Cooperation versus Competition in Agency

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1. Introduction

Our objective is to study an agency model in which the principal has a choice between getting two agents to cooperate and getting them to compete. We then discuss possible applications of this model to conglomerate mergers and issues in the design of incentives and the structuring of productive tasks within organizations.

We consider a principal’s choice of how to motivate two distinct agents. Each agent has an effort choice (unobservable to the principal) that stochastically affects his output. Each agent’s output is also affected by an exogenous noise, and noise terms across the two agents may be (conditionally) correlated. This correlation is allowed to vary from zero to unity. The principal has three distinct ways she can organize the tasks that these two agents perform. One is to keep the agents independent of each other and make each agent’s wage depend on just his own output. We call this an “independent” organization of tasks. A second way is for the principal to keep the agents independent but to use relative performance evaluation so that each agent is rewarded based in part on how well he does relative to the other.

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agent. In this case the two agents play a noncooperative game. We call this a "competitive" organization of tasks. A third approach the principal could take is to induce cooperation. The production processes controlled by the two agents could be organized in such a way that it is possible for each agent to facilitate the task performed by the other agent (i.e., each agent can choose to expend part of the effort involved in performing each of the two tasks). Moreover, given this team effort, the agents may also pool their wages and share them as they see fit. In other words, the agents cooperate in choosing both effort levels and payoffs. The principal continues to observe the individual output of each task, so that wages can be conditioned on these individual outputs as well as the total output. We call this a "cooperative" organization of tasks.

Our main result is that there is a critical value of the conditional correlation between the outputs of the two agents such that a cooperative organization of tasks is preferred by the principal when the conditional correlation is below this critical value and a competitive organization is preferred by the principal when the conditional correlation is above this critical value. Moreover, even when the outputs from the two tasks are conditionally uncorrelated, the optimal incentive contract makes each agent's wage depend on the outputs of both tasks. Since this is a case in which relative performance evaluation (see Bainan and Demski; Green and Stokey; Holmstrom, 1982; Mookherjee; Nalebuff and Stiglitz) is shown to be not the preferred mode of task organization and motivation, the intuition for this result differs from the usual reason for making a agent's compensation depend on the output related to a task to which he is not directly assigned.

There is an extensive and growing literature on multiple agents (e.g., Demski and Sappington, Ma et al., Mookherjee and Reichelstein). The main message of this literature is that, given a set of incentive contracts for the agents, there are often multiple Nash equilibria with disparity in the preference ordering of these equilibria across the principal on the one hand and the agents on the other. That is, agents could cooperate to reach an outcome other than that desired by the principal.

Cooperation by agents can take other forms as well. They can coalesce to pool their payoffs (side contracting) and share them for coinsurance purposes, and they can help each other out in the performance of their tasks. On the sidecontracting issue, much of the focus has been on the pernicious effects of such "collusion" among agents (see, e.g., Tirole, 1986, 1988). We examine conditions under which such side contracting hurts the principal and when it helps. Models in which an agent can allocate effort to many tasks have been examined by Drago and Turnbull (1987, 1988) and Lazear, for example. These papers focus on the incentive effects of specific forms of exogenously specified compensation schemes. Our objective is to derive the optimal incentive contract and explore the potential benefits to the principal of inducing cooperation among agents who join together in performing their tasks.

Our notion of the principal inducing cooperation—involving agents sharing tasks as well as pecuniary payoffs—is akin to the formation of (non-
hierarchial) worker peer works in the framework of Williamson. These groups involve collective and usually cooperative activity within organizations. Cooperation in our context entails the group performing an integrated set of tasks. Williamson alludes to the potential organizational benefits of the formation of groups in such cases:

Members of common or integrated task groups not only know the requisite attributes to look for in admitting a new member but are able, as a by-product of their working relationships, to mutually monitor one another—virtually automatically, at little incremental monitoring expense. Peer group affiliation may thus be sought in part because the group, rather than the individual, is better able to bear risks and because, in relation to the market, the group has superior ex ante screening and ex post monitoring capabilities. (44)

We capture these aspects of cooperation by assuming that cooperating agents can monitor each other more effectively than noncooperating agents. This relative monitoring advantage permits the capturing of risk-sharing benefits emanating from pooling pecuniary payoffs. An interesting example of such group-monitoring schemes is that of the Grameen Bank in Bangladesh, discussed in Varian (1990a).1

An alternative to inducing cooperation among agents may be “selective intervention” by the principal. For instance, the principal could physically locate the agents in such a way as to permit mutual observability of some performance-relevant inputs, prohibit the agents from pooling either efforts or pecuniary payoffs, and design contracts that induce each agent to report on the other. However, our premise is that such lack of cooperation among the agents will impede their observability of each other’s effort inputs, thereby diminishing the attractiveness of such a scheme relative to cooperation. Moreover, with such a scheme one is unable to avail of the risk-sharing benefits that stem from cooperation.2

The articles most closely related to ours are Holmstrom and Milgrom (1990), Varian (1990a), Itoh (1990, 1991), and Ramakrishnan and Thakor (1984b).3 Like us, Holmstrom–Milgrom and Varian show that payoff pooling

1. Another striking example of the benefits of peer group formation is provided by Akerlof. He provides evidence that suggests that the aggregate productivity of a peer group of cash posters was significantly higher than that which would have obtained if these cash posters had been segregated, with each poster’s output metered separately.

2. If agents who are precluded from cooperating could still observe each other’s efforts perfectly, then we could achieve first best if the contractual mechanism to induce each to report truthfully involves no distortions. An interesting example of such a mechanism is provided by Varian (1990b) in the context of a general problem involving externalities, where a regulator can implement the socially efficient outcome through a scheme in which agents report on each other. In our context, however, it is likely that the “selective intervention” scheme will involve noisy effort observability by the noncooperating agents and distortions in coxing truthful revelation of their observations. Sufficient noise in mutual effort observability will render cooperation a superior format for task organization in cases where it also dominates competition.

3. These other articles were done independently of ours and were brought to our attention while we were revising this article. We thank Bengt Holmstrom for initially bringing some of these other articles to our attention.
by agents lowers the principal's welfare if the agents cannot observe each other's effort. Whereas we permit agents to pool payoffs and efforts with cooperation, Itoh (1991) compares independent risk assignment with cooperation without payoff pooling and mutual monitoring. Although he considers only conditionally uncorrelated task outcomes, his notion of effort pooling is more general than in our model where the two agents' efforts are perfect substitutes. It is shown that the principal may benefit from this type of effort pooling alone. Itoh's earlier work (1990) is closer to our article in that both payoff pooling and mutual monitoring are allowed with cooperation. Only conditionally uncorrelated task outcomes are considered, but cooperation with payoff pooling and mutual monitoring is also compared to cooperation in the sense of Itoh (1991) (i.e., without payoff pooling and mutual monitoring). Thus, unlike we do, Itoh does not assess the principal's gains from cooperation versus competition. This question is taken up in Holmstrom and Milgrom. Ramakrishnan and Thakor (1984b) also show, in the context of a model of financial intermediation, that the principal benefits by encouraging agents to cooperate in groups. That article proves that an infinitely large cooperative group achieves first best even when the principal cannot observe any agent's effort. Unlike our model here, however, outputs of agents are uncorrelated, so that the trade-off between cooperation and competition is not addressed.

We explore two applications of our model. The first is to conglomerate mergers. These include "pure" conglomerate mergers of firms in completely unrelated businesses as well as mergers of firms with imperfectly correlated prospects. There is an extensive empirical literature that indicates that the total gains from conglomerate mergers are, on average, significantly positive and large (see, e.g., Malatesta, Jensen and Ruback, Dennis and McConnell). An explanation for these gains has remained elusive, however. Our model provides a possible explanation based on the result that a merger can help reduce incentive costs. The second application is to the general issue of the design of incentive schemes within organizations. Much of the recent discussion of factors contributing to high productivity at top U.S. corporations like MCI, Honeywell, Xerox, etc., and at Japanese firms has been riveted on the merits of team-building within organizations (see Dertouzos et al, Hackman and Oldham, Lincoln and McBride). We discuss conditions under which team development is beneficial to the organization and when it is not. We also

4. The recent literature has examined how the total gains are divided between acquirers and targets. The evidence suggests that shareholders of acquiring firms earn slightly positive average returns in mergers (e.g., Bradley et al.). Møck et al. find that for the 1975–1987 period, announcement period returns for acquiring firms are predominantly negative when the acquisition is aimed at diversification or acquiring a rapidly growing target, and when the acquiring firm's manager performed poorly prior to the acquisition. They conclude that poor managerial objectives drive many bad acquisitions. Such mergers are not our focus. Moreover, we also do not seek to explain how the total gains from mergers are divided between acquirers and targets. Rather, our theory sheds light on why these total gains may be significant.
examine the question of designing specific incentive schemes such as those for salesmen (see, e.g., Lal and Staelin).

The article is organized as follows. The basic agency model is examined in Section 2 with a discussion of the possible ways of motivating multiple agents. In Section 3, competitive and cooperative task allocations are compared under the assumption that the outputs of the two tasks are conditionally uncorrelated. The general probabilistic structure for firms with imperfectly conditionally correlated outputs is developed in Section 4, and competitive and cooperative task allocations are again compared. Applications are discussed in Section 5. Section 6 is the conclusion. All proofs are in the Appendix.

2. The Basic Agency Model

2.1 The Model

The agency model used is similar to Grossman and Hart and Holmstrom (1979), and in particular, the multi-agent extensions of Holmstrom (1982) and Mookherjee. The economy lasts for one period and agents are in elastic supply. At the beginning of the period, the principal negotiates an incentive contract with each agent. This contract induces the agent to take some action that, along with the realization of a random variable representing an exogenous source of noise in the production process, determines the output from the task at the end of the period. The agent’s action choice is unobservable ex post to the principal, implying that the agent’s compensation can be based only on task output.

For simplicity, any task output can take only one of two values, “high” (H) and “low” (L). Agents are risk averse and identical, and each has a von Neumann–Morgenstern utility function over wealth, represented by \( U(\cdot) \), with \( U'(\cdot) > 0 \), and \( U''(\cdot) < 0 \), where primes signify derivatives. We shall also use subscripts for partial derivatives of functions with more than one argument. Moreover, agents have disutility for effort. That is, if \( \Omega \) represents the agent’s feasible (nondenumerable) action space, and \( \omega \) the agent’s effort, then the agent’s total utility is (with \( m \) representing wealth) \( J(m, \omega) = U(m) - V(\omega) \), with \( V'(\omega) > 0 \), \( V''(\omega) > 0 \ \forall \omega \in \Omega \).

Let \( \pi \) be the task output. The probability mass function of \( \pi \) is given by \( \text{Prob}[\pi = H \mid \omega] = q(\omega) \), and \( \text{Prob}[\pi = L \mid \omega] = 1 - q(\omega) \), with \( q'(\omega) > 0 \), \( q''(\omega) < 0 \ \forall \omega \in \Omega \), and if \( \omega_{\text{max}} \) is the maximal element of \( \Omega \), then \( q(\omega_{\text{max}}) < 1 \). The agent’s incentive contract is a function, \( \phi : \{H, L\} \to \mathbb{R} \) (the real line), that pays the manager a dollar amount \( Z \) if \( \pi = H \) and \( X \) if \( \pi = L \). Define \( U(Z) = z \) and \( U(X) = x \). Throughout, lowercase letters denote utilities and capitals represent the corresponding dollar payoffs. Thus, \( J(\phi, \omega) = zq(\omega) + x[1 - q(\omega)] - V(\omega) \).

Assume that the principal is risk neutral toward the agent’s compensation. Define \( r(\cdot) = U^{-1}(\cdot) \). Note that \( r(\cdot) \) exists because \( U(\cdot) \) is strictly increasing and continuous on \( \mathbb{R} \) and hence invertible. Moreover, \( r'(\cdot) > 0 \) and \( r''(\cdot) > 0 \).

