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COLLATERAL AND RATIONING: SORTING EQUILIBRIA IN MONOPOLISTIC AND COMPETITIVE CREDIT MARKETS*

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1. INTRODUCTION

Lenders usually know less than borrowers about payoff-relevant borrower attributes. These attributes may be a personal characteristic as in Jaffee-Russell [1976] or some parameter of an earnings distribution as in Stiglitz-Weiss (S-W) [1981]. In either case, the informational asymmetry is likely to affect the credit market equilibrium.

The principal objective of this paper is to explore the role of market structure in credit allocation when there is such an informational asymmetry. The questions to which we seek answers are: Why do lenders sometimes ration credit even when deposit availability is relatively unconstrained? What is the economic function of collateral and how is its usefulness affected by credit market structure? What is the impact of collateral on credit rationing? Why do we observe co-signers?

These issues are analyzed under two market structures. In Section 2, we assume that a bank acts as a price-setting monopolist in the loan market. Two principal results are obtained. First, collateral will not be used unless it is sufficiently valuable to the bank to make the loan riskless. Second, in some cases, the bank's credit policy discourages high-risk borrowers from applying for credit. The bank need not explicitly reject these applicants; it simply raises the loan interest rate to induce them to exit the market.[2]

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[2] In our model, the distribution of returns to a low-risk borrower exhibits first-order stochastic dominance over the distribution of returns to a high-risk borrower. As will be seen, this implies that, as the loan interest rate increases, high-risk borrowers drop out of the market before low-risk borrowers, in contrast to the model of S-W [1981]. In that model, the distribution of returns to high-risk borrowers is a mean-preserving spread of the distribution of returns to low-risk borrowers, which implies that low-risk borrowers drop out first. Altering our analysis so that the return distribution of one borrower type is a mean-preserving spread of the other would imply that the concentration set of the return distribution would shift with the borrower's type. Consequently, a borrower's type would be observable *ex post* in the good state (assuming that the bad state return for both types is still zero). This would permit sorting with costless contingent contracts of the kind discussed in Bhattacharya [1980].

In Section 3, a perfectly competitive market is analyzed. We assume that each bank faces a perfectly elastic deposit supply schedule and that banks compete for loans as well as deposits. Competition for loans results in every borrower being offered a contract that maximizes its expected utility subject to the constraint that the bank breaks even. Here we find that collateral plays a useful role. By designing credit contracts with inversely related interest rates and collateral requirements, banks can sort borrowers into risk classes. Low-risk borrowers choose contracts with low interest rates and high collateral requirements whereas high-risk borrowers choose contracts with high interest rates and low collateral requirements. We also show that insufficient borrower wealth endowments may result in some applicants facing a nonzero fractional probability of being denied credit. Thus, equilibrium credit rationing is possible even when collateral is available and deposit supply is perfectly elastic.[3] We then demonstrate that the presence of a co-signer who increases collateral availability — thereby eliminating the possibility of rationing — always strictly improves borrower welfare. Interestingly, the equilibrium amount of collateral offered by the low-risk borrower with a co-signer is less than the amount it would offer if it simply had sufficient collateral-eligible wealth of its own to eliminate rationing.

In Section 4, we compare the welfare properties of the equilibria under monopoly and perfect competition and show that, under some circumstances, expected social welfare under monopoly exceeds that under perfect competition. Section 5 concludes. All proofs are in the Appendix.

2. CREDIT MARKET EQUILIBRIUM UNDER MONOPOLY

Consider a universally risk-neutral economy in which each investor has a known end-of-period endowment W. In addition, an initial endowment can be invested, providing a nonstochastic terminal payoff b. Alternatively, the investor may borrow \$1 from a bank, add this to the initial endowment, and invest the total in a risky project that yields R if successful and zero otherwise. The safe and risky projects are assumed to be mutually exclusive, so b represents an opportunity cost to the investor of undertaking the risky project. The real-valued positive scalars W, b, and R are common knowledge.

The probability that the risky project succeeds is δ. The bank faces a fixed pool of observationally identical borrowers consisting of two types, δ_1 and δ_2, where $0<\delta_1<\delta_2<1$. Each borrower knows its own δ, but the bank cannot distinguish among borrowers. The bank does know that a fraction γ of these borrowers are high-risk ($\delta=\delta_1$) types and that $1-\gamma$ are low-risk ($\delta=\delta_2$) types.

The bank's credit policy consists of the probability π of granting credit, the interest factor, α (one plus the loan interest rate), and the amount of collateral,

[3] Contrast this with S-W [1981], where constrained deposit supply plays a key role in engendering credit rationing, and with Bester [1985], where collateral availability eliminates rationing.

C. As in Barro [1976], we assume a disparity in collateral valuation by the borrower and the bank by defining the bank's valuation as βC, with $\beta \in [0, 1]$.[4] This disparity reflects the transactions costs the bank faces in taking possession of and liquidating collateral.

The bank's expected profit per borrower is given by

$$(1) \qquad \gamma\pi_1\{\delta_1\alpha_1+[1-\delta_1]\beta C_1-r\} + [1-\gamma]\pi_2\{\delta_2\alpha_2+[1-\delta_2]\beta C_2-r\},$$

where r represents the bank's cost of funds (one plus the deposit interest rate). We assume that the bank's deposits are elastically supplied at r. Throughout the analysis we focus on the case in which the monopolist induces the borrower to default in the unsuccessful state (i.e., $C_i<\alpha_i$).[5] Thus, the monopolist's revenue in the unsuccessful state is βC_i.

The bank's credit policy is a vector $\{\alpha_i, C_i, \pi_i\}$, $i\in\{1, 2\}$. The incremental expected utility of a borrower is the increase in the borrower's expected utility from taking a bank loan relative to the option of investing in the safe project and obtaining a sure terminal wealth of $W+b$. The incremental expected utility of a type-i borrower taking a credit policy indexed by $j\in\{1, 2\}$ is $\pi_j\{\delta_i[R-\alpha_j]-[1-\delta_i]C_j-b\}$. The revelation principle implies that the bank can restrict its attention to credit policies which induce applicants to truthfully reveal their success probabilities.[6] The bank's credit policy must, therefore, satisfy the incentive compatibility constraints

$$(2a) \qquad \pi_1\{\delta_1[R-\alpha_1] - [1-\delta_1]C_1-b\} \geq \pi_2\{\delta_1[R-\alpha_2] - [1-\delta_1]C_2-b\}.$$

$$(2b) \qquad \pi_2\{\delta_2[R-\alpha_2] - [1-\delta_2]C_2-b\} \geq \pi_1\{\delta_2[R-\alpha_1] - [1-\delta_2]C_1-b\}.$$

In addition, credit contracts must be individually rational, i.e.,

$$(3) \qquad \pi_i\{\delta_i[R-\alpha_i] - [1-\delta_i]C_i-b\} \geq 0 \qquad i\in\{1, 2\}.$$

The bank's optimization problem is to choose $\{\alpha_i, C_i, \pi_i\}$, $i\in\{1, 2\}$, to maximize

[4] Chan-Kanatas [1985] show that a disparity in the valuation of collateral is not a prerequisite for collateral to be useful; it is sufficient that the borrower and the lender are asymmetrically informed about the former's project. In fact, they establish that the amount of collateral used is an increasing function of the extent of the divergence between the borrower and the lender in their evaluation of the project.

