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## Moral Hazard, Agency Costs, and Asset Prices in a Competitive Equilibrium

Ram T. S. Ramakrishnan and Anjan V. Thakor

### 1. Introduction and Summary

The behavior of economic agents in the presence of uncertainty about exogenous events and imperfect information about the endogenously influenced actions of other agents with whom they contract has been receiving growing attention. In particular, the economic theory of agency explicitly recognizes that when agents enter into synergistic relationships, each agent will act in a manner consistent with the maximization of its personal welfare, thus giving rise to a phenomenon called moral hazard. Harris and Raviv [8], Holmström [10], and Shavell [21] have analyzed the nature of Pareto-optimal contractual mechanisms designed to ameliorate moral hazard and achieve efficient risk sharing. Jensen and Meckling [12], Grossman and Hart [6], and Thakor and Gorman [22] have explored the impact of moral hazard on the capital structure decisions of firms. Arrow [1] explained the absence of complete contingent claims markets on the basis of moral hazard, and Harris and Raviv [7] have examined the impact of moral hazard on the structure of health insurance contracts.

What is missing, however, is an analysis of how the *values* of assets in a competitive capital market are affected by moral hazard. Since the owners of capital frequently employ agents to manage productive assets, a dichotomy between management and ownership is an integral part of modern economies, and moral hazard can be expected to be pervasive in such settings. Unfortunately, the extant literature on equilibrium asset pricing ignores the potential effects of moral hazard on the prices of managed assets. The major impediment to theorizing about such effects has been the lack of an equilibrium theory

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that relates asset prices to corporate production-investment decisions. The asset pricing models developed in finance give us only a consistency relationship between current prices and the distribution of future prices. Since the latter distribution is *exogenous* to the model, managerial actions do not enter anywhere in the pricing equation.

Our objective in this paper is to develop a framework within which an appreciation can be gained for the qualitative differences between the traditional notion of valuation and asset pricing under moral hazard. We will focus on how differences in the *information structures* which define agency relationships in the capital market impinge on the market prices of assets. To study these effects, we will partially *endogenize* the distribution of the cash flows generated by an asset by assuming that managerial actions can affect these cash flows.

We consider a fairly simple economy. Throughout, all potential managers of assets are assumed to be identical, risk averse, and in elastic supply.<sup>1</sup> Principals, who are prospective owners of assets, are assumed to be risk neutral. Thus, the price of any asset in a competitive capital market is simply the discounted present value of the net expected terminal cash flow accruing to its owner. This net flow is the expected value of the gross end-of-period revenue yielded by the asset less the contractually agreed upon compensation for the manager. The predetermined managerial compensation formula directly influences managerial actions which in turn (partially) determine asset returns. In this framework, we develop a model in which it is assumed that the output resulting from an agent's expenditure of effort is costlessly observable without error *ex post*, and that *only one agent* is employed to manage any given asset. We use this model to prove a theorem which establishes sufficient and necessary conditions under which changes in the existing accounting system can affect the distribution of asset prices in the economy. Specific distributional

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<sup>1</sup>This 'identical agents' assumption may appear overly restrictive, especially in light of the fact that we later allow assets to be heterogeneous with respect to the technologies with which they are endowed. However, we easily can allow agents to have variegated skills--if each asset technology requires a different type of skill, all that we need is a very large number of agents associated with *each* type of skill. A competitive equilibrium will then result. Although the assumption that agents have identical preferences is not quite that inconsequential to the analysis, it is by no means unduly heroic. In a partial equilibrium analysis of the market for agents, Ross [20] has shown that if there is an elastic supply of two types of agents who differ in their risk attitudes, the less risk-averse agents will drive the more risk-averse agents out of the market. Although Ross does not consider effort aversion on the part of the agent, his analysis can be easily extended to include the agent's aversion to effort if it is assumed that effort is freely observable *ex post*.

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and managerial risk preference assumptions are utilized to *explicitly* solve for Pareto-optimal contracts and illustrate the theorem. A significant fact that emerges from the analysis is that advances in information systems, particularly as they relate to the design of performance evaluation mechanisms, can have a substantive impact on managerial productivity. In traditional valuation models, which usually focus on changes induced by purely technological innovation, this appears to be a neglected source of influence.

In a later section, we relate our analysis to the now-familiar concept of *agency costs*, first introduced by Jensen and Meckling [12]. Although there is a growing literature in finance on the influence of agency costs on corporate capital structure decisions, the utility-based foundations of the concept itself remain somewhat nebulous (see [22]), partly because of a lack of sufficient integration with the fairly well-developed literature on the economic theory of agency ([10], [21], etc.). Our analysis clearly highlights the relevant issues and identifies the parameters which impinge on the magnitude of agency costs, as well as the manner in which these costs can be precisely measured. The formal results are also explicated *graphically*, to facilitate comparison with the predominantly graphical arguments of Jensen and Meckling [12]. To do this we have to restrict the manager's feasible action space and use a rather simple production function. However, the model remains rich enough to serve our purpose effectively.

The paper is organized in four remaining sections. In Section II, we develop the basic model for valuing assets in the presence of moral hazard. The model is followed by a derivation of the conditions under which changes in the accounting system can influence asset prices. Section III contains an illustration that clearly highlights the role of each of the conditions stipulated in the theorem. In Section IV, we present a simplified version of the model with the accompanying graphical development. Finally, our concluding remarks are presented in Section V.

## II. The Basic Model

In a competitive capital market the value (or price) of an asset can be expressed as

$$(1) \quad P = L[Q(x)],$$

where  $L[\cdot]$  is a positive valuation operator and  $Q(x)$  is the cumulative distribution function of the (possibly intertemporal) net cash flows yielded by the asset. This is a general valuation expression that subsumes all the asset

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pricing models developed in finance as special cases. The specific form of the valuation operator will, of course, depend on the assumed market structure and could be influenced by investor preferences.

Irrespective of the properties of  $L[\cdot]$ ,  $Q(x)$  is invariably assumed to be exogenous to the valuation process itself and beyond the control of either the owners or the managers of the asset. While it is reasonable to assume that mutual fund managers assemble portfolios of stocks and bonds without exercising any influence over the return distributions of these securities, the plausibility of extending the assumption to the case of those who manage productive assets is questionable. Billions of dollars are spent annually to compensate managers for the use of their technical and administrative expertise. In part, such an expenditure reflects the belief that differences in management quality can induce significant differences in the benefits that can be extracted from assets.

Armed with this argument, we will allow the distribution of cash flows of a given asset to be partially endogenized by assuming that managerial actions can change this distribution. The single period cash flow,  $x$ , yielded by the asset is, therefore, a function,  $X(\alpha, \theta)$ , of the manager's choice of action (effort)  $\alpha$ , as well as the realization of some exogenous uncertainty,  $\theta$ . It is assumed, for analytical tractability, that  $\alpha$  is a scalar, but it should be understood that it subsumes all managerial actions that can affect  $x$ . No restrictions are placed on  $X(\alpha, \theta)$  except that  $\partial X / \partial \alpha > 0$  for each  $\theta \in \Theta$  (a possibly nondenumerable set of states). This implies that in any *given* state, a higher level of effort by the manager results in a higher cash flow. Further, the manager's choice of action is made before the realization of  $\theta$ .

Consider now a simple economy which consists of two types of economic entities--principals and agents. All principals are risk neutral and are assumed to be endowed with capital. They are, therefore, existing and prospective owners of assets in the economy. The principals, however, do not possess the expertise to profitably manage any asset. This necessitates the contracting of the services of managers. Managers, who are risk averse, are precluded from investing in any assets,<sup>2</sup> and, consequently, they can satisfy

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<sup>2</sup>We have, therefore, created an impenetrable barrier between principals and agents. Admittedly, this is a significantly abstract version of the real world where managers frequently have ownership claims to assets managed by other managers as well as by themselves. This implies that the theoretically convenient dichotomy between owners and managers is often very fuzzy in practice. However, given our objectives, the simple setting proposed here is not only adequate, but also helps to avoid highly complex and seemingly intractable mathematical formulations.

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their consumption needs only by selling their services. This restriction on managerial activities means that the implications of the manager reducing risk through diversification are ignored. In a state preference framework, it means that the manager is prevented from setting his or her personal marginal rates of substitution for wealth in different states equal to state prices, even in a complete capital market. The assumption of principal risk neutrality is consistent with the assumption that the states that affect the firm's output are all *firm specific* and do not affect the aggregate output of the economy. Thus, even if principals are risk averse, they will contract as if they were risk neutral if the firm-specific risk is diversifiable. For simplicity, all managers are assumed to have identical preferences and skills. Principals, as well as managers, are in elastic supply, which means a competitive equilibrium will result.

