficient investment. This preference for ratings accuracy does not impact the choice of CRA in the first period because all CRAs are ex ante identical. However, in the second period, an estimate of the accuracy of a CRA's ratings is based on its reputation for ratings accuracy developed on the basis of its first-period ratings. The second consideration in a firm's choice of CRA is the rating offered by the CRA. The firm gets a preview of the ratings that will be issued by different CRAs before it chooses a CRA. The firm prefers a higher rating to a lower rating. The rationale is that CRAs' ratings are noisy indicators of credit quality and if two CRAs that have the same reputation for accuracy report different ratings, then a higher rating will allow the firm to raise financing at a lower cost and have higher shareholder wealth.[18]

[17] See Caves (1998).

[18] We would get similar results if we assume that firms' choices of CRAs are fixed but

The following specification incorporates these motives of firms in their choices of CRAs. Each firm considers a set of $C(n)$ candidate CRAs. This set is random except that the CRA chosen by the firm in the previous period may be included in the set. Firm j chooses CRA i from its set of candidate CRAs in period t with probability $\omega_{i,j}^t$, given by

$$\omega_{i,j}^t = \frac{1}{C(n)}(1 + k_1(R_i^t - \overline{R}_j^t) + k_2(FV_{i,j} - \overline{FV}_j)), \tag{25}$$

where, k_1 and k_2 are positive constants, R_i^t is the reputation of CRA i at the beginning of period t, $FV_{i,j}$ is the expected value of a rating from CRA i to firm j's shareholders, and $\overline{R}_j^t$ and $\overline{FV}_j$ are cross-sectional averages of CRA reputation and value of rating to firm across firm j's set of candidate CRAs.[19] The reputation of CRA i at the beginning of period 2 is the average social value of the ratings it issued in the first period:

$$R_i^2 = \frac{n}{M}\left(\sum_{\text{firm } j \text{ rated by CRA } i \text{ in period } 1} SV_j^1\right). \tag{26}$$

Each CRA's objective is to maximize its expected current and future discounted fees. Let λ be the per-period discount factor. The objective of CRA i in reporting a rating for firm j in the first period is to maximize

$$f^1\omega_{i,j}^1 + \lambda\phi(n)f^2\left(\sum_{\text{CRA } i \text{ is firm } j\text{'s candidate CRA in period 2}} \omega_{i,j}^2\right). \tag{27}$$

Substituting Eq. (25) and Eq. (26) in the above expression, CRA i chooses a rating for firm j to maximize

$$\frac{1}{C(n)}\left(1 - \frac{1}{C(n)}\right)\left[\lambda C(n)\phi(n)f^2k_1SV_j^1 + f^1k_2FV_{i,j}\right]. \tag{28}$$

The above expression shows that the CRA's objective is to maximize a weighted average of the social value of ratings and the value of the rating to

the fee they pay for a rating is increasing in the CRA's reputation and in the expected shareholder wealth from the rating.

[19] The value of a rating to the firm depends on reported rating and the true project quality. The firm does not directly observe the true project quality needed to value a rating, but estimates it by observing the equilibrium ratings reported by several CRAs.

the firm's shareholders where the weight α attached to social value and the weight β attached to rating's value to the firm's shareholders are given by

$$\begin{aligned} \alpha &= \left(1 - \frac{1}{C(n)}\right) \lambda\phi(n) f^2 k_1, \\ \beta &= \frac{1}{C(n)} \left(1 - \frac{1}{C(n)}\right) f^1 k_2. \end{aligned} \tag{29}$$

The ratio of the two weights is given by

$$\alpha/\beta = \frac{\lambda\phi(n)C(n)f^2 k_1}{f^1 k_2}. \tag{30}$$

Given the weights k_1 and k_2 on the social value of ratings and on the expected value of the rating to shareholders, respectively, chosen by the firms in their CRA choices, α and β depend on three factors: (i) the magnitudes of the current and future rating fees (f^1 and f^2), (ii) the probability that the CRA survives a period ($\phi(n)$), and (iii) the number of candidate CRAs that a firm considers before choosing a CRA ($C(n)$). These factors capture different facets of competition in the CRA industry. Greater industry competition is likely to exert a downward pressure on rating fees, resulting in a decline in the ratio f^2/f^1. Increasing industry competition is also likely to lower the survival probability, $\phi(n)$, of a CRA as n increases. Both these factors reduce the ratio α/β. As for the third factor, an increase in the number of CRAs (n) induces each firm to consider a larger set of candidate CRAs in choosing the CRA from which it buys rating. That is, an increase in n can increase $C(n)$ and thereby increase the ratio α/β. The intuition is that as the number of CRAs increases, an average CRA's current market share decreases, but its potential for growth in market share increases because firms cast a wider net when comparing CRAs.