[5] If $C_i\geq\alpha_i$, the borrower will repay the loan in both states of nature; i.e. the loan will be riskless. We assume that the borrower faces no costs in liquidating collateral at the end of the period, although liquidation may be costly at the start of the period because collateral is tied to productive activity during the period. If $\delta_1 R-b-W>0$ (a condition which hereafter will be assumed), a monopolistic bank can be shown to prefer a risky credit policy ($C_i<\alpha_i$ for $i=1, 2$) to a credit policy in which at least one type receives a riskless loan. The derivation of this sufficient condition (available on request) involves solving for the optimal incentive compatible risky credit policy (the problem characterized in this section) and comparing profits under this policy to the profits under the optimal incentive compatible policies in which (i) both types receive riskless loans, (ii) one type receives a riskless loan while the other type receives a risky loan.

[6] See, for example, Myerson [1979] or Harris-Townsend [1981].

(1) subject to (2a), (2b), (3) and

(4) $$0 \leq \pi_i \leq 1, \qquad i \in \{1, 2\}.$$

(5) $$0 \leq C_i \leq W, \qquad i \in \{1, 2\}.$$

(4) requires that π_i be a probability while (5) restricts collateral to be non-negative and no greater than the borrower's terminal endowment.

To provide a benchmark, we first state the solution to the bank's problem under full information. In this case, the bank maximizes $\pi_i\{\delta_i\alpha_i+[1-\delta_i]\beta C_i-r\}$ subject to (3)–(5). It is straightforward to verify that the solution to this problem (denoted by superscript o) is:[7]

$$\alpha_i^0 = R - b[\delta_i]^{-1}, \qquad i \in \{1, 2\}.$$

$$C_i^0 = 0, \qquad i \in \{1, 2\}.$$

$$\pi_i^0 = \begin{cases} 1 \text{ if } \delta_i R - b - r \geq 0, \\ 0 \text{ otherwise,} \end{cases} \qquad i \in \{1, 2\}.$$

Hereafter, we assume that $\delta_1 R - b - r > 0$, so that $\pi_1^0 = \pi_2^0 = 1$.

Under full information, the bank never requires a borrower to secure a loan with collateral because, with $\beta < 1$, collateral is costly. The bank can more efficiently extract borrower surplus by specifying an interest factor $R - b[\delta_i]^{-1}$ equal to the return R in the successful state less the imputed cost, $b[\delta_i]^{-1}$, of undertaking the risky project. Note that low-risk borrowers pay a higher interest rate than high-risk borrowers because, under the full information optimum, the monopolist extracts all borrower surplus, and low-risk borrowers have a greater surplus than high-risk borrowers.

We now state the bank's optimal policy under asymmetric information.

PROPOSITION 1. *Under asymmetric information about borrower types, the monopoly bank's optimal credit policy (denoted by asterisks) is given by:*

$$\alpha_i^* = \alpha^* = \pi_1^*\{R - b[\delta_1]^{-1}\} + [1 - \pi_1^*]\{R - b[\delta_2]^{-1}\}, \qquad i \in \{1, 2\}.$$

$$C_i^* = 0, \qquad i \in \{1, 2\}.$$

$$\pi_1^* = \begin{cases} 1 \text{ if } \delta_1 R - b - r \geq b[\delta_2 - \delta_1]\delta_1^{-1}[1-\gamma]\gamma^{-1}. \\ 0 \text{ otherwise.} \end{cases}$$

$$\pi_2^* = 1.$$

The bank's optimal policy can be described by two cases. If expected social surplus for a high-risk borrower is sufficiently large ($\delta_1 R - b - r \geq b[\delta_2 - \delta_1]\cdot$

[7] The sufficient condition $\delta_1 R - b - W > 0$ that guarantees the optimality of risky loans under asymmetric information also guarantees the optimality of risky loans under full information. Thus, the policy described below dominates any riskless credit policy the monopolist could implement.

$\delta_1^{-1}[1-\gamma]\gamma^{-1}$), both types receive loans at the interest factor $R-b[\delta_1]^{-1}$, the full information optimal interest factor for a high-risk borrower. Neither borrower puts up collateral. The difference between this solution and the full information solution is that a low-risk borrower pays a lower interest rate. This case entails no deadweight loss because the reduction in the interest rate to the low-risk borrower constitutes a pure transfer between that borrower and the monopolist.

If expected social surplus, $\delta_1 R-b-r$, for a high-risk borrower is less than $b[\delta_2-\delta_1]\delta_1^{-1}[1-\gamma]\gamma^{-1}$, the monopolist prices such borrowers out of the market by quoting an interest factor equal to the full information optimal interest factor for a low-risk borrower. Such price rationing is inefficient (relative to the first best) because positive social surplus would be generated if loans were granted to the high-risk borrowers. This surplus cannot be realized, however, because the monopolist cannot distinguish between borrower types.

A noteworthy feature of the optimal credit policy is that the collateral of each borrower type is zero. As we have seen, collateral is an inefficient tool for extracting borrower surplus. However, one might think that collateral would be an efficient sorting device. Why not sort borrowers by offering lower interest rates to borrowers that provide more collateral? The reason this is not optimal is that if the bank, starting from the full information solution, simultaneously raises the collateral C_1 and lowers the interest rate α_1 offered to high-risk borrowers, the low-risk types would have even more of an incentive to choose the high-risk contract instead of the low-risk contract. The low-risk borrowers really like the lower interest rate α_1 but are not as concerned about the higher collateral requirement because their probability of failure is low. Thus, collateral is not an efficient sorting device and is optimally set equal to zero.[8]

3. EQUILIBRIUM IN A PERFECTLY COMPETITIVE CREDIT MARKET

3.*A*. *Characterization of the Equilibrium.* In a perfectly competitive credit market, banks announce credit contracts and compete *ex ante* on the terms of these contracts. All entry occurs *ex ante.* Once banks commit to contracts, borrowers apply for loans under the terms announced. For simplicity we assume that a borrower can apply to only one bank during the period under consideration. We focus exclusively on Nash equilibria. A Nash equilibrium is a set of credit contracts such that each contract earns non-negative profits for the bank, and there exists no other set of contracts which, when offered in addition to that set, earns positive profits in the aggregate and non-negative profits individually.

Under full information, the Nash equilibrium credit policy maximizes a borrower's expected utility subject to the constraint that the bank earns zero profits on that borrower. It is straightforward to verify that the full information

[8] This result obtains even when borrowers' success probabilities lie in a continuum, $[\delta_1, \delta_2]\subseteq [0, 1]$. We have formally analyzed the case of a continuum and found that the results are essentially unchanged.

competitive equilibrium is given by:[9]

$$\alpha_i^0 = r\delta_i^{-1},\ C_i^0 = 0,\ \pi_i^0 = 1.$$

The full information equilibrium collateral, C_i^0, and probability, π_i^0, are identical under perfect competition and monopoly. The interest rates differ in the two regimes, reflecting the fact that under perfect competition the borrower receives the entire expected social surplus $\delta_i R - b - r$, while under monopoly the bank receives this surplus.