Since in an agency theoretic framework it makes little sense to discuss firms as separate from the individuals who contract to create them, the usual microeconomic distinction between firms and their employees will not be made. Firms are merely *shells* and their existence as legal entities is of no particular interest here. Instead, we will focus on assets in the economy and explore the implications, for their valuation, of principals and managers entering into contractual relationships to share the economic rents accruing from these assets. Associated with each asset is a (possibly unique) technology and thus the cash flow function for the  $i^{\text{th}}$  asset can be expressed as

$$(2) \quad x_i = X_i(\alpha, \theta) \equiv X(\alpha, \theta, T_i)$$

where  $T_i$  is the "technology" related to the  $i^{\text{th}}$  asset. So, when purchasing an asset, the principal (or group of principals) also buys the technology that comes with it. Throughout, it will be assumed that principals and managers have *homogeneous beliefs* about  $\theta$ . To manage any asset the principal must hire a manager who, for taking an action  $\alpha \in A$  (the feasible action space of the manager), will be compensated by the principal according to some predetermined fee schedule or incentive contract  $\langle \phi \rangle$ ; i.e., the manager is paid an amount  $\phi$  and the principal gets  $x - \phi$ . Throughout, we will assume that only one manager is required to manage a given asset. Every manager is assumed to be an expected utility maximizer who possesses a von-Neumann-Morgenstern utility function  $U(\alpha, \phi)$ . It is assumed that  $\partial U / \partial \alpha < 0$ ,  $\partial U / \partial \phi > 0$ , and  $\partial^2 U / \partial \phi^2 < 0$ --every manager is risk averse and has a positive marginal utility for his share of the payoff. In addition, the manager has an aversion toward higher effort. This introduces *moral hazard* in the model because the principal is interested

in maximizing the expected returns accruing to him from the asset, while the manager simply maximizes his *personal expected utility*.

Actions taken by managers, and consequently the efficiency with which assets are managed, may not be observable or verifiable by principals *ex post*. This will generally create the need for some system under which information can be generated about the activities of managers and measures for evaluating their performance can be constructed. We shall call this the accounting system and represent it by  $\Omega$ . Mathematically,  $\Omega$  is assumed to be a space of (bounded and measurable) functions and when the action taken by the manager is unobservable *ex post*, principals may choose a function  $\omega(\alpha) \in \Omega$  which we shall call a *monitor* of the manager's effort; i.e.,  $\omega(\alpha)$  conveys information about managerial activities that could be used as a basis for computing the manager's compensation.<sup>3</sup> The information conveyed by  $\omega(\alpha)$  could be imperfect and will depend on the properties of  $\Omega$ .

In a competitive capital market, the price of the  $i^{\text{th}}$  asset,  $P_i^*$ , will be<sup>4</sup>

$$(3) \quad P_i^* = \text{Max}_{\substack{\langle \phi_i \rangle \in \Phi \\ \alpha_i \in A \\ \omega_i(\alpha_i) \in \Omega}} \iint (x - \phi_i(x, \omega_i) - \xi(\omega_i)) q_i(x, \omega_i; \alpha_i) dx d\omega_i$$

subject to

$$(4) \quad \iint U(\alpha_i, \phi_i(x, \omega_i)) q_i(x, \omega_i; \alpha_i) dx d\omega_i = \bar{C}$$

$$(5) \quad \alpha_i \in \underset{\alpha_i \in A}{\text{argmax}} \iint U(\alpha_i, \phi_i(x, \omega_i)) q_i(x, \omega_i; \alpha_i) dx d\omega_i$$

where

$q_i(., .; .)$  is the joint density function of the output and the monitor for the  $i^{\text{th}}$  asset, conditional upon the effort  $\alpha_i$  chosen by the  $i^{\text{th}}$  asset's

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<sup>3</sup>Note that  $\Omega$  could consist of a variety of functions in addition to monitoring functions, and, in practice of course, the accounting system is used for considerably more than merely monitoring managers. However, for our purpose it suffices to focus on the "monitoring subset" of  $\Omega$ .

<sup>4</sup>The notation "argmax" means the set of arguments that maximizes the objective function. Since the expectation in (5) need not be concave in  $\alpha$ , the use of this notation is necessary. If the set of optimal actions,  $\{\alpha_i^*\}$ , is not a singleton, it will be assumed that the manager will choose an  $\alpha_i^* \in \{\alpha_i^*\}$  so as to maximize the principal's welfare.

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manager;<sup>5</sup>

$\xi(\omega_i)$  is the cost of utilizing the monitor and could depend on the properties of the monitor used;

$\Phi$  is the space of bounded and measurable feasible fee schedules;<sup>6</sup>

$\phi_i(\cdot, \cdot)$ , the compensation of the manager of the  $i^{\text{th}}$  asset, depends on the output as well as some *ex post* measure of effort;<sup>7</sup> and

$\bar{c}$  is the manager's reservation utility level. It is assumed throughout that the terminal cash flow,  $x$ , generated by the management of the asset under the agency relationship, is costlessly observed *without error ex post*.

Essentially, this mathematical formulation implies that in a *competitive* market the price of any asset will be expected value of the *net cash flow* accruing to the (prospective) owner or owners of the asset, *assuming that the asset will be optimally managed*.<sup>8</sup> By optimal asset management, we mean that the principal will select a (cost-effective) monitor and a fee schedule that will induce the manager to take an action that will maximize the principal's welfare subject to the constraint that the manager's expected utility (given his or her optimal choice of action in response to the fee schedule chosen) is equal to a certain minimum reservation level  $\bar{c}$ .<sup>9</sup> Note that  $\bar{c}$  will be the outcome of an equilibrium in the managerial labor market. It is clear that the formal statement of the valuation problem is considerably simplified by

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<sup>5</sup>We have suppressed the dependence of  $x$  on  $\theta$  and have employed the distribution of  $x$ . This is done to avoid differentiability problems caused by the assumed boundedness of the fee schedules. A discussion of this issue appears in [13] and [9].

<sup>6</sup>If  $\Phi$  is a family of bounded functions and is equicontinuous or consists of functions of bounded variation, a solution to the maximization problem in (3), (4), and (5) can be guaranteed. However, if  $\Phi$  is expanded to contain all bounded and measurable functions, no general existence proof is available. In that case, it will be necessary to assume that there exists an optimal solution  $(\alpha_i^*, \phi_i^*(x, \omega_i), \omega_i^*(\alpha_i))$  such that  $\alpha_i^* (\neq 0) \in \text{Int } A$ .

<sup>7</sup>Thus, if  $\alpha_i$  is observable without error *ex post*,  $\omega_i(\cdot) \equiv \alpha_i$ , and if  $\Omega$  is empty,  $\omega_i(\cdot)$  will be the null element. In general,  $\omega_i(\cdot)$  will be a random variable whose (marginal) density function will depend on  $\alpha_i$ .

<sup>8</sup>It is assumed, without loss of generality throughout this paper, that the discount factor (the riskless rate of interest) for principals is zero.

<sup>9</sup>In general, a weak inequality ( $\geq$ ) should be used in (4) to allow for possible monopolistic elements in the managerial labor market. However, the equality implies that this market is assumed to be competitive. In fact, in a subsequent section, we will formally prove that with a competitive labor market, this inequality will be a binding equality.



the assumed risk neutrality of principals--with risk-averse principals and an incomplete market, at least one serious problem that would have to be resolved is that of a possible lack of unanimity among the (prospective) owners of the assets about its value (see Baron [2]). In general, the monitoring functions contained in  $\Omega$  can be classified as informative and noninformative, efficient and inefficient. These classifications are defined below. Since in each definition the asset is fixed, the subscript  $i$  is dropped.<sup>10</sup>

Definition 1:

Consider a specific asset. A monitor  $\omega$  of  $\alpha$  is called *informative* with respect to this asset if there exist at least two measurable sets  $M^+$  and  $M^-$  in the range of  $\omega$  such that

$$(6) \quad \frac{q_{\alpha}(x, M^+; \alpha)}{q(x, M^+; \alpha)} \neq \frac{q_{\alpha}(x, M^-; \alpha)}{q(x, M^-; \alpha)}$$

where

$$q(x, M^+; \alpha) = \int_{M^+} q(x, \omega; \alpha) d\omega, \quad q(x, M^-; \alpha) = \int_{M^-} q(x, \omega; \alpha) d\omega,$$

$$q_{\alpha}(x, M^+; \alpha) = \int_{M^+} q_{\alpha}(x, \omega; \alpha) d\omega,$$

and

$$q_{\alpha}(x, M^-; \alpha) = \int_{M^-} q_{\alpha}(x, \omega; \alpha) d\omega.$$

$\omega$  is called *noninformative* if  $\frac{q_{\alpha}(x, \omega; \alpha)}{q(x, \omega; \alpha)}$  is constant for almost every  $\omega$ .

