The Properties of Product Line Prices

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February 2014

Abstract. We study multi-product Cournot suppliers who offer menus of differentiated qualities. Under conditions that include linear and constant-elasticity demand specifications, the equilibrium price for each quality is equal to its price in a single-product Cournot market. A corollary is that a monopolist who naively ignores the substitutability of her products successfully implements the optimal discriminatory scheme. Under other reasonable specifications, the equilibrium prices are close to the single-product prices. We also evaluate the impact of minimum quality standards.

Students of economics are taught that maximizing the profit from a single product requires the equalization of marginal revenue and marginal cost. Advanced students are told that optimizing an entire menu of quality-differentiated products is more complex, because the substitutability of the products means that a change in the supply of one quality influences the profits earned from the other qualities.

If a multi-product monopolist were to ignore this substitutability, then would she be harmed? That is, are her profits reduced by using only the rudimentary techniques of single-product profit maximization? Relatedly, do the equilibrium prices in a multi-product oligopoly differ from the prices that we would see in single-product markets?

Under certain “textbook” conditions for demand, the answer to all of these questions is: No. Specifically, the equilibrium prices that emerge in a multi-product Cournot oligopoly (a monopoly is a special case) are equal to those in single-product markets. Even if the conditions are violated, the multi-product and single-product prices remain very close. The situations in which those prices differ more often coincide with those in which there is little loss to a supplier from restricting to a single-product menu.

The pricing equivalence result holds when the Mills ratio (the reciprocal of the hazard rate) of the distribution of buyers’ types (a buyer’s type is his willingness to pay for increases in quality) is linear in the type. This condition holds when types are either uniformly or Pareto distributed, for example, and is equivalent to a linear relationship between the marginal revenue of a single product and its price. This holds if the inverse demand curve for a product exhibits constant curvature (this is the elasticity of its slope) as it does when demand is linear or exhibits constant elasticity.

To develop intuition, consider a monopolist airline selling economy and business travel classes. A business-class ticket can be considered to be a bundle of an economy ticket and...
a supplemental quality upgrade. This perspective is useful because the bundle’s components are neither complements nor substitutes (although business and economy tickets are certainly substitutes). That is, a local change in the upgrade price influences the demand for the upgrade to business class (which is purchased in addition to an economy seat), but this change does not influence the marginal buyer of an economy ticket. Similarly, a local change to the economy price does not influence the marginal buyer of the upgrade.

Because the “baseline” economy product and the “upgrade” to business class are neither substitutes nor complements, to solve her pricing problem the monopolist may optimize separately in each of the baseline and upgrade markets. The correct product-line price for business class is the sum of the economy and upgrade prices.

Now consider the abolition of economy class. This forces the supplier to sell the baseline product and the upgrade together as a compulsory (or pure) bundle. The marginal cost of this bundle is the sum of the marginal costs of its components. At the original optimum, this sum equals the sum of the marginal revenue terms from the economy and upgrade markets. If there is a linear relationship between marginal revenue and price then the marginal revenue for the economy-plus-upgrade combination is equal to the sum of the separate economy and upgrade marginal revenue terms, when evaluated at the sum of the original component prices. This implies that the original business-class price remains optimal in what is now a business-class-only single-product market. Similarly, abolishing business class does not change the optimal economy-class ticket price. This reasoning also holds in a Cournot oligopoly, and so (if the Mills ratio is linear) the equilibrium multi-product prices are the same as the prices that emerge from separate single-product Cournot markets.

We also investigate the impact on prices and profits when the Mills ratio is not linear, considering numerically both beta and log-normal specifications. Although the non-linearity implies that the equilibrium multi-product prices do differ from the corresponding single-product prices, the difference is often very small. Additionally, the limited circumstances under which the price differences are larger are situations in which a supplier gains the least from selling multiple products rather than a single quality.