Under asymmetric information, Nash equilibria are never pooling (see Rothschild-Stiglitz [1976] and Wilson [1977]). Thus, in a competitive equilibrium (if such an equilibrium exists) two distinct credit contracts will be offered by banks.[10] This equilibrium pair of credit contracts is incentive compatible. The equilibrium credit policy $\{\alpha_i^*, C_i^*, \pi_i^*\}$, $i \in \{1, 2\}$ solves Maximize $\gamma\pi_1\{\delta_1[R-\alpha_1]-[1-\delta_1]C_1-b\}+[1-\gamma]\pi_2\{\delta_2[R-\alpha_2]-[1-\delta_2]C_2-b\}$.
Subject to:

$$\text{(6a)} \qquad \pi_1\{\delta_1[R-\alpha_1] - [1-\delta_1]C_1 - b\} \geq \pi_2\{\delta_1[R-\alpha_2] - [1-\delta_1]C_2 - b\}.$$

$$\text{(6b)} \qquad \pi_2\{\delta_2[R-\alpha_2] - [1-\delta_2]C_2 - b\} \geq \pi_1\{\delta_2[R-\alpha_1] - [1-\delta_2]C_1 - b\}.$$

$$\text{(7)} \qquad 0 \leq \pi_i \leq 1, \qquad i \in \{1, 2\}.$$

$$\text{(8)} \qquad 0 \leq C_i \leq W, \qquad i \in \{1, 2\}.$$

$$\text{(9)} \qquad \delta_i\alpha_i + [1-\delta_i]\beta C_i = r, \qquad i \in \{1, 2\}.$$

We label this maximization problem as Ω.

As can be easily verified, the full information solution is not incentive compatible under asymmetric information. A high-risk borrower would covet the contract designed for the low-risk type. Thus, a new equilibrium must be found. In deriving this equilibrium, we consider two cases, $C_2^* < W$ and $C_2^* = W$.

Case 1: *W does not impose a binding constraint on collateral.* We show in the next proposition that when the borrower's terminal endowment exceeds C_2^*, the equilibrium credit policy involves no rationing.

PROPOSITION 2. *If*

$$\text{(10)} \qquad W \geq r[\delta_2 - \delta_1]\{\delta_2[1-\delta_1] - \beta\delta_1[1-\delta_2]\}^{-1},$$

then the Nash equilibrium under asymmetric information (if it exists) is given

[9] If $W \geq r$, there is another equilibrium in which $\alpha_i = C_i = r$, i.e., loans are riskless. To rule out this uninteresting equilibrium, we assume $W < r$. To verify that this condition eliminates riskless equilibria under either full or asymmetric information, suppose that $W < r$ but that the equilibrium credit contract for type i is riskless. Zero profits imply $\alpha_i = r$. In order for a borrower not to default we must have $C_i \geq r$. But feasibility calls for $C_i \leq W$ which implies $W \geq r$, a contradiction.

[10] The existence of a Nash equilibrium is discussed in footnote 12.

by:[11]

$$\alpha_1^* = r[\delta_1]^{-1},\ \alpha_2^* = r[\delta_2]^{-1} - [1-\delta_2]\beta C_2^* \delta_2^{-1}.$$

$$C_1^* = 0,\ C_2^* = r[\delta_2-\delta_1]\{\delta_2[1-\delta_1] - \beta\delta_1[1-\delta_2]\}^{-1}.$$

$$\pi_i^* = 1, \qquad i \in \{1, 2\}.$$

Proposition 2 indicates that, in contrast to monopoly, the competitive equilibrium involves collateral as a sorting device. Borrowers with a low success probability pay a higher interest rate, but post no collateral. Those with a high success probability provide collateral but pay a lower interest rate. As in Bester [1985], collateral requirements are inversely related to interest rates and a sorting equilibrium is attainable without credit rationing. Thus, when terminal wealth endowments are sufficiently large, credit rationing is eliminated.[12,13]

At this point it is useful to discuss why collateral is used under perfect competition but not under monopoly. Because the monopolist would like to charge higher interest rates to borrowers with a higher willingness to pay for a loan, the monopolist's incentive problem is to deter low-risk (high willingness to pay) borrowers from claiming to be high-risk (low willingness to pay) borrowers. This incentive can be counteracted by making the low-risk contract more favorable or the high-risk contract less favorable to the low-risk types. However, increases in collateral always harm low-risk borrowers less than high-risk borrowers, and collateral is, therefore, not a useful sorting device under monopoly. By contrast, under perfect competition, interest rates must just cover loan costs. This means that high-risk borrowers must pay higher interest rates than low-risk borrowers and must, therefore, be deterred from choosing the contract designed for low-risk types. This can be accomplished by requiring a positive collateral as part of the low-risk contract. Collateral in this case sorts effectively because it is more onerous to high-risk borrowers than to low-risk borrowers.

[11] Note that as in other Nash equilibria under asymmetric information (e.g., Wilson [1977] or Spence [1978]) the cross-sectional distribution of types, γ, does not affect the equilibrium contracts.

[12] The requirement $r[\delta_2-\delta_1]\{\delta_2[1-\delta_1]-\beta\delta_1[1-\delta_2]\}^{-1} \leq W$ ensures that an equilibrium with collateral does not involve rationing. It is not inconsistent with the earlier assumption stated in footnote 8 that $W<r$, since $r[\delta_2-\delta_1]\{\delta_2[1-\delta_1]-\beta\delta_1[1-\delta_2]\}^{-1}<r$.

[13] As is well known, a Nash equilibrium may not exist under asymmetric information (see Rothschild-Stiglitz [1976] and Wilson [1977]). The problem is that a pooling contract might Pareto dominate the separating contracts. A necessary and sufficient condition for the existence of a stable separating Nash equilibrium is

$$\text{(F.1)} \qquad \delta_2[\bar{\delta}]^{-1} \geq 1 + [1-\delta_2][1-\beta][\delta_2-\delta_1]\{\delta_2[1-\delta_1] - \beta\delta_1[1-\delta_2]\}^{-1},$$

where $\bar{\delta} \equiv \gamma\delta_1 + [1-\gamma]\delta_2$. This condition can be shown to hold whenever δ_2 is sufficiently large relative to δ_1. Even if this condition does not hold, the solution described in Proposition 2 represents a Riley [1979] reactive equilibrium. On the other hand, if (F.1) does not hold, the Wilson anticipatory equilibrium is a pooling contract with no collateral and an interest factor $r\bar{\delta}^{-1}$. For a detailed discussion of the existence issue and alternative equilibrium concepts, see Besanko-Thakor [1985].

Case 2: W imposes binding constraint on collateral.

If (10) does not hold, the credit policy described in Proposition 2 will be infeasible. The optimal solution to Ω thus involves $C_2 = W$. In this case, banks respond by rationing credit in equilibrium.