Let us imagine that the principal has two tasks that she would like performed. She must hire two agents. Each agent operates a production process
as described above. There are three ways that the principal can organize these two tasks.

(i) Keep the agents independent and compensate each agent independently of how the other agent does ("independent" task assignment). Assuming that the principal cannot observe the agent's effort, we refer to the outcome in this case as the "second best" solution.

(ii) Let the agents form a team so that they can cooperate in the performance of their tasks and in pooling their payoffs and sharing them ("cooperative" task assignment).

(iii) Keep the agents independent in terms of their tasks but let them compete in a tournament to determine each agent's wage ("competitive" task assignment).

Note that in case (i), since an agent's compensation cannot in any way depend on how the other agent performs his task, the conditional correlation in the outputs of the agents does not affect the solution. Hence, we will view this as our benchmark case and analyze it first.

2.2 Independent Task Assignment

Suppose the principal wants each agent to choose some action \( \omega \). This action is arbitrarily chosen and hence need not be optimal for the second best case.\(^5\) The principal must design the incentive contract such that the following incentive compatibility (IC) constraint is satisfied\(^6\) for each agent:

\[
q'(\hat{\omega})[z - x] - V'(\hat{\omega}) = 0.
\]  

(1)

Since choosing \( \omega \) is equivalent to choosing \( q \), we will henceforth drop the argument \( \omega \) and write \( q \) to denote \( q(\omega) \), \( \hat{q} \) to denote \( q(\hat{\omega}) \), \( \hat{V} \) to denote \( V(\hat{\omega}) \), and so on. In addition, the contract must guarantee each agent at least some minimum reservation utility, \( R \). Thus, \( \phi \) must be individually rational (IR); that is,

\[
\hat{q}z + [1 - \hat{q}]x - \hat{V} \geq R.
\]  

(2)

The principal must therefore

\[
\text{Minimize } \hat{q}t(z) + [1 - \hat{q}]r(x),
\]  

(3)

subject to (1) and (2).

---

5. We will hold \( \hat{\omega} \) fixed throughout. In comparing two arrangements, we will use the same \( \hat{\omega} \) for both arrangements. Even though \( \hat{\omega} \) may not be optimal for either arrangement, if the shareholders experience lower expected contracting costs with one arrangement, it must be the superior one since \( \hat{\omega} \) was arbitrarily chosen.

6. The first-order condition approach is valid here since the Grossman and Hart CDFC condition is satisfied.
Let the pair \((\tilde{x}, \tilde{z})\) represent the optimal solution to the above problem. Using standard techniques, this solution can be shown to be

\[
z = R + \hat{V} + [1 - \hat{q}] \hat{V}'[\hat{q}']^{-1},
\]

(4)

\[
\tilde{x} = R + \hat{V} - \hat{q}\hat{V}'[\hat{q}']^{-1}.
\]

(5)

If \(\hat{\omega}\) is observable ex post, the principal could achieve the first best solution by paying each agent \(t(R + \hat{V})\) dollars if \(\omega = \hat{\omega}\) is observed and nothing otherwise. This would result in a cost to the principal of \(t(R + V(\hat{\omega}))\). The second best solution that results in (4) and (5) imposes on the principal an expected cost per agent of

\[
\hat{q}t(\tilde{z}) + [1 - \hat{q}]t(\tilde{x}) > t(\hat{q}\tilde{z} + [1 - \hat{q}]\tilde{x}) \quad \text{[since } t(\cdot) \text{ is convex]}
\]

\[
= t(R + \hat{V}) \quad \text{[from (4) and (5)].}
\]

(6)

Hence, in the first best case, the principal's welfare, with each agent choosing the \(\omega \) optimal for that case, must be strictly greater than the highest second best welfare. Holmstrom (1979) proves a similar result. We assume throughout that principal bears the total agency cost. However, the derived Pareto optimal contracts do not change in form even if this cost is shared.

3. Task Assignments with Conditionally Uncorrelated Outputs

3.1 Cooperative Task Assignment

Suppose that there are two identical tasks (1 and 2) whose outputs are uncorrelated for every pair \((\omega_1, \omega_2)\). Each task requires an agent to be assigned to it. The principal has organized the two agents in a cooperative task assignment format. Here \(\omega_i\) is the effort allocated by the (cooperating) agents in the group to task \(i\). Note that \(\omega_i\) is not necessarily the effort input of the agent directly assigned to task \(i\). Rather, \(\omega_1 + \omega_2\) is the total amount of effort the two agents jointly decide to supply to the two tasks. How this total effort is shared by the two agents is addressed shortly. Each task's output, but not the action choice of either agent, is individually observable ex post. We assume now that the two agents can costlessly and perfectly monitor each other. This ensures cooperation between them. However, the principal must still contend with moral hazard.

The sequence of events is as follows. The principal first offers contracts to the agents. The agents then decide among themselves: (i) how they will share the total payoff generated by these contracts (i.e., they choose the appropriate side payments), (ii) the level of effort to be supplied to each task, and (iii) the contribution of each agent to the effort supplied to each task.

7. Note that \(\hat{\omega}\) was arbitrarily chosen for the second best case. It could, for instance, be a second best optimum. Clearly, shareholders may desire a different managerial action in the first best case. But if the shareholders are strictly better off in the first best case with a suboptimal \(\hat{\omega}\), they must be strictly better off with the optimal \(\hat{\omega}\).
Even with a cooperative task assignment, there are two ways the principal can contract with the agents. One is to negotiate separate contracts with the two agents, in which case each agent is told that he will be paid $Z if $x = H$ for his task and $X$ if $x = L$ for his task. The other arrangement is to negotiate joint contracts, in which case the two agents are given a combined payment that depends on the outputs of both tasks, with the sharing of this payment left to the agents.

3.1.1 Separate Contracts. Since the agents are risk averse and are cooperating, they will co-insure each other and agree to pool payoffs. In the lemma below, we show that because they are identical and can perfectly monitor each other, they will equally share the total work load as well as the total (pooled) payoff. In this lemma, we use a more general version of contracts than the separate contracts mentioned above. The contract given below is the same as separate contracts if we set $Y = [X + Z]/2$.

**Lemma 1.** Suppose the principal firm offers a symmetrical contract of the following form to the agents:

\[
\text{Fee} = 2Z, \quad \text{if } \pi_1 = \pi_2 = H,
\]

\[
= 2Y, \quad \text{if } \pi_i \neq \pi_j,
\]

\[
= 2X, \quad \text{if } \pi_1 = \pi_2 = L.
\]

Then the agents will expend equal effort and they will share the payoffs equally under each outcome.

How should the principal now write incentive contracts? Our objective is to compare the outcome here with the second best. We note first that the second best contracts might not “work” when agents cooperate. Suppose the second best contract $\phi = \{\bar{x}, \bar{z}\}$ given by (4) and (5) is used. Each agent’s personal effort contribution will be $[\omega_1 + \omega_2]/2$, and his expected utility will be

\[
J(\bar{\phi}, \omega_1, \omega_2) = q_1 q_2 \bar{z} + \{q_1[1 - q_2] + q_2[1 - q_1]\} \bar{y}
\]

\[
+ \{[1 - q_1][1 - q_2]\} \bar{x} - V([\omega_1 + \omega_2]/2),
\]

\[
\bar{y} \equiv U([\bar{X} + \bar{Z}]/2) > [\bar{x} + \bar{z}]/2 = \bar{y},
\]

\[
\bar{x} = U(\bar{X}), \quad \bar{z} = U(\bar{Z}).
\]

Differentiating (7) with respect to $\omega_1$, setting $\omega_1 = \omega_2 = \hat{\omega}$, and utilizing (4) and (5) yield $J_{\omega_1}(\bar{\phi}, \hat{\omega}, \hat{\omega}) = \check{q}'(1 - 2\hat{q}) (\bar{y} - \bar{y})$, where $\hat{\omega}$ is the desired action for the second best case. Since $\check{y} > \bar{y}$ and $\check{q}' > 0$, if $\check{q} > 0.5$, we obtain $J_{\omega_1}(\bar{\phi}, \hat{\omega}, \hat{\omega}) < 0$. This means that if the agents are awarded the second best contracts, each agent may find it privately optimal to choose an effort lower than $\omega$.

8. If $\check{q} < 0.5$, the managers will chose $\omega_1 > \hat{\omega}$. 
when agents are allowed to cooperate. The following theorem shows that, if this is done, cooperation among agents enhances the wealth of the principal.

**Theorem 1.** Suppose the outputs of the two tasks are uncorrelated and for each task, the function \( q(\omega) \) is such that \( q'(\omega)/(1 - q(\omega)) \) is nonincreasing in \( \omega \). Then, if the agents assigned to these two tasks cooperate, there exist contracts that improve the principal’s welfare and that reward each manager based only on the output of the task he is assigned to.

The reason why cooperation makes the principal better off is that it permits incentive contracts to be altered in a manner that reduces expected contracting costs. This happens because the nature of incentive constraints with a cooperative task assignment is different from that for the second best. With cooperation, effort inputs (and side payments between agents) jointly maximize the sum of the payoffs of the two agents. By contrast, in the second best case, each agent chooses his effort to maximize his own expected payoff (i.e., the agents play noncooperatively).

There are three distinctions between the cooperative task assignment and the second best cases that deserve emphasis. First, in the former, cooperative behavior between agents facilitates the principal’s task of soliciting the desired effort inputs relative to the second best. That is, incentive problems are attenuated because each agent’s individual compensation can be made dependent on the outputs of both tasks in a manner that diminishes incentives for opportunistic behavior by both agents. Second, cooperation enables agents to pool their individual payoffs and then share this pool. This sort of coinsurance is unavailable in the second best case. Thus, better risk sharing is achieved with cooperation. Third, cooperation allows the agents to pool efforts. That is, with identical agents, each supplies \((\omega_1 + \omega_2)/2\) effort, where \(\omega_1\) and \(\omega_2\) are the effort inputs to tasks 1 and 2, respectively. Such effort pooling does not affect the solution if the outputs of the two tasks are (conditionally) uncorrelated. With uncorrelated outputs, the effort allocated to each task is equal, so one can imagine that each agent supplies all of the effort allocated to his task and none to the other task. We will see later on that this is not true when outputs are conditionally correlated. In that case, the principal prefers inducing unequal effort allocations to the two tasks. Since agents are identical in all respects, effort pooling allows them each to supply the same effort even though the tasks they are managing are allocated different effort inputs. To summarize this discussion, in the case in which outputs are conditionally uncorrelated, only payoff pooling is necessary for cooperation to improve welfare. On the other hand, if there was effort pooling but no payoff pooling, the outcome with cooperation would be no different from the second best. When effort pooling and payoff pooling are combined, there is a strict welfare improvement with cooperation. That is, when outputs are conditionally corre-

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9. Payoff pooling improves welfare only if the agents can monitor each other, as we have assumed they can with cooperation; otherwise, it is bad for welfare.
lated, we will need both aspects of a cooperative task assignment to come into play: payoff pooling and effort pooling.

Another point worth noting is that the first best is unattainable with a cooperative task assignment even though the agents are able to monitor each other perfectly. The reason is that the principal’s ability to infer each individual agent’s action is not enhanced by the cooperation among the agents. The fact that agents can monitor each other only implies that they can cooperate to implement any effort allocation strategy they desire. The principal’s ex post informational disadvantage still leaves room for the agents to jointly exploit her by undersupplying effort relative to the first best.

3.1.2 Joint Contracts. A joint contract gives the two agents in a cooperative task assignment a combined payment of

$$
\begin{align*}
2Z, & \quad \text{if } \pi_1 = \pi_2 = H, \\
2Y, & \quad \text{if } \pi_1 \neq \pi_2, \\
2X, & \quad \text{if } \pi_1 = \pi_2 = L, 
\end{align*}
$$

(8)

where $U(Y) = y$. The sharing of this combined payoff is left to the agents. Is such an arrangement better than separate contracts for the principal?