Subscripts denote partial derivatives.

Definition 2:

Consider a specific asset. A monitor  $\omega$  of  $\alpha$  is called *efficient* with respect to this asset if  $P^* > P^{\circ}$ , where  $P^*$  is the price of the asset if the monitor is used and  $P^{\circ}$  is its price if the monitor is not used. The monitor is called *inefficient* if  $P^* \leq P^{\circ}$ .

These definitions are now used, in the theorem given below, to establish the contention that when moral hazard is explicitly considered in asset valuation, changes in the accounting system can affect the probability distribution of asset cash flows and, consequently, asset prices. The key to this result is the *ex post* unobservability of managerial actions--with perfect observability forcing contracts can be employed to eliminate the moral hazard

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<sup>10</sup>Definition 1 is due to Holmstrom [10].

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problem, as demonstrated by Harris and Raviv [8].<sup>11</sup> In the proof of the theorem, the assumptions that the output,  $x$ , is observed without error *ex post* and that only one manager is engaged in the productive activity related to the asset play an important role.

Theorem 1:

Consider some specific asset. Assume that managerial actions cannot be observed without error *ex post*, and that the current accounting system,  $\Omega^0$ , contains only noninformative and inefficient monitors.<sup>12</sup> Then, the (minimum) *necessary* conditions for a change in  $\Omega^0$  to cause an increase in the price of the asset are:

- (i) managers are risk averse;
- (ii) the probability density function of the asset's cash flow does<sup>13</sup> not have a compact support that moves with  $\alpha$ ; and
- (iii) the change in  $\Omega^0$  is affected through the addition of at least one informative monitor.

The above conditions are *sufficient* if the informative monitor added is also efficient.

Proof:

See Appendix.

The Proof demonstrates how a change in the accounting system can alter the manager's choice of action (in the Proof, the *monitor* induces the manager to change his effort) and, thereby, the price of the asset. Although it is assumed that the initial starting point is an accounting system that contains only inefficient monitors, the theorem easily can be extended to the case where an accounting system with inefficient monitors is augmented by more efficient monitors.

A point to be noted is that the fluctuations in the equilibrium price of the asset across different account regimes are not due to any variation in the *intrinsic* value of the asset, but merely reflect differences in the *efficiency* with which it is *managed*. This notion is now illustrated through an example in the next section.

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<sup>11</sup>Forcing contracts were first referred to by Ross [18].

<sup>12</sup>It will be proven in this theorem that any noninformative monitor, irrespective of its cost, will be inefficient.

<sup>13</sup>When the distribution of an asset's cash flow has a compact support that moves with  $\alpha$ , it means that there is a positive probability of detecting any deviation from a prespecified action.

### III. An Illustration

Consider an economy in which all principals are risk neutral and all managers have utility functions of the form  $U(\alpha, \phi(x)) = \sqrt{\phi(x)} - \alpha$  and  $\bar{C} = 0$ . Consider two assets--B and D. Asset B's cash flow is generated by the production function  $x_B = \alpha \theta_B$ , where  $\theta_B$  is lognormal with  $E(\log \theta_B) = 0$  and  $\text{var}(\log \theta_B) = \sigma^2$ . Asset D's cash flow is generated by the production function  $x_D = \alpha \theta_D$ , where  $\theta_D$  is uniformly distributed over  $[\exp(\sigma^2/2) - k, \exp(\sigma^2/2) + k]$ , with  $k \in (0, \exp(\sigma^2/2))$ , a fixed scalar. We want to compute the prices of these two assets under different information structures.

#### Solution:

First note that for asset B

$$E(\log x_B) = \log \alpha, \quad \text{var}(\log x_B) = \text{var}(\log \theta_B) = \sigma^2,$$

$$q(x_B | \alpha) = \{\sqrt{2\pi} \sigma x_B\}^{-1} \exp\{-[(\sqrt{2}\sigma)^{-1} \log(x_B/\alpha)]^2\},$$

$$q_\alpha(x_B | \alpha) = q(x_B | \alpha) \cdot (\alpha \sigma^2)^{-1} \log(x_B/\alpha), \quad \text{and}$$

$$E(x_B) = \exp[\log \alpha + \sigma^2/2] = \alpha \psi, \quad \text{where } \psi \equiv \exp(\sigma^2/2).$$

Similarly, for asset D

$$E(x_D) = \alpha \psi \quad \text{and} \quad \text{var}(x_D) = k^2/3.$$

Now let  $\lambda$  be the multiplier for (4) and  $\mu$  the multiplier for (5). If only  $x$  is observable *ex post*, (5) will have to be used. Assuming for the moment that  $\Omega$  is empty and optimizing the Lagrangian pointwise gives us the following characterization for  $\phi_i^*(x)$  ( $i = B, D$ ):

$$(9) \quad \{\partial U(\phi_i(x))/\partial \phi_i(x_i)\}^{-1} = \lambda_i + \mu_i \{\partial q(x_i; \alpha_i)/\partial \alpha_i\} \{q(x_i; \alpha_i)\}^{-1}$$

where  $U$  is assumed to be separable ( $U(\alpha, \phi(x)) \equiv U(\phi(x)) - V(\alpha)$ ) as in the proof of Theorem 1 and  $\alpha_i$  is obtained from (5). Note that  $\lambda_i$  is obtained as a solution to (4) for a given  $\bar{C}$  and  $\mu_i$  is obtained as a solution to

$$(10) \quad \int (x_i - \phi_i(x_i)) (\partial q(x_i; \alpha_i)/\partial \alpha_i) dx_i + \mu_i \left\{ \int U(\phi_i(x_i)) (\partial^2 q(x_i; \alpha_i)/\partial \alpha_i^2) dx_i - \partial V^2/\partial \alpha^2 \right\} = 0.$$

However, if both  $x$  and  $\alpha$  are observable *ex post*, (5) can be dropped and the optimal contract will be equivalent to

$$\phi_i^*(\alpha) = \begin{cases} t & \text{if } \alpha = \alpha_i^* \\ 0 & \text{otherwise} \end{cases}$$

where  $t$  (a constant) is obtained as a solution to

$$(11) \quad \{\partial U(t)/\partial t\}^{-1} = \lambda.$$

Initially assume that both  $x$  and  $\alpha$  are observable without error *ex post*. Solving for the optimal contracts gives

$$\text{and} \quad \begin{aligned} \phi_B^*(\alpha) &= \begin{cases} (\psi/2)^2 & \text{if } \alpha = \psi/2 \\ 0 & \text{otherwise} \end{cases} \\ \phi_D^*(\alpha) &= \begin{cases} (\psi/2)^2 & \text{if } \alpha = \psi/2 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

where  $\phi_B^*(\cdot)$  and  $\phi_D^*(\cdot)$  are the optimal contracts for assets B and D, respectively. It is clear that  $\alpha_B^* \equiv \alpha(\langle \phi_B^* \rangle) = \psi/2$  and  $\alpha_D^* \equiv \alpha(\langle \phi_D^* \rangle) = \psi/2$ , and the prices of the two assets, in a competitive capital market, should be the same, namely  $(\psi/2)^2$ .

Next, assume that only  $x$  is observable *ex post* and  $\Omega$  is empty. Using the Euler equation we see that the optimal contract for asset B is of the form

$$(12) \quad \hat{\phi}_B(x_B) = \zeta_1^2 [1 + \log(x_B/\zeta_1)]^2$$

where  $\zeta_1 \equiv [2(1+\sigma^2)]^{-1}\psi$ , and the manager's optimal action is  $\hat{\alpha}_B \equiv \alpha(\langle \hat{\phi}_B \rangle) = \zeta_1$ . Lack of observability of managerial actions reduces the price of the asset to  $(\psi/2)^2(1+\sigma^2)^{-1}$ . Thus, only if  $\sigma^2=0$  will the price of asset B be unaffected by the principal's inability to costlessly verify the manager's actions. This, of course, simply confirms a widely known result--in a world of certainty, information about  $\alpha$  is of no value because it can be directly inferred from  $x$  anyway.