PROPOSITION 3. *When the borrower's terminal endowment imposes a binding constraint on collateral, then (subject to a parametric restriction on b*[14]*) the Nash equilibrium under asymmetric information (if it exists) is given by*

$$\hat{\alpha}_1^* = r\delta_1^{-1},\ \hat{\alpha}_2^* = [r-[1-\delta_2]\beta W]\delta_2^{-1}.$$

$$\hat{C}_1^* = 0,\ \hat{C}_2^* = W.$$

$$\hat{\pi}_1^* = 1,\ \hat{\pi}_2^* = \{\delta_1[R-\hat{\alpha}_1^*]-b\}\{\delta_1[R-\hat{\alpha}_2^*] - [1-\delta_1]W-b\}^{-1}.$$

(The use of a hat in conjunction with an asterisk denotes the equilibrium when the collateral constraint is binding).

Because the collateral needed for self-selection exceeds W, a collateral requirement of W is insufficient to deter high-risk borrowers from choosing the low-risk contract. The bank responds to this incentive compatibility problem by reducing the probability of extending credit to a low-risk borrower, thereby randomizing its credit policy. The low-risk contract is still acceptable a low-risk borrower because the interest rate is low. This lower interest rate is of lesser value to a high-risk borrower because the probability of actually paying it is lower. Thus, high-risk borrowers are coaxed away from the low-risk contract.[15]

With $0 < \hat{\pi}_2^* < 1$, rationing exists even with collateral. Each low-risk borrower faces some likelihood of being explicitly denied credit even though the bank's supply of loanable funds is unconstrained. It is important to note that this result does not imply that, in competitive credit markets, banks will prefer to ration lower risk borrowers. If the bank can perfectly sort borrowers into distinct risk classes based on *observable* differences alone, then there would not be any rationing. A more likely situation is one in which observable borrower characteristics permit borrowers to be grouped into *coarse* risk classes. Each group may contain

[14] This restriction is

$$br[\delta_2-\delta_1]\{\delta_2[\delta_1 R-r]-b\delta_1[1-\delta_2]+\beta\delta_2]\}^{-1} \leq W$$

which requires in essence that b is small relative to W. If this restriction does not hold, and if the inequality in (10) does not hold, then one can show that the solution to Ω involves $\pi_1 = \pi_2 = 0$. Note that for any set of values of δ_1, δ_2, R, r, β and W, this restriction is always satisfied in a neighborhood of $b=0$. Also note that this restriction is not inconsistent with the condition $W \leq r[\delta_2-\delta_1]\{\delta_2[1-\delta_1]-\beta\delta_1[1-\delta_2]\}^{-1}$ that gives rise to Case 2.

[15] Again, the question of existence arises. One can show that if

$$\{\bar{\delta}[\delta_1 R-r-b]\}\{\delta_1[\bar{\delta}R-r-b\bar{\delta}\delta_2^{-1}]\}^{-1} > \{\Delta-[1-\delta_1]\delta_2\delta_1^{-1}W\}\{\Delta-W[1-\delta_2]\}^{-1}$$

where $\Delta \equiv \delta_2 R-r+\beta W[1-\delta_2]-b$, then
the separating allocation of Proposition 3 will strictly dominate any pooling allocation, thus guaranteeing existence.

borrowers of two or more types, with further *ex ante* sorting frustrated by the lack of sufficiently many observable differences. In this case, our analysis indicates that, *within* a given undifferentiated group, rationing —when it occurs— will involve the less risky borrowers. Our results are consonant with the observation that the rationed borrowers within a group *may* have higher risk than borrowers in some other (observationally distinguishable) group who are not rationed. Whether there is rationing within a group is driven by the degree of heterogeneity within the group. Our analysis indicates that rationing is more likely when there is greater *intra-group* heterogeneity.

We have assumed borrowers' wealth endowments are identical. If, as in the S – W [1981] section on collateral, W varied across borrowers unsystematically (so that a borrower's W conveyed no information about its type), then we would find that, among the low-risk borrowers, those who get rationed are the less wealthy borrowers.

3.*B*. *An Economic Rationale for a Co-Signer*. Since credit rationing results from the low-risk borrower being unable to post enough collateral, we now examine whether the borrower can be made better off by a co-signer who increases the collateral available. We assume the co-signer knows the borrower better than the bank does. If this were not true, a co-signer could not benefit the borrower.

Suppose the condition in Proposition 3 hold, so that $\hat{\pi}_2^* < 1$. To simplify notation, let $b = 0$. Suppose the co-signer loans the borrower collateral-eligible assets worth S, which are offered by the borrower to the bank as additional collateral. The co-signer is assumed to know the borrower's δ, which is an extreme characterization of the stipulation that the co-signer is better informed than the bank about the borrower's project.

Let $\{\alpha_2^c, W+S\}$ designate the contract designed by the bank for the low-risk borrower when the borrower can avail of a co-signer. It is assumed that S is large enough to eliminate rationing. Suppose the co-signer's loan of S to the borrower is at an interest factor $\zeta > 1$.[16] The co-signer will be valuable if the sum of the expected gain to the borrower from having a co-signer and the expected profit of the co-signer is positive.

In an equilibrium with a co-signer, the high-risk contract is the same as in Propositions 2 and 3, and incentive compatibility is assured if the high-risk borrower is just indifferent between that contract and using a co-signer to take the low-risk contract i.e.,

$$\delta_1 R - r = \{\delta_1[R - \alpha_2^c] - [1 - \delta_1]W - \delta_1 \zeta S\} + \delta_1 \zeta S - rS. \tag{11}$$

Note that the term in the braces in (11) is the expected utility of the high-risk borrower from mimicking and the rest of the terms on the right side of (11) represent the

[16] We do not specify the precise value of ζ because we do not wish to constrain the sharing rule between the borrower and the co-signer.

co-signer's expected profit.[17] The bank's zero profit condition on the low-risk contract is

(12) $$\delta_2\alpha_2^c + [1-\delta_2]\beta[W+S] = r.$$

Combining (11) and (12) yields

(13) $$S = \theta[C_2^* - W]$$

where $\theta \equiv \{[1-\delta_1]\delta_2 - \delta_1[1-\delta_2]\beta\}\{r\delta_2 - \delta_1[1-\delta_2]\beta\}^{-1} < 1$, and C_2^* is defined in Proposition 2. Thus, the total collateral required by the bank from the low-risk borrower is

(14) $$S + W = \theta C_2^* + [1-\theta]W < C_2^*.$$

Interestingly, therefore, to eliminate the likelihood of being rationed, the low-risk borrower must post more collateral when its own wealth is large enough to fully accomodate the bank's collateral requirement than it must when collateral replenishment by a co-signer is necessary.

We can take the expressions for C_2^* and $\hat{\pi}_2^*$ from Proposition 2 and 3 respectively, and rearrange to obtain

(15) $$1 - \hat{\pi}_2^* = A[C_2^* - W][\delta_1 R - r\delta_1\delta_2^{-1} - AW],$$

where $A \equiv [1-\delta_1] - \delta_1\delta_2^{-1}[1-\delta_2]\beta$.

