The highs and the lows: A theory of credit risk assessment and pricing through the business cycle

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Abstract

This paper develops a theory of how risk is assessed and priced through the business cycle by developing an intuitive model in which there is uncertainty about whether outcomes depend on the risk-management skills of banks or are just based on luck, in the spirit of Piketty’s (1995) model of “left-wing” and “right-wing” dynasties. Periods of sustained banking profitability cause all agents to rationally elevate their estimates of bankers’ skills, despite the uncertainty about what is driving outcomes. Everybody consequently becomes sanguine about bank risk, credit spreads decline, and banks choose increasingly risky assets. Conditional on subsequently observing unexpectedly high defaults, beliefs can endogenously shift to put more weight on outcomes being luck-driven, and this causes credit spreads to widen and may be enough to trigger a crisis. Regulatory implications of the analysis are extracted.

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1. Introduction

1.1. Motivation

An interesting stylized fact about banking is how the evaluation and pricing of risk by bankers and other market participants changes over the business cycle. For example, Krainer (2004) documents that credit spreads shrink during expansions and widen during economic contractions/recessions. There are various reasons why spreads, and even equity risk premia, vary over the business cycle.
For example, Bekkers et al. (2013) provide evidence that this is driven at least in part by changes in risk aversion that are correlated with monetary policy changes. Moreover, monetary policy and other shocks can work through the “balance-sheet channel” to change the distribution of wealth that can cause a variation in spreads in general equilibrium.\(^1\) Nevertheless, there is substantial empirical evidence that lending standards vary over the business cycle, and that this is also a cause of variation in spreads. Asea and Blomberg (2014) examine quarterly data on two million commercial loans over the 1977–1993 time period and document systematic patterns in lending standards. They find that lending standards decline—credit terms become easier for borrowers—during economic expansions and tighten in recessions. Similarly, Bassett et al. (2014) also document that lending standards started becoming more lax in the U.S. in late 2003 and this easing of standards continued until early 2006, with tightening that started in early 2007. Further evidence is provided by Dell’Ariccia et al. (2012) who show that a relaxation of credit standards, triggered by an increased demand for loans, contributed to the boom and the ensuing financial crisis of 2007–2009.

The phenomenon is not just limited to U.S. banking. Gizycki's (2001) empirical analysis shows that Australian banks weakened their credit standards during a rapid expansion of aggregate credit, which typically goes hand in hand with economic booms; Jiménez et al. (2012) provide similar evidence for Spain. And this collective optimism about credit risk during expansions is not just limited to bankers. As Brunnermeier (2009) notes, credit rating agencies appeared to have overly optimistic forecasts about the creditworthiness of structured finance products (prior to the 2007–2009 financial crisis) at the same time that lending standards were becoming more lax and the market was awash in cheap credit. Moreover, the U.S. government’s Financial Crisis Inquiry Commission (FCIC) report blames regulators—specifically the Federal Reserve—of being too supportive of industry growth objectives and not being vigilant enough in “reining in” risk taking.

Many have noted that these lax credit standards during expansions sow the seeds of future crises as loans extended on easier terms come back to haunt banks when there is a downturn (see, for example, Asea and Blomberg (2014) and Acharya and Naqvi (2012)). During these contractions, the pricing of risk changes quite dramatically, credit standards become more stringent, and risky lending is eschewed. Bassett et al. (forthcoming) view such a tightening of credit standards as a shock to credit supply and document a significant decline in lending as a result.

1.2. Research question

Why does the evaluation and pricing of risk change like this over the business cycle and what does this teach us about the risk management practices of bankers and portfolio managers? The main goal of the paper is to develop a theory to address this question. The theory is aimed at understanding sudden reassessments of risk in a world in which rational agents are engaged in Bayesian learning, with “rations optimism” prevailing when things are going well, and a precipitous (seemingly non-Bayesian) decline in this optimism when some “bad news” arrives.

1.3. The analysis

The basic version of the two-period model used to develop the theory has three key building blocks. The first is that banks can choose between a relatively safe loan and a potentially more profitable risky loan, with public observability of the type of loan chosen. The probability of success (repayment) of the loan depends on the realization of a macroeconomic state: there is a high probability of a “skill” macroeconomic state in which outcomes are influenced by the a priori unknown skills of banks and a small probability of a “luck” macroeconomic state in which these outcomes are purely exogenous.\(^2\) No one knows at the outset which state governs the economy, but there are common prior beliefs about the probabilities of the luck and skill states. If outcomes are driven only by luck, then banks

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\(^{1}\) See Bernanke and Gertler (1995).

\(^{2}\) An alternative interpretation is that there are two systematic risk regimes: high risk and low risk. In the high risk regime, default risk is deemed to be so high that investors will fund only low-risk loans even if banks are viewed as being highly skilled, whereas in the low-risk regime, riskier loans are funded if the bank is viewed as being skilled enough.
prefer to invest in (and can raise financing for) only safe loans. If outcomes are skill-driven, banks will prefer to and be able to fund more profitable risky loans if these banks are viewed as being skilled enough. This building block generates a setting in which banks initially invest in safe loans, and after experiencing successful repayment on these loans they switch to more risky loans, conditional on agents believing that outcomes depend on bankers’ skills. That is, initial success leads to an upward revision in the skills of bankers via rational learning and these higher skill assessments enable riskier lending. There is symmetric information at all dates about whether outcomes are luck-driven or skill-driven and how skilled the bank is if outcomes are skill-driven.

The second building block is that there are multiple time periods and the realization of the macro uncertainty can change from one period to the next. That is, conditional on initial success with loan repayment and agents believing that the success was skill-driven, banks are able to invest in riskier loans. However, if beliefs about whether outcomes are skill-driven or luck-driven remain intertemporally static, then investors will always continue to fund successful banks and risk-taking will keep rising with no financial crisis. It is only when beliefs can switch from a skill regime to a luck regime that previously-sanctioned risk taking can fall out of favor with investors, so this possibility of regime switching is an essential aspect of sudden changes in risk assessments.

While adding this second building block is necessary, it is not enough to generate a drop in lending associated with a change in risk assessments. The reason is that if agents shift their beliefs about whether outcomes are skill-driven or luck-driven, then banks will also switch at that point to loans that investors will support given the new beliefs. By making their loan portfolio choices move lock-step with the beliefs of their financiers, banks can avoid a reduction in lending.

This is why we need the third building block – interim refinancing risk. For each time period, it is endogenously shown that the bank will fund itself with debt that is of shorter maturity than its loans. The reason for this is the important contribution of Calomiris and Kahn (1991) which rationalized demand deposits as a source of discipline on bankers. Withdrawal risk for the bank then arises from the regime shift pertaining to the macro uncertainty occurring before the loans for that period mature. Because each bank faces withdrawal risk only due to the realization of a systematic-risk variable, the inability of one bank to refinance itself is accompanied by a similar experience for all banks that have invested in that asset, so this risk reassessment actually generates a crisis.

In the basic model, there are only two time periods and two types of loans. When the model is extended to have multiple periods and multiple levels of loan risk, in order to accommodate more realism, we get the result that initially banks invest in safe loans and if these are successfully repaid, they switch to loans of moderate risk. The presence of banks in the market for risky products creates liquidity that invites other institutions to enter, which further enhances liquidity. In this analysis, we encounter a somewhat surprising result: if repayments are experienced on these moderately risky loans for long enough, banks switch to the riskiest loans available, bypassing all available levels of risk between the moderately-risky and most-risky loans. Thus, we get a form of “three-asset separation” in terms of the bank’s asset choice—when there is a sufficiently long sequence of good outcomes, risk taking rises in steps, going from safe loans to moderately risky loans and then to the riskiest loans.

The analysis assumes that there is a time-invariant probability that outcomes are skill-determined in every period, and that the realization of this macro uncertainty in any period tells investors whether outcomes are skill-based or luck-based. That is, outcomes could be viewed as being skill-based in one period and luck-based in the next. But this leaves unanswered the important question of how beliefs are determined. That is, following say a long sequence of realizations that outcomes are skill-based, what could cause a belief realization that they are luck-based? To address this question, an extension of the model is developed in which there is a true underlying reality and it is either that outcomes are skill-based or that they are luck-based. However, no one knows what the reality is, and all agents share common prior beliefs about the probability that outcomes are skill-based. Over time, there is updating of beliefs about the probability distribution of the macro uncertainty based on the number of banks that experienced successful repayments on their loans and the number of banks that failed.

Because of the symmetric information assumption, banks always fund the loans most profitable to their shareholders that investors are willing to finance.
Thus, there are two types of learning now. One is about the macro uncertainty, which is based on observing aggregate bank failures. The other is about the skill of an individual bank, which is based solely on observing the loan repayment or default outcome related to that bank. Two cohorts/generations of banks – whose lending is not synchronized in time – are modeled, with the fortunes of both cohorts being affected by the same macro uncertainty. The analysis shows that unexpectedly large defaults in one cohort – which may be no more than the statistical realization of a low-probability event – can cause beliefs about the macro uncertainty to change sufficiently to cause interim withdrawals and liquidations of banks in the other cohort.\textsuperscript{4}

This modeling approach, wherein there is a regime in which people believe outcomes are skill-dependent and another in which they believe they are just luck, is reminiscent of Piketty’s (1995) model of social mobility and redistributive politics. In his model, there are two groups of agents – those in “left-wing dynasties” who believe outcomes are exogenous (luck) and those in “right-wing dynasties” who believe they depend on individual effort. The left-wing dynasties support higher redistributive taxation and supply less effort, while right-wing dynasties support lower redistribution and work harder. Instead of two groups of agents, in the model in this paper there are two states/regimes, which could be thought of as regimes distinguished by different types of agents representing the predominant majority.\textsuperscript{5}

It is important to note the causal chain of events predicted by this theory. It is when agents believe outcomes are skill-dependent and there is a sufficiently long experience of good outcomes that risky credit spreads shrink, risky lending is elevated and there is an economic boom.\textsuperscript{6} That is, risk-taking will tend to be “underpriced” by all concerned during economic booms. Moreover, since all banks are \textit{ex ante} identical in the model, when there is a shift in beliefs that outcomes are due to luck, it takes down all banks. This should not be taken too literally. In a setting with cross-sectional heterogeneity, say one in which banks have different capital levels, for example, we would expect the higher-capital banks to be more likely to survive.\textsuperscript{7}

\subsection*{1.4. Related literature}

This paper is related to previous theories of how banks change credit standards during the business cycle. Rajan (1994) and Ruckes (2004) develop theories in which banks soften their lending standards in response to competitive pressures during booms. Acharya and Naqvi (2012) build a model in which excess liquidity in banks leads to excessive credit volume because loan officers are incented to grow loan volume. This excess lending during the good times then sows the seeds of a crisis in the future. Unlike these papers, in the model in this paper there is no excess supply of liquidity or competitive pressure that induces banks to engage in riskier lending during booms. Rather, it is a reassessment of risk based on rational learning about the skills of bankers to manage these risks. This learning essentially changes credit spreads, making riskier loans cheaper to finance when banks have experienced a long string of successes (no defaults). Moreover, in the model developed here, it is not just banks but all agents—including regulators and financiers of banks—who become more sanguine about risk as banks experience success.