Our note contributes to the established techniques for quality-based and quantity-based price discrimination (Mussa and Rosen, 1978; Maskin and Riley, 1984). We use an upgrades approach (Johnson and Myatt, 2003, 2006a,b) which views higher qualities as combinations of incremental upgrades. Most notably, Itoh (1983) used such an approach. Many of our (non-numerical) results concerning monopoly are re-discoveries of his important but insufficiently widely known results. We contribute by developing the implications of his results for pricing naiveté, by extending to Cournot oligopolies, and by showing numerically that, even when consumer types deviate from common textbook specifications, there is often little difference between multi-product and single-product prices. This note joins other recent work (Cowan, 2007, 2012; Anderson and Dana Jr, 2009; Aguirre, Cowan, and Vickers, 2010) as part of a resurgent interest in price discrimination. The curvature of demand is an important feature; the treatment of this and its pass-through properties (Bulow and Pfleiderer, 1983) has been developed recently by Weyl and Fabinger (2013).
Supply and Demand for Multiple Qualities

**Demand.** A buyer with type \( \theta \sim F(\theta) \) is willing to pay at most \( \theta q \) for a single unit of a product with quality \( q \in [0, \infty) \). The continuous distribution \( F(\cdot) \) has a strictly positive density \( f(\cdot) \). The Mills ratio \( M(\cdot) \) is the reciprocal of the hazard rate:

\[
M(\theta) \equiv \frac{1 - F(\theta)}{f(\theta)}.
\]  

An equivalent definition is:

\[
M(\theta) \equiv \frac{\theta - \theta_L}{\theta_H - \theta_L}.
\]

A buyer \( \theta \) purchases a single unit of the product that offers him the greatest positive surplus.

\[ 1 - F(p) \] is the quantity demanded at price \( p \) for a standalone product with normalized quality \( q = 1 \), and \( P(z) \equiv F^{-1}(1 - z) \) is the associated inverse demand function at \( z \in (0, 1) \). In an \( m \)-supplier Cournot single-product oligopoly selling quality \( q = 1 \), the marginal revenue gained from an output expansion in a symmetrically divided market is

\[
\text{Marginal Revenue} = P(z) + \frac{ZP'(z)}{m} = p - \frac{M(p)}{m},
\]

where \( p = P(z) \).

A particular case of interest is when \( M(\theta) \) is linear, so that there is a linear relationship between marginal revenue and price. For example, if \( \theta \sim U[\theta_L, \theta_H] \) then

\[
F(\theta) = \frac{\theta - \theta_L}{\theta_H - \theta_L} \Rightarrow M(\theta) = \theta_H - \theta \Rightarrow \text{Marginal Revenue} = \frac{(m + 1)p - \theta_H}{m},
\]

Another case of interest is the generalized (type II) Pareto distribution:

\[
F(\theta) = 1 - \left( \frac{\theta - \mu}{\sigma/\xi} \right)^{-1/\xi} \Rightarrow M(\theta) = \xi(\theta - \mu) \Rightarrow \text{Marginal Revenue} = \frac{(m - \xi)p + \xi \mu}{m},
\]

where the location, scale, and shape are determined by \( \mu, \sigma, \) and \( \xi \), and where \( \theta \geq \mu + (\sigma/\xi) \). Setting \( \mu = 0 \) generates a demand function with constant elasticity.

The linearity of the Mills ratio also imposes structure on the inverse demand for a product. For example, under the generalized Pareto specification, \( P(z) \equiv F^{-1}(1 - z) \) is

\[
P(z) = \mu + \frac{\sigma z^{-\xi}}{\xi} \Rightarrow -\frac{zP''(z)}{P'(z)} = 1 + \xi.
\]

This final expression is the curvature (that is, the elasticity of the slope) of the inverse-demand function. Robinson (1933) calls this the “adjusted concavity” of inverse demand.

**Definition (Constant Curvature Demand).** The demand for quality has constant curvature if the elasticity of the slope of the inverse demand curve for a single product is constant. This holds if and only if there is a linear relationship between marginal revenue and price, which is true if and only if the Mills ratio is linear. This property is satisfied by the linear and constant-elasticity specifications.

If demand has constant curvature then, under both monopoly and Cournot oligopoly, there is a constant pass-through rate for changes in marginal cost (Bulow and Pfleiderer, 1983).

Although the class of constant-curvature demand specifications includes leading textbook examples, it can be restrictive. If a distribution has an increasing hazard rate and unbounded
support, then necessarily the Mills ratio is non-linear.\(^3\) Many common specifications (such as the unimodal beta, the normal, and gamma distributions) have a convex Mills ratio. Nevertheless, and as we show later, common specifications often exhibit approximate linearity.