For asset D, however, the first-best solution can be obtained even under uncertainty if  $3k < \psi$ . This is possible because the distribution of this asset's cash flow has a compact moving support. To see this, note that with the action  $\psi/2$ , the lowest possible output is  $(\psi/2)(\psi-k)$ . Thus, a contract of the form

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$$(13) \quad \hat{\phi}_D(x_D) = \begin{cases} (\psi/2)^2 & \text{if } x \geq (\psi/2)(\psi-k) \\ 0 & \text{otherwise} \end{cases}$$

will do the trick if,

$$\hat{\alpha}_D \equiv \alpha(\langle \hat{\phi}_D \rangle) = \psi/2.$$

Suppose the manager picks an action  $\alpha_D^0 \neq \hat{\alpha}_D$ . In this case,

$$P(x_D > (\psi/2)(\psi-k)) = \{2\alpha_D^0 k\}^{-1} \{\alpha_D^0(\psi+k) - (\psi/2)(\psi-k)\},$$

and the manager's expected utility is

$$[\psi/2] P(x_D \geq (\psi/2)(\psi-k)) - \alpha_D^0,$$

with the symbol  $P(\cdot)$  denoting the probability measure. Note that this term can be positive (and thus greater than  $\bar{c} = 0$ ) only if  $\alpha_D^0 < \psi/2$ . This means that if the manager decides to take an action other than  $\psi/2$ , the manager must necessarily choose a *lower* action. Further, for the manager to have a nonzero probability of escaping detection (that his or her action differs from  $\alpha_D^0$ ), we must have  $\alpha_D^0(\psi+k) > (\psi/2)(\psi-k)$ , which implies  $\alpha_D^0 > (\psi/2)(\psi-k)(\psi+k)^{-1}$ . Thus, if  $\alpha_D^0 \neq \hat{\alpha}_D$ , it must be true that  $\alpha_D^0 \in ((\psi/2)(\psi-k)(\psi+k)^{-1}, \psi/2)$ . Now if  $\alpha_D^0$  is the manager's optimal choice, it must satisfy the first-order condition

$$(\psi/2)\{[4(\alpha_D^0)^2 k]^{-1}[\psi(\psi-k)]\} - 1 = 0,$$

which means  $\alpha_D^0 = \sqrt{\psi(\psi-k)(8k)^{-1}}$ . It is easy to see that if  $\psi > 3k$ , then  $\alpha_D^0 > \psi/2$ , and a first-best solution is not attainable. Obviously, then, as long as the distribution of  $\theta_D$  satisfies the condition  $\psi > 3k$ , the manager's optimal choice of action,  $\alpha_D^0$ , will be  $\psi/2$ , and the contract described in (13) will yield a first-best solution.<sup>14</sup>

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<sup>14</sup> An alternative means of obtaining a first-best solution is to replace the zero in (13) with a penalty that is large enough to force the agent to take the desired action  $\psi/2$ . However, a commonly stated objection to penalties is that they may have to be infeasibly large. If there are constraints on managers' wealth, which restrict the extent to which managers may be penalized, it may be impossible to resort to this scheme for achieving first-best efficiency.

## The Role of Monitoring

It is clear that if the  $\theta_D$  distribution satisfies the stipulated restriction, the principal has no incentive to acquire costly monitoring for asset D. On the other hand, monitoring could prove valuable for asset B. Suppose a monitor  $\omega$  is available and its relationship to  $\alpha$  can be expressed as  $\omega = \alpha e$ , where  $e$  is a lognormal random variable with  $\text{cov}(e, \theta) = 0$ ,  $E(\log e) = 0$ , and  $\text{var}(\log e) = \sigma_\omega^2$ . Assume initially that the monitor is costless. It is obvious then that it is efficient.

For the moment, let us drop the subscript B for notational convenience. Since  $q_\alpha(x, \omega; \alpha) = q(x, \omega; \alpha) \cdot \Delta$ , where  $\Delta \equiv (\alpha \sigma^2)^{-1} \log(x/\alpha) + (\alpha \sigma_\omega^2)^{-1} \log(\omega/\alpha)$ , the Euler equation can be used to show that the optimal contract is of the form

$$(14) \quad \phi^*(x, \omega) = \zeta_2^2 [1 + \{\sigma_\omega^2 / (\sigma^2 + \sigma_\omega^2)\} \log(x/\zeta_2) + \{\sigma^2 / (\sigma^2 + \sigma_\omega^2)\} \log(\omega/\zeta_2)]^2$$

where

$$\zeta_2 \equiv \psi [\sigma^2 + \sigma_\omega^2] \{2(\sigma^2 + \sigma_\omega^2 + \sigma^2 \sigma_\omega^2)\}^{-1}.$$

With this contract, the manager's optimal choice of action is  $\alpha^* = \zeta_2$ , and the price of the asset is

$$(15) \quad P^* = \psi^2 (\sigma^2 + \sigma_\omega^2) \{4(\sigma^2 + \sigma_\omega^2 + \sigma^2 \sigma_\omega^2)\}^{-1}.$$

An examination of (14) reveals an intuitively appealing property--the weight assigned to the monitor in the optimal contract is *inversely* proportional to the variance of the monitor relative to that of the output. In the limit, as this variance goes to infinity, the weight attached to the monitor goes to zero. Of course, this observation is predicated upon the assumption that the cost of using the monitor,  $\xi = 0$ . Otherwise, if  $\xi$  and  $\sigma_\omega^2$  are also inversely related, the monotonicity of the (inverse) relationship between the variance of the monitor and the extent of its use in the optimal contract may be violated. Also note that  $\lim_{\sigma_\omega^2 \rightarrow \infty} P^* = \psi^2 \{4(1 + \sigma^2)\}^{-1}$ , which is the same as

the second-best solution with no monitoring, and  $\lim_{\sigma_\omega^2 \rightarrow 0} P^* = \psi^2/4$ , which is identical to the first-best solution when  $\alpha$  is observable. Therefore, if the

monitoring technology has a positive cost  $\xi$ , using the monitor with asset B would be of value if  $P^* - \xi > \psi^2 \{4(1 + \sigma^2)\}^{-1}$ , where  $P^*$  is given by (15).

The above illustration reemphasizes the potential dependence of asset values on the observability of contract variables *ex post* and helps to clarify

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an apparent misunderstanding about the role of accounting. Numerous empirical studies of the efficient markets hypothesis have found that a significant portion of the information contained in accounting statements is impounded in security prices even before these statements are released. This appears to have led to the conclusion that at any instant in time the distribution of future security prices is independent of the accounting system and somehow determined by other factors in the economy. For example, Beaver [3] argues that accounting information is valuable because it helps the individual investor to assess the systematic risk of securities. Gonedes [5] claims that newly-generated accounting information provides signals on the true underlying distribution of returns. If the basic premise of this paper (that managers can affect asset cash flows) is accepted, then it is clear that such theorizing, which ignores the stewardship role of accounting, is at best incomplete. The reason is that principals can very rarely observe managerial actions *ex post* and, therefore, in the absence of any monitoring technology, incentive contracts would depend only on  $x$ . However, accountants (particularly auditors) provide principals with information about managerial actions in addition to that provided by the realized output of the firm. This permits the use of monitors and more efficient contracts and, consequently, creates a variation in the distribution of asset cash flows via a change in managerial actions. For example, in the numerical illustration, the open interval  $(\psi^2\{4(1+\sigma^2)\}^{-1}, \psi^2/4)$  corresponds to the possible distribution of asset (B) prices associated with varying degrees of efficiency of  $\Omega$ . In fact, assuming for the moment that  $\xi$  is independent of  $\sigma_\omega^2$ , one can parameterize the efficiency of  $\Omega$  by  $\sigma_\omega^2$ . Since  $\partial P^*/\partial \sigma_\omega^2 < 0$ , we can say that as the accounting system becomes more efficient (that is, as  $\sigma_\omega^2$  declines), the price of asset B increases. In other words, associated with *each* degree of efficiency in  $\Omega$  is a *different competitive equilibrium* price for asset B. This implies that the distribution of asset prices is not independent of the accounting system, as is so often claimed, but is at least partially determined by it.