While the main goal of the analysis is to explain how sudden shifts in risk assessments can occur and the implications of these for risk has implications for financial crises. In the version of the model with more than two periods, a financial crisis occurs when banks are making the riskiest loans and...
market participants realize that outcomes depend only on luck. In other words, credit risk is the highest when people think it is the lowest. This is reminiscent of Minsky (1975) who argued that optimism increases during economic expansions, driving up speculation and leverage, thereby making the financial system more fragile. While Minsky (1975) based his argument on irrationality, the theory developed here shows that this can happen even with rationality when there is a regime-switching possibility. Thus, this paper can be viewed as Minsky (1975) meeting Piketty (1995) in a financial crisis setting. This aspect of this paper connects it to the vast literature on financial crises.

Historical accounts of financial crises that have occurred over the years appear in Bordo et al. (2001), Gorton (1988), Kindleberger (1978), and Reinhart and Rogoff (2008). The real consequences of financial crises are documented in Atkinson et al. (2013), Boyd et al. (2005), Kupiec and Ramirez (2013), and Laeven and Valencia (2012). Bernanke (1983) discusses the real effects of the banking crisis during the Great Depression. Wilms et al. (2014) provide a review of the real effects of crises.

Papers that review the crisis literature include Allen and Gale (2007), Lo (2012), Rochet (2008), and Thakor (forthcoming). The events surrounding the 2007–2009 subprime crisis are described in great detail in Brunnermeier (2009) and Gorton (2010), among others. Many papers have been written on the causes of financial crises (e.g. Acharya and Yorulmazer (2008), De Jonghe (2010), and Wagner (2010)). Some have pointed to political forces (e.g. Johnson and Kwak (2010), Rajan (2010), and Stiglitz (2010)), whereas monetary policy in the U.S. and Europe has been viewed as a major contributor by others (e.g. Maddaloni and Peydro (2011) and Taylor (2009)). Yet others have pointed to incentive conflicts of various sorts (e.g. Kane, 1990, forthcoming; Farhi and Tirole, 2012). More recently, complexity, interconnectedness and innovation have appeared in theories of financial crises and contagion. Interconnectedness and complexity are modeled in Allen et al. (2012) and Caballero and Simsek (2013). Gennaioli et al. (2012) point to “neglected risks” in innovative financial products, and Thakor (2012) models the role of disagreement over the value of innovative financial products as causal factors in crises. The neglected risk modeled in Gennaioli et al. (2012) is a behavioral bias. For other behavioral theories, see Gennaioli et al. (2015) and Thakor (2015). In both these papers, agents irrationally overreact to good news, which leads to excessively risky lending. Finally, the view that seems to be gathering the most support is that the combination of excessive leverage and maturity mismatching by financial institutions have contributed significantly to financial crises. For theories along these lines, see Acharya and Thakor (2014), Shleifer and Vishny (2010) and Farhi and Tirole (2012). Excessive leverage also plays a role in the theory developed by Goel et al. (2014), but in that theory, it is the correlated nature of leverage on banks’ and households’ balance sheets that creates fragility and makes the system vulnerable to a crisis. They also provide supporting evidence.

In contrast to these papers, the focus in this paper is not on incentive conflicts, leverage, complexity or innovation. The central idea in this paper, and one that has not been previously examined as a causal factor in sudden reassessments of risk that sometimes lead to financial crises, is that experience-based learning, whereby outcomes are attributed primarily to skill but have a non-zero probability of being viewed in the future as being just luck, can create an environment for banks to take successively higher levels of risk that eventually leads to a crisis. Leverage-financed lending booms followed by a sudden contraction of liquidity due to a sharply higher (seemingly non-Bayesian) spike in perceptions of risk are features common to many crises. This paper explains that such sharp revisions in assessments of risk can occur even with Bayesian rational agents.

I do not mean to suggest that learning is the only factor that—operating through bank risk management—affects the likelihood of banking crises. The impact of government-provided safety nets—like deposit insurance—on risk taking cannot be overlooked; see Thakor (2014) for a discussion of these issues in the context of the impact of bank capital on financial stability. Demirgiic-Kunt and

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8 Rajan (2010) points out that politicians were interested in expanding home ownership in the U.S., and therefore strengthened the Community Reinvestment Act (CRA). Agarwal et al. (2012) provide evidence that this encouraged banks to pursue riskier mortgage lending.

9 The argument here is that easy-money policies adopted by central banks in the U.S. and Europe contributed to housing price bubbles in various countries.

10 Thakor (2015) develops a regime-switching model, but one in which the availability heuristic bias causes agents to ignore the likelihood that outcomes could be due to luck. This leads to excessive exuberance during periods of sustained economic booms.
Detragiache (1999) study 61 countries and document that explicit deposit insurance seems to be detrimental to bank stability. Calomiris and Haber (2014) attribute this to the moral hazard created by safety nets with a growing tendency to shift the losses of bank failures to taxpayers, and they suggest that banking fragility reflects the structure of a country’s fundamental political institutions. A striking aspect of this that Calomiris and Haber (2014) highlight is that some countries have had numerous banking crises and others (Canada, New Zealand, Australia, Singapore, Malta and Hong Kong) have had none, a puzzle typically not addressed in theories of banking crises. As an illustration of this, they compare the U.S.—which has had 12 major banking crises since 1840—with Canada, which has had none. They point to a number of factors, with political economy roots, to explain this. In particular, they point out that the Canadian banking system had no branching restrictions and no deposit insurance, factors that led to consolidation, diversification and a focus on prudent risk management and solvency among Canadian banks. These arguments suggest that we should think of the interaction between learning and political economy generating differences in risk management across banking systems in different countries. Supporting this view is the emphasis in Calomiris and Haber (2014) on the differences in underwriting standards across U.S. and Canadian banks prior to the 2007–2009 crisis in the U.S.; underwriting standards declined significantly at U.S. banks, but not at Canadian banks.¹¹ In other words, if bank structure and regulation provide strong incentives for banks to avoid risky lending, then the impact of learning in whetting the risk appetites of banks and investors is retarded.

The rest is organized as follows. The two-period (five dates) model is developed in Section 2, and analyzed in Section 3. An analysis of the bank’s loan portfolio and funding choices with more than two periods and more than two types of loan appears in Section 4, and it shows that a sufficiently prolonged sequence of good outcomes eventually leads to the bank financing the riskiest loans available. The analysis of how beliefs about the macroeconomic uncertainty can change based on the observation of aggregate defaults appears in Section 5. Section 6 is devoted to a discussion of the implications of the analysis for financial crises and regulatory policy, and the empirical prediction of the model. Section 7 concludes. All proofs are in the Appendix A.

2. The model

This section describes the base model with five dates and two time periods. I begin with a description of agents and preferences. I then discuss the sequence of events, followed by the investment opportunity set. This is followed by a statement of the observability assumptions and description of how beliefs are revised. I conclude with a summary of the timeline and a discussion of why the various elements of the model are needed.

2.1. Preferences of key players

Everybody is risk neutral and the riskless rate is zero. The key players in the base model are financial intermediaries and institutional investors who can provide financing for these intermediaries. The liabilities of the intermediaries are uninsured,¹² so even though I will refer to these intermediaries as “banks” henceforth, it should be understood that they encompass a broad array of financial intermediaries that raise their funding via non-traded and other claims in the capital market, including investment banks and commercial banks. For commercial banks, this would be deposit funding (including uninsured deposits), and for investment banks it could be traded debt, repos, as well as longer-term borrowing from other institutions. Thus, the financial market here includes the shadow banking system.

¹¹ Calomiris and Haber (2014), p. 324 write: “Canada’s banking system avoided the 2007–2009 banking crisis that crippled the United States and much of Western Europe. The relaxation of underwriting standards that we discuss in Chapter 7 produced a quintupling of residential mortgage arrears in the United States. In contrast, Canadian residential mortgage arrears displayed characteristic, and enviable, dullness.” Their characterization of lax underwriting standards prior to the crisis at U.S. and European banks is consistent with the theory developed here.

¹² The model goes through with partially insured liabilities.
2.2. Sequence of events: agents, information signals and types of loans

There are five payoff-relevant dates: \( t = 0, 1, 2, 3 \) and \( 4 \) that cover two time periods over which banks operate. At \( t = 0 \), there are \( N_0 \) banks in the market. Each bank can choose to invest $1 at \( t = 0 \) in either a prudent loan (\( P \)) or a risky loan (\( R \)). Both loans mature at \( t = 2 \). For simplicity, the entire $1 is raised at \( t = 0 \) in the form of (uninsured) debt financing through non-traded debt supplied by institutional investors; the bank’s choice between such debt and direct capital market financing will be considered later. The debt is competitively priced to yield investors a zero expected return. Debt maturity will be endogenized.

2.3. Bank manager and loan monitoring

The bank’s manager needs to monitor the loan in order to produce a positive payoff. If the loan is not monitored at \( t = 0 \), it produces a payoff of 0 at \( t = 2 \), regardless of whether it is a \( P \) or an \( R \) loan. The same is true for the second-period loan. If not monitored at \( t = 2 \), it produces a zero payoff at \( t = 4 \). The bank manager’s utility for each period can be written as

\[
U(i,j) = W + \alpha \left[ \text{value of bank equity with loan } i \text{ and monitoring decision } j \right] - C_j + Bi_{\text{survive}}
\]

where \( W \) is the manager’s fixed up-front wage each period, \( \alpha \) is a positive constant, \( j \in \{1, 0\} \) where \( j = 1 \) if the bank monitors and \( j = 0 \) if the bank does not monitor, \( C > 0 \) is the manager’s personal cost of monitoring, \( B \) is the personal benefit to the manager from the bank surviving until the end of the period, and \( i_{\text{survive}} \) is the indicator function which is 1 if the bank survives until the end of the period and zero otherwise. It will be assumed that:

\[
B > C
\]

so that the manager’s personal benefit from survival exceeds his personal monitoring cost.

The debt investors who finance the bank receive a signal at the interim date \(( t = 1)\) for the first-period loan and \(( t = 3)\) for the second-period loan that informs them whether the bank monitored its loan or not. If the loan was not monitored, then its value is \( Ln \in (0, 1) \) if it is liquidated at the interim date \(( t = 1) \) or \(( t = 3)\), where \( L_n \) is an arbitrarily small positive quantity. Since the value of an unmonitored loan is zero at the end of the period, debt investors will wish to liquidate the loan at the interim date if they discover it was not monitored.