Now the sum of the low-risk borrower's expected utility and the co-signer's expected profit is

(16) $$\delta_2[R - \alpha_2^c] - [1-\delta_2]W - rS,$$

whereas the low-risk borrower's expected utility without a co-signer is

(17) $$\hat{\pi}_2^*[\delta_2\{R - \alpha_2^*\} - \{1-\delta_2\}W].$$

The difference between (16) and (17), denoted by B, is the net gain from employing a co-signer. Using the expressions for α_2^c from (12), S from (13), $1-\hat{\pi}_2^*$ from (15), and $\hat{\alpha}_2^*$ from Proposition 3, we can verify that $B > 0$ always. This entire discussion is summarized as

PROPOSITION 4. *An equilibrium involving a co-signer strictly Pareto dominates an equilibrium not involving a co-signer when the latter equilibrium entails rationing. In equilibrium, only the low-risk borrowers use co-signers and the collateral offered to the bank by these borrower lies strictly between the collateral levels in the rationing and no-rationing equilibria without co-singers.*

4. WELFARE ANALYSIS OF MONOPOLY AND PERFECT COMPETITION

In this section we explore the comparative welfare properties of the monopoly

[17] We could as well specify the incentive compatibility condition in terms of just the borrower's expected utility if all of the surplus created by the co-signer is assumed to accrue to the borrower. The important thing is that the *total* surplus must be captured on the right side of (11).

and competitive equilibria. For this purpose, expected social welfare, E, is defined as the sum of the expectations (across states and types) of bank profit and borrower utility, i.e.,

$$E = \gamma\pi_1\{\delta_1 R - [1-\delta_1][1-\beta]C_1 - b - r\} + [1-\gamma]\pi_2\{\delta_2 R - [1-\delta_2][1-\beta]C_2 - b - r\}.$$

Under full information, because the monopolist perfectly discriminates between types, both the monopoly and competitive equilibria yield the first-best expected welfare, E^0. Under asymmetric information, expected welfare under monopoly and perfect competition will in general differ. Perfect competition always involves a welfare loss induced by asymmetric information. To see this, consider the case in which $C_2^* < W$. The equilibrium expected welfare, E_C^*, is

$$(18) \quad E_C^* = E^0 - [1-\gamma][1-\delta_2][1-\beta]r[\delta_2-\delta_1]\{\delta_2[1-\delta_1] - \beta\delta_1[1-\delta_2]\}^{-1}.$$

As shown previously, however, the monopoly equilibrium under asymmetric information will achieve the first-best welfare if both borrower types receive loans. This occurs when

$$(19) \quad \delta_1 R - b - r \geq b[\delta_2 - \delta_1]\delta_1^{-1}[1-\gamma]\gamma^{-1}.$$

When (19) is satisfied, the monopoly equilibrium yields a larger expected welfare than the perfectly competitive equilibrium. Expression (19) is more likely to hold when: (i) the return R in the good state is large, (ii) the borrower's opportunity cost b is small, and (iii) the proportion, γ, of high risks is large.

When (19) is not satisfied, asymmetric information entails a welfare loss under monopoly too. The difference, $E_M^* - E_C^*$, between the welfare under monopoly and the welfare under perfect competition is given by

$$(20) \quad E_M^* - E_C^* = -\gamma[\delta_1 R - b - r] + [1-\gamma][1-\delta_2][1-\beta]r[\delta_2-\delta_1]\{\delta_2[1-\delta_1] - \beta\delta_1[1-\delta_2]\}^{-1}.$$

In general, the sign of the right side of (20) is ambiguous. For appropriately chosen parameter values, however, this sign will be positive,[18] indicating that, with asymmetric information, the deadweight loss under perfect competition could exceed that under monopoly.

5. CONCLUDING REMARKS

Since credit rationing has important implications for the functioning of credit markets and the conduct of monetary policy (see Blinder-Stiglitz [1983] and Smith [1983]), it is not surprising that the credit rationing literature is extensive (see Baltensperger [1978], Cukierman [1978], Freimer-Gordon [1965], Fried-Howitt

[18] For example, if $\delta_1 = .1$, $\delta_2 = .9$, $\gamma = .05$, $b = .5$, $R = 20$, $r = 1.10$, and $\beta = .7$, (19) will not hold, but the right side of (20) will be positive.

[1980], Jaffee [1971], Jaffee-Modigliani [1969], Koskela [1983], Samuelson [1952], and Thakor-Callaway [1983]). The papers by Jaffee-Russell (J-R) [1976], Stiglitz-Weiss (S-W) [1981, 1983], and Bester [1985] are of special relevance to us.

In J-R, "pathologically" honest borrowers cannot be distinguished from economically rational borrowers. Rationing emerges because restricted loan sizes, resulting in excess demand, induce a lower fraction of defaults. This approach has two problems. First, a (stable) market equilibrium does not exist. Second, the rationing that occurs is likely to be transitory since default is an *ex post* choice of the borrower and reputations will therefore develop. By contrast, the rationing described here could persist even if the bank learns about its customers, provided only that some payoff-relevant information remains unavailable to the bank. The similarity between the J-R analysis and ours is that a competitive market is viewed as one involving unrestricted competition for borrowers by banks that obtain elastically supplied deposits. A difference is that, in J-R, the loan size is a choice variable. This leads to a type of rationing that we have not analyzed, one involving the bank lending a lower amount to the borrower than requested.

In S-W [1981], rationing occurs because a bank's expected profit decreases in the quoted loan interest rate beyond some point due to adverse selection and incentive effects. Thus, the interest rate at which demand equals supply may not be that at which the bank's expected profit is maximized. There are three basic differences between S-W's [1981] work and ours. First, since the success probability is not a matter of borrower choice in our models, prices have no incentive effects. We believe that asymmetric information is more fundamental in bank lending than the incentive effects of loan interest rates.[19] Second, S-W [1981] assume a constraint on the bank's supply of loanable funds, which may compel loan applicant rejection at the equilibrium interest rate. We establish the possibility of rationing without such a constraint. And third, as discussed in footnote 2, a key modeling difference between S-W's work and ours is that they distinguish between borrower types through a mean-preserving spread, whereas we distinguish between them through a first-order stochastic dominance ordering.

Our results depend on the model feature that risk is described by a single parameter and that this parameter is revealed (under competition) by the borrower's collateral choice. In reality, such precise risk identification may not be possible because the number of risk dimensions may exceed the number of credit instruments available to lenders. S-W [1981] make the general claim that rationing should be expected whenever unknown borrower attributes outnumber available credit instruments. We show that rationing is possible even when the number of sorting instruments exceeds the number of borrower attributes. In light of the

[19] Term loans, for instance are frequently tied to the purchase of specific capital equipment and the acquisition of such equipment is costlessly observable to the bank *ex post*. Loans to finance working capital needs can also be easily monitored because working capital is readily observable *ex post*. Similar reasoning applies to consumer lending, automobile loans and home mortgage loans.

S-W observation, therefore, there may be more rationing in reality than is indicated by our analysis.