One might claim that the issue of whether the compensation of one agent in a cooperative task assignment should depend on the output of the other task is a red herring. After all, since the agents share the total compensation, it does not seem relevant to identify some part of it as accruing to one agent. While this is true, separate and joint contracts should be distinguished as follows. With separate contracts, the payment to an agent in the event that one task succeeds while the other fails is halfway between what he gets when both succeed or when both fail. With a joint contract, the payment is not so restricted.

We prove that the compensation of each agent should be made contingent on the other task’s output.10 The intuition is that the agents will (jointly) decide to supply a task with a “high” effort not only because it stochastically increases the output of the task and thus the compensation received for managing the task, but also because doing so increases the expected compensation for managing the other task. Therefore, motivating factors are strengthened and incentive costs shrink. We state this finding as follows.

Theorem 2. Suppose the outputs of the two tasks are uncorrelated for every $(\omega_1, \omega_2)$ and these tasks are organized in a cooperative task assignment format. Then, joint contracts strictly Pareto dominate separate contracts, unless the agents have logarithmic preferences. With logarithmic preferences, separate contracts and joint contracts are equivalent.

10. This result should be contrasted with Proposition 2 in Mookherjee. See also Gjesdal.
By using joint contracts, a cooperative task assignment reduces the exogenous uncertainty facing agents. This should enable their reservation utilities to be satisfied at lower cost. The proof of Theorem 1 formally establishes this by showing that even separate contracts result in an expected contracting cost lower than second best. By Theorem 2, we know that the optimal joint contracts generally result in even lower expected costs than separate contracts.

Another result suggested by this analysis is that the gains from cooperation may be related to the uncertainty present in the task outputs. In the next theorem, we show that this is true, at least in some cases.

Theorem 3. Suppose \( U(Z) = \sqrt{Z} \) and \( q[1 - q]V'(q) \) is quasiconvex in \( q \). Then the total value enhancement due to cooperation between agents whose tasks have uncorrelated outputs will increase as the variance of the outputs of the tasks increases.

Suppose we define \( z - x \) as the incentive spread in the agent's compensation. Then, an implication of our earlier analysis is that the incentive spread with a cooperative task assignment is higher than in the second best case.

Corollary 1. Suppose the effort level desired by the principal, \( \omega \), is such that \( q(\omega) > 0.5 \). Then the incentive spread in the incentive contract is higher with a cooperative task assignment (for both separate and joint contracts) than it is in the second best case.

Recall that the condition \( q(\hat{\omega}) > 0.5 \) is precisely what is needed for the second best contracts to induce effort less than \( \hat{\omega} \) with a cooperative task assignment.

Hereafter, we shall use only joint contracts and refer to them as JCC(\( r \)) (joint contracts with cooperation and a correlation factor of \( r \) between task outputs). Relevant for this section are JCC(0) arrangements.

3.2 Competitive Task Assignment

As an alternative to cooperation, the principal can try to curtail agency costs by obtaining information about agents' actions through comparison of their outputs with those of similar agents. In this case, however, because of uncorrelated outputs, performance comparison conveys no information and the Performance Comparison Separate Tasks (PCST) contracting costs are equal to the second best costs.

Theorem 4. Suppose the outputs of two tasks are uncorrelated for every \( (\omega_1, \omega_2) \). Then the principal prefers a cooperative assignment of these tasks to a competitive assignment.

In the next section more will be said about the implications of this theorem.
4. The Probabilistic Structure for Correlated Outputs and Analysis of Assignment of Tasks with (Conditionally) Correlated Outputs

4.1 A General Probabilistic Structure

We now characterize the joint distribution of the outputs of two tasks managed by two agents. Conditional on the effort level chosen for each task, the two outputs are correlated in general. Thus, our preceding analysis is a special case of what follows.

The output of a task depends on two factors: the effort allocated to it and an exogenous uncertainty that may be correlated with that of another task. The outputs of two tasks are correlated conditional on some effort allocations. However, this correlation arises from a correlation in the random elements affecting outputs and hence the marginal distribution of the output of a particular task is independent of the effort allocated to another task. We also assume that the q function for every task is the same. This assumption is innocuous since observable differences between tasks have only a “scaling effect” and do not affect the results.

Let \( \omega_i \) be the effort allocated to task \( i \) and \( \pi_i \) be the output, where \( i = 1, 2 \). Define \( P_{ij} = \text{Prob}[\pi_1 = i, \pi_2 = j \mid \omega_1, \omega_2], \; i,j \in \{H,L\} \). Then, the joint earnings probabilities must satisfy

\[
P_{HH} + P_{HL} = q_1, \tag{9}
\]

\[
P_{LH} + P_{LL} = 1 - q_1, \tag{10}
\]

\[
P_{HH} + P_{LH} = q_2, \tag{11}
\]

\[
P_{HL} + P_{LL} = 1 - q_2, \tag{12}
\]

\[
P_{HH}, P_{HL}, P_{LH}, P_{LL} \in [0,1]. \tag{13}
\]

It is easy to also derive the following:

\[
E(\pi_i) = L + q_i (H - L), \quad \text{Var}(\pi_i) = [H - L]^2 q_i (1 - q_i),
\]

\[
\text{Cov}(\pi_1, \pi_2) = [H - L]^2 \tilde{C}_{12},
\]

\[
\tilde{C}_{12} = [1 - q_1][1 - q_2]P_{HH} - q_2[1 - q_1]P_{HL} - q_1[1 - q_2]P_{HL} + q_1q_2P_{LL}. \tag{14}
\]

11. What we have in mind is information correlation, not production correlation. That is, there is information about some exogenous uncertainty that affects the earnings of both divisions, and when this information is received, it reveals something about each firm’s earnings. Thus, observing one firm’s earnings provides information about the other firm’s earnings.
and $E(\cdot)$ is expectation, $\text{Var}(\cdot)$ is variance, and $\text{Cov}(\cdot, \cdot)$ is covariance. Since we are interested in (conditionally) correlated outputs, let us define $\varrho$ as the statistical correlation coefficient between $\pi_1$ and $\pi_2$, so that

$$
\varrho = \frac{\text{Cov}(\pi_1, \pi_2)}{\sqrt{\text{Var}(\pi_1) \text{Var}(\pi_2)}}^{-1},
$$

which means

$$
\varrho = \tilde{C}_{12} \left\{ \sqrt{q_1 q_2 [1 - q_1] [1 - q_2]} \right\}^{-1}, \tag{15}
$$

with $\tilde{C}_{12}$ given by (14).

Suppose $\omega_1 \leq \omega_2$, so that $q_1 \leq q_2$. In this case, define the "correlation factor" $r$ as

$$
r = \frac{\varrho \sqrt{q_2 [1 - q_1]} \left\{ \sqrt{q_1 [1 - q_2]} \right\}^{-1}}. \tag{16}
$$

As shown in the Appendix, (9)–(13) can now be used to derive, for $\omega_1 \leq \omega_2$,

$$
P_{HH} = rq_1 + [1 - r]q_1 q_2, \tag{17}
$$

$$
P_{HL} = [1 - r]q_1 [1 - q_2], \tag{18}
$$

$$
P_{LH} = r[q_2 - q_1] + [1 - r]q_2 [1 - q_1], \tag{19}
$$

$$
P_{LL} = r[1 - q_2] + [1 - r][1 - q_1][1 - q_2]. \tag{20}
$$

Feasibility restrictions on $r$ and general expressions that apply for both $\omega_1 \leq \omega_2$ and $\omega_1 \geq \omega_2$ are derived in the Appendix. In general, $r$ is neither the correlation coefficient nor the covariance, although $r = \varrho$ when $q_1 = q_2$ and/or $\varrho = 0$. Although this specification admits negatively correlated outputs, we restrict attention to $r \in [0,1]$ for analytical convenience.

4.2 Tasks with Outputs that Have $r \in (0,1)$

In considering the case in which $r \in (0,1)$, both agents should be viewed as taking the value of $r$ as exogenously given. They do recognize, however, that, given that value of $r$, their action choices affect the joint output probabilities through (17)–(20) and the output correlation coefficient as well.

4.2.1 Cooperative Task Assignment. Suppose two tasks whose outputs have a correlation factor of $r$ between them are organized in a cooperative task assignment format. The principal will use a JCC($r$) arrangement and reward the two agents on the basis of the combined output as described in (8). Note that, when $\pi_1 \neq \pi_2$, (8) implies that it is optimal to pay the agents the same amount regardless of which task has the high output. The reason is that it is not incentive compatible to distinguish between the $\{H,L\}$ and $\{L,H\}$ states
when inducing equal efforts in both tasks is optimal. We show later that equal effort allocation is not generally optimal (except when \( r = 0 \)), so that the optimal contract distinguishes between the \( \{H,L\} \) and \( \{L,H\} \) states. Determining the optimal allocation of total effort across the two tasks is difficult. However, if we assume, for now, that effort inputs will be equal in the two tasks, we have the following result.

**Theorem 5.** Suppose two tasks whose outputs have a correlation factor of \( r \) between them are organized in a cooperative task assignment and the principal wants to ensure that some desired action choice, \( \hat{\omega} \), is made for each task. Then the expected compensation costs to the principal decrease monotonically as \( r \) decreases. Moreover, in the special case in which \( r = 1 \), the expected contracting costs to the principal with a cooperative task assignment are equal to the second best costs.

As long as we hold fixed an effort allocation rule that says that both tasks receive equal effort inputs, expected contracting costs with cooperation increase monotonically from a base level intermediate between the first and second best costs to a maximum given by the second best cost as the output correlation factor rises from zero to unity. In the special case in which \( r = 1 \), \( P_{LH} = P_{HL} = 0 \) (i.e., \( \pi_1 \neq \pi_2 \) has probability zero) since the implemented contracts guarantee \( \omega_1 = \omega_2 = \hat{\omega} \). Each agent’s expected utility is

\[
J(\phi; \omega_1 = \omega_2 = \hat{\omega}) = \hat{q} z + [1 - \hat{q}] x - \hat{V},
\]

which equals the agent’s second best expected utility with \( \hat{z} = z \) and \( \hat{x} = x \).

However, when \( r \in (0,1] \), it is not necessarily optimal to induce equal effort inputs in the two tasks. The reason is that unequal effort inputs can alter the correlation coefficient [see (16)] in a way that is possibly welfare-improving. Our next result formalizes this for the case \( r = 1 \).

**Lemma 2.** Suppose \( r = 1 \). Then the principal will wish to induce \( \omega_1 \neq \omega_2 \) in a cooperative task assignment.

An important implication of Theorem 5 and Lemma 2 is that, when \( r = 1 \), expected costs with cooperation will be below second best costs. The intuition behind the optimality of asymmetric effort allocations for \( r > 0 \) is as follows. Starting from the second best contract, a small change in effort creates a second-order loss as the effort allocated to one task increases above \( \hat{\omega} \) and the effort allocated the other task decreases below \( \hat{\omega} \). There is, however, a first-order gain as this asymmetric effort allocation decreases the extent to which the outputs of the two tasks are correlated (i.e., \( P_{HH} \) and \( P_{LL} \) are reduced) and provides improved risk sharing with cooperation. At \( r = 0 \), there is no correlation and both effort inputs are equal with maximum risk sharing. As \( r \) increases, the efforts allocated to the two tasks diverge to an increasing extent from \( \hat{\omega} \) to offset partially the effect of an increasing \( r \). We now establish that
expected costs with cooperation will lie strictly between first and second best
costs.

**Lemma 3.** There does not exist any $r \in [0,1]$ for which the expected
contracting costs with a cooperative task assignment are as low as the first best
contracting costs.