2.4. Investment opportunities: \( P \) and \( R \) loans

The bank chooses at \( t = 0 \) and then again at \( t = 2 \) between loans \( P \) and \( R \), a mutually-exclusive set of relationship loans.\(^{13}\)

2.4.1. \( P \) loans

Loan \( P \) is either good (\( G \)) or bad (\( B \)) but no one can determine for sure \textit{a priori} whether the loan is \( G \) or \( B \). Conditional on being monitored, a loan of type \( G \) pays off \( X_p > 1 \) with probability (w.p.) 1, and a \( B \) loan pays off \( X_p \) w.p. \( b \in (0, 1) \) and 0 w.p. \( 1 - b \).

It is commonly believed at \( t = 0 \) that there is a state of nature, \( m \), in any given period, which could be interpreted as a macroeconomic state. The realization of this state at date \( t \), denoted, \( m_t \), has two possible values: “skill” and “luck”. In the “luck” state (probability \( \lambda \in (0, 1) \)), outcomes are pure luck, so the probability that the loan is type \( G \) is fixed at \( r \in (0, 1) \) for all banks. In the “skill” state (probability \( 1 - \lambda \)), outcomes are skill-dependent, so the probability that the loan is type \( G \) depends on the skill/talent of the bank in monitoring the loan after it is made, and more talented banks are viewed as having higher probabilities of making good loans. Specifically, there are two possible types of banks:

\(^{13}\) For a theory of relationship lending, see Boot and Thakor (1994, 2000). Relationship lending allows the banks to earn relationship-related rents, as in the model here.
talented (τ) and untalented (u). A type-τ bank monitors the P loan with perfect efficiency and is thus able to ensure that the loan is G w.p. 1. A type-u bank, however, has no monitoring ability and thus ends up with a B loan w.p. 1. The common prior belief at \( t = 0 \) is that the probability that any given bank is type-τ is r, and the probability that it is type u is \( 1 - r \). The banks themselves share these beliefs, so there is no private information about the bank's type.

In any given period, the realization of the skill or luck state applies to \( P \) loans made by all banks, but across the two periods, these state variables are identically and independently distributed (i.i.d.) random variables. That is, the realization of \( m_i \) does not provide any new information about the likelihood of a particular realization of \( m_{i+1} \).

The macro state – skill or luck – is independently and identically distributed (i.i.d.) across time periods, and all agents share the common prior belief that the probability of the luck state in any period is \( \hat{\lambda} \). In the base model, it is assumed that, at the interim date \( (t = 1 \) for the first period and \( t = 3 \) for the second period), which state has been realized for the period can be costlessly discovered by all, including the bank's creditors. Later, this information will be permitted to be discovered only at a cost.

While one may interpret the skill and luck states as representing two different objective physical regimes, they may also represent nothing more than subjective assessments of agents about whether economic outcomes are driven by skill/effort or just luck. That is, the skill and luck macro states can be interpreted as states of “aggregate investor sentiment”, in the spirit of Piketty’s (1995) model, with the luck state being one in which market sentiment is dominated by agents in Piketty's “left-wing dynasties”, and the skill state being one in which market sentiment is dominated by agents in Piketty’s “right-wing dynasties”. A shift from one state to the other may be triggered by events in the real economy or changes in political initiatives/rhetoric. For example, higher-than-expected defaults on sub-prime mortgages may cause investors to shift from beliefs that loan repayment outcomes are determined by the screening skills of bankers to the belief that they are driven largely by the bad luck experienced by subprime borrowers. Alternatively, there may be an objective underlying reality, as in the extended version of the model in Section 5.

At \( t = 0 \) then, the prior belief about the probability of success of the \( P \) loan at \( t = 2 \) is

\[
r_o^P = r + [1 - r]b
\]

It is assumed that

\[
r_o^P X_p > 1
\]

so \( P \) can be financed with debt given these prior beliefs.

However,

\[
b X_p < 1
\]

so a bank known to be type-u almost surely (in the state in which the probability of having a type-G loan is skill-dependent) would never be able to raise financing for a \( P \) loan.

2.4.2. R loans

The \( R \) loan can be either good (\( \hat{G} \)) or bad (\( \hat{B} \)). No one can determine a priori whether a given loan is \( \hat{G} \) or \( \hat{B} \). A loan of type \( \hat{G} \) pays off \( X_R \) w.p. \( q \in (0, 1) \) and 0 w.p. \( (1 - q) \), whereas a \( \hat{B} \) loan pays off 0 w.p. 1. It is assumed that the \( \hat{G} \)-type \( R \) loan has a higher expected value to financiers than the \( \hat{G} \)-type \( P \) loan, i.e.,

\[
\frac{C + \alpha}{\alpha} > q X_R + Z > X_p + Z
\]

14 This assumption is merely to reduce notational clutter, and one could assume instead that a type-u bank ends up with a \( B \) loan w.p. less than 1.

15 If the macro uncertainty exhibited positive serial correlation, the results will get even stronger, although the algebra will become more messy. A macro uncertainty realization of skill in one period will indicate a higher (than prior) probability of the same state in the second period, making elevated risk-taking even more attractive. In Section 5, a variant of the model will be presented in which there is only one underlying reality (either outcomes are luck-based or they are skill-based) across all time periods, and economic agents learn over time, based on observing aggregate defaults, updating their beliefs about whether outcomes are based on luck or skill.
where $Z > 0$ is a non-pledgable payoff that is available to the bank’s shareholders if the loan stays on the books until it matures, is repaid and is not prematurely liquidated.\footnote{The principal role of $Z$ is to create a wedge between the interests of the bank’s creditors and those of its shareholders. Thus, creditors may wish to liquidate the bank even when shareholders would like to continue, which permits a focus on liquidations that are generated by the short-term nature of debt funding rather than voluntary bank closures by its shareholders. Instances in which the interests of shareholders and bondholders diverge \textit{ex post} are not hard to find. For example, if continuation merely creates a mean-preserving spread of the payoff distribution with liquidation, creditors will prefer liquidation even though shareholders prefer continuation.}

In (6), the first inequality, $\frac{Z}{r} > q[X_R + Z]$, means that the manager’s equity ownership is never sufficient to induce him to monitor the loan, in the absence of a liquidation threat by creditors. The second inequality, $q[X_R + Z] > X_P + Z$, means that the $R$ loan has a higher expected value to financiers than the $P$ loan.

As in the case of the $P$ loan, there are two possible macroeconomic states of relevance with the $R$ loan in any given period. In the “luck” state (which has probability $\lambda$), the loan is type $G$ with an exogenously fixed probability $r$. In the “skill” state (w.p. $1 - \lambda$), the probability that the loan is type $G$ is dependent on the bank’s monitoring skill. In this case, the common belief is that a type-$\tau$ bank will be able to ensure w.p. 1 that it is a $G$ loan, whereas a type-$u$ bank will ensure w.p. 1 that it is a $B$ loan.

It will also be assumed, for later use, that the difference in loan repayment (success) probabilities across the good and bad loans is greater for the $R$ loan than for the $P$ loan, i.e.,

$$q > 1 - b$$  \hspace{1cm} (7)

This is motivated by the observation that $R$ is a more complex loan for which the importance of the bank’s skill/talent is greater than for $P$.

At $t = 0$ then, the prior probability of success of the type $R$ loan at $t = 2$ is:

$$r^R_0 = rq$$  \hspace{1cm} (8)

and it is assumed that

$$r^R_0[X_R + Z] < 1$$  \hspace{1cm} (9)

so the $R$ loan cannot be financed with debt or equity given the prior beliefs.

A summary of these loans and payoffs is provided in the table below.

| Table 1
Loan payoffs. |
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<td>Loan</td>
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<td><strong>Exogenous loan payoff distribution in the luck macro state (probability $\lambda$)</strong></td>
</tr>
<tr>
<td>$P$</td>
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<tr>
<td>$R$</td>
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</tbody>
</table>

| Loan | Bank type | Probability of bank type | Loan type, conditional on bank type | Date 2 payoff |
| **Endogenous loan (bank-type-dependent) payoff distribution in the skill macro state (probability $1 - \lambda$)** | | | |
| $P$ | $\tau$ | $r$ | $G$ w.p. 1 | $X_P > 1$ w.p. 1 |
| | $u$ | $1 - r$ | $B$ w.p. 1 | $X_P$ w.p. $b$ |
| | | | 0 w.p. $1 - b$ |
| $R$ | $\tau$ | $r$ | $\tilde{G}$ w.p. 1 | $X_{\tilde{G}} > 1$ w.p. $q$ |
| | $u$ | $1 - r$ | | 0 w.p. $1 - q$ |
| | | | | 0 w.p. $1$ |
2.5. Observability and knowledge assumptions

Each bank’s loan choice at any given date is commonly observable. So, investors know the bank’s choice (P or R) before providing financing. Also publicly observable is whether the bank’s loan repaid or defaulted at the end of the period (t = 2 for the first period and t = 4 for the second period). For now, $m_t$ (skill or luck) is observable to all once it is realized. There are common beliefs about each bank’s type at each date $t$, so no bank knows more about its type than others do.

2.6. Non-pledgeable payoffs

The payoff $Z$ is non-pledgeable in the sense that it cannot be pledged by the bank as repayment to creditors. Each loan also has an additional social value $J_{P0}$; that is available only if the loan is successfully repaid. This social value is another non-pledgeable payoff. It can be interpreted as a benefit to society from the success of the borrower’s project due to its positive employment, consumer welfare and tax-revenue-enhancing wealth generation consequences.

2.7. Financing from institutional investors

Institutional investors provide financing in a competitive private debt market, so that bank debt is priced to yield investors an expected return of zero. If investors liquidate a (monitored) loan at $t = 1$ or $t = 3$, they collect $L < rq[X_R + Z]$, where $rq[X_R + Z] < 1$. The assumption that $L < rq[X_R + Z]$ means that a bank liquidation is inefficient ex post.

2.8. Definition of a financial crisis

A financial crisis occurs if all banks in the system are prematurely liquidated at either $t = 1$ or $t = 3$.

2.9. Summary of timeline

At $t = 0$, there are $N_0$ banks. Each bank chooses to invest $1 in either a prudent (P) loan or a risky (R) loan. Each loan is entirely funded by uninsured debt and each matures at either $t = 1$ or $t = 2$. The debt will be called “short maturity” if it matures at $t = 1$ and “long maturity” if it matures at $t = 2$. At $t = 1$, based on their discovery of the realized value of $m_t$, investors who funded the bank at $t = 0$ may wish to collect their repayment at $t = 1$ or renew funding until $t = 2$. In the event that funding is not renewed, the bank must either repay investors in full or liquidate. If the bank survives until $t = 2$ without liquidation, then it collects its loan payment (if available) and pays off investors. Any positive profit at $t = 2$ on the first-period loan is paid out as a dividend to the bank’s shareholders. The lending cycle then restarts at $t = 2$ and the bank makes a new choice of loan and funds it entirely with new debt. This debt is “short maturity” if it matures at $t = 3$ and “long maturity” if it matures at $t = 4$. The “skill” or “luck” macro state realization for the second period, $m_3$, occurs at $t = 3$, and this is observed at that time by all in the second period. The terminal payoff is realized at $t = 4$.