**Supply.** The constant marginal cost of producing quality \(q\) is \(c(q)\). This satisfies \(c(0) = 0, c'(q) > 0,\) and \(c''(q) > 0\): there are decreasing returns to enhanced quality.

There are \(n\) quality levels \(0 < q_1 < \cdots < q_n\).\(^4\) \(c_i \equiv c(q_i)\) is the constant marginal cost of quality \(q_i\), \(\Delta q_i \equiv q_i - q_{i-1}\), \(\Delta c_i \equiv c_i - c_{i-1}\), \(q_0 \equiv 0\) and \(c_0 \equiv 0\). The properties of \(c(q)\) ensure that \(0 < \Delta c_i / \Delta q_i < \cdots < \Delta c_n / \Delta q_n\). Finally, \(0 < F'(\Delta c_i / \Delta q_i)\) (it is inefficient to supply everyone) and \(F'(\Delta c_n / \Delta q_n) < 1\) (the efficient supply of the highest quality is positive).

The market is served by \(m\) multi-product Cournot oligopolists, who simultaneously choose their supplies of the \(n\) different qualities. To keep our notation simple, we mainly focus on an industry in which all suppliers have identical production technologies. However, our results are robust to some differences in their technological capabilities (Proposition 2).

As a benchmark, we consider a Cournot game in which the suppliers sell a single product with quality \(q_i\). Given our assumptions, the equilibrium is symmetric. The total equilibrium industry output \(z_i^\uparrow\) equates marginal revenue to marginal cost in the usual way:

\[
P(z_i^\uparrow) + \frac{z_i^\uparrow P'(z_i^\uparrow)}{m} = \frac{\Delta c_i}{\Delta q_i}. \tag{6}
\]

Equivalently, the marginal buyer’s type is \(\theta_i^\uparrow\) where \(z_i^\uparrow = 1 - F(\theta_i^\uparrow)\). He is indifferent, and so

\[
p_i^\uparrow = \theta_i^\uparrow q_i, \quad \text{where} \quad \theta_i^\uparrow - \frac{M(\theta_i^\uparrow)}{m} = \frac{c_i}{q_i}. \tag{7}
\]

If the Mills ratio is linear then this yields a simple closed-form solution to the Cournot price.

**Lemma 1** (Single-Product Cournot Prices). *If demand has constant curvature, so that \(M(\theta) = \alpha + \beta \theta\) for some \(\alpha\) and \(\beta < m\), then the equilibrium price in a Cournot market for quality \(q_i\) is

\[
p_i^\uparrow = \frac{mc_i + \alpha q_i}{m - \beta}. \tag{8}
\]

**Equilibrium Prices in a Multi-Product Cournot Oligopoly**

In a multi-product Cournot oligopoly, \(m\) symmetric suppliers simultaneously choose their outputs of the \(n\) different qualities, where \(z_{ik}\) is supplier \(k\)’s output of quality \(q_i\). At the industry level, \(z_i\) is the total output of quality \(q_i\), \(p_i\) is its price, and we define \(\Delta p_i \equiv p_i - p_{i-1}\).

**Equilibrium.** Market-clearing prices ensure that higher types purchase higher qualities. The marginal buyer of quality \(q_i\) is the type \(\theta_i\) with \(Z_i \equiv \sum_{j=i}^n z_j\) others above him, and so \(Z_i = 1 - F(\theta_i)\). He is indifferent between qualities \(q_i\) and \(q_{i-1}\), and so \(\Delta p_i \equiv p_i - p_{i-1} = \theta_i \Delta q_i\).

\(^3\)An increasing hazard implies a decreasing Mills ratio. If \(M(\theta)\) were linear then it would eventually cross zero.

\(^4\)There is no limit to \(n\), and our results can also be applied to a continuum-of-qualities specification.
$Z_i$ is the fraction of products sold with quality $q_i$ or higher. This can be interpreted as the supply of upgrades to this quality level (and beyond). Similarly, $\Delta p_i$ is the price of an upgrade from quality $q_{i-1}$ to the next step in the product line.

Supplying $n$ different qualities is equivalent to selling $n$ upgrades. From an upgrades perspective, quality $q_i$ is a combination of the $i$ upgrades from $\Delta q_1$ to $\Delta q_i$. An upgrade’s price $\Delta p_i$ depends only on its own supply, and so the advantage of thinking in terms of upgrades is that they are neither substitutes nor complements.