The importance of this lemma becomes apparent in the next two subsections. In Theorem 5, we established the monotonicity of expected contracting costs in $r$ for the suboptimal class of contracts which induce efforts to be
allocated equally across the two tasks for $r > 0$. However, it is possible to
examine what happens with the optimal effort allocation scheme deployed for
$r \in (0,1)$. It is interesting that at $r = 1$, if we start with equal efforts allocated
to both tasks, then a small perturbation that makes effort allocations slightly
asymmetric across the two tasks does not affect the gross expected output, but
it does reduce expected contracting costs (see the proof of Lemma 2). This
delivers the result that it is optimal for the principal to induce efforts that are
asymmetric across the two tasks at $r = 1$. Moreover, it permits us to focus
exclusively on the effect that changing effort allocations has on expected
contracting costs. This result depends on the fact that, for any $r \in [0,1]$, the
expected output $EO$ less the social cost of producing that expected output,
$2V([\omega_1 + \omega_2]/2)$, is at a stationary point at $q_1 = q_2 = \hat{q} \equiv q(\hat{\omega})$, and is
maximized at that point. To see this, note that the first best involves choosing
$\omega_1$ and $\omega_2$ to maximize

$$EO = [q_1 + q_2][H - L] + 2L - 2V([\omega_1 + \omega_2]/2), \quad \text{for any } r \in [0,1].$$

It is easy to see that the relevant first-order conditions now imply $\omega_1 = \omega_2 =
\hat{\omega}$. Moreover, note that the expected output, $EO = [q_1 + q_2][H - L] + 2L$.
Thus, if we write $q_1 = q(\hat{\omega} - \varepsilon)$ and $q_2 = q(\hat{\omega} + \varepsilon)$, for $\varepsilon > 0$ small, we have
$\partial EO/\partial \varepsilon|_{\varepsilon = 0} < 0$ and $\partial^2 EO/\partial \varepsilon^2|_{\varepsilon = 0} < 0$. Thus, in small neighborhoods of
$\hat{\omega}$, effort perturbations have no material output ramifications.

However, to show that the principal is better off under cooperation when $r$
decreases under the optimal effort allocation scheme, we must take asymmetric
effort allocations as our starting point. That is, suppose at some $r \in (0,1]$, the
optimal effort allocation is such that $q_2 > q_1$. Then any change in this effort
allocation will also affect the expected output. This implies that we can no
longer focus exclusively on the principal’s expected cost of contracting with
the agents. Rather, we must consider the principal’s welfare, as defined by the
difference between the gross expected output, $EO$, and the expected contract-
ing cost, $EC$. Another point to take into account is that the principal will now
distinguish between the $\{L,H\}$ and $\{H,L\}$ states in paying the agents. That is,
the principal now pays the team $2t(z)$ if $\pi_1 = \pi_2 = H$, $2t(y_1)$ if $\pi_1 = H$ and $\pi_2 = L$, $2t(y_2)$ if $\pi_1 = L$ and $\pi_2 = H$, and $2t(x)$ if $\pi_1 = \pi_2 = L$. This leads us to
the following result.
Theorem 6. Given the optimal allocations of effort under a cooperative task assignment, the principal's welfare increases monotonically as \( r \) decreases.

The intuition behind this result is roughly as follows. If the principal's sole objective was to maximize expected output less the effort disutility in producing it, she would induce the agents to set \( \omega_1 = \omega_2 = \hat{\omega} \), for any \( r \). However, as \( r \) increases beyond zero, the symmetric efforts allocation scheme increasingly jeopardizes risk sharing between the cooperating agents. To satisfy their participation constraint, the principal bears an escalating expected contracting cost to implement the symmetric efforts allocation scheme. She, therefore, deviates from the first best efforts allocation scheme to permit better risk sharing. As \( r \) declines from any positive level at which the optimal scheme involves \( \omega_2 > \hat{\omega} > \omega_1 \), the agents avail of improved risk sharing even if the efforts allocation scheme is unchanged. This means that the principal can provide the same risk sharing at lower cost.

4.2.2 Competitive Task Assignment. Intuitively, the principal's gain from using PCST contracts should increase with \( r \), reaching a maximum at \( r = 1 \). In fact, we will show that a first best Nash equilibrium of \((\hat{\omega}, \hat{\omega})\) can be achieved at \( r = 1 \). Unfortunately, every other pair \((\omega, \omega)\) is also a Nash equilibrium. Another problem is coordination. Since the use of PCST contracts is common knowledge ex ante, both agents know that the welfare of each depends on what the other does. This creates an incentive for them to coordinate to implement \( \omega_i = \omega_j = 0 \). This ensures \( \pi_i = \pi_j \) with probability 1, yielding each agent an expected utility of \( R + V(\hat{\omega}) - V(0) \), which exceeds the utility from choosing \( \omega_i = \omega_j = \hat{\omega} \).

Thus, PCST contracts must destroy the viability of such coordination. Note, however, that when we talk of coordination we mean that the agents privately negotiate some deal among themselves that is not in the principal's best interest. We do not permit agents to write legally binding contracts to coordinate and choose inferior actions. Suppose that for two tasks \( i \) and \( j \), \( r \in (0, 1) \). If the principal uses a competitive task assignment, she can use the following contract to motivate the agent assigned to task \( i \). This contract makes coordination (between the agents assigned to tasks \( i \) and \( j \)) suboptimal:

\[
\phi_i(\pi_i, \pi_j) = \begin{cases} 
  t(z), & \text{if } \pi_i = \pi_j = H, \\
  t(y_1), & \text{if } \pi_i = H, \pi_j = L, \\
  t(y_2), & \text{if } \pi_i = L, \pi_j = H, \\
  t(x), & \text{if } \pi_i = \pi_j = L.
\end{cases}
\] (22)

The principal will also contractually motivate the manager to choose \( \omega = \hat{\omega} \) as part of a dominant strategy equilibrium (in which coordination is not viable). When \( r = 1 \), (22) reduces to

\[
\phi_i(\pi_i, \pi_j) = \begin{cases} 
  t(z*), & \text{if } \pi_i > \pi_j, \\
  t(y*), & \text{if } \pi_i = \pi_j, \\
  t(x*), & \text{if } \pi_i < \pi_j,
\end{cases}
\] (23)
where

\[ z^* = R + \hat{V} + \hat{V}'[\hat{q}']^{-1}, \]  
\[ y^* = R + \hat{V}, \]  
\[ x^* = R + \hat{V} - \hat{V}'[\hat{q}']^{-1}. \]  

(24a)  
(24b)  
(24c)

We now have the following result.\(^{12}\)

**Theorem 7.** Suppose there are two tasks whose outputs have a correlation factor of \( r \) between them. If the principal organizes these tasks in a competitive task assignment format and employs PCST contracts [functionally identical to (22)], expected contracting costs decrease monotonically as \( r \) increases. Moreover, when \( r = 1 \), there is a dominant strategy equilibrium with the contract described in (23), (24a)–(24c), such that the expected contracting cost equals the first best cost.

At \( r = 0 \), if competitive task assignment is used, performance comparison is useless and we have \( t(z) = Z = Y_1 = \hat{Z} \) and \( Y_2 = X = \hat{X} \). At \( r = 1 \), we have \( t(y_2) = Y_2 < Z = X = t(R + V(\omega)) < Y_1 \). For \( r \in (0,1) \), we have \( Y_2 < X \). This means that the agent's compensation when only his task has a low output is lower than when both tasks have low outputs. The main message of this theorem is that as \( r \) increases, the expected contracting costs decrease monotonically with PCST contracts.\(^{13}\)

4.2.3 Implications for the Task Assignment Decision. From Lemma 3 and Theorem 6 we know that the expected contracting costs with cooperation rest between first best and second best costs, with the exact location depending on the correlation factor. From our earlier analysis and Theorem 7, we know that the expected costs with competition start from the second best cost at \( r = 0 \) and fall to the first best cost at \( r = 1 \). Moreover, with cooperation, expected costs rise monotonically from JCC(0) at \( r = 0 \) to the second best cost at \( r = 1 \). Thus, there will exist some critical value of \( r \), say \( r_c \in (0,1) \), such that the expected contracting costs for cooperation and competition intersect. Hence, cooperation is optimal for \( r \leq r_c \), whereas competition is beneficial for \( r > r_c \).

---

12. The value of rank-order tournaments when outcomes are correlated has also been noted by Lazer and Rosen. More specifically, Theorem 6 is stronger than Proposition 6 in Mookherjee. The reason is that we have a more specific model than Mookherjee's.

13. We can establish monotonicity with performance comparison because inducing equal effort inputs in both firms is indeed optimal. From (16) we see that \( \varrho \leq r \) and \( \varrho \) is maximized when \( \omega_1 = \omega_2 \), implying \( q_1 = q_2 \). With PCST contracts, it is optimal to make \( \varrho \) as high as possible.
5. Applications

5.1 Mergers

It has been somewhat puzzling that the total gains from mergers are so large. Many theoretical attempts have been made to rationalize these gains, based on a variety of factors such as taxes, valuation discrepancies created by exogenous shocks (Gort), market power consolidation, and technological economies of scale. Because of limitations on tax loss carryovers, there are probably some tax gains from mergers. The remaining explanations, however, are unsuitable for conglomerate mergers, which are a significant portion of all mergers. We define a conglomerate merger as a consolidation of two separate companies that constitute two distinct divisions in the merged firm. What we have in mind are essentially M-form conglomerates in which divisions have imperfectly correlated prospects. This would include divisions whose businesses are completely unrelated as well as those whose businesses are partly related. We do not address mergers of firms in identical industries, such as a widget manufacturer merging with another widget manufacturer to increase market share. However, product or market extensions are not precluded, such as a widget manufacturer merging with the manufacturer of earthmoving equipment in which widgets are used.

It is well-known that the “traditional” justification for conglomerate mergers—they enable shareholders to diversify—is flawed. Personal diversification by shareholders is a more than adequate substitute for corporate diversification (see, e.g., Fama and Miller). However, our analysis of the trade-off between cooperation and competition can resurrect a variant of this diversification rationale. Suppose that a merger of two firms permits their managers to cooperate rather than compete. This cooperation takes the form of these managers “helping each other out” through effort pooling and also involves each manager’s compensation depending on the output of his division as well as that of the whole firm. It is reasonable to assume that such an arrangement

14. This was particularly true in the 1960s. See Celler-Kefauver Act: Sixteen Years of Enforcement: Antitrust Sub-committee, Committee on the Judiciary, (Washington, D.C.: 1967, p. 7, Table 5). Moreover, Salter and Weinhold find that roughly half the mergers during 1975–1978 involving more than $100 million constituted “unrelated diversification.” In the subsequent decades, however, many of these mergers failed. One reason for these failures may be that many of the conglomerate mergers in the 1960s were of the “Go-Go” variety [see, e.g., Malkiel’s account of these mergers], which should be distinguished from the M-form conglomerates that we consider in this article.

15. The fact that we define conglomerate mergers in include firms with imperfectly correlated prospects distinguishes our work from Aron’s agency model in which conglomerate mergers are sought for diversification. Aron defines a conglomerate merger rather narrowly to include only firms with uncorrelated prospects. Thus, a key difference is that firm outputs are pairwise uncorrelated in her article, whereas we examine the entire spectrum of possibilities from uncorrelated to (conditionally) perfectly correlated outputs. Consequently, performance comparison plays no role in her model and she does not obtain any of our results regarding the relationship between merger value and correlation between the earnings of the merger candidates. A second key difference is that a merger in Aron’s model does not lead to cooperation between managers as it does in our article. Thus, she does not deal with the cooperation versus competition issues that we study.
is unavailable if these firms do not merge. Our view is that transactions between independent firms are market transactions, whereas those between divisions in a merged firm are internal organization transactions. Williamson (1975) points out two important advantages of internal organization over market modes of contracting. One is that managements of divisions within the firm are more amenable to calls for cooperation. The other is that internal organization has an auditing advantage relative to interfirm organization. This is consistent with our assumption that a merger allows managers to cooperate to exchange income and mutually observe effort inputs.