A summary of the sequence of events is provided in Fig. 1. In Fig. 2, there is a pictorial summary of the probability distributions of the projects.

2.10. Discussion of model

The model has numerous moving parts, so it is useful briefly review the role these elements play in the model and why they are needed.

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17 It makes little difference to the analysis if the bank was partly financed with equity, as long as it is not predominantly financed with equity. Non-trivial amounts of debt financing are essential for fragility.

18 It will be shown in the analysis that the short maturity of the bank’s debt emerges endogenously.

19 This is for simplicity. If the bank could use its first-period profit to fund part of its second-period loan, it would reduce the reliance on outside debt. However, the analysis is qualitatively unchanged if the bank has to fund the loan even partly from outside debt.
Two periods – five dates

Two periods are the minimum needed to accommodate learning about banks on the part of both banks and their investors, so that funding renewals can be made contingent on observed outcomes. The five dates are needed to allow for interim withdrawal risk in each period in addition to the loan granting and repayment dates in each period.

Bank monitoring of loans at a private cost to banker

This is needed to endogenize maturity mismatching by the bank in a manner similar to the market discipline argument in Calomiris and Kahn (1991). While it is analytically most convenient to model this as post-lending monitoring in the Holmstrom and Tirole (1997) sense, one can also interpret it as pre-lending screening in the Ramakrishnan and Thakor (1984) sense. This means that the loans in this model can be viewed either as business loans where post-lending monitoring matters a great deal, or mortgages and other kinds of consumer loans where screening is more important.20

Fig. 1. Sequence of events.

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20 For empirical evidence on the importance of screening for mortgage lending, see Keys et al. (2010) and Purnanandam (2011).
2.10.3. P and R loans

These two types of loans are needed to give the bank an asset portfolio choice. The potential dependence of the loan payoffs on the bank’s skill is at the heart of the learning model.

2.10.4. Macro state uncertainty

This uncertainty is the core of the analysis because it permits discontinuous changes in risk assessments with Bayesian rational agents.

3. Analysis of the base model

The main goal of this section is to analyze the model developed in the previous section to examine how banks fund themselves, their debt maturity and their loan portfolio choices.

3.1. Bank debt maturity

In this subsection, the question of the bank’s debt maturity is examined. The following parametric restrictions will be imposed on the model.

\[ W > C > x(qX_{8} - 1) \]
the first part of the restriction merely says that the manager’s up-front wage is enough to cover his loan monitoring cost, which is sufficient for his participation constraint to be satisfied. The second part of the restriction says that, even if the bank was identified unambiguously as type \( \tau \) and it invested in loan \( R \), the net expected payoff to the manager (a fraction \( \alpha \) of the payoff to the bank’s shareholders) is exceeded by his personal cost of monitoring. This creates a moral hazard that the bank’s creditors must account for. The following result is now immediate.

**Proposition 1.** In each period the bank will finance its loan with short-maturity debt. That is, in the first period, it will issue debt at \( t = 0 \) that matures at \( t = 1 \) and has to be replaced with short-maturity debt that matures at \( t = 2 \). In the second period, it will issue short-maturity debt at \( t = 2 \) that matures at \( t = 3 \) and is replaced with short-maturity debt that matures at \( t = 4 \).

The intuition is straightforward. In the absence of any refinancing risk – which would be the case with long-term debt – the bank manager would not monitor. This would make it impossible for the bank to raise financing. Thus, short-term debt is the bank’s only viable financing alternative.

3.2. Bank loan market: analysis of outcomes at \( t = 2 \)

In the usual backward induction manner, I start by analyzing the second period, beginning at \( t = 2 \), first and then the first period beginning at \( t = 0 \).

3.2.1. Default/repayment on first-period loan

Suppose the bank made a \( P \) loan at \( t = 0 \). There are now two possible states for the bank at \( t = 2 \); (i) its loan defaults at \( t = 2 \), or (ii) its loan pays off.

**State (i): Repayment failure (\( P \) loan defaults at \( t = 2 \)):** In this case, the common posterior belief about the bank’s type at \( t = 2 \) is:

\[
\tau_2^f(\text{skill}) = \Pr(\tau|\text{default on first-period loan}, m_1 = \text{skill}) = 0
\]

if it was observed that \( m_1 = “\text{skill}” \). And it is

\[
\tau_2^f(\text{luck}) = \Pr(\tau|\text{default on first period loan}, m_1 = \text{luck}) = r
\]

if it was observed that \( m_1 = “\text{luck}” \). We can thus compute the expected success probabilities on the second-period projects, conditional on having observed first-period failure. Using notation \( P_j^i \) and \( R_j^i \), \( j \in \{S, D\}, i \in \{1, 2\} \) to designate the event \( j \) of either successful repayment (\( S \)) or default (\( D \)) in period \( i \) (which is 1 or 2, corresponding to repayment outcomes observed at \( t = 2 \) for period 1 and \( t = 4 \) for period 2) on loans \( P \) and \( R \) respectively, one can write the posterior probabilities at \( t = 2 \) as:

\[
\Pr(P_S^2|P_D^1, m_1 = \text{luck}) = \Pr(P_S^2|P_D^1, m_1 = \text{luck}, m_3 = \text{luck})\Pr(m_3 = \text{luck}) \\
+ \Pr(P_S^2|P_D^1, m_1 = \text{luck}, m_3 = \text{skill})\Pr(m_3 = \text{skill})
\]

conditional on \( m_1 = \text{luck} \) having been observed. Using (12) yields:

\[
\Pr(P_S^2|P_D^1, m_1 = \text{luck}) = r x_0^p + (1 - \lambda) r_0^p = r_0^p = r + (1 - r)b
\]

(13)

Similarly, using (11):

\[
\Pr(P_S^2|P_D^1, m_1 = \text{skill}) = \lambda x_0^p + (1 - \lambda)b
\]

(14)

Note that \( \partial \Pr(P_S^2|P_D^1, m_1 = \text{skill})/\partial \lambda > 0 \). Thus, for \( \lambda \) small enough, it will be true that

\[
\Pr(P_S^2|P_D^1, m_1 = \text{skill})[X_p + |Z|] < 1
\]

(15)

which means that, subsequent to first-period default and the knowledge that \( m_1 = \text{skill} \), no financing will be available for the \( P \) loan in the second period. It will be assumed throughout that (15) holds. Similarly, using (11) and (12):
The bank’s repayment obligation to debt investors should be such that the amount raised from these investors at \( t = 0 \) (\$1) equals the expected payoff to them at \( t = 2 \), i.e.

\[
\bar{D}_p(Pr(P^2_S|P^1_S, m_1 = \text{skill}) = 1/Pr(P^2_S|P^1_S, m_1 = \text{skill}))
\]
I now turn to the $R$ loan. If we assume that the bank will be liquidated in the second-period if it invests in the $R$ loan at $t = 2$ and $m_3 = \text{luck}$ at $t = 3$, then the bank's expected profit from an $R$ loan is:

$$\tilde{\pi}_R^a = [1 - \lambda]qT_2^b (\text{skill}) \{X_R - \overline{D}_R(\Pr(R_2^2|P_1^1, m_1 = \text{skill})) + Z\}$$

(28)

where the bank’s repayment obligation is:

$$\overline{D}_R(\Pr(R_2^2|P_1^1, m_1 = \text{skill})) = [1 - \lambda L][\{1 - \lambda]qT_2^b (\text{skill})\}^{-1}$$

(29)

The following result can now be stated.

**Lemma 2.** For $\lambda \in (0, 1)$ and $b \in (0, 1)$ sufficiently small and $q \in (0, 1)$ sufficiently large, we have:

$$[1 - \lambda]qT_2^b (\text{skill})X_R > \Pr(P_2^2|P_1^1, m_1 = \text{skill})X_p + \Pr(P_2^2|P_1^1, m_1 = \text{skill}) - [1 - \lambda]qT_2^b (\text{skill})Z$$

(30)

The implication of (30) is that a bank that experienced repayment success on the first-period $P$ loan and observed $m_1 = \text{skill}$ will have such a high posterior probability of success on the $R$ loan in the second period that the bank’s shareholders’ expected payoff will be higher with an $R$ loan than with a $P$ loan. It will be assumed throughout that the conditions stated in Lemma 2 will be satisfied, so (30) holds.

The following result is now easy to derive.

**Proposition 2** (Bank’s Second-Period Lending). If its first-period $P$ loan repaid at $t = 2$ and $m_1 = \text{skill}$, the bank prefers to invest in the $R$ loan in the second period at $t = 2$. It raises $1 in financing at $t = 2$ to invest in the second-period loan at $t = 2$ and promises creditors a repayment of $L$ if investors collect on their credit at $t = 3$, and $\overline{D}_R(\Pr(R_2^2|P_1^1, m_1 = \text{skill}))$ if they provide renewed funding at $t = 3$ that requires repayment at $t = 4$. If the bank’s creditors verify at $t = 3$ that the bank monitored its loan, then they will renew funding at $t = 3$ if $m_3 = \text{skill}$ and liquidate the bank if $m_3 = \text{luck}$.

The intuition is as follows. Based on the previous analysis, if the bank’s first-period loan successfully repays, then the posterior belief about the bank’s probability of making a second-period loan that will repay is so high that it becomes profitable for the bank to make an $R$ loan and financiers will fund it. If there is no renewal of the loan at $t = 3$, the bank has to liquidate, so it collects $L$ to pay creditors. Conditional on the bank having monitored its loan, the creditors will wish to do this only if $m_3 = \text{skill}$, because in this case beliefs about repayment probabilities revert to prior beliefs. If $m_3 = \text{skill}$, then creditors get a higher expected payoff if they allow the bank to continue than if they liquidate it.

Since all banks are identical in this model, all those who experience success on their first-period $P$ loan will invest in $R$ loans in the second period. This is essentially the whole banking industry since banks that experienced first-period default have exited at $t = 2$. The exit of a bank that experiences defaults is an outcome of the parametric restrictions imposed, and the assumption that all loans of a particular type have perfectly correlated prospects. More generally, if a bank makes multiple loans of the same type ($P$ or $R$) with imperfectly correlated prospects, then whether a bank that experiences (some) first-period defaults exits or not will depend both on the number of defaults and the bank’s leverage. Banks with fewer defaults and higher capital levels may survive, but what will be true is that they will stick to safe lending and make only $P$ loans in the second period, regardless of the realization of $m_1$. Hence, the broader risk-management implication of Proposition 2 is that banks that experience first-period loan repayment success pursue riskier lending in the second period, whereas banks that experience (some) loan defaults stick to safe lending or are forced to exit.