The net impact of competition on the ratio α/β depends on the relative impact of the three factors discussed above. When the number of CRAs is relatively small and the industry is an oligopoly, an increase in competition is likely to have a modest impact on the fee ratio f_2/f_1 and also have a small impact on $\phi(n)$. The main effect will be an increase in the set of CRAs that issuers can choose from, i.e., $C(n)$ will increase as n increases. Thus, the ratio α/β is likely to go up with an increase in competition when the number of CRAs is small.

When there is a relatively large number of CRAs, however, a further increase in the number of CRAs will reduce the fee ratio f_2/f_1 as the market becomes more competitive, and $\phi(n)$ will decline as well. The theory developed in Satterthwaite (1979) suggests that in markets for "reputational goods," when the number of sellers becomes large enough and buyers face search costs, an increase in the number of sellers does not increase the number of sellers that any buyer compares to decide which seller to buy from, i.e., $C(n)$ becomes insensitive to n when n is large enough.[20] This would imply a decline in α/β when n increases from an already large value.

[20] In Satterthwaite (1979) this may cause prices to perversely rise with more competition (see Pauly and Satterthwaite (1981) for empirical support in the primary care physicians market). However, in their setting, prices charged are unobservable, as is physician's quality. With CRAs, prices are observable, regardless of n, but qualities may require search costs to uncover.

Appendix B: Proofs

Proof of Proposition 1: Suppose $\hat{q}(r) < \hat{q}(r')$, $\rho(r|q) > 0$ and $\rho(r'|q') > 0$ with $q, q' \in [Q_l, Q_h]$. Since $Z_{11} < 0$ and $Z(\hat{q}, q)$ is maximized at $h(q)$, Eq. (9) implies that,

$$\hat{q}(r')) > h(q) \geq q + \eta. \tag{31}$$

Taking an expectation and substituting in Eq. (8) yields $\hat{q}(r') - \hat{q}(r) > \eta$. $\square$

Proof of Proposition 2: N is a unique positive integer because (1) there is a trivial solution to Eq. (11a)-(11c) for $n = 1$ and (2) there is no solution for $n > 1 + (Q_h - Q_l)/\eta$ because $Z_{11} < 0$ and Eq. (11b) imply $a_{i+1} - a_i \geq \eta$ so $Q_h - Q_l \geq a_n - a_0 \geq (n-1)\eta$.

We now show that a solution to Eq. (10a)-(10c) exists for each $n \leq N$. For any solution $(a_0, a_1, ...)$ to Eq. (10a)-(10b), define $s(a_0, a_1, ...) = \max\{j \mid a_i \leq Q_h \forall i \leq j\}$. Consider a random variable x with a continuous probability distribution over $(0, \infty)$ such that $E[x] = \infty$ and a random variable y such that $y = q$ with probability θ, $0 < \theta < 1$, and $y = Q_h + x$ with probability $1 - \theta$. The probability distribution of y is proportional to the probability distribution of q for $y < Q_h$ so we can replace q with y in these equations as this does not change the solutions to Eq. (10a)-(10c). Since the probability distribution of y is continuous and $Z_{11} < 0$, given a_{i-1} and a_i, there is exactly one value of a_{i+1} that satisfies Eq. (10b). Since $a_0 = Q_l$ is fixed, given a_1, there is a unique a_2 that satisfies Eq. (10b) and given a_1 and a_2, there is a unique a_3 that satisfies Eq. (10b) and so on. Thus, there is exactly one solution to Eq. (10a)-(10b) for each value of a_1. Further, these solutions are continuous in a_1 because the distribution of y is continuous. So we can define $s(a_1) = s(a_0, a_1, ...)$ where $a_0, a_1, ...$ satisfy Eq. (10a)-(10b). By definition of N, there exists a value of a_1 at which $s(a_1) = N$. Moreover, $s(Q_h) = 1$. Since $(a_0, a_1, ...)$ are continuous in a_1, $s(a_1)$ changes by at most 1 when a_1 is varied so for each n, $1 \leq n \leq N$, there exists a_1 such that $s(a_1)$ changes discontinuously between n and $n-1$. This requires that $a_n = Q_h$ so $a_0, a_1, ..., a_n$ is a solution to Eq. (10a)-(10c).