We write $Z_{ik}$ for the quantity of the $i$th upgrade sold by supplier $k$. Supplier $k$ must choose her upgrades to satisfy $Z_{ik} \geq \cdots \geq Z_{nk} \geq 0 \equiv Z_{n+1,k}$. She seeks to maximize:

$$\text{Multi-Product Profit of Supplier } k = \sum_{i=1}^n z_{ik}(p_i - c_i)$$

$$= \sum_{i=1}^n (Z_{ik} - Z_{(i+1)k}) \left( \sum_{j=1}^i (\Delta p_j - \Delta c_j) \right)$$

$$= \sum_{i=1}^n Z_{ik}(\Delta p_i - \Delta c_i)$$

$$= \sum_{i=1}^n \Delta q_i Z_{ik} \left( P(Z_i) - \frac{\Delta c_i}{\Delta q_i} \right). \quad (9)$$

The $i$th element of this summation depends only upon $Z_{ik}$ and $Z_i$. Hence, the Nash equilibrium of a multi-product Cournot game replicates $n$ single-product Nash equilibria in each upgrade market, so long as these upgrade supplies, which we denote $Z_{ik}$, satisfy the monotonicity constraints $Z_{1k} \geq \cdots \geq Z_{nk}$ (so that $z_{ik}^* = Z_{ik} - Z_{(i+1)k} \geq 0$). These constraints are satisfied when there are decreasing returns to quality (Johnson and Myatt, 2003, 2006a,b).

**Lemma 2** (from Johnson and Myatt (2006a)). There is a unique Nash equilibrium of the multi-product Cournot oligopoly game. The total industry output $Z_i^*$ of the $i$th upgrade satisfies

$$P(Z_i^*) + \frac{Z_i^* p'(Z_i^*)}{m} = \frac{\Delta c_i}{\Delta q_i}, \quad (10)$$

and the equilibrium industry output of quality $q_i$ itself is $z_i^* = Z_i^* - Z_{i+1}^*$.

**Pricing Equivalence.** In equilibrium the marginal consumer of quality $q_i$ (and hence of the $i$th upgrade) has a type $\theta_i^* = P(Z_i^*)$. The equilibrium price of that upgrade is

$$\Delta p_i^* = \theta_i^* \Delta q_i \quad \text{where} \quad \theta_i^* - \frac{M(\theta_i^*)}{m} = \frac{\Delta c_i}{\Delta q_i}. \quad (11)$$

A special case is when demand exhibits constant curvature, so that $M(\theta) = \alpha + \beta \theta$ for some $\alpha$ and $\beta < m$. In this case, the equilibrium price for the $i$th upgrade is

$$\Delta p_i^* = \frac{m \Delta c_i + \alpha \Delta q_i}{m - \beta} \quad (12)$$

To obtain the equilibrium price of a complete quality $q_i^*$, we simply sum the equilibrium upgrade prices. If the demand for quality has constant curvature, then

$$p_i^* = \sum_{j=1}^i \Delta p_j^* = \frac{\sum_{j=1}^i (m \Delta c_j + \alpha \Delta q_j)}{m - \beta} = \frac{mc_i + \alpha q_i}{m - \beta} = p_i^\dagger, \quad (13)$$

which (from equation (8) of Lemma 1) is the equilibrium price in a single-product market.
**Proposition 1** (Pricing Equivalence). If the demand for quality has constant curvature, then the equilibrium price for a particular quality in a multi-product Cournot oligopoly coincides with the corresponding equilibrium price in a standalone single-product Cournot market.

This proposition holds for any product line. Thus, for any two different product lines containing the same quality, the price of that common quality must be the same.

**Corollary** (to Proposition 1). If the demand for quality has constant curvature, then the equilibrium price for each quality in a product line is independent of the other qualities that are available.

A special case is when $m = 1$. If a monopolist naively specifies single-product monopoly prices for her qualities and if her demand is linear or exhibits constant elasticity, then she replicates the fully optimal prices from Mussa and Rosen (1978).\(^5\) This simple result is not widely known, even though it follows straightforwardly from Proposition 5 of Itoh (1983).