Strictly within the model, since only banks that experienced (complete) loan repayment at $t = 2$ are able to continue, and these banks invest in $R$ loans in the second period, an observation of $m_3 = \text{luck}$ at $t = 3$ leads to a sharp reassessment of risk. We can interpret this as a discontinuous increase in the

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21 In the model, each bank makes only one loan, but this is the same as the bank making an arbitrarily large number ($N > 1$) of loans if the prospects of all loans of the same type are perfectly correlated.

22 This is consistent with the empirical evidence in Berger and Bouwman (2013) that banks with higher capital have a higher likelihood of surviving a financial crisis.
credit spread or risk premium on R loans, which leads to liquidations of all banks and a financial crisis. Note also that Lemma 2, which states the conditions under which banks switch to the R loan, requires that \( \lambda \in (0, 1) \) be sufficiently low. Since a crisis occurs only when the luck state is realized, this also implies that the ex ante probability of a crisis must be low enough for a crisis to occur ex post. If the probability of the luck state is sufficiently high, banks will never choose the R loan and investors will never fund it.

3.3. Analysis of outcomes at \( t = 0 \)

The following result is now immediate, given our earlier analysis.

**Lemma 3** *(Banks First-Period Lending).* At \( t = 0 \), all banks invest in P loans. Investors are promised a repayment of L at \( t = 1 \) and \( D_P(r^0_0) \) at \( t = 2 \), where

\[
D_P(r^0_0) = \left| r^0_0 \right|^{-1}
\]

and the bank's shareholders' expected profit is:

\[
\pi_1 = r^0_0[X_P - D_P(r^0_0) + Z]
\]

Investors do not liquidate the bank and agree to renew financing at \( t = 1 \) regardless of the realized value of \( m_1 \). There is no reassessment of risk in the first period.

The reason there is no reassessment of risk in the first period is that the realized value of \( m_1 \) does not cause investors to change their beliefs from their prior beliefs about the bank's ability since no repayment/default on any loan has been observed. The reason why the bank invest in P is that R cannot be financed with investors' prior beliefs about the bank's ability.

We see then that banks start out investing in P loans, and then those that experience successful repayment go on to make R loans in the second period, as long as the probability of \( m_3 = \text{luck} \) is small enough (see Lemma 2). But if \( m_3 = \text{luck} \) is indeed observed at \( t = 3 \), there is a sharp upward reassessment of risk. This reassessment leads to liquidations of banks, which are ex post inefficient both because of the non-pledgeable rents, \( Z \), that these banks' shareholders lose, but also due to the loss of additional social rents, \( J \). These liquidations affect all banks, so it is a crisis. The following result is now useful to note.

**Lemma 4** *(Social Welfare).* If the non-pledgeable social rent, \( J \), is large enough, then the bank may invest in R in the second period even though it is socially inefficient compared to investing in P.

The intuition is that P always has a higher repayment probability than R, and hence there is a lower probability that the social rent \( J \) will be lost with P than with R. This means the bank's shareholders may wish to invest in R and financiers will be willing to provide financing even though these loans are socially inefficient compared to the P loans. This can justify regulatory portfolio restrictions or capital requirements that reduce the attractiveness of R loans for the bank. The next result establishes that the restrictions on the deep (exogenous) parameters of the model do not lead to an empty set of permissible values.

**Proposition 4.** The set of exogenous parameters satisfying the restriction on the model ((2), (4), (5), (6), (7), (9), (10), (15) and (30)) is non-empty.

4. A T-period model: the effect of more time periods

Imagine now that the previous analysis represented the first two periods of an arbitrarily long time horizon, i.e., \( t = T > 4 \) is an arbitrarily large number. Also, instead of a single R loan, there is a large set of R loans, say \( \{R_1, R_2, ..., R_N\} \), with \( N \) arbitrarily large. The probability distribution for loan \( R_i \) is that a type-\( \tau \) bank will be able to ensure w.p. 1 that it is a \( \tilde{G}_i \) loan, where \( \tilde{G}_i \) loan pays off \( X^i_k \) w.p. \( q \), and \( 0 \) w.p. \( 1 - q \), and \( X^i_k > X^j_k \) if \( i > j \), whereas a type-\( \tau \) bank will ensure w.p. 1 that it is a \( \tilde{B}_i \) loan, where
a \( \tilde{B} \) loan generates a payoff of \( X^t_R \) w.p. \( b_R \) and a payoff of \(-[i - 1]k\) w.p. \( 1 - b_R \), where \( k > 0 \) is a positive constant and \( 0 < b_R < \min\{0.5, q\} < 1 \). Specifically, it is assumed that \( X^t_R = X^{t-1}_{R} + [i - 1]k \forall i \in \{2, \ldots, N\} \).

As in the previous analysis, \( rqX^t_R < 1 \forall i \). That is, it is assumed that \( X^t_R = X_R \), where \( X_R \) was defined in Section 2 (see Table 1). For simplicity, let us continue with the assumption that the bank's creditors can costlessly observe the realization of \( m \) at each odd date: \( t = 1, 3, \ldots \).

A parametric restriction will be imposed on \( r \), which says that \( r \) is small enough that

\[
|1 - r|^{-1} < |1 - 2b_R|^{-1}
\]

(33)

This restriction ensures that, evaluated at the prior belief that the bank is type \( \tau \), riskier loans are less attractive to the bank.

Let the liquidation value of any loan be \( L \in (L_1, L_2) \), where

\[
L_1 = E\{X^t_R + [N - 1]k|rq + (1 - r)b_R| - [1 - r]\}b_R|N - 1|k;
\]

\[
L_2 = \min\{L_1 + Z, X^t_R|rq + (1 - r)b_R|\}
\]

(34)

which means that all liquidations are inefficient. Note that, given (33), we know that \( L_1 < L_2 \).

A couple of points are worth noting. First, unlike the two-period model, the bad loan, \( \tilde{B} \), has a positive probability of success. This is needed in a multi-period setting to prevent the posterior belief that the bank is type \( \tau \), conditional on a successful outcome of a risky loan in the luck macroeconomic state, from becoming 1. Second, the bank's payoff on the risky loan in the default state is \(-[i - 1]k\), which can be viewed as the bank's loss given default, a measure of risk that goes up with \( i \). That is, viewed in this sense, loan \( R_i \) is riskier than loan \( R_j \), if \( i > j \).

Now the probability of success with a \( P \) loan at \( t = 0 \), \( r_0^P \), is given by \( r \), and we will assume (4) and (5) hold. The probability of success with a \( R_1 \) loan at \( t = 0 \) is:

\[
r_0^{R_1} = rq + [1 - r]b_R
\]

(35)

The analog of (6) is

\[
q[X^1_R + Z] > X_p + Z
\]

(36)

The analog of (7) is

\[
q - b_R > 1 - b
\]

(37)

and the analog of (9) is

\[
r_0^{R_1}|X^1_R + Z| < 1.
\]

(38)

Now, the value of risky loan \( R_i \), to financiers, evaluated at the prior beliefs is:

\[
EV_i = rq\left[X^1_R + Z + [i - 1]k\right] + [1 - r]\left\{b_R\left[X^1_R + Z + [i - 1]k\right] - [1 - b_R][i - 1]k\right\}
\]

(39)

The following result is useful.

**Lemma 5.** (i) \( \partial EV_i/\partial r > 0 \), (ii) \( \partial EV_i(r)/\partial i < 0 \), and (iii) \( \partial^2 EV_i/\partial i\partial r > 0 \).

The implications of this lemma are as follows. From (i), we see that as the (posterior) belief that the bank is type \( \tau \) increases, so does the value of any risky loan, \( R_i \). However, (ii) implies that evaluated at the prior belief \( r \), riskier loans are less valuable. And, (iii) says that the marginal impact of an increase in the posterior belief that the bank's type is \( \tau \) is greater for larger values of \( i \), i.e., for riskier loans.

We now need additional notation. We know that if the bank invests in a \( P \) loan at \( t = 0 \), then the posterior probability at \( t = 2 \) that the bank is of type \( \tau \), conditional on success at \( t = 2 \) and \( m_1 = \text{skill} \), is given by (18) as:

---

23 One may ask what it means to have a negative payoff for the bank in the loan default state. The idea here is that the bank makes numerous investments of tangible and intangible capital that can only be recovered if the loan repays. Default causes the bank to lose these investments.
\[ \tau^5_2 (\text{skill}) = \Pr(\tau = \text{skill} | \pi_1^1, m_1 = \text{skill}) = \frac{r}{r + b[1 - r]} \]

The posterior belief at \( t = 2 \) about the probability of success in the second period for \( R_1 \) (at \( t = 4 \)) is:

\[ \Pr(R_1^1 | \pi_1^1 * \text{bank}) = \lambda R_1^1 + [1 - \lambda] \{ q \tau^5_2 (\text{skill}) + [1 - \tau^5_2 (\text{skill})] b_R \} \]

(40)

and, it shall be assumed that:

\[ \Pr(R_1^2 | \pi_1^1) > \Pr(R_1^0 | \pi_1^1) \]

(41)

Moreover, let \( \tau^5_2 (\text{skill}) \) be small enough that we have the following analog of (33):

\[ \Pr(R_1^2 | \pi_1^1) \left[ 1 - \Pr(R_1^2 | \pi_1^1) \right]^{-1} < [1 - 2b_R][q]^{-1} \]

(42)

Essentially, the restrictions imposed by (41) and (42) guarantee that the bank will switch from the \( P \) to the \( R_1 \) loan at \( t = 2 \) if its first-period \( P \) loan succeeded; note that (42) is sufficient for \( R_1 \) to be preferred to any other \( R_i, i > 1 \).

Now, for an odd date \( t \), define \( \tau^5_2 (\text{skill}^\text{odd}, S_t^\text{even}) \) as the posterior probability at date \( t \) that the bank is type \( \tau \), where the vector \( \text{skill}^\text{odd} = (\text{skill}_1, \text{skill}_2, \text{skill}_3, \ldots, \text{skill}_i) \) indicates that the macroeconomic state was revealed to be \( m = \text{skill} \) at all odd dates up to (and including) date \( t \), and the vector \( S_t^\text{even} = (S_2, S_4, S_6, \ldots, S_{t-1}) \) indicates that the loans made by the bank successfully repaid at all even dates up to (and including) date \( t - 1 \). Then, we see that \( \tau^5_2 (\text{skill}^\text{odd}, S_t^\text{even}) \) is defined recursively by:

\[ \tau^5_2 (\text{skill}^\text{odd}, S_t^\text{even}) = \frac{q \tau_2^{5,2-1}}{b_R \tau^5_2 + b_R[1 - \tau^5_2 - 2]} \]

(43)

The following result can now be proved:

**Lemma 6.** There exists a large enough (odd) number, \( t^* \), such that \( \partial EV_i(\tau^5_2) / \partial i < 0 \forall t < t^* \) and \( \partial EV_i(\tau^5_2) / \partial i > 0 \forall t \geq t^* \), where \( \tau^5_2 \) is short-hand for \( \tau^5_2 (\text{skill}, S_t^\text{even}) \).