We now show that a solution to Eq. (10a)-(10c) is an equilibrium. The rating function is deterministic and hence a trivial probability distribution. To show that equilibrium condition 2 holds, it is sufficient to show that Eq. (9) holds. If $r = r'$, Eq. (9) holds. Suppose $r \neq r'$, project quality $q \in (a_{i-1}, a_i)$ and the corresponding equilibrium rating is r.

If $r > r'$,

$$\begin{aligned}
0 &= Z\left(E\left[q|a_i \le q \le a_{i+1}\right], a_i\right) - Z\left(E\left[q|a_{i-1} \le q \le a_i\right], a_i\right) \\
&\ge Z\left(E\left[q|a_i \le q \le a_{i+1}\right], q\right) - Z\left(E\left[q|a_{i-1} \le q \le a_i\right], q\right) \\
&= Z\left(E\left[q|a_i \le q \le a_{i+1}\right], q\right) - Z\left(q(r), q\right) \\
&\ge Z\left(q(r'), q\right) - Z\left(q(r), q\right), \qquad (32)
\end{aligned}$$

where the first equality follows from Eq. (10b), the first inequality holds because $Z_{12} > 0$ from Eq. (6), and the last inequality holds because $Z_{11} < 0$ from Eq. (6).

If $r < r'$

$$\begin{aligned}
0 &= Z\left(E\left[q|a_{i-1} \le q \le a_i\right], a_{i-1}\right) - Z\left(E\left[q|a_{i-2} \le q \le a_{i-1}\right], a_{i-1}\right) \\
&\le Z\left(E\left[q|a_{i-1} \le q \le a_i\right], q\right) - Z\left(E\left[q|a_{i-2} \le q \le a_{i-1}\right], q\right) \\
&= Z\left(q(r), q\right) - Z\left(E\left[q|a_{i-2} \le q \le a_{i-1}\right], q\right) \\
&\le Z\left(q(r), q\right) - Z\left(q(r'), q\right), \qquad (33)
\end{aligned}$$

where the first equality follows from Eq. (10b), the first inequality holds because $Z_{12} > 0$ from Eq. (6), and the last inequality holds because $Z_{11} < 0$ from Eq. (6). Thus, Eq. (9) holds. The equilibrium investment level and face value of debt are consistent with investors' rational beliefs about q.

Finally, we show that any equilibrium must be of the form characterized in the Proposition. Consider ratings r and r' that result in different inferred project qualities, $\hat{q}(r)$ and $\hat{q}(r')$. Assume $\hat{q}(r) < \hat{q}(r')$ without loss of generality. Since $Z_{11} < 0$, CRA prefers r to r' for q less than a threshold value and r' to r for q more than the threshold value. So the ranges of q corresponding to different ratings are nonoverlapping. Moreover, continuity of Z in Eq. (5) requires that if the CRA issues a rating for values q_1 and q_2 of q than it should issue that rating for all values of q between q_1 and q_2. Thus, ratings partition the range of q into disjoint intervals. The Proposition characterizes all equilibria in which ratings partition the range of q into disjoint intervals and which satisfy the CRA's IC constraint Eq. (9). □