**Asymmetric Oligopolies.** Proposition 1 also holds when suppliers have asymmetric capabilities, so long as they do not differ too much. Suppose that cost asymmetries are such that in equilibrium all suppliers offer complete product lines (so that $z_{ik}^* > 0$ for all $i$ and $k$). This is true if the suppliers’ production technologies are not too different. With complete product lines, each supplier satisfies the relevant first-order condition in each upgrade market. Writing $\Delta c_{ik}$ for the marginal cost of upgrade $i$ to supplier $k$, equilibrium prices satisfy

$$
\Delta p_i^* = \theta_i^* \Delta q_i \quad \text{where} \quad \theta_i^* = \frac{M(\theta_i^*)}{m} = \frac{\sum_{k=1}^m \Delta c_{ik}}{m \Delta q_i}.
$$

(14)

We can sum over the upgrades, just as we did in the derivation of equation (13).

**Proposition 2** (Pricing Equivalence in an Asymmetric Oligopoly). If, in equilibrium, all suppliers offer complete product lines then the claim of Proposition 1 holds in an asymmetric oligopoly.

**The Effect of Product-Line Changes**

We now study the response of equilibrium prices to changes in the menu of available qualities. For the monopoly case, this exercise was completed by Itoh (1983). A contribution here is to show that his results also hold for a Cournot oligopoly.

**Bundling Upgrades.** Consider two neighboring upgrades $j \in \{i, i+1\}$ in the product line. Their equilibrium prices satisfy equation (12). Combining the solutions,

$$
\frac{\Delta q_i \theta_i + \Delta q_{i+1} \theta_{i+1}}{\Delta q_i + \Delta q_{i+1}} - \frac{\Delta q_i M(\theta_i^*) + \Delta q_{i+1} M(\theta_{i+1}^*)}{m(\Delta q_i + \Delta q_{i+1})} = \frac{\Delta c_i + \Delta c_{i+1}}{\Delta q_i + \Delta q_{i+1}}.
$$

(15)

Now suppose that the two upgrades are forcibly bundled to form a two-step upgrade. The equilibrium price for this bundle is $\hat{\Delta p} = \hat{\theta}(\Delta q_i + \Delta q_{i+1})$, where $\hat{\theta}$ is the unique solution to

$$
\hat{\theta} - \frac{M(\hat{\theta})}{m} = \frac{\Delta c_i + \Delta c_{i+1}}{\Delta q_i + \Delta q_{i+1}}.
$$

(16)

\(^5\)The problem differs slightly from Mussa and Rosen (1978) because the product line is finite. However, the qualities intervals can be arbitrarily narrow, and our approach does work in the continuum-of-qualities case.
Now suppose that $M(\cdot)$ is strictly convex. Using equations (15) and (16),

$$
\hat{\theta} - \frac{M(\hat{\theta})}{m} = \frac{\Delta q_i \theta_i^* + \Delta q_{i+1} \theta_{i+1}^*}{\Delta q_i + \Delta q_{i+1}} - \frac{\Delta q_i M(\theta_i^*) + \Delta q_{i+1}^* M(\theta_{i+1}^*)}{m(\Delta q_i + \Delta q_{i+1})} \\
< \frac{\Delta q_i \theta_i^* + \Delta q_{i+1} \theta_{i+1}^*}{\Delta q_i + \Delta q_{i+1}} - \frac{1}{m} \left( \frac{\Delta q_i \theta_i^* + \Delta q_{i+1} \theta_{i+1}^*}{\Delta q_i + \Delta q_{i+1}} \right) \\
\Rightarrow \hat{\theta} < \frac{\Delta q_i \theta_i^* + \Delta q_{i+1} \theta_{i+1}^*}{\Delta q_i + \Delta q_{i+1}} \Rightarrow \Delta \hat{\theta} < \Delta p_i^* + \Delta p_{i+1}^*. \quad (17)
$$

The first inequality follows from the convexity of $M(\cdot)$, the implication holds because $\theta - \frac{M(\theta)}{m}$ is increasing in $\theta$, and the conclusion follows from the definitions of $\Delta \hat{\theta}$, $\Delta p_i^*$, and $\Delta p_{i+1}^*$.

**Lemma 3 (Combining Upgrades).** If the Mills ratio is convex (respectively, concave) then the price for a bundle of upgrades is less than (respectively, more than) the sum of the upgrade prices.