We can think of the output of each firm as its residual (idiosyncratic) earning after systematic components have been filtered out. Thus, we can imagine that the shareholders have two choices: they can either keep their firms separate and use PCST contracts (competitive task assignment) or they can merge and use JCC contracts (cooperative task assignment). To the extent that contracting costs are an important factor in the merger decision, they can compare the contracting costs across these two arrangements to help determine if the merger is worthwhile. Our model would predict that firms should merge if the correlation between their idiosyncratic returns is relatively low and should not merge if this correlation is relatively high. In addition, our model implies the following testable predictions when applied to conglomerate mergers.

(i) The incentive portion of a manager’s compensation package will increase subsequent to a conglomerate merger (Corollary 1).

(ii) Firms with higher unsystematic risk are likely to gain more from conglomerate mergers (Theorem 3).

16. One reason may be to protect the confidentiality of strategic information.

17. Williamson attributes this advantage to constitutional and incentive differences which operate in favor of the internal mode. We have chosen to model this auditing advantage as the agents being able to observe each other’s efforts. An added benefit of cooperation is that agents may be able to avail of improved information sharing. Millon and Thakor show in the context of financial intermediation that cooperation can improve welfare even if the agents cannot mutually observe efforts, as long as there is sufficiently valuable information that they can share.

18. Holmstrom (1982), Ramakrishnan and Thakor (1984a), and others have argued that the systematic component of a firm’s earnings has no incentive effects and is thus irrelevant for writing incentive contracts.

19. The competitive task assignment in the absence of a merger in practice differs from that in our model because there are two principals instead of one. Thus, when they practice relative performance evaluation, neither principal internalizes the externality of her contract on the other contract. We thank the referee for pointing this out to us.

20. In many firms, executive compensation is a nontrivial fraction of total expenses and income, so that the possibility of reducing it through mergers has some significance for shareholders. For example, we have examined the financial statements of some banks and bank holding companies over two separate three-year periods and found the following results. For 1980–1982, total compensation was approximately 45 percent of income prior to the deduction of noninterest expenses, 58 percent of total noninterest expenses, and 195 percent of net income. For 1983–1987, total compensation was approximately 41 percent of income prior to the deduction of noninterest expenses, 53 percent of total noninterest expenses, and 197 percent of net income.

21. The prediction that firms with highly correlated returns should not merge assumes away possible gains due to the creation of monopoly power.
(iii) Adjusting for all other factors, the total dollar value of executive compensation on average should be lower in firms that have undergone conglomerate mergers than in comparable nonconglomerate firms (implied by Theorem 2).

(iv) Within a multidivisional firm, the compensation of a divisional manager should not depend only on his division’s performance, but should also depend on the performance of other divisions (implied by Theorem 2).

(v) In a multidivisional firm, if two divisions have (close to) perfectly (positively) correlated earnings, their managers will be motivated with performance comparison contracts (Theorem 7).

(vi) In a firm in which the manager’s compensation is based on a performance comparison contract, the manager’s compensation when only his firm does poorly is lower than when his firm as well as the “benchmark” firm do poorly. (See the discussion following Theorem 7.)

(vii) Among firms that use performance comparison contracts, the average executive compensation should be a decreasing function of the correlation between the (residual) earnings of the firm and those of the “benchmark” (Theorem 7).

There is some empirical support for our predictions. In our model, the total gains from a merger are positive. While most empirical studies have documented positive excess returns for shareholders of target firms, Bradley et al. report positive excess returns to shareholders of acquiring firms in tender offers for the decades of the 1960s and the 1970s, and negative excess returns for the 1980s. However, they find that the excess returns for acquiring firms are positive for the 1963–1984 period, and the dollar amounts of the combined wealth change for target and acquiring firms are positive for each of the subperiods, including the 1980s. Moreover, the results of You et al. indicate that even when acquiring firms earned negative excess returns on average, there is a substantial fraction of these firms that earned positive excess returns.

As for the specific predictions listed above, empirical support for (i) has been provided by Jung. His results show that conglomerate mergers are accompanied by a statistically significant increase in the incentive portion of the compensation of top management in the acquiring firm, after all other confounding factors are controlled for. Further, Gibbons and Murphy provide empirical support that performance comparison contracts are widely used. Although an earlier article by Antle and Smith did not find evidence that executive compensation depends on an industry benchmark, Gibbons and

22. Note that this prediction says that mergers are good for the shareholders. This is in contrast with Amihud and Lev’s observation that managers may merge to diversify their own employment risk at the expense of shareholders. In their model, conglomerate mergers never benefit the shareholders, in which case it is difficult to see why shareholders simply do not block such mergers. In our model, managers may gain from a merger if shareholders do not alter their contracts after the merger. But we explicitly model shareholders as rational agents who alter managerial incentive contracts after a merger, so that managers continue to earn the same (reservation) utility they did prior to the merger, and the resulting savings in contracting costs are captured by the shareholders.
Murphy report that industry performance is partially filtered out of firm performance in determining executive compensation, although this effect diminishes when market movements are also included. Prediction (vi) of our model implies a negative correlation between the executive’s pay and the performance of the benchmark. Viewing the industry as the benchmark, the Gibbons and Murphy empirical results support this prediction. The remaining predictions are yet to be tested, we believe.

5.2 Design of Incentives Within Organizations

Human resources are being viewed by organizations as being increasingly important to their success. As we noted in the Introduction, much has been written and said about the key role of cooperation and teamwork among employees in an organization’s success. Indeed, the following quote seems to exemplify the prevailing viewpoint:

> We are driving decision making down to the lowest appropriate organizational level, training supervisors and management in interpersonal skills, and creating teams and task forces to help us become leaner and more aggressive in the marketplace. (James T. Ryan, Director of Human Resources, Welch Foods, Inc.)

Our analysis points to the value of inducing cooperation among agents within an organization. It suggests that if the outputs of two tasks are highly correlated, then it may be worthwhile to induce competition among the agents assigned to those tasks. Otherwise, cooperation should be preferred.

This result of our analysis has interesting implications for strategic controls and the management of human resources within multidivisional organizations. As Williamson points out, an M-form organization may indeed serve effectively as a miniature capital market. Thus, it would not be unusual to expect both cooperative and competitive task assignments to be encountered within a sufficiently diverse and large organization of this form. To the extent that top management controls over divisions in such organizations are optimally designed to be indirect rather than elaborately intrusive, the design of task assignments to achieve congruence between top management and divisional goals will be of paramount importance. A key feature of the task assignment decision is recognition of intradivisional incentive problems as well as interdivisional incentive problems. Within each division, there are usually numerous tasks that are sufficiently variegated to warrant inducing cooperation among the agents performing these tasks. Thus, a multidivisional firm might craft its incentive systems and provide a workplace to foster cooperation within each division. To the extent that there are some divisions whose outputs are highly correlated—for technological or other synergies—it would be optimal to induce competition among them. This way cooperation may be used to align the goals of divisional subordinates to those of the divisional manager, and competition among division managers may align divisional goals with those of top management. Of course, a conglomerate
will also have divisions whose outputs exhibit low correlation, so that it will often be optimal to induce multilitered cooperation, first among divisional employees and then across divisions.

There has recently been much discussion about human resources management in Japanese firms versus that in U.S. firms. The stylized facts about Japanese firms suggest that cooperation is encouraged (see, e.g., Aoki, Lincoln and McBride). Kagono et al. conclude from their survey that information sharing and interpersonal interactions are key organizational modes in Japanese firms. Frank posits that these organizational attributes cultivate emotions that lead to cooperation. The prevailing viewpoint seems to be that there is more cooperation in Japanese firms than in U.S. firms. There are probably numerous reasons for this, including differences in the organization of production processes and in cultural factors (Itoh, 1991). However, our analysis suggests that this may also be due to differences in how basic tasks are defined in Japanese and U.S. firms. We have taken the task definition process as exogenous, but it stands to reason that if basic tasks are defined so that their outputs are not too highly correlated, cooperation, rather than competition, will be the preferred mode of task organization. Thus, the problem of designing incentive systems in a conglomerate may be inextricably linked to the primitive problem of how individual tasks are defined.

A somewhat specific application of our analysis is to the design of salesforce compensation plans. Basu et al. and Lal and Staelin have developed agency-theoretic models of salesforce compensation plans. However, they do not examine how to motivate multiple salespersons. Our analysis implies that, in some cases, it may be a worthwhile to form teams of salespersons and assign them groups of sales territories in the spirit of a cooperative task assignment. On other occasions, it may be best to induce competition among salespersons. Cooperation is more valuable the greater is the uncertainty faced by each salesperson in achieving his sales target. In either case, it would be optimal to make a salesperson’s compensation depend not only on the sales from his territory but also on the sales from other territories.

6. Concluding Remarks

We have developed a simple model in which a principal can choose to organize productive tasks in a competitive or cooperative format. The outputs from these tasks, which depend on their managers’ actions, may be correlated through exogenous factors. Using two-state distributions, we have presented a general variance–covariance structure for correlated outputs for analyzing this agency problem. With this, we are able to examine a principal’s choice between getting agents to cooperate and getting them to compete. We have then applied our analysis to conglomerate mergers and the design of incentives within organizations. These applications suggest that understanding a principal’s choice of format for the assignment of productive tasks may be useful in analyzing the architecture of economic systems within firms as well as across firms in industries.
Appendix

Proof of Lemma 1

Suppose the agents together have decided to expend effort \( \omega_1 \) in task 1 and \( \omega_2 \) in task 2, with agent 1 contributing total effort \( W_1 \) to the two tasks and agent 2 total effort \( \omega_1 + \omega_2 - W_1 \) to the two tasks.

Since the agents can monitor each other perfectly, exact effort assignments across tasks are irrelevant. Let the sharing arrangement of the fees to the agents be as follows:

<table>
<thead>
<tr>
<th>Fee to</th>
<th>Agent 1</th>
<th>Agent 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>if ( \pi_1 = \pi_2 = H )</td>
<td>( Z_1 )</td>
<td>( 2Z - Z_1 )</td>
</tr>
<tr>
<td>if ( \pi_i \neq \pi_j )</td>
<td>( Y_1 )</td>
<td>( 2Y - Y_1 )</td>
</tr>
<tr>
<td>if ( \pi_1 = \pi_2 = L )</td>
<td>( X_1 )</td>
<td>( 2X - X_1 )</td>
</tr>
</tbody>
</table>

In the optimal contract, if \( \pi_i \neq \pi_j \), the effort shares of the two agents will be independent of effort assignments. The optimal sharing contract effort allocations can be obtained by maximizing the second agent’s utility subject to a minimum utility constraint for the first agent. This will be solved for a given contract and effort allocations across tasks, \( \omega_1 \) and \( \omega_2 \). This problem is

Maximize \( q(\omega_1)q(\omega_2)U(2Z - Z_1) + q(\omega_1)[1 - q(\omega_2)]U(2Y - Y_1) + [1 - q(\omega_1)]q(\omega_2)U(2X - X_1) - V(\omega_1 + \omega_2 - W_1), \)

subject to

\( q(\omega_1)q(\omega_2)U(Z_1) + q(\omega_1)[1 - q(\omega_2)]U(Y_1) + [1 - q(\omega_1)]q(\omega_2)U(X_1) - V(W_1) \geq R. \)

Adding a multiplier \( \lambda \) for the constraint and differentiating with respect to \( W_1, Z_1, Y_1, \) and \( X_1 \), we get

\[
\frac{V'(\omega_1 + \omega_2 - W_1)}{V'(W_1)} = \frac{U'(2Z - Z_1)}{U'(Z_1)} = \frac{U'(2Y - Y_1)}{U'(Y_1)} = \frac{U'(2X - X_1)}{U'(X_1)} = \lambda.
\]

If each agent’s utility is weighted equally, the multiplier \( \lambda = 1 \), and the first-order condition is satisfied at \( W_1 = [\omega_1 + \omega_2]/2, Z_1 = Z, Y_1 = Y, \) and \( X_1 = X \). The agents of the two tasks will put in equal effort and they will share the payoffs equally under each outcome. Q.E.D.