#### IV. Agency Costs and Competitive Asset Valuation

Our principal objective in this section is to highlight, through the use of a very simple model, the salient points covered in the preceding discussion and to graphically explain the significance and nature of agency costs in the determination of equilibrium asset prices under imperfect information.

##### A. The No-Monitoring Case

Let each manager's feasible action space be  $A \equiv \{0, 1\}$ , and assume that

the (stochastic) output,  $x$ , can take one of two values, 0 or 1. The action chosen by the manager can affect the probability of "success." That is, define the probability mass function

$$(16) \quad q(x|\alpha) = \begin{cases} s & \text{if } x=1, \alpha=1 \\ 1-s & \text{if } x=0, \alpha=1 \\ p & \text{if } x=1, \alpha=0 \\ 1-p & \text{if } x=0, \alpha=0 \end{cases}$$

where  $1 \geq s > p > 0$ .

Assume that every manager's preferences are described by the utility function

$$\hat{U}(\alpha, \phi) = U(\phi) - V(\alpha),$$

where

$$V(\alpha) = \begin{cases} \bar{a} > 0 & \text{if } \alpha=1 \\ 0 & \text{if } \alpha=0. \end{cases}$$

As usual, assume  $U'(\cdot) > 0$ ,  $U'' < 0$ . Since  $U(\cdot)$  is strictly increasing over its domain, it is invertible. So, define  $\phi(U) \equiv U^{-1}(\cdot)$  as the inverse of the  $U(\cdot)$  function. As in previous sections,  $\bar{c}$  denotes the managerial reservation utility level, and primes on *functions* will be used to signify partial derivatives.

Now the principal's objective is to find a contract with the least expected cost that motivates the manager to choose  $\alpha=1$ . If the manager's choice of action is unobservable *ex post*, this contract will be of the form

$$(17) \quad \phi(x) = \begin{cases} m' & \text{if } x=1 \\ n' & \text{if } x=0. \end{cases}$$

Let  $U(m') = m$  and  $U(n') = n$ . The manager's expected utilities can then be expressed as

$$(18) \quad E \hat{U}(\cdot, \cdot) = \begin{cases} sm + (1-s)n - \bar{a} & \text{with } \alpha=1 \\ pm + (1-p)n & \text{with } \alpha=0. \end{cases}$$

For *individual rationality* (IR), we need the expected utility in (18) to be at least as great as  $\bar{c}$ , and, for *incentive compatibility* (IC), we need the expected utility in (18) to be at least as large as that in (19). Thus, we can postulate the following optimization problem for the principal.



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$$(20) \quad \begin{array}{ll} \text{Minimize} & s\phi(m) + (1-s)\phi(n) \\ & m, n \end{array}$$

subject to

$$(21) \quad \text{IR: } sm + (1-s)n \geq a + \bar{c}$$

$$(22) \quad \text{IC: } (s-p)(m-n) \geq \bar{a}.$$

The following theorem establishes an important fact regarding the above problem.

Theorem 2:

The IR and IC constraints are binding equalities.

Proof:

Letting  $\lambda$  and  $\mu$  represent the Lagrange multipliers associated with the constraints (21) and (22), the first-order conditions with respect to  $m$  and  $n$ , respectively, yield

$$(23) \quad \phi'(m) = \lambda + \mu(s-p)/s$$

and

$$(24) \quad \phi'(n) = \mu - \mu(s-p)/(1-s).$$

Since  $\phi(\cdot)$  is increasing and strictly convex, and since  $m > n$ , we have  $\phi'(m) > \phi'(n)$ . From (23) and (24) it follows then that  $\mu > 0$ , because  $s > p$ . Further, from (24),  $\phi'(n) > 0$  implies that  $\lambda > 0$ . Thus, both (21) and (22) are binding equalities. *Q.E.D.*

We can now verify that the price of the managed asset will be lowered by the principal's inability to observe managerial actions *ex post*. Using equalities in (21) and (22) gives us

$$(25) \quad \bar{m} = \bar{c} + \bar{a}(1-p)(s-p)^{-1}, \quad \bar{n} = \bar{c} - \bar{a}p(s-p)^{-1}.$$

From (21) we have

$$s\bar{m} + (1-s)\bar{n} = \bar{c} + \bar{a},$$

and from Jensen's inequality for convex functions it follows that

$$s\phi(\bar{m}) + (1-s)\phi(\bar{n}) > \phi(\bar{c} + \bar{a}),$$

In Figure 1, we have depicted the above solution graphically. The utility (disutility) to the manager is plotted against the probability of realizing  $x=1$ , (the "good" output) in the top half of the figure. With  $\alpha=1$ , this probability is  $s$ , the manager's effort disutility is  $\bar{a}$  and his expected utility for wealth,  $U(\phi)$ , is  $\bar{a}+\bar{C}$ . With  $x=1$ , the manager's monetary compensation is  $m'$  and the utility derived from it is  $m$ . If  $x=0$ ,  $n'$  is the compensation and  $n$  the resulting utility. The two utility levels,  $m$  and  $n$ , are marked on two horizontal lines, the lower one corresponding to the probability of  $x=1$  being zero and the upper one corresponding to the (almost) sure realization of  $x=1$ . For any nonzero probability of realizing  $x=1$  that is less than one, the associated *expected* utility for the manager lies on the straight line joining  $m$  and  $n$ .

[illegible]

Now, if the manager chooses  $\alpha=1$ , the probability of realizing  $x=1$  will be  $s$  and the manager's expected utility will be given by the intersection of the line  $C_1 C_2$  (through  $s$ ) and the line  $mn$ . This is point A. To satisfy individual rationality (constraint (21)), the  $mn$  line should cut  $C_1 C_2$  to the right of A. If  $\alpha=0$  is chosen, the probability of  $x=1$  is  $p$  and the manager's expected utility for wealth is provided by the intersection of the  $mn$  line and  $D_1 D_2$ , which passes through  $p$ . To satisfy incentive compatibility (constraint (22)), this intersection should be at least  $\bar{a}$  utiles to the left of point A.

To compute the optimal values of  $m$  and  $n$ , we have to consider the principal's problem. The lower half of Figure 1 is a plot of the manager's utility,  $U$ , versus the monetary cost,  $\phi(U)$ , of providing this utility. If  $\bar{m}$  and  $\bar{n}$  are chosen as the optimal values of  $m$  and  $n$ , then  $\phi(\bar{m}) = \bar{m}'$  and  $\phi(\bar{n}) = \bar{n}'$  are the respective monetary equivalents in terms of fees to be paid to the manager. The expected cost to the principal is  $s\bar{m}' + (1-s)\bar{n}'$ . Since  $s = R_1 T/R_1 R_2 = FA/FE$ , and because  $H$ ,  $G$ , and  $L$  lie vertically below  $F$ ,  $E$ , and  $A$ , we have  $HL/HG = s$ . Thus,  $L$  is the relevant point on the segment  $HG$ , and  $JL$  is the expected cost to the principal of implementing the contract  $\langle \bar{m}, \bar{n} \rangle$ . From this we can deduce that it is in the principal's interest to keep the line  $\bar{m} \bar{n}$  corresponding to the optimal contract as far to the left as possible. Therefore,  $\bar{m} \bar{n}$  must pass through A. Moreover, the line  $\bar{m} \bar{n}$  should have the highest possible slope, because the chord  $GH$  will shift upward (and increase the principal's expected cost) as the slope of  $\bar{m} \bar{n}$  decreases. Thus, the expected utilities in the optimal contract,  $\bar{m}$  and  $\bar{n}$ , are obtained by simply extending  $BA$  in both directions till it meets the two horizontal lines corresponding to the probability of  $x=1$  being zero and one, respectively.

If the manager's actions can be observed perfectly *ex post*, the expected utility level of  $\bar{a}+\bar{c}$  can be guaranteed by offering the manager  $\phi(\bar{a}+\bar{c})$ , contingent upon his selecting  $\alpha=1$ . This remuneration corresponds to the point K. Thus,  $JK$  represents the first-best cost. Since  $\phi(U)$  is convex,  $K$  will lie below the chord  $HG$ , and, hence, below  $L$ , where  $LJ$  is the second-best cost. The additional cost  $KL$  is incurred by the principal only because the manager is risk averse and his or her actions cannot be observed. We refer to this as an *agency cost*.