This lemma says that if there is a sufficiently long sequence of successful loan outcomes along with a correspondingly long sequence of consecutive \( m_t = \text{skill} \) states, the posterior belief about the bank being of type \( \tau \) will be high enough to ensure that riskier loans will have higher expected values. This suggests that, after a long enough string of successful loan outcomes during a “bull market” economy, the bank will switch to the riskiest loan available. This leads us to the main result of this section.

**Proposition 3.** In a credit market equilibrium with \( T \) (arbitrarily large) periods and costless discovery of the macro state \( m_t \), the bank will invest in the \( P \) loan in the first period. Conditional on the first-period loan being successfully repaid and \( m_1 = \text{skill} \), the bank invests in the \( R_1 \) loan in the second period. Conditional on each \( R_i \) loan successfully paying off in each period and \( m_t = \text{skill} \) being realized at dates \( t = 3, 5, \ldots \), the bank will continue to invest in \( R_1 \) each period until date \( t < t^* \) (defined in Lemma 6). At any date \( t < t^* \), if investors observe that \( m_t = \text{luck} \) has occurred, they will not liquidate any banks. However, if the number of periods that the \( R_1 \) loan is successfully repaid in conjunction with \( m_t = \text{skill} \) being realized is greater than or equal to \( t^* \), then risk assessments will be favorable enough that the bank will switch to investing in \( R_N \) in the next period and will continue to do so in the periods that follow, conditional on each loan being successfully repaid and \( m_t = \text{skill} \) being realized. If \( m_t = \text{luck} \) is realized at any \( t > t^* \), investors will liquidate all banks with \( R_N \) loans and a financial crisis will ensue.

This proposition, depicted graphically in Fig. 3, makes a number of points. First, there is a “three-asset separation” here. The bank begins by investing in the \( P \) loan, then switches to \( R_1 \), conditional on some events, and then continues with \( R_1 \) until it switches to \( R_N \). There is no financial crisis even if \( m_t = \text{luck} \) at any time \( t \) prior to \( t^* \), when banks switch to \( R_N \), the riskiest loan. That is, if a sufficiently long sequence of good outcomes can lower the risk premium on the riskiest loan enough to make it attractive for the bank. For \( t > t^* \), an \( m_t = \text{luck} \) realization triggers a crisis. Thus, there has to be a sufficiently long “bull market run” – with the macroeconomic state being \( m_t = \text{skill} \) and banks experiencing good loan outcomes – before a crisis occurs. Prior to this, there is a very high level of
risk-taking by banks that is not considered very risky by banks, investors or regulators. That is, it is not only risk management within banks that tolerates risky loans, but also investors and regulators who do not view these as unacceptably risky.

The intuition for this result is that riskier loans have values that are more sensitive to the bank’s skill, and thus increase more rapidly as perceptions of the bank’s skill goes up. Thus, when skill perceptions are relatively low, either $P$ or $R_1$ is the preferred loan. Since these loans have values, evaluated at prior beliefs about bank skill, investors do not liquidate these loans even if $m_t = \text{luck occurs}$. However, when skill perceptions are sufficiently high, the credit spread narrows enough that the $R_N$ loan has the highest expected value, so banks switch to it. But, if $m_t = \text{luck occurs}$ in any period, skill perceptions drop down to the prior belief, the credit spread widens and a crisis occurs because the expected value of $R_N$, evaluated at the prior belief, is lower than the liquidation value. This intuition is illustrated in Fig. 4.

5. How and why do beliefs about outcome determinants shift?

An interesting question is: what determines whether investors believe that outcomes are skill-dependent or luck-dependent? Thus far it has been assumed that the realization of $m_t \in \{\text{luck, skill}\}$ is entirely exogenous. In this section, a slightly modified version of the model is presented in which there is learning about whether outcomes are determined by luck or by skill.
Consider a six-date world with \( t = 0, 1, 2, 3, 4, 5 \). There are two cohorts of banks. At \( t = 0 \), the first cohort, consisting of \( N_0 \) banks, enters the market and makes loans. Let \( N_t \) be the number of banks in this cohort at date \( t \). Then at \( t = 1 \), the second cohort, consisting of \( M_1 \) banks, enters the market and makes loans. Let \( M_t \) be the number of banks in this cohort at date \( t \geq 1 \). For simplicity, no new banks enter after \( t = 1 \), and any bank that fails at any given date exits the market after that date. Cohort sizes are thus weakly decreasing over time.

Each cohort of banks starts out with the same prior beliefs about each bank’s ability and a choice between the \( P \) and \( R \) loans as in the base model in Section 2. Instead of assuming that in each period \( m_t \) is realized and this determines whether outcomes are skill-dependent or luck-dependent, it will now be assumed that there is a “true” state: in every period outcomes are determined either by skill or by luck, i.e., \( m_t = m \in \{ \text{luck, skill} \} \). \( \forall t \) but no one knows whether it is luck or skill that determines outcomes. The common prior belief is that the probability that outcomes are determined by luck is \( \lambda_0 \) and the probability that outcomes are determined by skill is \( 1 - \lambda_0 \). This belief will be updated at date \( t \) based on how many banks in each cohort experience success. A loan made at \( t \in \{ 0, 1, 2, 3 \} \) matures at \( t + 2 \) unless prematurely liquidated at \( t + 1 \).

Here is the sequence of events. At \( t = 0 \), \( N_0 \) banks in cohort 1 make \( P \) loans since that is all investors will fund, given prior beliefs. At \( t = 1 \), \( M_1 \) banks in cohort 2 make \( P \) loans. At \( t = 2 \), \( N_0 - n_2 \) of the \( N_0 \) banks experience success and \( n_2 \) experience failure and exit, so \( N_2 = N_0 - n_2 \). Based on the observation of this, all agents update their prior beliefs about the common state, so the posterior probability that outcomes are determined by luck is now \( \lambda_2 \). At this stage, the financiers of the cohort 2 banks can determine whether to liquidate the loans or continue funding them. At \( t = 2 \), the surviving \( N_2 \) cohort 1 banks make new loans that mature at \( t = 4 \). At \( t = 3 \), \( M_1 - m_3 \) cohort 2 banks experience success and \( m_3 \) experience failure and exit. So \( M_3 = M_1 - m_3 \). Based on this, agents update their beliefs about the common state and the new posterior belief is that the probability that outcomes are determined by luck is \( \lambda_3 \). At this stage, financiers of the second set of loans made by the cohort 1 banks determine whether to liquidate the loans or allow the bank to continue. The surviving cohort 2 banks then make their second round of loans. At \( t = 4 \), \( N_2 - n_4 \) of the \( N_2 \) cohort 1 banks experience success, and \( n_4 \) experience failure. Based on this, a new posterior belief, \( \lambda_4 \), emerges about the common state and financiers of the cohort 2 banks decide whether to liquidate their banks or let them continue until \( t = 5 \). The model ends at \( t = 5 \). In this setting, note that relevant changes in beliefs about the macro state can occur only at \( t = 2, 3 \) or 4. See Fig. 5.

Based on the previous analysis in Section 3, we know that at \( t = 0 \) and \( t = 1 \) the cohort 1 and cohort 2 banks will make only \( P \) loans. Consider now what might happen at \( t = 2 \). The cohort 1 banks will include \( n_2 \) banks that fail and \( N_0 - n_2 = N_2 \) that succeed. This outcome does not change beliefs about the skills of the cohort 2 banks, and we know from the previous analysis that the failed cohort 1 banks exit, whereas the successful cohort 1 banks will invest in \( R \) loans (given the conditions in Lemma 2).

How will agents revise their beliefs and arrive at \( \lambda_2 \)? This is addressed below.

**Lemma 7.** Regardless of the number of cohort 1 banks that succeed at \( t = 2 \)

\[
\lambda_2 = \lambda_0 \quad (44)
\]

It should not be surprising that beliefs about the common state are unchanged by the observed number of failures and successes among the cohort 1 banks at \( t = 2 \). The reason is that with the \( P \) loan the expected probability of success is the same regardless of whether outcomes are determined by skill or luck, given prior beliefs. A similar result appears below.

**Lemma 8.** Regardless of the number of cohort 2 banks that succeed at \( t = 3 \),

\[
\lambda_3 = \lambda_0 \quad (45)
\]

The intuition for this lemma is the same as that for Lemma 7. The cohort 2 banks invested in \( P \) loans at \( t = 1 \). Moreover, the outcomes at \( t = 2 \) related to the loans made by cohort 1 banks reveal no information about the abilities of cohort 2 banks. Thus, \( \lambda_3 \) remains at \( \lambda_0 \). This indicates that the first updating that leads to a posterior belief that differs from the prior about the common state occurs at \( t = 4 \). We now have the main result of this section.
Proposition 5. At \( t = 4 \), if a sufficiently large number of the \( R \) loans made by cohort 1 banks fail, the posterior belief about the common state changes to \( \lambda_4 < \lambda_0 \), and this may cause the financiers of the cohort 2 banks that invested in \( R \) loans at \( t = 3 \) to force the liquidation of these banks, precipitating a crisis.

The \( R \) loans made at \( t = 2 \) were made by banks that experienced success on their first-period \( P \) loans. Thus, these banks have sufficiently high posterior beliefs about their types (conditional on \( m = \text{skill} \)), \( \tau_\text{2}^2 \) (skill), that they could finance \( R \) loans (see (18) for \( \tau_\text{2}^2 \) (skill)). If a very large fraction of these \( R \) loans fail, it is in a state of the world in which a relatively small fraction of loans are expected to fail if outcomes were truly determined by skill. Financiers infer, therefore, that it is more likely that outcomes are determined by luck and lower the probability they assign to outcomes being determined by skill. This drop causes the expected probability of success on an \( R \) loan – with the expectation being taken over the skill and luck macro state – to decline relative to its value at \( t = 3 \) when the cohort 2 banks made their \( R \) loans. If this decline is sufficiently large, then the expected success probability becomes so low that financiers decide to liquidate all the \( R \) loans made by cohort 2 banks at \( t = 3 \).