Proof of Theorem 1

Equal payoff and effort sharing implies that the agents together will maximize each agent’s utility, which can be written as
\( J(\phi, \omega_1, \omega_2) = q(\omega_1)q(\omega_2)U(Z) + q(\omega_2)[1 - q(\omega_2)]U(Y) \\
+ [1 - q(\omega_2)]q(\omega_2)U(Y) + [1 - q(\omega_1)][1 - q(\omega_2)]U(X) \\
- V((\omega_1 + \omega_2)/2). \)

Suppose the two agents are considering a cooperatively determined change in the effort for each task from \( \hat{\omega} \), the principal’s choice. Let \( b > 0 \) parameterize this change, with \( K_1 \) and \( K_2 \) as scalars. Thus, \( \omega_1(b) = \hat{\omega} + bK_1, \omega_2(b) = \hat{\omega} + bK_2 \); so that \( \omega_1(0) = \omega_2(0) = \hat{\omega} \) and, differentiating with respect to \( b \),

\[
J_b = [K_1q'(\omega_1)q(\omega_2) + K_2q'(\omega_2)q(\omega_1)]U(Z) \\
+ [K_1q'(\omega_1)[1 - 2q(\omega_2)] + K_2q'(\omega_2)[1 - 2q(\omega_1)]]U(Y) \\
- [K_1q'(\omega_1)[1 - q(\omega_2)] + K_2q'(\omega_2)[1 - q(\omega_1)]U(X) \\
- [(K_1 + K_2)/2]V'(\omega_1 + \omega_2)/2.
\]

At \( b = 0 \), the derivative

\[
[K_1 + K_2][q'(\hat{\omega})[q(\hat{\omega})[U(Z)U(Y)] + [1 - q(\hat{\omega})][U(Y) - U(Z)]] - V'(\hat{\omega})/2
\]

should equal zero for every \( K_1 \) and \( K_2 \). This will be true only if the term multiplying \( [K_1 + K_2] \) is zero. Evaluating the second-order condition at \( b = 0 \),

\[
J_{bb}(\phi, \omega_1, \omega_2) = [K_1^2 + K_2^2]q''(\hat{\omega})[q(\hat{\omega})[U(Z) - U(Y)] + [1 - q(\hat{\omega})][U(Y) - U(Z)] + [2K_1K_2q'(\hat{\omega})][U(Z) - 2U(Y) + U(X)] - [K_1 + K_2]V''(\hat{\omega}).
\]

From earlier assumptions, \( q'(\omega) < 0 \) and \( V''(\omega) > 0 \). So the first and third terms are negative, while the second term has the sign of \( K_1K_2[U(Z) - 2U(Y) + U(X)] \). If the principal just offers \( Y = (Z + X)/2 \), then \( U(Z) - 2U(Y) + U(X) < 0 \). So if \( K_1 \) and \( K_2 \) are of the same sign, then \( J_{bb} < 0 \). Note that \( Z \geq Y \geq X \).

Suppose \( K_1 \) and \( K_2 \) are of opposite sign. If \( U(Z) - 2U(Y) + U(X) \leq 0 \), then for a given \( K_1 - K_2 \), \( J_{bb} \) is maximized if \( K_1 = -K_2 = K \). Then an upper bound for the first two terms of \( J_{bb} \) is \( 2K^2[q''(\hat{\omega})[1 - q(\hat{\omega})] + q'(\hat{\omega})^2][U(Y) - U(X)] \). So \( J_{bb} \) will be negative if \( q'(\omega)/(1 - q(\omega)) \) is nonincreasing in \( \omega \) [i.e., \( \log(1 - q(\omega)) \) is quasiconcave in \( \omega \)]. [This is satisfied for most common functional forms of \( q(\omega) \); that is, \( q(\omega) = 1 - \exp[-a\omega] \), for some \( a > 0 \), or \( q(\omega) = (a + b\omega)/(ac + bd\omega) \), for some \( a > 0 \), \( b > 0 \), \( c > d > 1 \).]

The principal’s problem, having fixed \( \hat{\omega} \), is

\[
\text{Minimize } q^2t(z) + 2q[1 - q]t(y) + [1 - q^2]t(x),
\]

\[ x,z \]
subject to

\[ \text{IR: } J(\phi, \omega, \omega) = \frac{\partial^2}{\partial z^2} + 2\frac{\partial}{\partial \eta} [1 - \eta] \eta + [1 - \eta]^2 x \geq R + V(\omega); \] (A3)

\[ \text{IC: } J_{bb}(\phi, \omega, \omega) = 0 \text{ or } \frac{\partial}{\partial z} [z - y] \eta + \{y - x\} [1 - \eta] = V'(\omega)/2, \] (A4)

\[ J_{bb}(\phi, \omega, \omega) < 0; \] (A4')

\[ \text{IS: } t(z) + t(x) - 2t(y) = 0; \] (A5)

where IS stands for "identity satisfaction," and (A4) is obtained by dividing (A1) by \([K_1 + K_2]\) and equating to zero. Because \(t'(\cdot) > 0\), (A5) tells us that \(z \geq y \geq x \text{ or } x \geq y \geq z\). But since \(q'(\cdot) > 0\) and \(V'(\cdot) > 0\), (A4) disallows \(x \geq y \geq z\). Thus, the strict convexity of \(t(\cdot)\) gives us

\[ z > y > x \quad \text{and} \quad z + x < 2y. \] (A6)

Adding Lagrange multipliers \(\mu, \eta,\) and \(\theta\) to (A3), (A4), and (A5), respectively, and differentiating with respect to \(z, y,\) and \(x\) produces

\[ t'(z) [1 - \theta(\eta)^{-2}] = \mu + \eta q'[\eta]^{-1}, \] (A7)

\[ t'(y) [1 + \theta(\eta) [1 - \eta]^{-1}] = \mu - \eta q'[2\eta - 1][2\eta(1 - \eta)]^{-1}, \] (A8)

\[ t'(x) [1 - \theta(1 - \eta)^{-2}] = \mu - \eta q'[1 - \eta]^{-1}. \] (A9)

From these first-order conditions, we can show that \(\mu > 0\) and \(\eta > 0\). This means that (A3) and (A4) hold as equalities, implying

\[ z(y) = [R + V(\omega) + \{1 - \eta]V'(\omega)/2\eta'] - [1 - \eta]y[\eta]^{-1}, \] (A10)

\[ x(y) = [R + V(\omega) - \{\eta V'(\omega)/2\eta'\} - \eta y[1 - \eta]'^{-1}, \] (A11)

which are optimal if the principal wants to induce a choice of \(\omega\). To compare this with the second best solution \((\tilde{x}, \tilde{z})\), suppose we set \(y = \tilde{y} = (\tilde{x} + \tilde{z})/2\). Then we get \(z(\tilde{y}) = \tilde{z}\) and \(x(\tilde{y}) = \tilde{x}\). This solution is infeasible because (A5) is violated. If \(y\) is increased from \(\tilde{y}\), we know from (A10) and (A11) that both \(z(y)\) and \(x(y)\) will decrease from \(\tilde{y}\) and \(\tilde{x}\), respectively. Thus, the function \(t(z) + t(x) - 2t(y)\) will decrease to zero, as required. Consequently, the best contract for each manager, \((\tilde{x}, \tilde{z})\), satisfies \(\tilde{x} < x\) and \(\tilde{z} < z\). Also, \(t(\tilde{x}) = (t(\tilde{x}) + t(\tilde{z}))/2\). Making these substitutions in (A2) gives us

\[ \tilde{q}t(\tilde{x}) + \tilde{q}[1 - \tilde{q}]t(\tilde{z}) + \tilde{q}[1 - \tilde{q}]t(\tilde{w}) + [1 - \tilde{q}]\tilde{t}(\tilde{x}) \]

\[ < \tilde{q}t(\tilde{x}) + [1 - \tilde{q}]t(\tilde{z}) \]

\[ (= \text{expected second best contracting cost}). \] Q.E.D.
Proof of Theorem 2

The principal’s problem with joint contracts mirrors that with separate contracts stipulated in (A2)–(A4), with the exception that (A5) is missing. Adding Lagrange multipliers \( \mu \) and \( \eta \) to (A3) and (A4), respectively, and differentiating with respect to the choice variables \( z, y, \) and \( x \) gives us the first-order optimality conditions that can be rearranged to obtain

\[
t'(z) - t'(y) = t'(y) - t'(x). \tag{A12}
\]

Moreover, since the optimality conditions imply \( \mu > 0 \) and \( \eta > 0 \), (A3) and (A4) hold as equalities, and in combination with (A13), they provide the optimal joint contract. It is apparent that the separate and joint contracts solutions will be identical iff

\[
t(z) - t(y) = t(y) - t(x) \Leftrightarrow t'(z) - t'(y) = t'(y) - t'(x).
\]

Easily verified is the fact that this holds only if \( t(\cdot) \) is exponential or \( U(\cdot) = \log(\cdot) \). [Note that since (A13) need hold only at the optimum, log utility is a sufficient, but not necessary, condition for equivalence.] For other preferences, joint contracts strictly dominate separate contracts because (A5) is binding.

Q.E.D.

Proof of Theorem 3

Let \( \omega^{-1}(q) \) be the inverse of \( q(\omega) \) and define \( V(\omega^{-1}(q)) = V(q) \) [i.e., \( V(q) \) is the disutility of obtaining probability \( q \)]. Also, define \( M = [H + L]/2 \) and \( H - L = 2\xi \). Thus, \( H = M + \xi \) and \( L = M - \xi \), and the variance of the output

\[
4\xi^2 q[1 - q].
\]

First consider the second best solution: For a given \( q \), the expected output, \( EO_s \), is \( q[M + \xi] + [1 - q][M - \xi] \)—that is, \( M + q[2\xi - 1] \). Also, since the contract is \( z(q) = R + V(q) + [1 - q]V'(q) \) and \( x(q) = R + V(q) - qV'(q) \), the expected contracting cost \( E\phi_s \), is \( qt(z(q)) + [1 - q] \times t(x(q)) \). And since \( r(z(q)) = [z(q)]^2 \), we have

\[
E\phi_s = [R + V(q)]^2 + [1 - q][V'(q)]^2.
\]

Let \( s(\xi) = \max_q [EO_s - E\phi_s] \) be the net expected payoff to the principal at the optimum, for a given \( \delta \). Using the Envelope Theorem, we get \( ds(\xi)/d\xi = 2q_s(\xi) \), where \( q_s(\xi) \) is the optimal probability for the given \( \xi \).