The upgrade bundle combines commonly shaped demand curves. The sum of the original marginal revenue terms equals the sum of the marginal costs. However, for the bundle the marginal revenue terms are now evaluated at an appropriately weighted average type. If marginal revenue is a concave function of price then, at this average point, marginal revenue is greater; this induces an output expansion and so a lower price.

**Product Line Changes.** The elimination of quality $q_i$ forces the bundling of the upgrades $\Delta q_i$ and $\Delta q_{i+1}$. The prices of qualities below $q_i$ are unaffected; such lower qualities are combinations of upgrades $\Delta q_j$ for $j < i$. However, if the price of the bundle $\Delta q_i + \Delta q_{i+1}$ falls then the price of $q_{i+1}$ must fall. There are no changes to upgrades above this and so other complete qualities above $q_{i+1}$ must experience the same change in price seen for $p_{i+1}$.\(^6\)

**Proposition 3 (The Effect of Product Line Changes).** The elimination of a quality does not change the equilibrium prices of lower qualities, but results in a common price change for higher qualities. If the Mills ratio is convex (respectively, concave) then prices of higher qualities fall (respectively, rise).

This proposition also reveals the relationship between equilibrium product-line prices and the corresponding single-product prices when the curvature of demand is non-constant. Consider the price of quality $q_i$. If all higher qualities are removed then its price is unaffected; however, if all lower qualities are removed and if the Mills ratio is convex then its price falls. Once all other qualities are removed then its price is the single-product price.

**Corollary** (to Proposition 3). If the Mills ratio is convex (respectively, concave) then the equilibrium price for a quality in a product line is higher (respectively, lower) than its standalone price. In both cases the gap between the multi-product and standalone prices is highest for higher qualities.

The final claim holds because higher qualities are the sum of a larger number of upgrade steps, and so their prices are influenced by more bundling operations.

\(^6\)This implies that the claims of Proposition 3 from Itoh (1983) apply in a Cournot environment. This is in the context of a multiplicative specification for consumers’ preferences, so that the willingness of type $\theta$ to pay for quality $q$ is $u(\theta, q) = \theta q$. However, for general $u(\theta, q)$ it is also true that the prices of qualities below $q_i$ are unaffected, and the prices of qualities $q_{i+1}$ change by the same amount. However, the size of the change depends upon the properties of $u(\theta, q)$ as well as the shape of the Mills ratio $M(\theta)$.
Quality Regulations. The regulation of a multi-product market might take several forms, including bounds to the qualities offered, or the requirement to offer only one quality.

A restriction to a single quality directly removes choice from buyers. However, if the price of the remaining product falls, and falls by enough, then some buyers may benefit. If the demand for quality exhibits constant curvature then (applying Proposition 1) the price of the remaining product is unchanged, and so nobody is helped. If the Mills ratio is concave, then the remaining single product has a higher price, and so everyone is harmed. The only situation in which some (but not necessarily all) buyers benefit is when the Mills ratio is convex. A regulatory restriction to a single quality helps an interval of buyer types that includes all of the original purchasers of the surviving quality.

Next consider a cap on the maximum quality that is offered. This is equivalent to removing the highest qualities in sequence, and has no effect on the prices of remaining qualities (Proposition 3). A quality cap, then, can only harm buyers.

Finally, consider a minimum quality regulation. As with other product removals, this can only be helpful if the Mills ratio is convex. The prices of all remaining higher qualities fall, and so higher types must benefit; lower types, however, are harmed.

Proposition 4 (Minimum Quality Standards). If the Mills ratio is concave, then all consumers are harmed by a minimum quality standard. If the ratio is convex, then there is a critical consumer type $\bar{\theta}$ such that types above $\bar{\theta}$ are helped by such a standard, whereas types below $\bar{\theta}$ are harmed.

A minimum quality may well be expected to lower suppliers’ profits. Given a free-entry condition, this induces exit and so raises prices. Thus, if the Mills ratio is close to linear (as it is for specifications studied in the next section) the net effect of the standard will be to raise all prices. This is (in such a free-entry world) unambiguously bad for welfare.

The Impact of Naive Pricing

If demand has constant curvature then a naive monopolist who chooses single-product prices for her product line nevertheless successfully replicates the optimal discriminatory scheme. Of course, this is not true if the demand for quality has variable curvature. Here we study the prices and profits of a monopolist when the Mills ratio of types is non-linear.