This result illustrates one mechanism by which beliefs can shift across the skill and luck macro states to dramatically alter risk assessments. In the base model, the transition in beliefs is stark and sudden. But the analysis in this section shows that it need not be so. Even a smoother belief transition can have the effect of substantially elevating risk assessment.

6. Welfare implications, related issues and policy implications

This section discusses a number of issues. First, I discuss the welfare implication of the model. This is followed by a discussion of the implications of the model for risk management in banks through the
cycle and financial crises. I end the section with a discussion of empirical predictions and regulatory policy implications.

6.1. Efficiency

A bank liquidation is always *ex post* inefficient in this model because the value of the bank, including the value of its non-pledgeable assets, is greater than the liquidation value. Nonetheless, creditors force liquidation because they do not have access to the non-pledgeable assets. Moreover, due to the loss of the social benefit, $J$, of bank continuation, liquidations and the crisis they precipitate are also *ex ante* inefficient.

6.2. Risk assessment and risk management through the cycle

The analysis shows that the longer banks do well in terms of experiencing low defaults, the more the risk premium on risky loans relative to safe loans shrinks and the more attractive risky lending becomes for banks. Risk managers in banks will tend to get “marginalized” and credit rating agencies will assign ratings that may seem overly optimistic with the benefit of post-crisis hindsight. Note that this happens due to Bayesian rational learning and does not require banks/investors to ignore (tail) risks, as in Gennaioli et al. (2012). Regulators too will sanction these risk management practices in banks, even in the absence of regulator-taxpayer conflicts, as in Boot and Thakor (1993) and Kane (1990).

However, when banks experience defaults or beliefs shift and investors believe outcomes are luck-dependent, risk assessments change in the opposite direction as credit spreads widen, risk management in banks becomes more conservative, and risky lending is eschewed. To the extent that loan defaults are higher in recessions, these risk assessments are more likely during recessions as well, so risk taking is likely to be substantially curtailed during such times. This suggests that risk-taking is likely to be procyclical, with banks making increasingly risky loans during economic booms and becoming more “risk averse” during downturns.

6.3. Financial crises

The Reinhart and Rogoff (2008) evidence suggests a systematic pattern of high leverage, correlated risky lending and asset price bubbles prior to financial crises. While there is no capital structure decision for the bank in this model, the analysis does show that a financial crisis is preceded by risky lending, and a high price for risky loans that looks like a price bubble when the crisis arrives.

Moreover, when we consider the triggering mechanism in Section 5, there is a similarity between that mechanism and what occurred during the 2007–2009 crisis. Unexpectedly large defaults on sub-prime mortgages and escalating risks in even highly-rated securities backed by those mortgages caused the beliefs of investors and institutions to shift and led them to conclude that the assets they thought were safe were no longer safe. The problem was exacerbated when institutions that had done so well for so long began to experience difficulties, which caused liquidity to evaporate and the Asset-Backed Commercial Paper (ABCP) market to collapse.

Crisis in other countries have followed similar patterns. For example, the financial crisis in Sweden in the early 1990s, which saw non-performing loans at Swedish banks increase to 11% of GDP in 1993, was also preceded by a substantial credit expansion and a real estate boom that saw house prices doubling between 1981 and 1991. Similarly, the Japanese financial crisis that started in 1991 was preceded by asset price bubbles in real estate and in the stock market.

Since only successful banks survive and all invest in $R$ after surviving, the pre-crisis lending decisions of banks are clearly correlated in our model.
6.4. Empirical predictions and policy implications

The first prediction of the analysis is that a financial crisis will occur only when all agents share the common belief ex ante that the probability of a crisis is very low, and when bank risk-taking is at its highest. In particular, bank risk-taking will proceed in three steps: relatively safe lending at first, followed by loans of moderate risk, and then if there is a sufficiently long period of success, banks engage in the riskiest lending available.

This result is based on Lemma 2 and Proposition 3. The reason why a low ex ante probability of occurrence of a crisis is a necessary condition for a crisis to occur is that it is only when the probability of the luck state occurring is low enough that banks are willing to invest in very risky loans, something that is a necessary precursor to a crisis. This is because it is only in that event that the elevated perceptions of bankers’ skills following many periods of success remain at relatively high levels, and it takes sufficiently high ability perceptions to make very risky loans both attractive to banks and acceptable for investors to fund. As for the second part of the implication, we have extensively discussed the “three-fund separation” result earlier.

The second prediction is that lending-related risk management in financial institutions will tend to be procyclical, with more aggressive risk taking in economic booms and more conservative (lower-risk) lending in downturns.

The third prediction is that crises are cyclical phenomena and are (almost unavoidably) highly likely to occur following a long period of relatively high banking profits.

I now turn to the regulatory implications of the analysis. The first implication follows readily from the earlier results. It is that even if regulators exhibit no agency problems vis-à-vis taxpayers, they will be less vigilant when banks are doing well, and unlikely to prevent investments in highly-risky assets, simply because they will believe, like everyone else, that banks are skilled enough to manage the risks.

The second implication is that regulators should attempt, on an ex ante basis, to put in place mechanisms that make risk-taking more expensive for the bank’s shareholders when the bank is doing well. One simple mechanism for doing this is higher (equity) capital requirements that are countercyclical. Recalling the discussion of the interaction of learning and political economy in the Introduction, scaling back on government safety nets and attending to banking industry structure will also facilitate the goal of arresting bank risk taking.

There are examples of countries that have adopted such measures. In recent years, macroprudential policies in India dynamically varied risk weights and provisioning requirements for certain classes of real estate loans, and Colombian rules specifically targeted loan market overheating in 2008 with higher capital and provisioning requirements, as well as other policy changes. Countries like Canada, Australia and New Zealand, that have avoided banking crises, have all had banking systems with a relatively small number of large banks with significant charter values, so the cost of risk taking—the possible loss of bank charter—has always been high for the bank’s shareholders, a feature that dampens risk taking, even with learning. Of course, other policies, such as relatively high interest rates that kept a house price bubble from forming, also contributed to these countries escaping the 2007–2009 crisis.

Since in my model the probability of success of the risky loan, $R_i$, is always less than the probability of success of the prudent loan, $P$, higher capital requirements make $R_i$ less attractive than $P$ for the bank’s shareholders. By raising the capital requirement when banks are doing well—and the longer the industry has done well, the higher the requirement should be—risk-taking is made more expensive precisely when it is more attractive for banks.25

In a nutshell, the analysis here provides a straightforward way to understand numerous stylized facts related to bank risk management through the cycle, crises, the apparent “marginalization” of risk managers in banks during periods of high banking profitability, high compensation for bank CEOs, and high pre-crisis liquidity in asset markets as well as seemingly lax regulatory monitoring of banks prior to crises. Moreover, the theory developed here explains why banks, regulators and investors all seem to simultaneously “underestimate” risk and why market participants do not adjust contracts to

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25 This argument does not rely on equity capital being more costly than debt financing. It is simply based on the idea that a bank knows that a riskier investment increases the odds of losing its capital in the event of default, as in monitoring model of Holmstrom and Tirole (1997) and the sorting model of Chan et al. (1992).
correct the apparent misalignment or increase credit spreads immediately prior to the crisis in anticipation of the impending consequences of such behavior. This theory suggests that many of the regulatory initiatives that emerged in response to the recent financial crisis may not succeed in preventing future crises. These initiatives include restrictions on executive compensation in banking, pricing safety nets like deposit insurance more accurately and making the pricing risk-sensitive, putting restrictions on regulatory forbearance (e.g. of the sort put in place after the FDICIA in 1991), aligning the incentives of regulators more closely with those of taxpayers, engaging in more active monitoring of systemically important financial institutions (SIFIs), etc.

7. Conclusion

This paper has proposed a theory of bank credit risk management through the business cycle. The theory is based on rational learning that leads to revisions in inferences of banking skills in an environment in which skills may or may not matter, depending on a macro state that could be shaped by investor sentiment or developments in the real sector. In such a setting, a sufficiently long sequence of favorable outcomes for banks leads all agents—banks themselves, their investors, and regulators—to assign relatively high probabilities to the abilities of banks to manage their own risks. This provides banks with access to low-cost funding, and encourages banks to engage in riskier lending. Consequently, either if agents can directly observe that outcomes are just due to luck or observation of aggregate defaults leads to such an inference, there is a sharp increase in the risk premia on risky assets. Under some conditions, a crisis occurs as debt investors withdraw their funding from banks.26

Future research should be directed at better understanding, in a formal way, the potential interaction between the learning modeled here and the political economy hypothesis of Calomiris and Haber (2014). In particular, the regulatory policy implications of this interaction should serve to illuminate a variety of vexing issues in bank regulation design and open up new questions for research.

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Appendix A

Proof of Proposition 1. Suppose investors provide the bank with a long-maturity debt at \( t = 0 \) that matures at \( t = 2 \). Then, given (15), the bank manager will prefer not to monitor the loan, since the probability that he will lose his personal benefit, \( B \), by not monitoring is zero. Hence the creditors’ expected payoff at \( t = 2 \) will be zero, which means they will be unwilling to extend credit. With short-term debt, if creditors discover at \( t = 1 \) that no loan monitoring was done, then they will liquidate the bank and the bank manager will lose \( B \). Given (2), it follows that he will prefer to monitor the loan in this case. This means short-term debt is the only viable financing instrument for the bank. □

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26 While the trigger for a change in beliefs in this paper is a sufficiently high number of aggregate defaults, there may be other triggers as well. For example, the adoption of new government policies may be another trigger.
Proof of Lemma 1. If \( m_1 = \text{skill} \) and the bank experiences default on its first-period \( P \) loan, then from (15) and (17), we know that neither the \( P \) nor the \( R \) loan can be funded in the second period, so the bank must exit. If \( m_1 = \text{luck} \), then beliefs about the success probabilities of the \( P \) and \( R \) loans revert to those that existed at \( t = 0 \). □

Proof of Lemma 2. The total value of a bank to its financiers, conditional on \( m_1 = \text{skill} \), successful repayment of the first-period \( P \) loan and investment in the \( R \) loan in the second period is:

\[
\lambda L + [1 - \lambda]q\tau_2^2(\text{skill})|X_R + Z|
\]

(A.1)

If we assume that the loan will be liquidated at \( t = 3 \) conditional on \( m_3 = \text{luck} \). If the bank chooses to invest in a \( P \) loan in the second period, then its value to its financiers is (recalling (21)):

\[
\Pr(P_2^p|P_1^p, m_1 = \text{skill})|X_R + Z| = |X_R + Z|\lambda \tau_2^p + [1 - \lambda]\tau_2^2(\text{skill}) + (1 - \tau_2^2(\text{skill}))b)
\]

(A.2)