Now consider the cooperative task assignment. (A12) implies \( z - y = y - x \), and (A10) and (A11) imply \( z(q) = R + V(q) + [1 - q]V'(q), y(q) = R + V(q) + V'(q)[1 - 2q][2]^{-1}, \) and \( x(q) = R + V(q) - qV'(q) \). After some simplification, the expected contracting cost, \( E\phi_c \), can be shown to be equal to \( [R + V(q)]^2 + [V'(q)^2 q[1 - q]/2] \). The expected output \( EO_c = M + q[2\xi - 1] \). With \( c(\xi) \) defined as the net expected payoff to the principal at the optimum, we can show that \( dc(\xi)/d\xi = 2q_c(\xi) \), where \( q_c(\xi) \) is the optimal probability for the given \( \xi \). Note now that \( q_s(\xi) \) is the maximizer of \( M + q[2\xi - 1] - [R + V(q)]^2 - q[1 - q][V'(q)]^2 \) and \( q_c(\xi) \) is the maximizer of \( M + \)}
\[ q[2\xi - 1] - [R + V(q)]^2 - \{q[1 - q][V'(q)]^2/2 \}. \] Since \(q[1 - q][V'(q)]^2\) is convex, we see that \(q_\xi > q_\xi(\xi)\), for any \(\xi\). Thus, \(d[c(\xi) - s(\xi)]/d\xi > 0\), which means that the benefits from cooperation increase as the variance of output goes up. QED

Proof of Corollary 1
From the proofs of Theorems 1 and 2, we know that the optimal cooperative task assignment contract (joint and separate) is attained by increasing \(y\) from the second best value, \(\bar{y}\). But if \(z(y)\) and \(x(y)\) in (A10) and (A11), respectively, are set at \(\bar{z}\) and \(\bar{x}\), respectively, then \(\partial z(y)/\partial y = -[1 - \bar{q}][\bar{q}]^{-1}\) and \(\partial x(y)/\partial y = -\bar{q}[1 - \bar{q}]^{-1}\), with both derivatives evaluated at \(y = \bar{y}\). For \(\bar{q} > 0.5\) we have \(\bar{q}[1 - \bar{q}]^{-1} > [1 - \bar{q}][\bar{q}]^{-1}\), which means that in moving from the second best contract to the optimal cooperative task assignment contract, \(x\) reduces more than \(z\). Thus, the incentive spread, \(z - x\), widens. Q.E.D.

Proof of Theorem 4
Obvious. Q.E.D.

Details Concerning Derivation of Probabilistic Structure for Correlated Outputs
Using the relationships between the joint output probabilities given by (9)--(13), we can rewrite (14) as

\[
\tilde{C}_{12} = P_{HH}[1 - q_1][1 - q_2] - q_2[1 - q_1][q_1 - P_{HH}]
- q_1[1 - q_2][q_2 - P_{HH}] + q_1q_2[1 - q_2 - q_1 + P_{HH}].
\] (A13)

Combining (15) and (A13), we can obtain the following solution:

\[
P_{HH} = q_1q_2 + \phi \sqrt{q_1q_2[1 - q_1][1 - q_2]}.
\] (A14)

Using (A14) in conjunction with (9)--(12) and assuming \(\omega_1 \geq \omega_2\), we obtain

\[
P_{HL} = q_1[1 - q_2] - \phi \sqrt{q_1q_2[1 - q_1][1 - q_2]},
\] (A15)

\[
P_{LH} = q_2[1 - q_1] - \phi \sqrt{q_1q_2[1 - q_1][1 - q_2]},
\] (A16)

\[
P_{LL} = [1 - q_2][1 - q_1] + \phi \sqrt{q_1q_2[1 - q_1][1 - q_2]}.
\] (A17)

Thus, the joint output probabilities must satisfy (A14)--(A17) and the feasibility constraint (13). Symmetric solutions exist for \(\omega_1 \leq \omega_2\). Note that (13) restricts the range of feasible values for \(\phi\). Using (A14)--(A17) with (13) shows that the range is

\[
\{ -[\sqrt{q_1q_2/[1 - q_1][1 - q_2]} \land \sqrt{[1 - q_1][1 - q_2]/q_1q_2}],
\sqrt{q_2[1 - q_1]/q_1[1 - q_2]} \},
\] (A18)
where “\(^\wedge\)" is the "min" operator. Later we shall use "\(^\vee\)" as the "max" operator. When \(q = -\sqrt{q_1q_2/[(1 - q_1)(1 - q_2)]}\), we have \(P_{HH} = 0\); and when \(q = -\sqrt{[1 - q_1]1 - q_2]/q_1q_2}\), we have \(P_{LL} = 0\). Similarly, when \(q = \sqrt{q_2[1 - q_1]/q_1[1 - q_2]}\), we have \(P_{LH} = 0\). Now define the "correlation factor" \(r\) as

\[
r = q_1[1 - q_2] \{\sqrt{q_2[1 - q_1]}\}^{-1},
\]

(A19)

for the case in which \(\omega_1 \geq \omega_2\), and

\[
r = \sqrt{q_2[1 - q_1] \{\sqrt{q_1[1 - q_2]}\}^{-1}},
\]

(A20)

for the case in which \(\omega_1 \leq \omega_2\). Then, we can write the following general solutions for the joint output probabilities that satisfy (13), (A14)–(A17), and are applicable for \(\omega_1 \geq \omega_2\) as well as for \(\omega_1 \leq \omega_2\):

\[
P_{HH} = r[q_1 \wedge q_2] + [1 - r] q_1q_2,
\]

(A21)

\[
P_{HL} = q_1 [1 - q_2][1 - r] + r[\{q_1 - q_2\} \vee 0],
\]

(A22)

\[
P_{LH} = q_2 [1 - q_1][1 - r] + r[\{q_2 - q_1\} \vee 0],
\]

(A23)

\[
P_{LL} = [1 - r][1 - q_1]q_2 + r[1 - \{q_1 \vee q_2\}].
\]

(A24)

Using (A18) and (A19), we can see that, for \(\omega_1 \geq \omega_2\),

\[
r \in [-\{q_1[1 - q_2]^{-1} \wedge [1 - q_2]\}q_2^{-1}], 1],
\]

(A25)

and using (A18) and (A20) we have, for \(\omega_1 \leq \omega_2\),

\[
r \in [-\{q_2[1 - q_2]^{-1} \wedge [1 - q_1]q_1^{-1}], 1].
\]

(A26)

Proof of Theorem 5

Let \(\delta_1 = 1 - q_1, \delta_2 = 1 - q_2\). Assume \(q_1 \leq q_2\). Using (8) and (17)–(20), the agent's expected utility, \(J(\phi, \omega_1, \omega_2)\), is

\[
r[q_1z + \{q_2 - q_1\}y + \delta_2x] + [1 - r][q_1q_2z + \{q_1\delta_2 + q_2\delta_1\}y + \delta_1\delta_2x] - V(\omega_1 + \omega_2/2).
\]

(A27)

Let \(\omega\) be the "desired" effort allocation to each task, and \(\omega_1(b) = \omega + bK_1, \omega_2(b) = \omega + bK_2\), where \(K_1\) and \(K_2\) are constants. Substituting this in (A27) and differentiating with respect to (w.r.t.) \(b\) yields the following evaluation of \(J_b(\phi, \omega, \omega)\) at \(b = 0\):
\[ \dot{q}'K_1\{r + [1 - r]\dot{q}\\{z - y\} + [1 - r][1 - \dot{q}]\{y - x\}\] 
\[ - [(K_1 + K_2)/2]V'(\hat{\omega}) + \dot{q}'K_2\{(1 - r)\dot{q}\{z - y\}\] 
\[ + \{r + [1 - r][1 - \dot{q}\}\{y - x\}\} = 0. \quad (A28) \]

(A28) holds for any \( K_1 \) and \( K_2 \) if the following equations hold:

\[ \dot{q}'\{r + [1 - r]\dot{q}\\{z - y\} + [1 - r]\{1 - \dot{q}\}\{y - x\}\] 
\[ - V'(\hat{\omega})/2 = 0, \quad (A29) \]

\[ \dot{q}'\{(1 - r)\dot{q}\{z - y\} + \{r + [1 - r][1 - \dot{q}]\}\{y - x\}\] 
\[ - V'(\omega)/2 = 0. \quad (A30) \]

For \( r > 0 \), (A29) and (A30) jointly imply

\[ y = [x + z]/2. \quad (A31) \]

Now, substituting \( \omega_1 = \omega_2 = \hat{\omega} \) and (A31) in (21), the IR constraint, \( J(\phi, \omega, \omega) \geq R \), becomes

\[ J(\phi, \hat{\omega}, \hat{\omega}) = \dot{q}z + [1 - \dot{q}]x \geq V(\hat{\omega}) + R. \quad (A32) \]

Substituting (A31) in (A29) or (A30), we get

\[ z - y = V'(\hat{\omega})[2\dot{q}']^{-1}. \quad (A33) \]

The principal’s objective is now to minimize

\[ L(\phi, \hat{\omega}, r) = r[\dot{q}t(z) + [1 - \dot{q}]r(x)] 
\[ + [1 - r][\dot{q}^2t(x) + 2\dot{q}(1 - \dot{q})t((x + z)/2) 
\[ + [1 - \dot{q}]^2t(x)], \quad (A34) \]

subject to (A32) and (A33). Using the Envelope Theorem, we now have

\[ \frac{dL(\phi, \hat{\omega}, r)}{dr} = \dot{q}[1 - \dot{q}]\left[t(x) + t(z) - 2t\left(\frac{x + z}{2}\right)\right], \quad (A35) \]

which is strictly positive since \( t(\cdot) \) is convex. This completes our proof for \( r \in (0, 1) \). The proof for \( r = 1 \) is obvious from the discussion following Theorem 5.

Q.E.D.

Proof of Lemma 2

Without loss of generality, let \( \omega_2 \geq \omega_1 \). Let \( \hat{\omega} \) be a candidate optimal effort, so that \( \omega_1 = \omega_2 = \hat{\omega} \). Now suppose the principal wants to induce \( \omega_1 = \hat{\omega} - \gamma \) and \( \omega_2 = \hat{\omega} + \gamma \) with \( \gamma > 0 \) a scalar. Let \( q(\omega_1) = q_1 < q(\omega_2) = q_2 \) and let \( q(\hat{\omega}) = \dot{q} \). Now, the joint output probabilities, when \( r = 1 \), are \( P_{HH} = q_1, P_{HL} \)
\[ q_1 z + [q_2 - q_1] y + [1 - q_2] x - V((\omega_1 + \omega_2)/2). \]  

(A36)

The optimal choice of \( \omega_2 \) is given by

\[ y - x = V'((\omega_1 + \omega_2)/2)[2q'(\omega_2)]^{-1}, \]  

(A37)

and the optimal choice of \( \omega_1 \) is given by

\[ z - y = V'((\omega_1 + \omega_2)/2)[2q'(\omega_1)]^{-1}. \]  

(A38)

At \( \omega_1 = \omega_2 = \hat{\omega} \), we have \( y - x = z - y = V'(\hat{\omega})[q'(\hat{\omega})]^{-1} \). The IR constraint is that the expression in (A36) be equal to \( R \). Solving for \( z, y, \) and \( x \) from the IR constraint and the IC constraints (A37) and (A38), we have

\[ z(\gamma) = R + V\left(\frac{\omega_1 + \omega_2}{2}\right) + [1 - q_1]V'\left(\frac{\omega_1 + \omega_2}{2}\right)[2q_1']^{-1} \]
\[ + [1 - q_2]V'\left(\frac{\omega_1 + \omega_2}{2}\right)[2q_2']^{-1}, \]  

(A39)

\[ y(\gamma) = R + V\left(\frac{\omega_1 + \omega_2}{2}\right) - q_1V'\left(\frac{\omega_1 + \omega_2}{2}\right)[2q_1']^{-1} \]
\[ + [1 - q_2]V'\left(\frac{\omega_1 + \omega_2}{2}\right)[2q_2']^{-1}, \]  

(A40)

\[ x(\gamma) = R + V\left(\frac{\omega_1 + \omega_2}{2}\right) - q_1V'\left(\frac{\omega_1 + \omega_2}{2}\right)[2q_1']^{-1} \]
\[ - q_2V'\left(\frac{\omega_1 + \omega_2}{2}\right)[2q_2']^{-1}. \]  

(A41)

At \( \gamma = 0 \), we have

\[ z(0) = R + V(\hat{\omega}) + [1 - \hat{q}]V'(\hat{\omega})[q'(\hat{\omega})]^{-1}, \]  

(A42)

\[ y(0) = R + V(\hat{\omega}) + [1 - 2\hat{q}]V'(\hat{\omega})[2q'(\hat{\omega})]^{-1}, \]  

(A43)

\[ x(0) = R + V(\hat{\omega}) - \hat{q}V'(\hat{\omega})[q'(\hat{\omega})]^{-1}. \]  

(A44)

The expected contracting cost for the principal per agents is

\[ E\Phi(\gamma) = q(\hat{\omega} - \gamma)\mathbb{I}(z) + [q(\hat{\omega} + \gamma) - q(\hat{\omega} - \gamma)]\mathbb{I}(y) \]
\[ + [1 - q(\hat{\omega} + \gamma)]\mathbb{I}(x). \]
Thus,
\[
\frac{dE\phi(y)}{dy} \bigg|_{y=0} = -q'(\omega)t(z(0)) + 2q'(\omega)t(y(0))
- q'(\omega)t(x(0)) - q[t'(z)x_v] + [1 - q][t'(x)x_v].
\]

Note, however, that $z_\gamma = x_\gamma = 0$ at $\gamma = 0$. Thus,
\[
\frac{dE\phi(y)}{dy} \bigg|_{y=0} = -q'(\omega)[-t(z(0)) + 2t(y(0)) - t(x(0))] < 0,
\]
since $t(\cdot)$ is convex and $y(0) = [x(0) + z(0)]/2$. Thus, the principal experiences a decrease in expected contracting costs by increasing $\gamma$ beyond zero. Further, the principal’s expected gross output as a function of $\gamma$ is $EO(\phi) = [q(\omega + \gamma) + q(\omega - \gamma)][H - L] + 2L$, which means
\[
\frac{dEO(\phi)}{d\gamma} \bigg|_{\gamma=0} = [q'(\omega + \gamma) - q'(\omega - \gamma)][H - L] = 0 \text{ at } \gamma = 0.
\]
This means that the principal can reduce her expected contracting costs without affecting her expected payoff when she increases $\gamma$ beyond zero. Thus, $\omega_1 \neq \omega_2$ at an optimum.