We study two flexible specifications, the beta distribution and the log-normal distribution. We find that the impact of naiveté is often small, and that if the impact is larger then there is little lost by selling only a single product.

Beta Distributed Types. Suppose that types and qualities lie in $[0, 1]$. Furthermore:
\[
c(q) = \frac{q^2}{2} \quad \text{and} \quad f(\theta) \propto \theta^{\delta-1}(1-\theta)^{\left(1-\gamma\right)\delta-1} \quad \text{for} \quad \gamma \in (0, 1) \quad \text{and} \quad \delta > 0. \quad (18)
\]
Hence $\theta$ follows a beta distribution with mean $E[\theta] = \gamma$ and variance $\text{var}[\theta] = \gamma(1-\gamma)/(\delta+1)$; the uniform distribution is obtained when $\gamma = 0.5$ and $\delta = 2$. Three examples are illustrated.

\[\text{Although we focus on a monopolist here, the findings readily extend to the properties of Cournot prices.}\]
in Figure 1. The relevant part of the Mills ratio is where $M(\theta) < \theta$ (this is where marginal revenue is positive). The examples all exhibit substantial convexity in that region.

Figure 1c reports the discriminatory and standalone prices for $\gamma = 0.5, \delta = 20$, and $n = 100$ equally spaced qualities. The price sequences are different but close. The price difference is greatest for the highest quality (the Corollary to Proposition 3 predicts this). The standalone price $p^*_i$ is 7% adrift of the optimal price $p^n$, and at the mid-point quality it is only 2.3% below. Overall, the incorrect use of single-product prices causes a loss of only 1.7% of potential profit. Note that $\delta = 20$ yields modest but non-negligible dispersion in buyers’ types: the 95% confidence interval for $\theta$ is approximately $[0.32, 0.68]$.

Now we explore what happens as consumer heterogeneity varies, via a change in $\delta$. For fixed mean $\gamma$, when $\delta$ is very small, the distribution of types is approximately uniform. In contrast, when $\delta$ becomes very large, buyers clump tightly around the mean.
Figure 1d compares the profit from optimal pricing to the profit from naive pricing, for the case of $\gamma = \frac{1}{2}$ and $n = 20$ equally spaced qualities. For this figure, $\text{var}[\theta]$ is smallest for $\delta = 100$, which yields a 95% confidence interval of approximately $[0.42, 0.58]$.

For small $\delta$, there is no meaningful penalty from the use of naive single-product prices. The reason is that the Mills ratio becomes approximately linear for such $\delta$, because the distribution becomes approximately uniform. In contrast, precisely because there is significant heterogeneity amongst consumers, restricting to only a single product can lead to larger losses. For example, even with $\delta = 10$, the reduction in profits exceeds 10%.

As buyers become homogeneous, the convex Mills ratio means that naiveté is more costly. Interestingly, for large $\delta$, the monopolist is better off if she sells only a single (optimally chosen and priced) product, rather than selling the full range at naive prices.

Selling a range of products priced naively rather than a single product priced optimally presents a trade-off. On the one hand, offering multiple products increases the potential to extract revenue from consumers. On the other hand, the sub-optimal pricing means that consumers are offered substitution possibilities that limit the monopolist’s profit. The magnitude of these effects depends on the underlying dispersion of consumers.

For high dispersion (that is, low $\delta$), there is a significant potential gain from offering a menu of products. The Mills ratio is approximately linear here, and so the naive prices approximately equal the optimal ones. On the other hand, when consumers are very homogeneous, there is little to be gained from selling multiple products. Furthermore, the naive price for a product extracts nearly all surplus from the market if it is the only product offered. This means that each option in the menu, naively priced, in a multi-product setting leaves the typical consumer with approximately zero surplus. In turn, this means that which product most consumers actually buy is very sensitive to small changes in the price, so that there is a significant loss from mis-pricing. Offering the most-profitable single product (optimally priced) is safer and does extremely well.

Log-normally Distributed Types. We now consider a log-normal specification for types, so that $\log \theta \sim N(\mu, \sigma^2)$. This satisfies $\log \mathbb{E}[\theta] = \mu + \frac{\sigma^2}{2}$ with a Gini coefficient equal to $2\Phi(\sigma/\sqrt{2}) - 1$ where $\Phi(\cdot)$ is the standard normal distribution function. Equivalently,

$$\sigma = \sqrt{2}\Phi^{-1}\left(\frac{\text{Gini} + 1}{2}\right) \quad \text{and} \quad \mu = \mathbb{E}[\theta] - \frac{\sigma^2}{2}. \quad (19)$$

For our exercises here we fix the mean valuation $\mathbb{E}[\theta]$ (changes in it do not effect the relative performance of optimal and naive prices) and we use the Gini coefficient as a convenient scale-free measure of the dispersion of consumers’ preferences.