Label the expression in (A.1) as \( \xi_2^{p} \) and the expression in (A.2) as \( \eta_2^{p} \). It can be seen that (30) is sufficient for \( \eta_2^p > \xi_2^p \). Moreover, \( \partial(A.1)/\partial \lambda < 0 \) and \( \partial(A.1)/\partial q > 0 \). Further, we also see that \( \partial(A.2)/\partial q > 0 \). Thus, by continuity, (30) holds for \( \lambda \) and \( b \) sufficiently small and \( q \) sufficiently large, given that (30) holds trivially when \( q = 1, \lambda = b = 0 \). □

Proof of Proposition 2. Suppose the bank experiences successful repayment of its first-period loan at \( t = 2 \) and \( m_1 = \text{skill} \). Then, for the second-period loan choice, compare \( \pi_2^p \), the expected profit from an \( R \) loan with \( \pi_2^p \), the expected profit from a \( P \) loan, where:

\[
\pi_2^p = \Pr(P_2^p|P_1^p, m_1 = \text{skill})|X_R - D_P(\Pr(P_2^p|P_1^p, m_1 = \text{skill})) + Z|
\]

(A.3)

where \( D_P(\Pr(P_2^p|P_1^p, m_1 = \text{skill})) \) is by (27). Thus, substituting (27) in (A.3) yields:

\[
\begin{align*}
\pi_2^p &= \Pr(P_2^p|P_1^p, m_1 = \text{skill})|X_R - 1 + \Pr(P_2^p|P_1^p, m_1 = \text{skill})Z \\
&< \Pr(R_2^p|P_1^p, m_1 = \text{skill})|X_R + Z| - 1 \text{ given Lemma 2.} \\
&= \Pr(R_2^p|P_1^p, m_1 = \text{skill})|X_R - D_R(\Pr(R_2^p|P_1^p, m_1 = \text{skill})) + Z| \\
&= \pi_2^r.
\end{align*}
\]

Thus, the bank prefers an \( R \) loan to a \( P \) loan in the second period, conditional on first-period success. Since refinancing at \( t = 3 \) is possible with probability 1, investors who provided short-term financing at \( t = 2 \) that matures at \( t = 3 \) will view their claim as riskless. If creditors discover at \( t = 3 \) that \( m_3 = \text{skill} \), then they assess their expected payoff from refinancing the bank and getting repaid at \( t = 4 \) as \( q\tau_3^2(\text{skill})D_R(\Pr(R_2^p|P_1^p, m_1 = \text{skill})) \), which exceeds the payoff with liquidation at \( t = 3 \). If \( m_3 = \text{luck} \), then investors prefer to not renew funding and force liquidation since \( L > rqX_R \). □

Proof of Lemma 3. The proof is obvious given earlier arguments. □

Proof of Lemma 4. We know that the bank will invest in \( R \) in the second period, conditional upon successful repayment of \( P \) in the first period and \( m_1 = \text{skill} \). In this case its total value (including social value) is:

\[
\lambda L + [1 - \lambda]q\tau_2^2(\text{skill})|X_R + Z| + J
\]

(A.4)

If it invests in \( P \), the total value is:

\[
\Pr(P_2^p|P_1^p, m_1 = \text{skill})|X_R + Z| + J
\]

(A.5)

For (A.5) to exceed (A.4), we need

\[
J[\Pr(P_2^p|P_1^p, m_1 = \text{skill}) - [1 - \lambda]q\tau_2^2(\text{skill})] > \lambda L + [1 - \lambda]q\tau_2^2(\text{skill})|X_R + Z| - \Pr(P_2^p|P_1^p, m_1 = \text{skill})|X_R + Z|
\]

(A.6)
We know that
\[
\Pr(P^2_s|P^2_s, m_1 = \text{skill}) = \lambda r^0 + [1 - \lambda] \tau^0_2(\text{skill}) + [1 - \lambda] \tau^0_2(\text{skill}) b > [1 - \lambda] \tau^0_2(\text{skill}) > [1 - \lambda] q \tau^0_2(\text{skill})
\]
And that the right-hand side (RHS) of (A.6) is positive (but finite) by Lemma 2. Thus, for \( J \) large enough, (A.6) will hold. □

**Proof of Lemma 5.** Using (39) we see:
\[
\partial EV_i / \partial r = [q - b_R]_X^1 + Z + [i - 1]k + [1 - b_R][i - 1]k > 0 \text{ since } q > b_R \tag{A.7}
\]
Next, using (39), we have:
\[
\partial EV_i / \partial i = r q k + [1 - r] b_R k - [1 - b_R] k = k [rq + [1 - r] b_R - [1 - b_R][1 - b_R]] < 0 \text{ given } (33)
\]
Finally, using (A.7),
\[
\partial^2 EV_i / \partial i \partial r = [q - b_R] k + [1 - b_R] k > 0. \tag{□}
\]

**Proof of Lemma 6.** Since \( \partial EV_i(r) / \partial i < 0 \), we know that by continuity, \( \exists \tau^0_i > r \) such that \( \partial EV_i(\tau^0_i) / \partial i < 0 \). Since from (43) we know that \( \partial \tau^0_i / \partial t > 0 \), and since \( \partial^2 EV_i / \partial i \partial r > 0 \), we can assert \( \partial EV_i(\tau^0_i) / \partial i < 0 \) for \( t < t^* \), for some \( t^* \). Now, it follows that \( \lim_{t \to \infty} \tau^0_i = 1 \). Moreover, writing (33) as \( r < [1 - 2b_R][1 - r][q]^{-1} \), it can be seen that, the left-hand side converges to 1 as \( r \to 1 \), whereas \( \lim_{r \to 1} [1 - r][1 - 2b_R][q]^{-1} = 0 \). Thus, for \( t^* \) large enough, the inequality in (33) is reversed and \( \partial EV_i(\tau^0_i) / \partial i > 0 \) for \( \theta t > t^* \). □

**Proof of Proposition 3.** The proof that the bank will invest in the \( P \) loan in the first period follows from previous arguments. Moreover, since investors in the bank and the bank agree on values and the investors always price debt so as to break even, the previous analysis indicates that we can simply make the loan choice determination based on its total expected value. The fact that the back switches at \( t = 2 \) from the \( P \) loan to the \( R_1 \) loan follows from the restrictions (41) and (42) since \( R_1 \) is preferred to \( R_j \) when the probability is \( \tau^0_1(\text{skill}) \) that the bank is type \( \tau \).

Using Lemma 6, we know that (33) holds with \( \tau^0_1 = r \), and in that case, the economic value of \( R_1 \) exceeds the value of any \( R_j, j \geq 2 \). When \( t \geq t^* \), (33) is reversed and \( EV_i / \partial i > 0 \) so \( R_N \) has the highest value, and the bank switches to \( R_N \). □

**Proof of Proposition 4.** The following exogenous parameter values satisfy all of the restrictions:
\[
B = 1.5, \quad C = 1, \quad Z = 0.02, \quad M = 0.008, \quad r = 0.6, \quad b = 0.6, \quad W = 2, \quad \alpha = 0.3, \quad \lambda = 0.05, \quad X_p
\]
\[
= 1.2, \quad q = 0.77, \quad X_R = 2.1. \quad \text{□}
\]

**Proof of Lemma 7.** At \( t = 2 \), suppose \( N_2 \) banks out of \( N_0 \) are successful. Recall that the expected probability of success on a \( P \) loan made at \( t = 0 \) is \( r + [1 - r] b \). Thus, \( \Pr(m = \text{skill}|N_2 \text{ out of } N_1 \text{ banks experienced success}) \)
\[
= \binom{N_0}{N_2} \binom{N_0 - N_2}{N_2} [r + [1 - r] b]^N_2 (1 - r)[1 - b]^{-N_0 + N_2} [1 - \lambda_0] \]
\[
+ \binom{N_0}{N_2} \binom{N_0 - N_2}{N_2} [r + [1 - r] b]^N_2 (1 - r)[1 - b]^{-N_0 + N_2} \lambda_0 \tag{A.8}
\]
\[
= [1 - \lambda_0] \quad \text{□}
\]
Proof of Lemma 8. The proof is similar to that of Lemma 7. □

Proof of Proposition 5. If $N_A$ out of $N_2$ banks succeed at $t = 4$, then we can write:

$$\Pr (m = \text{skill} | N_A \text{ out of } N_2 \text{ banks succeeded at } t = 4)$$

$$= \frac{\binom{N_2}{N_A} \left[ \tau_2^f (\text{skill}) q \right]^{N_A} \left[ 1 - \tau_2^f (\text{skill}) q \right]^{N_2 - N_A} [1 - \lambda_0]}{\binom{N_2}{N_A} \left[ \tau_2^f (\text{skill}) q \right]^{N_A} \left[ 1 - \tau_2^f (\text{skill}) q \right]^{N_2 - N_A} [1 - \lambda_0]}$$

$$+ \binom{N_2}{N_A} \left[ rq \right]^{N_A} \left[ 1 - rq \right]^{N_2 - N_A} \lambda_0$$

(A.9)

where we recognize that the probability of success of an $R$ loan, conditional on $m = \text{skill}$, is $\tau_2^f (\text{skill})$, given by (18), and the probability of success of an $R$ loan, conditional on $m = \text{luck}$, is $rq$. Now note that, ignoring the integer problem, $\exists N_A$ small enough such that

$$\Pr (m = \text{skill} | N_A \text{ out of } N_2 \text{ banks succeeded at } t = 4)$$

$$< 1 - \lambda_0$$

(A.10)

To see this, note that the satisfaction of (A.10) requires that

$$\left\{ \tau_2^x (\text{skill}) q \right\}^{N_A} \left[ 1 - \tau_2^x (\text{skill}) q \right]^{N_2 - N_A} < \left[ rq \right]^{N_A} \left[ 1 - rq \right]^{N_2 - N_A}$$

(A.11)

Suppose $N_A = 0$. Then (A.11) becomes

$$1 < \left[ \frac{1 - rq}{1 - \tau_2^x (\text{skill}) q} \right]^{N_2}$$

(A.12)

which clearly holds since $\tau_2^x (\text{skill}) > r$. Thus, it has been shown that (A.10) holds for $N_A = 0$. By continuity, and ignoring the integer problem, it holds for $N_A$ small enough. Then the bank's financiers assess the expected payoff from continuation as

$$\lambda_4 rqX_R + \left[ 1 - \lambda_4 \right] q \tau_2^x (\text{skill}) \tilde{D}_R$$

(A.13)

where $\tilde{D}_R$ is the repayment obligation (see the proof of Proposition 2). We know that $rqX_R < L$. Thus, for $1 - \lambda_4$ small enough, it follows by continuity that:

$$\lambda_4 rqX_R + \left[ 1 - \lambda_4 \right] q \tau_2^x (\text{skill}) \tilde{D}_R < L$$

in which case financiers liquidate all banks and a crisis ensues. □

References