#### B. The Impact of Monitoring

Suppose a monitor,  $\omega$ , of the manager's effort is available to the principal, and has the probability mass function

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$$(26) \quad g(\omega|\alpha) = \begin{cases} s & \text{if } \omega=1, \alpha=1 \\ 1-s & \text{if } \omega=0, \alpha=1 \\ p & \text{if } \omega=1, \alpha=0 \\ 1-p & \text{if } \omega=0, \alpha=0. \end{cases}$$

Assume that  $x$  and  $\omega$  are stochastically independent.

It is apparent that the optimal contract will be *symmetric* in  $x$  and  $\omega$ , and will be of the form

$$\phi(x, \omega) = \begin{cases} m' & \text{if } x=1, \omega=1 \\ f' & \text{if } x=0, \omega=1 \text{ or if } x=1, \omega=0 \\ n' & \text{if } x=0, \omega=0. \end{cases}$$

For the moment, assume that the direct cost of using the monitor,  $\xi$ , is zero. The following theorem characterizes the usefulness of the proposed monitor.

Theorem 3:

Using the monitor  $\omega$  increases the equilibrium price of the asset.

Proof:

Let  $U(m') = m$ ,  $U(n') = n$ , and  $U(f') = f$ . Define the expected utilities achievable from the *new* contract as  $\bar{m}$ ,  $\bar{n}$ , and  $\bar{f} = (\bar{m} + \bar{n})/2$ , respectively, where  $\bar{m}$  and  $\bar{n}$  are the expected utilities resulting from the optimal contract *without* the monitor.

We will first show that the triplet  $\langle \bar{m}, (\bar{m} + \bar{n})/2, \bar{n} \rangle$  is *feasible*. With  $\alpha=1$ , the manager's expected utility for wealth is

$$EU = s^2 \bar{m} + 2s(1-s)(\bar{m} + \bar{n})/2 + (1-s)^2 \bar{n} = s\bar{m} + (1-s)\bar{n} = \bar{a} + \bar{c} \quad (\text{from (25)}).$$

With  $\alpha=0$ , the manager's expected utility for wealth is

$$EU = p^2 \bar{m} + 2p(1-p)(\bar{m} + \bar{n})/2 + (1-p)^2 \bar{n} = p\bar{m} + (1-p)\bar{n} = \bar{c} \quad (\text{from (25)}).$$

Thus, the proposed contract with the monitor is both individually rational and incentive compatible.

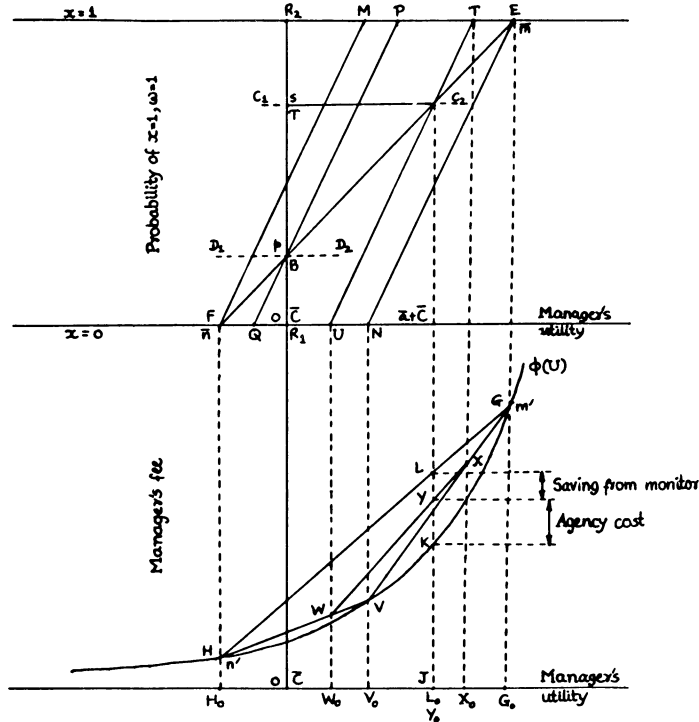
Next, we will establish that the expected cost to the principal is lower with the monitor. This expected cost is

$$\begin{aligned}
&= s^2 \phi(\bar{m}) + 2s(1-s)\phi((\bar{m}+\bar{n})/2) + (1-s)^2\phi(\bar{n}) \\
&< s[s\phi(\bar{m}) + (1-s)\{\phi(\bar{m})/2 + \phi(\bar{n})/2\}] \\
&+ (1-s)[(1-s)\phi(\bar{n}) + s\{\phi(\bar{m})/2 + \phi(\bar{n})/2\}] \\
&\text{(by Jensen's inequality)} \\
&= s\phi(\bar{m}) + (1-s)\phi(\bar{n}),
\end{aligned}$$

which is the expected cost without the monitor. *Q.E.D.*

The solution of the valuation problem with monitoring is sketched in Figure 2. In the top half, we mark  $\bar{m}$  and  $\bar{n}$  (the optimal utility levels without monitoring) at points E and F, as we did in Figure 1. Let the points M and N correspond to the utility level  $(\bar{m}+\bar{n})/2$ . Draw QP and UT parallel to FM through B and A, respectively. Then,  $MP/ME = FQ/FN = p$  and  $MT/ME = FU/FN = s$ . This implies that the point P corresponds to an expected utility for wealth of  $p\bar{m} + (1-p)\bar{f}$ , the point T to an expected utility of  $s\bar{m} + (1-p)\bar{f}$ , the point Q to an expected utility of  $p\bar{f} + (1-p)\bar{n}$ , and the point U to an expected utility of  $s\bar{f} + (1-s)\bar{n}$ . The expected utilities from choosing  $\alpha=1$  and  $\alpha=0$  are,

FIGURE 2



therefore, the utility levels corresponding to the points A and B, respectively. In other words, a contract yielding the utility triplet  $\langle \bar{m}, (\bar{m}+\bar{n})/2, \bar{n} \rangle$  is *feasible* as long as A and B are on UT and QP at  $s$  and  $p$ , respectively.

In the lower half of Figure 2, the respective monetary costs can be obtained from the points H, V, and G on the  $\phi(U)$  function. Points W and X correspond to the expected utility levels  $s\bar{f} + (1-s)\bar{n}$  and  $s\bar{m} + (1-p)\bar{f}$ , respectively. Thus, point W involves an expected cost of  $s\phi(\bar{f}) + (1-s)\phi(\bar{n})$  and point X involves an expected cost of  $s\phi(\bar{m}) + (1-p)\phi(\bar{f})$ . This means that the *actual* expected cost to the principal corresponds to a point on the chord WX at a distance  $sWX$  from W. This point is Y, which is directly below L on AL. To see why this is true, consider the vertical projections of the various points down to the utility axis and note that

$$H_O L_O / L_O G_O = H_O W_O / W_O V_O = V_O X_O / X_O G_O = s/(1-s),$$

which implies

$$(H_O L_O - H_O W_O) / (L_O G_O - X_O G_O) = W_O L_O / Y_O X_O = s/(1-s).$$

Also observe that the chord WX will always be below the chord HG. Thus, the expected cost is lower when the monitor is employed and LY represents the savings to the principal due to the monitor. The new agency cost is YK.

The above analysis now can be readily compared to the basic model of Jensen and Meckling [12]. The benefits of *monitoring and bonding* activities to which they refer are captured by LY, the reduction in agency costs due to these activities. If we introduce a positive direct cost of using the monitor, then  $\xi$  can be labeled as the monitoring and bonding component of total agency costs. The *residual loss* defined by Jensen and Meckling is analogous to YK, the welfare loss resulting from moral hazard in spite of monitoring.

By now it should be transparent that the agency cost in our model is created by managerial risk aversion, the presence of an exogenous uncertainty affecting the output, and imperfect (noisy) monitoring of the manager's actions. In the (implicit) certainty framework of Jensen and Meckling [12], agency costs arise due to an *exogenous restriction* limiting the feasible contract space to just stocks and bonds.

One final issue remains. In our discussion of monitoring thus far, we have used a contract that was arbitrarily picked rather than optimally chosen. The nature of the optimal incentive contract *with* monitoring is the subject of our last theorem.

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Theorem 4:

With the monitor characterized in (26), the Pareto-optimal incentive contract satisfies

$$(28) \quad [\phi'(m) - \phi'(f)] / [\phi'(f) - \phi'(n)] = p(1-s)/s(1-p).$$

Moreover, both the individual rationality and incentive compatibility constraints are binding as equalities.