S-W [1983] examine the incentive effects of terminations in a multiperiod framework and establish conditions under which a bank would deny credit to a borrower with a history of default.[20] The idea is that the threat of rationing discourages default, avoiding some of the negative incentive and sorting effects that alternative measures such as an interest rate increase might elicit. This insight is loosely related to the explanation for rationing in our competitive model. For type-δ_2 borrowers, credit granting may have to be randomized to ensure that type-δ_1 borrowers do not covet the contract designed for type-δ_2 borrowers. Thus, credit rationing becomes a threat strategy to ensure incentive compatibility.

The work most closely related to ours is Bester's [1985]. Like us, Bester assumes that collateral is costly to use[21] and characterizes a perfectly sorting competitive equilibrium with collateral, although in his model higher borrower risk implies a mean-preserving spread of the return distribution. The pivotal modeling difference between Bester's analysis and ours is that Bester assumes sufficient collateral is always available to achieve perfect sorting and thus does not model the rationing probability as a meaningful sorting instrument. This leads Bester to a conclusion contradictory to ours, namely that there is never any equilibrium credit rationing under competition. Our analysis has shown that, when the collateral constraint is binding, lenders cannot sort borrowers out based on their collateral choices alone. We have demonstrated that the threat of rationing is an effective sorting device in this case. Because the bank in Bester's model faces no collateral constraints, it can always sort without rationing.

To conclude, our analysis describes the partially attenuating influence of collateral on credit rationing, suggesting research that links rationing to specific financial contracting practices. For example, rationing encourages the development of forward lending markets which, in turn, mitigate the cost of rationing. Ultimately, of course, the aim is to explain the design and workings of rental markets, among which financial markets are a special case.

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[20] Another multiperiod credit market analysis appears in Spatt [1983] where borrowers' payoffs are intertemporally correlated in a two period model and the borrower's endogenous default decision signals its future return distribution. In equilibrium, those with good first period realizations signal their future creditworthiness by repaying whereas those with poor realizations default despite adequate liquidity.

[21] Although our analysis assumes $\beta<1$, we have also considered the case of $\beta=1$. This changes nothing under monopoly. Under perfect competition, $\beta=1$ leads to multiple equilibria with full information. With asymmetric information, the set of equilibria shrinks, although multiple Nash equilibria are possible. Further, (i) collateral is always used in equilibrium, (ii) the equilibrium involves rationing when the collateral constraint is binding and (iii) a Nash equilibrium always exists. Thus, the essence of our results is uninfluenced by the precise value β takes in [0, 1]. See Besanko-Thakor [1985] for a detailed discussion.

APPENDIX

PROOF OF PROPOSITION 1. In solving the monopolist's problem, we initially ignore constaints (2a) and (3) for $i=2$. In this "less-constrained" problem we first treat the π_i as parameters and optimize with respect to α_i and C_i. We then optimize with respect to π_i. Having solved the "less-constrained" problem, we show that this solution indeed satisfies (2a) and (3) for $i=2$.

The Lagrangian for the "less-constrained" problem is:

$$\begin{aligned} L = {} & \gamma\pi_1\{\delta_1\alpha_1+[1-\delta_1]\beta C_1-r\} + [1-\gamma]\pi_2\{\delta_2\alpha_2+[1-\delta_2]\beta C_2-r\} \\ & + \mu\{\pi_2\{\delta_2[R-\alpha_2]-[1-\delta_2]C_2-b\}-\pi_1\{\delta_2[R-\alpha_1]-[1-\delta_2]C_1-b\} \\ & + \lambda\{\pi_1\{\delta_1[R-\alpha_1]-[1-\delta_1]C_1-b\}\} + \tau_1 C_1 + \tau_2 C_2, \end{aligned}$$

where μ is the Lagrange multiplier for (2b), λ is the multiplier for (3) when $i=1$, and τ_i are the multipliers for the non-negativity constaints on C_i.

The first-order condition yield the following implications:

(A.1) $$\partial L/\partial\alpha_2 = 0 \Longrightarrow \mu = [1-\gamma].$$

(A.2) $$\partial L/\partial C_2 = 0 \Longrightarrow \tau_2 = \pi_2[1-\gamma][1-\delta_2][1-\beta].$$

(A.3) $$\partial L/\partial\alpha_1 = 0 \Longrightarrow \lambda = \gamma + [1-\gamma]\delta_2\delta_1^{-1}.$$

(A.4) $$\begin{aligned} \partial L/\partial C_1 = 0 \Longrightarrow \tau_1 = {} & \pi_1\{\gamma[1-\delta_1][1-\beta] \\ & + [1-\gamma]\{[1-\delta_1]\delta_2\delta_1^{-1}-[1-\delta_2]\}\}. \end{aligned}$$

Conditions (A.2) and (A.4) imply that $\tau_i>0$ when $\pi_i>0$. Thus, when $\pi_i>0$, $C_i^*=0$ for $i\in\{1, 2\}$. (When $\pi_i=0$, the corresponding values of C_i and α_i are not determined by the first-order conditions and can be set to whatever values are necessary to choke off demand by type-i borrowers). Condition (A.3) implies that (3) is binding for $i=1$, so

$$\alpha_1^* = R - b\delta_1^{-1},$$

whenever $\pi_1>0$. Condition (A.4) implies that (2b) is binding, so

(A.5) $$\alpha_2^* = R - b\delta_2^{-1} + \pi_1\pi_2^{-1}b[\delta_2^{-1}-\delta_1^{-1}].$$

Substituting these optimal values into the bank's objective function gives bank profit Z as a function of π_1 and π_2

$$Z = \gamma\pi_1\{\delta_1 R-b-r-b[\delta_2-\delta_1]\delta_1^{-1}[1-\gamma]\gamma^{-1}\} + [1-\gamma]\pi_2\{\delta_2 R-b-r\}.$$

Differentiating with respect to π_1 and π_2 yields

$$\partial Z/\partial\pi_2 = [1-\gamma]\{\delta_2 R-b-r\} > 0,$$

which implies $\pi_2^*=1$.

$$\partial Z/\partial\pi_1 = \gamma\{\delta_1 R-b-r-b[\delta_2-\delta_1]\delta_1^{-1}[1-\gamma]\gamma^{-1}\},$$

which implies

$$\pi_1^* = \begin{cases} 1 \text{ if } \delta_1 R - b - r \geq b[\delta_2 - \delta_1]\delta_1^{-1}[1-\gamma]\gamma^{-1}. \\ 0 \text{ otherwise.} \end{cases}$$

Substituting $\pi_2^* = 1$ into (A.5) yields the interest factors α_i^* that solve the "less-constrained" problem

$$\alpha_i^* = \pi_1^*\{R - b\delta_1^{-1}\} + [1-\pi_1^*]\{R - b\delta_2^{-1}\}.$$

To verify that the solution to the "less-constrained" problem satisfies (2a) note that

$$\pi_1^*\{\delta_1[R-\alpha_1^*]-[1-\delta_1]C_1^*-b\} - \pi_2^*\{\delta_1[R-\alpha_2^*]-[1-\delta_1]C_2^*-b\}$$
$$= \begin{cases} b[1-\delta_1\delta_2^{-1}] > 0 & \text{for} \quad \pi_1^* = 0 \\ 0 & \text{for} \quad \pi_1^* = 1. \end{cases}$$

To verify that (3) is satisfied for $j = 2$, note that

$$\pi_2^*\{\delta_2[R-\alpha_2^*]-[1-\delta_2]C_2^*-b\} = \begin{cases} 0 & \text{for } \pi_1^* = 0. \\ b[1-\delta_1\delta_2^{-1}] > 0 & \text{for } \pi_1^* = 1. \end{cases}$$

Q.E.D.