Proof of Corollary 1: With the functional-form assumptions that have been made, Eq. (10b) and Eq. (11b) reduce to

$$a_{i+1} = 2a_i - a_{i-1} + 4\delta. \qquad (34)$$

Substituting Eq. (11a), the solution to this difference equation is $a_i = Q_l + 2i(i-1)\delta$. Since N is the highest value of n that satisfies Eq. (11c), N is highest n such that $Q_l + 2n(n-1)\delta \leq Q_h$ or $(n-1/2)^2 - 1/4 \leq (Q_h - Q_l)/\delta$. That is, N is the largest integer not exceeding $(\sqrt{1 + 2(Q_h - Q_l)/\delta} + 1)/2$. For statement a., note that the solution to Eq. (34) and Eq. (10a) is

$$a_i = ia_1 - (i-1)Q_l + 2i(i-1)\delta. \tag{35}$$

Substituting Eq. (10c), we get $Q_h = na_1 - (n-1)Q_l + 2n(n-1)\delta$. Substituting a_1 from this equation in Eq. (35), we get $a_i = Q_l + (Q_h - Q_l)i/n - 2i(n-i)\delta$. Statement b. follows from Eq. (16) and statement a. □

Proof of Proposition 3: In Eq. (34), obtained from Eq. (11b), a_{i+1} is increasing in δ so the largest value of n satisfying Eq. (11a)-(11c) is decreasing in δ. Thus, N, the number of credit rating categories in the the equilibrium with most credit rating categories is decreasing in δ. Moreover, $\delta = \{\beta(1-p)c\}/\{2pb(\alpha+\beta)\}$ is decreasing in α and pb and increasing in β and$(1-p)c$. □

Proof of Lemma 1: First consider part a. Suppose $\hat{q}(r^i) < \hat{q}(r'^i)$, $\rho(r^i|s^i) > 0$ and $\rho(r'|s'^i) > 0$ with $s^i, s'^i \in [Q_l, Q_h]$. The incentive compatibility of CRA i's credit rating requires that the credit rating it assigns maximize the CRA's objective in Eq. (21). That is,

$$\begin{aligned}(\hat{q}(r^i) - E[q|s^i] - \delta)^2 \quad &\leq \quad (\hat{q}(r'^i) - E[q|s^i] - \delta)^2 \quad \forall s^i, r^i, r'^i, s'^i \\ &\quad\quad \text{if} \quad \rho^i(r^i|s^i) > 0, \rho(r'|s'^i) > 0. \end{aligned} \tag{36}$$

This simplifies to

$$E[q|s^i] \geq (\hat{q}(r^i) + \hat{q}(r'^i))/2 - \delta. \tag{37}$$

Taking an expectation from the perspective of investors, who observe r^i but not s^i, and substituting $\hat{q}(r^i) = E[E[q|s^i]|r^i]$ (from the rationality of the investors' inference), we get $\hat{q}(r^i) \leq (\hat{q}(r^i) + \hat{q}(r'^i))/2 - \delta$ or $\hat{q}(r'^i) - \hat{q}(r^i) \geq 2\delta$.

Next, we show that any equilibrium must be of the form characterized in the Proposition. Consider ratings r^j and r'^j issued by CRA j that result in different inferences of project qualities, $\hat{q}(r^j, r^k)$ and $\hat{q}(r'^j, r^k)$. Assume $\hat{q}(r^j, r^k) < \hat{q}(r'^j, r^k)$ without loss of generality. Consider a value s^{j*} of the CRA j's signal such that the CRA is indifferent between issuing ratings r^j and r'^j. From Eq. (22), CRA j will prefer r^j to r'^j for $s^j < s^{j*}$ and r'^j

to r^j for $s^j > s^{j*}$. So values of q for which CRA j issues different ratings do not overlap. Moreover, Eq. (6) and Eq. (9) require that if the CRA issues a rating for values s_1^j and s_2^j of s^j than it should issue that rating for all values of s^j between s_1^j and s_2^j. Thus, ratings partition the range of s^j into disjoint intervals. The Proposition characterizes all equilibria in which ratings partition the range of s^j into disjoint intervals and which satisfy CRA j's IC constraint Eq. (22). Finally, for existence, a trivial equilibrium with $n_i^A = n_i^B = 1$ satisfies Eq. (24a)-(24c). $\square$