Proof:

The principal's problem is

$$(29) \quad \begin{array}{l} \text{Minimize} \quad s^2\phi(m) + 2s(1-s)\phi(f) + (1-s)^2\phi(n) \\ m, f, n \end{array}$$

subject to

$$(30) \quad \text{IR: } s^2m + 2s(1-s)f + (1-s)^2n \geq \bar{a} + \bar{c}$$

$$(31) \quad \text{IC: } (s^2 - p^2)m + 2f[s(1-s) - p(1-p)] + n[(1-s)^2 - (1-p)^2] \geq \bar{a}.$$

Appending the multipliers  $\lambda$  and  $\mu$  to (30) and (31) respectively, the usual first-order conditions yield

$$(32) \quad \phi'(m) = \lambda + \mu(s-p)(s+p)/s^2,$$

$$(33) \quad \phi'(f) = \lambda + \mu(s-p)(1-s-p)/s(1-s),$$

and

$$(34) \quad \phi'(n) = \lambda + \mu(s-p)(s+p-2)/(1-s)^2.$$

Thus,

$$(35) \quad \phi'(m) - \phi'(f) = \mu p(s-p)/s^2(1-s)$$

and

$$(36) \quad \phi'(f) - \phi'(n) = \mu(s-p)(1-p)/s(1-s)^2.$$

The desired result, (28), now follows from (35) and (36).

Now, from (35) we can see that since  $s > p$ ,  $m > f$ , and  $\phi(\cdot)$  is increasing and convex,  $\mu$  must be strictly positive. Further, in (34) note that the term multiplying  $\mu$  is negative. Therefore, since  $\phi'(n) > 0$ , we must have  $\lambda > 0$ . Thus, both the IR and IC constraints are binding as equalities. The optimal  $m$ ,  $f$ , and  $n$  can now be obtained by using (28) in conjunction with (30) and (31) as equalities. *Q.E.D.*

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A *numerical example* will help to clearly summarize the major points made in this section. Suppose  $U(\phi) = \sqrt{\phi}$ ,  $\bar{a} = 0.20$ ,  $\bar{c} = 0.20$ ,  $s = 0.75$ , and  $p = 0.25$ . Then,  $\phi(U) = U^2$ .

Initially, assume  $\alpha$  can be observed *ex post*. In this case, the resulting first-best solution involves  $U(\phi) = \bar{a} + \bar{c} = 0.40$  and  $\phi(U) = 0.16$ . The equilibrium price of the asset is  $s - \phi(U) = 0.59$ .

If  $\alpha$  cannot be observed *ex post* and no monitor is available, the second-best solution can be found by using (25) to evaluate  $\bar{m}$  and  $\bar{n}$ . These values are  $\bar{m} = 0.50$  and  $\bar{n} = 0.10$ . Thus,  $\bar{m}' = (\bar{m})^2 = 0.25$  and  $\bar{n}' = (\bar{n})^2 = 0.01$ . The expected cost to the principal is  $s\bar{m}' + (1-s)\bar{n}' = 0.19$ , and the equilibrium price of the asset in a competitive capital market will be  $s - s\bar{m}' - (1-s)\bar{n}' = 0.56$ .

Consider now the monitor and the contract which was used in the proof of Theorem 3 and the graphical analysis in Figure 2. The utility levels are  $\bar{m} = 0.50$ ,  $\bar{n} = 0.10$ , and  $\bar{f} = (\bar{m} + \bar{n})/2 = 0.30$ . The corresponding monetary compensations are  $\bar{m}' = 0.25$ ,  $\bar{n}' = 0.01$ , and  $\bar{f}' = 0.09$ , and the expected cost to the principal is  $s\frac{2}{m} + 2s(1-s)\bar{f}' + (1-s)^2\bar{n}' = 0.1750$ . The price of the asset is 0.575.

Finally, if the optimal contract stated in Theorem 4 is used, we have

$$(37) \quad [\phi'(m) - \phi'(f)] / [\phi'(f) - \phi'(n)] = (m-f)/(f-n) = p(1-s)/s(1-p) = 1/9,$$

which means  $10f - n = 9m$ .

Utilizing (31) with an equality yields

$$(38) \quad m = 0.4 + n,$$

and combining (37) and (38) gives

$$(39) \quad f = 0.36 + n.$$

Substituting (38) and (39) in (30) allows us to obtain  $n = 0.04$ , which means  $m = 0.44$  and  $f = 0.40$ . Thus, the expected cost to the principal is  $s^2\phi(m) + 2s(1-s)\phi(f) + (1-s)^2\phi(n) = 0.1690$ , and the equilibrium price of the asset is 0.5810. Not surprisingly, the optimal contract with monitoring results in a higher asset value than the arbitrarily picked contract, but a lower value than that attainable in the first-best case. Of course, in a capital market in which asset prices are set competitively, only the optimal contract will be used, and in equilibrium the price of the asset will be 0.5810 if the monitor is available.



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#### V. Concluding Remarks

Agency theoretic models have facilitated our understanding of a variety of institutional and market phenomena. Little has been written, however, about the manner in which agency relationships affect equilibrium security prices.<sup>15</sup> Our paper should, therefore, be viewed as an initial attempt to provoke further discussion on the subject.

We have scratched merely the surface of what promises to be a fruitful area for future research. The framework developed in Section IV is likely to lend itself to the introduction of multiple managers in the agency relationship, so that issues related to the impact of moral hazard in groups on security prices can be addressed.<sup>16</sup> Further, there may be useful insights to gain from relaxing our assumption that the output can be costlessly observed *ex post*. Accounting manipulations can make reported income completely divorced from true economic income, and, thus, the valuation implications of assuming that  $x$  cannot be observed may be important.<sup>17</sup> Finally, a significant extension of our model would be to allow principals to be risk averse and use one of the more familiar asset pricing models like the CAPM or the arbitrage pricing model of Ross [19]. In a companion paper [17] we have done this, and have derived optimal incentive contracts as well as the welfare implications of permitting managers to trade claims against the outputs of their own firms. In *that* model, however, we have not incorporated monitoring. Thus, the task of simultaneously dealing with noisy monitoring and risk-averse principals still remains.

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<sup>15</sup>Recently, Diamond and Verrechia [4] have independently analyzed a model similar to ours. However, the issues they address are somewhat different.

<sup>16</sup>See [11] for a promising start in this direction.

<sup>17</sup>See [14] and [15] for models that use this assumption.

Proof of Theorem 1:

Throughout, the manager's utility function will be assumed to have the separable form  $U(\alpha, \phi) \equiv U(\phi) - V(\alpha)$  with  $U'(\cdot) > 0$ ,  $U''(\cdot) < 0$ ,  $V'(\alpha) > 0$ .

If managers are risk neutral, Harris and Raviv [8] have shown that optimal fee schedules are of the form  $\phi^*(x) = x - \bar{k}$ , where  $\bar{k}$  is a constant. With such 'pure rental' type contracts (with zero probability of default), the principal is indifferent to the agent's choice of action and thus changes in the accounting system can have no impact on managerial actions. This means that managerial risk aversion is necessary for the accounting system to affect asset prices. Further, it is also well known that if the density function  $q(x, \alpha)$  has a compact moving support, and if arbitrarily large penalties can be levied on the manager, a first-best solution can be achieved. If such penalties are not feasible, some additional restrictions will have to be imposed on  $q(x, \alpha)$  to obtain a first-best solution. In either of these cases, information revealed about  $\alpha$  *ex post* by the accounting system is redundant.

Next, assume that managers are risk averse and that the asset cash flow is unbounded. In this case, Holmström [10] has shown that the addition of a noninformative monitor to  $\Omega^0$  cannot affect the price of the asset even if the cost,  $\xi$ , of using this monitor is zero. That is, a noninformative monitor cannot be efficient.

We have, therefore, established that if any of the three conditions mentioned in the theorem is violated, a change in  $\Omega^0$  will not affect the price of the asset. We now will show that when all three conditions are satisfied, the price of the asset will increase if the cost of the (informative) monitor is sufficiently low. This claim is essentially similar to Proposition 3 in Holmström [10], but its proof is given here because Holmström's proof is flawed, as we pointed out in [16].

