Welfare gains of the poor: 
An endogenous Bayesian approach with spatial random effects

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Abstract

We introduce a Bayesian instrumental variable procedure with spatial random effects that handles endogeneity, and spatial dependence with unobserved heterogeneity. We account for endogeneity through a simultaneous equations system where conditional correlation between stochastic errors capture endogeneity, and exclusion restrictions are used to treat endogenous regressors. In addition, we propose a Bayesian hierarchical spatial framework to model spatial dependence and heterogeneity. A Gibbs sampling algorithm is used to draw samples of all our conditional posterior distributions. Finally, a Bayesian framework permits to easily perform statistical inference related to complicated non-linear functions of parameter estimates. We apply our method to analyze welfare effects on the poorest households generated by a process of electricity tariff unification. In particular, we deduce an Equivalent Variation measure from a logarithmic demand function and a budget constraint for a two-tiered pricing scheme. We find the posterior distribution of the Equivalent Variation, and estimate the welfare implications in a context where electricity tariffs decreased by as much as 17.53%. We find that the poorest municipalities reached welfare gains above 2% of their initial income.

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Keywords: Bayesian Estimation, Endogeneity, Simultaneous Equations, Spatial Random Effects, Welfare Analysis

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Introduction

We propose a Bayesian simultaneous equations system with spatial random effects suited to handle spatial dependence and heterogeneity, endogeneity and statistical inference associated with complicated non-linear functions of parameter estimates. In particular, we define a simultaneous equations system where conditional correlation between stochastic errors capture endogeneity, and instrumental variables are used to model endogenous variables. In addition, we employ a Bayesian hierarchical spatial framework, based on the Conditional Autoregressive (CAR) spatial prior, to structure the unobserved heterogeneity and the spatial dependence. After model specification, we find the conditional posterior distributions of all the parameter sets, thus we can use Gibbs sampling algorithms to draw simulations of all our posterior distributions. Finally, establishing a Bayesian approach allows to perform statistical inference relate to functions of parameter estimates, using simple rules of probability theory.

We apply our methodology to evaluate welfare implications on poor households, measured through the Equivalent Variation, caused by the electricity price changes, which took place in the province of Antioquia (Colombia), after Empresas Públicas de Medellín (EPM) acquired Empresa Antioqueña de Energía (EADE) in 2006. We should mention that the Equivalent Variation is a non-linear function of parameter estimates of a demand function, which we estimate using data at municipality level. Therefore, in our empirical exercise, we should take into consideration spatial effects, endogeneity between price and electricity demand, and unobserved heterogeneity due to latent economical, cultural and geographical factors. Finally, we want to perform statistical inference regarding the Equivalent Variation. This application is interesting by itself because electricity services represent a significant share of households’ budget, and this fact is prominent on the poor population (Gomez-Lobo, 1996, Ruijs, 2009, You and Lim, 2013). As a consequence, small variations in electricity prices may have relevant impacts on the welfare of households.
Consideration of spatial effects to perform statistical inference based on cross-sectional areal data has a long tradition in spatial statistics (Cressie, 1993, Ripley, 2004), and more recently in spatial econometrics (Anselin, 1988). Most of the methods in spatial econometrics have being based on the Frequentist approach; although there are some remarkable exceptions founded on the Bayesian framework (LeSage, 1997, 2000, Parent and LeSage, 2008, LeSage and Pace, 2009, LeSage and Llano, 2013).

Regarding the issue of endogeneity, this emerges naturally due to the presence of the spatial lag of the dependent variable in Spatial Autoregressive (SAR) models, and spatial econometric estimators have taken this problem into consideration since its beginning (Anselin, 1990, Kelejian and Prucha, 1998). However, the treatment of endogeneity due to other regressors is being recently analyzed (Rey and Boarnet, 2004, Kelejian and Prucha, 2004, Fingleton and Gallo, 2008, Crukker et al., 2013, Liu and Lee, 2013). Thus, these new kind of spatial estimators take into consideration spatial dependence and feedback endogeneity simultaneously. At the same time, they fail to take into account unobserved heterogeneity, and have to rely on asymptotic methods, like the Delta method, to perform statistical inference regarding complicated non-linear functions of parameter estimates.