Proof of Proposition 2. Let $\pi_i^* = 1 \forall i$. (We will show that this is indeed optimal). We conjecture that the optimal solution is such that (6b) is slack. We will solve the optimization problem subject only to (6a) and then show that the solution satisfies (6b).

Substituting for α_i from (9) and ignoring the feasibility restriction on C_i, the Lagrangian can be expressed as

$$\begin{aligned} L = {} & \gamma\{\delta_1 R - r - [1-\beta][1-\delta_1]C_1 - b\} \\ & + [1-\gamma]\{\delta_2 R - r - [1-\beta][1-\delta_2]C_2 - b\} \\ & + \mu[\delta_1 R - r - [1-\beta][1-\delta_1]C_1 - \\ & \qquad \{\delta_1 R - \delta_1[\{r - [1-\delta_2]\beta C_2\}\delta_2^{-1}] - \{1-\delta_2\}C_2\}] \end{aligned}$$

where μ is the Lagrange multiplier associated with (6a). Differentiating the Lagrangian with respect to C_2 gives

$$\partial L/\partial C_2 = -[1-\gamma][1-\beta][1-\delta_2] - \mu\delta_1\{[1-\delta_2]\beta\delta_2^{-1}\} + \mu[1-\delta_2] = 0.$$

Thus,

$$\mu = [1-\gamma][1-\beta]/[1-\delta_1\delta_2^{-1}\beta] > 0.$$

This implies that if $0 < C_2^* < W$, (6a) is binding. Differentiating the Lagrangian with respect to C_1 yields

$$\partial L/\partial C_1 = -\gamma[1-\beta][1-\delta_1] - \mu[1-\beta][1-\delta_1] < 0.$$

Hence, $C_1^*=0$. Solving (6a) for C_2^* produces

$$C_2^* = r[\delta_2-\delta_1]\{\delta_2[1-\delta_1]-\beta\delta_1[1-\delta_2]\}^{-1}.$$

The denominator in the above expression can be shown to be positive, so our conjecture that $C_2^*>0$ is confirmed.

To prove that the solution does not violate (6b), note that since (6a) is binding, we have

$$\delta_1[R-\alpha_1^*] - [1-\delta_1]C_1^* = \delta_1[R-\alpha_2^*] - [1-\delta_1]C_2^*,$$

which implies

(A.6)
$$\delta_2[R-\alpha_2^*] - [1-\delta_2]C_2^* = \delta_1[R-\alpha_1^*] - [1-\delta_1]C_1^* + [\delta_2-\delta_1][R-\alpha_2^*+C_2^*].$$

Moreover,

(A.7)
$$\delta_1[R-\alpha_1^*] - [1-\delta_1]C_1^* = \delta_2[R-\alpha_1^*] - [1-\delta_2]C_1^* - [\delta_2-\delta_1][R-\alpha_1^*+C_1^*].$$

Substituting (A.6) in (A.7) we obtain

(A.8)
$$\delta_2[R-\alpha_2^*] - [1-\delta_2]C_2^* - \{\delta_2[R-\alpha_1^*]-[1-\delta_2]C_1^*\} = [\delta_2-\delta_1][\alpha_1^*-\alpha_2^*+C_2^*-C_1^*] > 0,$$

because $C_2^*>C_1^*=0$, and $\alpha_1^*=\alpha_1^0>\alpha_2^0>\alpha_2^*$. Thus (6b) is slack as conjectured.

To complete the analysis we must demonstrate that $\pi_1^*=\pi_2^*=1$ is, in fact, optimal with $C_2^*<W$. Note that π_1^* should never be less than 1 because (6b), which is already slack, will still be slack as π_1 drops below 1, but (6a), which is tight, will be violated. Therefore, making $\pi_1<1$ will precipitate an incentive compatibility problem.

To prove that $\pi_2^*=1$ is optimal, let $C_2(\pi_2)$ and $\alpha_2(\pi_2)$ denote the optimal collateral and interest factor for an arbitrary π_2. Note that $C_2(\pi_2)$ and $\alpha_2(\pi_2)$ satisfy (6a) as an equality and also satisfy (9). One can show that

(A.9)
$$\pi_2C_2(\pi_2) = \{\pi_2[\delta_1R-r\delta_1\delta_2^{-1}-b] - [\delta_1R-b-r]\}\{[1-\delta_1]-[1-\delta_2]\delta_1\delta_2^{-1}\beta\}^{-1}.$$

Expected borrower surplus, U, as a function of π_2 is given by:

$$U = \gamma\{\delta_1R-r-b\} + [1-\gamma]\{\pi_2[\delta_2R-r-b] - [1-\delta_2][1-\beta]\pi_2C_2(\pi_2)\}.$$

Differentiating U with respect to π_2, using (A.9) and rearranging yields:

$$\partial U/\partial\pi_2 = [1-\gamma]\{\delta_2[1-\delta_1]- \beta\delta_1[1-\delta_2]\}^{-1}[\delta_2-\delta_1]\{\delta_2R-r-b+[1-\delta_2][1-\beta]b\} > 0.$$

Thus, $\pi_2^*=1$.

Q. E. D.

PROOF OF PROPOSITION 3. As before we will solve Ω subject to only (6a) and then verify that (6b) is indeed slack. Substituting (9) into the objective function yields the Lagrangian

$$\begin{aligned} L(\pi_1, \pi_2, C_1, \lambda) = {} & \gamma\pi_1\{\delta_1 R - r - [1-\beta][1-\delta_1]C_1 - b\} \\ & + [1-\gamma]\pi_2\{\delta_2 R - r - [1-\beta][1-\delta_2]W - b\} \\ & + \mu[\pi_1\{\delta_1 R - r - [1-\beta][1-\delta_1]C_1 - b\} \\ & - \pi_2\{\delta_1 R - r\delta_1\delta_2^{-1} + \delta_1\delta_2^{-1}[1-\delta_2]\beta W - [1-\delta_1]W - b\}. \end{aligned}$$

Noting that $\mu \geq 0$ we have

$$\partial L/\partial C_1 = -\gamma\pi_1[1-\beta][1-\delta_1] - \mu\pi_1[1-\beta][1-\delta_1] < 0,$$

which implies $\hat{C}_1^* = 0$. With $\hat{C}_1^* = 0$, we have

$$\partial L/\partial \pi_1 = [\gamma+\mu][\delta_1 R - r - b] > 0$$

since $\delta_1 R - r - b > 0$ by assumption. Hence, $\hat{\pi}_1^* = 1$. As a next step we show that $\mu > 0$ and $\hat{\pi}_2^* < 1$. Note that

$$\begin{aligned} \partial L/\partial \pi_2 = {} & [1-\gamma]\{\delta_2 R - r - [1-\beta][1-\delta_2]W - b\} \\ & - \mu\{\delta_1 R - r\delta_1\delta_2^{-1} - \delta_1\delta_2^{-1}[1-\delta_2]\beta W - [1-\delta_1]W - b\}. \end{aligned}$$

It must be true that $\mu > 0$. If not, $\partial L/\partial \pi_2 > 0$,[22] which would imply that $\hat{\pi}_2^* = 1$. But it can be easily shown that when the inequality (10) does not hold, the credit policy, $\hat{\alpha}_2^* = \{r - [1-\delta_2]\beta W\}\delta_2^{-1}$, $\hat{C}_2^* = W$, $\hat{\pi}_2^* = 1$ does not satisfy (6a).