Proof of Proposition 4: Consider an equilibrium with only one rating agency, CRA A. Suppose CRA A assigns ith rating (r_i^A) if $q^A \in [a_{i-1}^A, a_i^A]$. We now show that in any equilibrium in which both CRA A and CRA B report credit ratings, as characterized in Lemma 1, if CRA A's ith rating is identical to the rating (r_i^A) in single CRA equilibrium, then the next higher rating, $(i+1)$th rating, must reflect a larger range of q^A than in the single rating equilibrium.

$$\begin{aligned}
&\frac{E\left[q \mid q^A \in [a_i^A, a_{i+1}^A], q^B\right] - \left\{E\left[q \mid q^A = a_i^A, q^B\right] + \delta\right\}}{\left\{E\left[q \mid q^A = a_i^A, q^B\right] + \delta\right\} - E\left[q \mid q^A \in [a_{i-1}^A, a_i^A], q^B\right]} \\
&\leq \frac{\beta_h \left\{E\left[q \mid q^A \in [a_i^A, a_{i+1}^A]\right] - E\left[q \mid q^A = a_i^A\right]\right\} - \delta}{\delta + \beta_l \left\{E\left[q \mid q^A = a_i^A\right] - E\left[q \mid q^A \in [a_{i-1}^A, a_i^A]\right]\right\}} \quad \text{from Assumption 5} \\
&\leq \frac{\beta_h \left\{E\left[q \mid q^A \in [a_i^A, a_{i+1}^A]\right] - \left(E\left[q \mid q^A = a_i^A\right] + \delta\right)\right\} + \beta_h \delta - \delta}{\delta - \beta_l \delta + \beta_l \left\{E\left[q \mid q^A = a_i^A\right] - \left(E\left[q \mid q^A \in [a_{i-1}^A, a_i^A]\right] + \delta\right)\right\}}
\end{aligned} \tag{38}$$

From Eq. (10b) and Eq. (20), $E\left[q \mid q^A \in [a_i^A, a_{i+1}^A]\right] - \left(E\left[q \mid q^A = a_i^A\right] + \delta\right) = E\left[q \mid q^A = a_i^A\right] - \left(E\left[q \mid q^A \in [a_{i-1}^A, a_i^A]\right] + \delta\right)$. Denote the common value by w. Clearly $w \leq (Q_h - Q_l)/2$. Substituting w in the above inequality, we get

$$\begin{aligned}
&\frac{E\left[q \mid q^A \in [a_i^A, a_{i+1}^A], q^B\right] - \left\{E\left[q \mid q^A = a_i^A, q^B\right] + \delta\right\}}{\left\{E\left[q \mid q^A = a_i^A, q^B\right] + \delta\right\} - E\left[q \mid q^A \in [a_{i-1}^A, a_i^A], q^B\right]} \\
&\leq \frac{\beta_h w + \beta_h \delta - \delta}{\delta - \beta_l \delta + \beta_l w} < 1
\end{aligned} \tag{39}$$

where the last inequality follows from $w \leq (Q_h - Q_l)/2$ and Assumption 5. The above inequality shows that investors' project quality inference when they believe $q^A \in [a_i^A, a_{i+1}^A]$, is closer to the CRA's preferred inference (when $q^A = a_i^A$) than investors' project quality inference when they believe $q^A \in [a_{i-1}^A, a_i^A]$. With a quadratic objective function in Eq. (20), if $q^A = a_i^A$, CRA

A strictly prefers to report a rating that indicates $q^A \in [a_i^A, a_{i+1}^A]$ than a rating that indicates $q^A \in [a_{i-1}^A, a_i^A]$ so CRA's rating strategy is not incentive compatible for q^A slightly lower than a_i^A. Incentive compatibility is achieved if investors' project quality inference from reporting the $(i+1)$th rating is higher than their inference when they believe $q^A \in [a_i^A, a_{i+1}^A]$. This is possible only if rating $(i+1)$ corresponds to $q^A \in [a_i^A, a']$ where $a' > a_{i+1}^A$.

Finally, the welfare associated with the most informative equilibrium when there is only one CRA can be trivially implemented when there are two CRAs in an equilibrium where one CRA implements the rating strategy from the single-CRA equilibrium while the other CRA reports a rating with only one (uninformative) rating category. Equilibria in which both rating agencies provide informative ratings can enhance welfare. □

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