**Q.E.D.**

**Proof of Lemma 3**

We have already proved the claim for $r = 0$. For any $r \in (0,1]$, we know from Theorem 5 that contracting costs decline monotonically as $r$ decreases, as long as $\omega_1 = \omega_2 = \omega$. Since the first best is not achieved at $r = 0$, contracting costs must be higher for $r > 0$, assuming $\omega_1 = \omega_2 = \omega$. When $\omega_1 \neq \omega_2$ is allowed, contracting costs with cooperation for $r \in [0,1]$ are lower. However, if $\omega$’s are observable ex post, the first best solution involves $\omega_1 = \omega_2 = \omega$, yielding both agents their reservation utilities. Since $\omega_1 \neq \omega_2$ in the case of cooperation with $r \in (0,1]$, the first best is not achieved.

**Q.E.D.**

**Proof of Theorem 6**

Suppose the optimal contract at correlation factor $r$ is $[z,y_1,y_2,x]$ and induces $\omega_1, \omega_2$ in the two tasks with $\omega_1 < \omega_2$. Each agent’s expected utility is
\[
J([z,y_1,y_2,x],\omega_1,\omega_2)
= [q_1q_2 + rq_1(1 - q_2)]z + [q_1(1 - q_2) - rq_1(1 - q_2)]y_1
+ [(1 - q_1)q_2 - rq_1(1 - q_2)]y_2 + [(1 - q_1)(1 - q_2)]x
+ rq_1(1 - q_2)x - V(\omega_1 + \omega_2)/2).
\]

Suppose the correlation factor decreases to $r' < r$. Let the new contract at this correlation factor of $r'$ be $z' = z$, $y_2' = y_2$, $x' = x$, and
\[ y_1' = y_1 + (r - r')/(1 - r') \{ z + x - y_1 - y_2 \}. \]

The expected utility to each agent under the new contract at \( r' \) is

\[
J([z', y_1', y_2', x'], \omega_1, \omega_2) = [q_1 q_2 + r q_1 (1 - q_2) + (r' - r) q_1 (1 - q_2)] z \\
+ [q_1 (1 - q_2) - q_1 (1 - q_2) + (r' - r) q_1 (1 - q_2)] y_1 \\
+ [(1 - q_1) q_2 - r q_1 (1 - q_2) + (r' - r) q_1 (1 - q_2)] y_2 \\
+ [(1 - q_1) (1 - q_2) + r q_1 (1 - q_2) + (r' - r) q_1 (1 - q_2)] x \\
+ q_1 (1 - q_2) (1 - r') \left( \frac{r - r'}{1 - r'} \right) [z + x - y_1 - y_2] - V \left( \frac{\omega_1 + \omega_2}{2} \right) \\
= J([z, y_1, y_2, x], \omega_1, \omega_2).
\]

So the divisional efforts allocation scheme will be identical across the two contracts. Note that the principal’s expected output, \( EO = [q_1 + q_2][H - L] + 2L \), does not depend directly on \( r \). Thus, if agents continue with the same efforts, the principal’s expected output will be the same. The contracting cost with correlation factor \( r \) is

\[
EC = [q_1 q_2 + r q_1 (1 - q_2)] t(z) + [q_1 (1 - q_2) - r q_1 (1 - q_2)] t(y_1) \\
+ [(1 - q_1) q_2 - r q_1 (1 - q_2)] t(y_2) \\
+ [(1 - q_1) (1 - q_2) + r q_1 (1 - q_2)] t(x),
\]

and with correlation factor \( r' \) is

\[
EC' = [q_1 q_2 + r q_1 (1 - q_2) + (r' - r) q_1 (1 - q_2)] t(z) \\
+ [q_1 (1 - q_2) - r' q_1 (1 - q_2) ] \\
\times t \left( y_1 + \left( \frac{r - r'}{1 - r'} \right) [z + x - y_1 - y_2] \right) \\
+ [(1 - q_1) q_2 - r q_1 (1 - q_2) + (r' - r) q_1 (1 - q_2)] t(y_2) \\
+ [(1 - q_1) (1 - q_2) + r q_1 (1 - q_2) + (r' - r) q_1 (1 - q_2)] t(x).
\]

Thus, we have

\[
EC - EC' = (r - r') q_1 (1 - q_2) [t(z) + t(x) - t(y_2)] \\
+ q_1 (1 - q_2) [(1 - r) t(y_1) - (1 - r')] \\
\times t \left( y_1 + \left( \frac{r - r'}{1 - r'} \right) [z + x - y_1 - y_2] \right) \\
= (r - r') q_1 (1 - q_2) [t(z) + t(x) - t(y_2)] \\
+ q_1 (1 - q_2) [(1 - r) t(y_1) - (1 - r')]
× \left( y_1 \left\{ \frac{1 - r}{1 - r'} \right\} + \left\{ \frac{r - r'}{1 - r'} \right\} [z + x - y_2] \right) \\
> (r - r')q_1(1 - q_2)[t(z) + t(x) - t(y_2)] \\
+ q_1(1 - q_2)[(1 - r)t(y_1) - (1 - r')] \\
× \left\{ \left\{ \frac{1 - r}{1 - r'} \right\} t(y_1) + \left\{ \frac{r - r'}{1 - r'} \right\} t(z + x - y_2) \right\} \\
= (r - r')q_1(1 - q_2)[t(z) + t(x) - t(y_2) - t(z + x - y_2)] \\
= (r - r')q_1(1 - q_2) \\
× \left\{ \left\{ \frac{z - y_2}{z - x} \right\} t(z) + \left\{ \frac{y_2 - x}{z - x} \right\} t(x) - t(z + x - y_2) \right\} \\
+ \left\{ \left\{ \frac{y_2 - x}{z - x} \right\} t(z) + \left\{ \frac{z - y_2}{z - x} \right\} t(x) - t(y_2) \right\} \\
> 0 \quad \text{(use Jensen's inequality three times).} \\
\]

So the principal’s expected contracting cost is strictly lower at $r'$ than at $r$. Since expected outputs at the two correlation factors are identical under this scheme, the principal’s welfare increases as the correlation factor $r$ decreases.

Q.E.D.

Proof of Theorem 7

Let the agent assigned to task 1 choose $\omega_1$ and the agent assigned to task 2 choose $\omega_2$ and assume initially $r \in (0,1)$. Let $J^- (\phi, \omega_1, \omega_2)$ and $J^+ (\phi, \omega_1, \omega_2)$ be the expected utilities of the agent assigned to task 1 from choosing $\omega_1 \leq \omega_2$ and $\omega_1 > \omega_2$, respectively. These utilities can be evaluated using (22) and (A21)–(A24). Defining $D_1 = \{1 - r\} [q_2[z - y_2] + \delta_2[y_1 - x]]$, we have the following derivatives w.r.t. $\omega_1$:

\[
J^-_{\omega_1}(\phi, \omega_1, \omega_2) = q'[r[z - y_2] + D_1] - V'(\omega_1) = 0, \\
J^+_{\omega_1}(\phi, \omega_1, \omega_2) = q'[r[y_1 - x] + D_1] - V'(\omega_1) = 0.
\]

To obtain a dominant strategy equilibrium involving each agent choosing $\hat{\omega}$, we must ensure that $J^-_{\omega_1}$ and $J^+_{\omega_1}$ are independent of $\omega_2$. This is possible only if

\[
z - y_2 = y_1 - x = V'(\hat{\omega})[q'(\hat{\omega})]^{-1}. \tag{A45}
\]

The IR constraint is $J(\phi, \hat{\omega}, \hat{\omega}) \geq R$, where

\[
J(\phi, \hat{\omega}, \hat{\omega}) = r[q^2z + \{1 - \hat{q}\}x] + [1 - r][q^2z + \hat{q}\{1 - \hat{q}\}]\{y_1 + y_2\} \\
+ \{1 - \hat{q}\}^2x] - V(\hat{\omega}). \tag{A46}
\]

The principal must choose $z$, $x$, $y_1$, and $y_2$ to minimize $G(\phi, \hat{\omega}, \hat{\omega})$ subject to (A37) and the IR constraint, where $G(\phi, \hat{\omega}, \hat{\omega})$ is
\[ r(q_t(z) + [1 - q]t(x)) + [1 - r][q^2t(z) + q(1 - q)[t(y_1) + t(y_2)] + [1 - q]^2t(x)]. \] (A47)

After numerous manipulations, it can be proved that the optimal solution requires \( y_1 > z > x > y_2 \). Now define \( \{\hat{z}, \hat{y}_1, \hat{y}_2, \hat{x}\} \) as the contract that induces an effort choice of \( \hat{\omega} \) when \( r = \hat{r} \). As \( r \) increases above \( \hat{r} \), suppose we continue to use the old contract optimal for \( \hat{r} \). Clearly, (A45) will continue to hold. And substituting (A45) in (A46) yields \( J(\phi, \hat{\omega}, \hat{\omega}) = \hat{q} \hat{z} + [1 - \hat{q}] \hat{x} - V(\hat{\omega}), \) which is independent of \( r \). Thus, both constraints of the minimization program are satisfied and we are assured that \( \{\hat{z}, \hat{y}_1, \hat{y}_2, \hat{x}\} \) will be feasible for \( r > \hat{r} \). Differentiating \( G \) in (A47) w.r.t. \( r \) yields \( G_r(\phi, \hat{\omega}, \hat{\omega}) = \hat{q}[1 - \hat{q}][t(\hat{z}) - t(\hat{y}_1) + t(\hat{x}) - t(\hat{y}_2)], \) which is negative because \( \hat{y}_1 - \hat{z} = \hat{x} - \hat{y}_2, \hat{y}_1 > \hat{x} \), and \( t(\cdot) \) is convex. Thus, as \( r \) increases from \( \hat{r} \), the contract optimal at \( \hat{r} \) (for a given \( \hat{\omega} \)) continues to be feasible and reduces contracting costs. Thus, the contract optimal for \( r > \hat{r} \) can do no worse. Similar steps can be repeated to prove that the contract in (23) and (24a)–(24c) yields a dominant strategy equilibrium with \( (\hat{\omega}, \hat{\omega}) \) such that \( \pi_1 = \pi_2 \) with probability 1 and the principal bears the first best contracting cost, \( t(R + V(\hat{\omega})) \). Q.E.D.

References