Consider an informative monitor,  $\omega$ , of  $\alpha$ , and let  $\phi(x)$  be Pareto-optimal within the class of fee schedules depending only on  $x$ . Let  $\bar{\alpha}$  be such that

$$(A-1) \quad \int U(\phi(x)) q_{\alpha}(x; \bar{\alpha}) dx - V_{\alpha}(\bar{\alpha}) = 0$$

and

$$(A-2) \quad \int U(\phi(x)) q_{\alpha\alpha}(x; \bar{\alpha}) dx - V_{\alpha\alpha}(\bar{\alpha}) < 0.$$

Since  $\omega$  is informative, with positive scalars  $b$  and  $c$ , a variation  $b\delta\phi(x, \omega) + bc$  can be constructed to satisfy

$$(A-3) \quad \delta\phi(x, M^-) q(x, M^-; \bar{\alpha}) + \delta\phi(x, M^+) q(x, M^+; \bar{\alpha}) = 0.$$

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$$(A-4) \quad \delta\phi(x, M^-)q_\alpha(x, M^-; \bar{\alpha}) + \delta\phi(x, M^+)q_\alpha(x, M^+; \bar{\alpha}) = 1$$

where  $M^-$  and  $M^+$  satisfy (6). This type of variation was first used by Shavell [21].

$$(A-5) \quad \text{Let } Z(\alpha, b, c) = \int \int U(\phi(x) + b\delta\phi(x, \omega) + bc)q_\alpha(x, \omega; \alpha) d\omega dx - V(\alpha(b, c)).$$

For a given  $b$  and  $c$ , let  $\alpha(b, c)$  be the solution to the manager's maximization problem, i.e.,

$$(A-6) \quad Z_\alpha(\alpha(b, c), b, c) = \int \int U(\phi(x) + b\delta\phi(x, \omega) + bc)q_\alpha(x, \omega; \alpha(b, c)) d\omega dx \\ - V_\alpha(\alpha(b, c)) = 0$$

and the second-order condition

$$(A-7) \quad Z_{\alpha\alpha}(\alpha(b, c), b, c) < 0$$

also holds. Note that  $\alpha(0, c) = \bar{\alpha}$ .

To find the effect of positive variations in  $b$ , note that

$$Z_b(\alpha(b, c), b, c) = Z_b(\alpha, b, c) + \alpha_b(b, c) Z_\alpha(\alpha, b, c) \\ = Z_b(\alpha, b, c) \text{ since from (A-6), } Z_\alpha(\alpha, b, c) = 0.$$

At  $b=0$ ,

$$Z_b(\bar{\alpha}, 0, c) = \int U'(\phi(x)) \int \delta\phi(x, \omega) q(x, \omega; \bar{\alpha}) d\omega dx \\ + c \int \int U'(\phi(x)) q(x, \omega; \bar{\alpha}) d\omega dx \\ = c \int \int U'(\phi(x)) q(x, \omega; \bar{\alpha}) d\omega dx \quad \text{from (A-3).}$$

So for any  $c > 0$ ,  $Z_b(\bar{\alpha}, 0, c)$  is positive.

Differentiating (A-6) with respect to  $b$  we get

$$Z_{\alpha\alpha}(\alpha(b, c), b, c) \cdot \alpha_b(b, c) + Z_{\alpha b}(\alpha(b, c), b, c) = 0 \\ \text{or} \\ (A-8) \quad \alpha_b(b, c) = - \frac{Z_{\alpha b}(\alpha(b, c), b, c)}{Z_{\alpha\alpha}(\alpha(b, c), b, c)}.$$

Moreover,

$$Z_{\alpha b}(\alpha(b,c), b, c) = \iint (\delta\phi(x,\omega) + c) U'(\phi(x) + b\delta\phi(x,\omega) + bc) q_{\alpha}(x, \omega; \alpha(b,c)) dx d\omega.$$

At  $b=0$ ,

$$(A-9) \quad Z_{\alpha b}(\bar{\alpha}, 0, c) = \int U'(\phi(x)) dx + c \int U'(\phi(x)) q_{\alpha}(x, \omega; \bar{\alpha}) d\omega dx \text{ (using (A-4))}.$$

Irrespective of the sign of  $\int q_{\alpha}(x, \omega; \bar{\alpha}) d\omega$ ,  $Z_{\alpha b}(\bar{\alpha}, 0, c)$  is positive if  $c$  is sufficiently small, since the first term,  $\int U'(\phi(x)) dx$  is positive. Further, since  $Z_{\alpha\alpha}(\bar{\alpha}, 0, c) < 0$ , (A-8) implies that  $\alpha_b(0,c) > 0$ . This means that for a sufficiently small  $c$ , introduction of the proposed variation will induce the manager to increase his effort, at least in the (positive) neighborhood of  $b=0$ . Since for any  $c>0$ ,  $Z_b(\bar{\alpha}, 0, c) > 0$ , the manager's expected utility also goes up with the new fee schedule.

For the principal, define

$$F(b, c) = \iint (x - \phi(x) - b\delta\phi(x,\omega) - bc - \xi) q_{\alpha}(x, \omega; \alpha(b,c)) d\omega dx.$$

$$F_b(b,c) = -\iint (\delta\phi(x,\omega) + c) q_{\alpha}(x, \omega; \alpha(b,c)) d\omega dx$$

$$+ \iint (x - \phi(x) - b\delta\phi(x,\omega) - bc - \xi) q_{\alpha}(x, \omega; \alpha(b,c)) \cdot \alpha_b(b,c) d\omega dx.$$

At  $b=0$ , using (A-3) and the fact that  $\iint q_{\alpha}(x, \omega; \bar{\alpha}) d\omega dx = 1$  and  $\iint q_{\alpha}(x, \omega; \bar{\alpha}) d\omega dx = 0$ , we have

$$(A-10) \quad F_b(0,c) = -c + [\iint (x - \phi(x)) q_{\alpha}(x, \omega; \bar{\alpha}) d\omega dx] (\alpha_b(0,c)).$$

Note that  $\alpha_b(0,c) > 0$  and  $\iint (x - \phi(x)) q_{\alpha}(x, \omega; \bar{\alpha}) d\omega dx$  represents the effect of an increase in  $\alpha$  on the principal's welfare. If this double integral is positive, the second term on the right-hand side (RHS) of (A-10) will also be positive. With a small enough  $c$  we can then make  $F_b(0,c) > 0$ . On the other hand, if  $\iint (x - \phi(x)) q_{\alpha}(x, \omega; \bar{\alpha}) d\omega dx < 0$ , we can go back and let the new fee schedule be  $\phi(x) - b\delta\phi(x,\omega) + bc$ . The first term on the RHS of (A-9) then becomes  $-\int U'(\phi(x)) dx$  and by making  $c$  small enough, we can ensure  $Z_{\alpha b}(\bar{\alpha}, 0, c) < 0$  irrespective of the sign of  $\int q_{\alpha}(x, \omega; \bar{\alpha}) d\omega$ . This means  $\alpha_b(0,c) < 0$ , which in turn implies that the second term on the RHS of (A-10) can once again be guaranteed to be positive. Therefore,  $F_b(0,c)$  can be made positive in either case, by appropriately adjusting  $c$ . It is fairly straightforward to verify that  $\iint (x - \phi(x)) q_{\alpha}(x, \omega; \bar{\alpha}) d\omega dx = 0$  is impossible, for if it were true, we could perturb  $\phi(x)$  with some function  $r(x)$  and make the agent strictly better

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off without worsening the principal's lot. This would violate the presumed Pareto optimality of  $\phi(x)$ . To ensure that the principal will be better off in spite of the cost  $\xi$ , we must have

$$\int \int (x - \phi(x) - b^* \delta \phi(x, \omega) - b^* c^* - \xi) q(x, \omega; \alpha(b^*, c^*)) d\omega dx > \int (x - \phi(x)) q(x; \bar{\alpha}) dx$$

or

$$\xi < [ \int \int (x - \phi(x) - b^* \delta \phi(x, \omega)) q(x, \omega; \alpha(b^*, c^*)) d\omega dx \\ (A-11) \quad - \int (x - \phi(x)) q(x; \bar{\alpha}) dx - b^* c^* ]$$

where  $b^*$  and  $c^*$  are the optimal choices in the variation. In other words, the informative monitor,  $\omega$ , should be efficient. Since, in the above proof, the manager was assumed risk averse, asset cash flows were not constrained to have a compact moving support, and the monitor employed was informative, the sufficiency of (i), (ii), and (iii) is established. *Q.E.D.*

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