Because $\mu > 0$, (6a) holds as an equality, and is given by

$$\hat{\pi}_2^* = \{\delta_1[R - \hat{\alpha}_1^*] - b\}\{\delta_1[R - \hat{\alpha}_2^*] - [1-\delta_1]W - b\}^{-1}.$$

To complete the analysis we need to check that (6b) holds. (6a) can be shown to imply

$$\text{(A.10)} \qquad \begin{aligned} & \hat{\pi}_2^*\{\delta_2[R - \hat{\alpha}_2^*] - [1-\delta_2]W - b\} \\ & \qquad = \delta_1 R - b - r + \hat{\pi}_2^*[\delta_2 - \delta_1]\{R - \hat{\alpha}_2^* + W\}. \end{aligned}$$

Moreover,

$$\text{(A.11)} \qquad \delta_1 R - b - r = \delta_2\{R - \hat{\alpha}_1^*\} - b - [\delta_2 - \delta_1]\{R - \hat{\alpha}_1^*\}.$$

Combining (A.10) with (A.11) implies that (6b) holds if and only if

$$\text{(A.12)} \qquad \hat{\pi}_2^*\{R - \hat{\alpha}_2^* + W\} - \{R - \hat{\alpha}_1^*\} \geq 0.$$

[22] To verify that $\delta_2 R - r - [1-\beta][1-\delta_2]W - b > 0$, note that because $W < C_2^*$, it follows that $\delta_2 R - r - [1-\beta][1-\delta_2]W - b > \delta_2 R - r - [1-\beta][1-\delta_2]C_2^* - b$. Now, substituting the expressions for α_i^* and C_i^* from Proposition 2 into (A-6) and using the assumption that $\delta_1 R - r - b > 0$, one can demonstrate that $\delta_2 R - r - [1-\beta][1-\delta_2]C_2^* - b > 0$.

Tedious algebra establishes that (A.12) is equivalent to

$$W \geq br[\delta_2 - \delta_1]\{\delta_2[\delta_1 R - r] - b\delta_1[[1-\delta_2] + \beta\delta_2]\}^{-1},$$

which is the parametric restriction that is needed in order for a nontrivial solution to Ω to exist when Case 2 obtains (see footnote 14). Q. E. D.

REFERENCES

BALTENSPERGER, E., "Credit Rationing: Issues and Questions," *The Journal of Money, Credit and Banking*, 10 (May, 1978), 170–183.

BARRO, R. J., "The Loan Market, Collateral and Rates of Interest," *The Journal of Money, Credit and Banking*, 8 (November, 1976), 439–456.

BESANKO, D. AND A. V. THAKOR, "Collateral and Rationing Under Monopoly and Perfect Competition," BRC Working Paper (1985).

BESTER, H., "Screening vs. Rationing in Credit Markets with Imperfect Information," *The American Economic Review*, 75–4 (September, 1985), 850–855.

BHATTACHARYA, S., "Nondissipative Signalling Structures and Dividend Policy," *Quarterly Journal of Economics*, 95 (August, 1980), 1–24.

BLINDER, A. S. AND J. E. STIGLITZ, "Money, Credit Constraints, and Economic Activity," *The American Economic Review*, 73 (May, 1983), 297–302.

CHAN, Y. AND G. KANATAS, "Asymmetric Valuations and the Role of Collateral in Loan Agreements," *Journal of Money, Credit and Banking*, 17 (February, 1985), 84–95.

CUKIERMAN, A., "The Horizontal Integration of the Banking Firm, Credit Rationing and Monetary Policy," *Review of Economic Studies*, 139 (February, 1978), 165–178.

FREIMER, M. AND M. J. GORDON, "Why Bankers Ration Credit," *Quarterly Journal of Economics*, 79 (August, 1965), 397–416.

FRIED, J. AND P. HOWITT, "Credit Rationing and Implicit Contract Theory," *The Journal of Money, Credit and Banking*, 12 (August 1980), 471–487.

HARRIS, M. AND R. TOWNSEND, "Resource Allocation Under Asymmetric Information," *Econometrica*, 49 (1981), 33–64.

JAFFEE, D., *Credit Rationing and the Commercial Loan Market* (New York: John Wiley and Sons, 1971).

——— AND F. MODIGLIANI, "A Theory and Test of Credit Rationing," *The American Economic Review*, 59 (December, 1969), 850–872.

——— AND T. RUSSELL, "Imperfect Information, Uncertainty, and Credit Rationing," *Quarterly Journal of Economics*, 90 (November, 1976), 651–666.

KOSKELA, E., "Credit Rationing and Non-Price Loan Terms: A Re-Examination," *Journal of Banking and Finance*, 7 (1983), 405–416.

MYERSON, R. B., "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 47 (January, 1979), 61–74.

RILEY, J. G., "Infromational Equilibrium," *Econometrica*, 47 (March, 1979,) 331–359.

ROTHSCHILD, M. AND J. E. STIGLITZ, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90 (November, 1976), 629–649.

SAMUELSON, P. A., Statement in "Monetary Policy and Management of the Public Debt: Hearings Before the Subcommittee on General Audit Control and Debt Management," Joint Committee on the Economic Report, 2nd Congress and Session, (1952), 697.

SMITH, B., "Limited Information, Credit Rationing, and Optimal Government Lending," *The American Economic Review*, 73 (June, 1983), 305–318.

SPATT, C., "Credit Reputation Equilibrium and the Theory of Credit Markets," GSIA Working Paper No. 40–81–82, Carnegie-Mellon University (May, 1983).

Spence, A. M., "Product Differentiation and Performance in Insurance Markets," *Journal of Public Economics*, 10 (1978), 427–447.

Stiglitz, J. E. and A. Weiss, "Credit Rationing in Markets with Imperfect Information," *The American Economic Review*, 71 (June, 1981), 393–410.

——— and ———, "Incentive Effects of Terminations: Applications to the Credit and Labor Markets," *The American Economic Review*, 71 (December, 1983), 912–927.

Thakor, A. V. and R. Callaway, "Costly Information Production Equilibria in the Bank Credit Market with Applications to Credit Rationing," *Journal of Financial and Quantitative Analysis*, 18–2 (June ,1983), 229–56.

Wilson, C. A., "A Model of Insurance Markets with Incomplete Information," *Journal of Economic Theory*, 16 (1977), 167–207.