Figure 2 uses the normalization $\mathbb{E}[\theta] = 1$. In Figure 2a each line reports the Mills ratio $M(\theta)$ across the interval of types $[\theta_L, \theta_H]$ where $F(\theta_L) = 0.02$ and $F(\theta_H) = 0.98$. The highest four

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8Profit is quasi-convex, or “U-Shaped” in $\delta$, and so is maximized for very homogeneous (high $\delta$) or very heterogeneous (low $\delta$) buyers. This is a prediction from Johnson and Myatt (2006b, Proposition 1).

9The Gini coefficient is half of the expected absolute difference between two random draws (that is, half of the Gini mean difference) expressed as a fraction of the mean: $\text{Gini} = \mathbb{E}[|\theta - \theta'|]/2\mathbb{E}[\theta]$
Gini coefficients are chosen to correspond very roughly to inequality in countries such as Denmark (0.25), the United Kingdom (0.35), the United States (0.45), and Brazil (0.55). The lowest value 0.15 represents more homogeneity than is actually exhibited by any country. If a consumer’s willingness-to-pay for a product is a fixed fraction of his income, then this range contains reasonable values for the distribution of consumers’ types.

The relevant region is \( M(\theta) < \theta \) (below the 45° line), where marginal revenue is positive. For most specifications the Mills ratio is approximately linear in this region. Hence, naive pricing yields approximately the same prices and profit as optimal pricing. Only when types are rather more homogeneous (Gini = 0.15) does a significant non-linearity exist. This is more pronounced when \( \theta \) is small, corresponding to a situation in which most of the market is served. Hence, the divergence between optimal multi-product prices and single-product prices will be small unless very low production costs induce high output.

When consumer valuations are very similar, the gain from offering multiple products is smaller (Figure 2b). We see a repetition of the lesson from the beta distribution case. If the Gini coefficient is small, so that potential buyers are similar, then there is a significant difference between the multi-product and single-product prices, but in these cases it is optimal to serve most consumers the same product. This suggests that a monopolist does quite well by selling only a single product, so long as she selects and prices that product correctly.

A final observation from Figure 2b is that profit is no longer purely U-shaped in the dispersion of types: for very high inequality, profit drops with further increases in inequality.\(^\text{10}\) For very high Gini coefficients most mass shifts toward zero, but the distribution has a very long upper tail. To capture surplus from that tail requires products of higher quality. However, for Figure 2b we limit to a grid of \( n = 20 \) qualities on the interval \( q_i \in [0, 4] \). The upper bound prevents the monopolist from reacting optimally to high Gini coefficients.

\(^\text{10}\)Noting that \( \theta \sim N(\mu, \sigma^2) \), a higher Gini coefficient clockwise rotates the demand curve via an increase in \( \sigma^2 \), but also shifts the demand curve leftward via a lower \( \mu \). This shift violates the condition on the rotation quantity in Proposition 1 of Johnson and Myatt (2006b), and so we cannot guarantee that monopoly profit is a quasi-convex function of the Gini coefficient. Figure 2b illustrates a violation of quasi-convexity.
Concluding Remarks

As teachers of economics, we emphasize the conceptual differences between single-product and multi-product (discriminatory) pricing. But we do not typically explain whether more sophisticated tools are associated with significant changes to prices or final profits.

The early literature established how the removal of an option from a monopolist’s product line affects the prices of higher qualities; the shape of the Mills ratio is determinative (Itoh, 1983). From here, it is a simple step to observe that, in situations corresponding to common “textbook” specifications, discriminatory prices must be equal to single-product prices. A role of this note is to highlight this observation; we suggest that it is something that students of economics should know. Furthermore, our results show that this message carries over to a Cournot environment, and it holds in spirit under a wider range of demand specifications. Finally, the circumstances in which the price equivalence result breaks down are those in which there is little to be gained by offering a multi-product menu.

References