Unobserved heterogeneity is another issue that may characterize cross-sectional areal data (Parent and LeSage, 2008). Unfortunately, to the best of our knowledge, there is limited spatial econometric literature regarding this issue. This fact may obey to the difficulty of introducing unobserved heterogeneity in cross-sectional areal data using Frequentist methods. However, LeSage (2000), Smith and LeSage (2004), LeSage et al. (2007), Parent and LeSage (2008), Seya et al. (2012) and LeSage and Llano (2013) have tackled unobserved heterogeneity using a Bayesian approach. These references involved simultaneously spatial effects and unobserved heterogeneity, but they do not take into consideration recursive endogeneity. An issue that has been considered from a Bayesian perspective in Drèze (1976), Zellner (1998), Kleibergen and Van Dijk (1998), Kleibergen
Applications in literature that examine welfare effects of price changes in utilities, either through price variations within an existing structure, or different pricing schemes, are plentiful, but fail to take into account some important factors. Acton and Mitchell (1983) for example, look at the welfare effects on both consumers and producers from moving to a time-of-use price structure. The authors conclude that consumers attain a positive welfare gain from the new scheme, especially if they are highly price responsive. Another conclusion is that effects are greater for those with larger levels of consumption. Their results, however, are based on the Consumer Surplus welfare measure and therefore leave income effects out of the analysis. Gomez-Lobo (1996) estimate welfare implications of tariff rebalancing for British energy markets. The author presents some scenarios of price variations and shows that increased competition in the gas market can benefit most households but also comes to the conclusion that the households in lower income groups stand to lose the most. Although the study makes use of Heckman’s two-step estimator to correct for selection bias of households’ gas connectivity and some of the non-linearity in prices, the endogeneity between consumption and price is not resolved. This fact remains true even when using data at the household level. Furthermore, the use of the Compensating Variation as a measure of welfare, given that it ranks price alternatives incorrectly, can lead to problems with the main conclusions. Finally, when working with pooled household data instead of panel data, controlling for unobserved heterogeneity is difficult and can significantly damage estimated parameters and inference. The study by You and Lim (2013) suffers from the same issue with unobserved heterogeneity, and although they reject the use of Instrumental Variables in favor of Ordinary Least Squares, the set of instruments they propose is very limited. The authors evaluate the welfare of households in different income deciles through several alternative price schemes. Non-linearity in prices for the Equivalent Variation is taken into account in a fashion similar to ours for a linear demand function. The analysis concludes that price structures with lesser
blocks are preferred when society is concerned about inequality, but flat rates perform better if lower-income households can be compensated through other means. Ruijs (2009) also establishes the Equivalent Variation measure with increasing-block pricing present in water consumption in Brazil, through a linear demand function. The author uses aggregate data for 39 municipalities of the Metropolitan Region of São Paulo, estimates the aggregated demand function and finds the welfare effects associated with several possible scenarios, including price changes in both blocks. He emphasizes the effects that different policies can have on different income groups and how important it is to look at these instead of simple average welfare implications. He also notes how substantial the effects can be in terms of disposable income for the poorest households, even with slight price variations. These, and other studies, however, exhibit how welfare implications based on estimates that account for spatial characteristics, endogeneity and unobserved heterogeneity are scarce in literature (for example Dodonov et al., 2004, Lundgren, 2009).

As a conclusion of our literature review, we find that few authors have studied welfare effects due to changes within block price systems, and even less researchers introduce spatial random effects within an endogenous framework to analyze these welfare implications. From a theoretical standpoint, the main consideration of adopting the Bayesian approach is that it allows us to establish a statistical framework that simultaneously unifies decision theory, statistical inference and probability theory under a single philosophically and mathematically consistent structure. From an empirical perspective, the Bayesian approach has some advantages compared with the Frequentist framework in the present setting. In particular, we can easily make statistical inference associated with the Equivalent Variation, which is a complicated function of parameter estimates, using simple rules of probability theory, which could prove difficult with a Frequentist statistical approach. In addition, our econometric approach takes into account the endogeneity issue that is present, and the Bayesian paradigm allows us to identify the structural parameters from the reduced form, even using weak instruments, as in this empirical exercise. The Fre-
quentist procedures, however, deal with identification problems in the presence of weak instruments. Finally, a Bayesian framework permits us to introduce spatial random effects in our cross-sectional areal data structure, and control by the unobserved heterogeneity and autocorrelation that can arise in spatial settings. On the other hand, a Frequentist approach does not allow us to easily take this phenomenon into account.

Using data at the municipality level for the province of Antioquia, and different spatial contiguity criteria, we find that the posterior mean of the price, income, substitute and urbanization rate demand elasticities are -0.88, 0.30, 0.12 and 0.57, respectively. In addition, the posterior mean of the semi-elasticity of electricity demand associated with a sea level dummy, which is equal to one when municipality is located less than 1,000 meters above sea level and zero otherwise, is approximately 0.14. With these estimates as inputs, we calculate the posterior distribution of the Equivalent Variation welfare measure as a share of income for each municipality. We deduce this measure using a logarithmic demand function, and taking into account a budget constraint for a two-tiered pricing scheme. We find that the average household enjoys a mean welfare gain of approximately 0.87% of their initial income, which can be considerable when taking their socioeconomic situation into account. These results however, heavily depend on whether the municipalities are part of the Metropolitan Area or not, their average electricity consumption levels and other geographical and economic factors. For example, Medellín, the capital of the province, and its main economic activity center, presented a mean welfare gain equal to 0.14%, which is approximately equal to the average improvement for all Metropolitan Area municipalities. On the other hand, municipalities located outside of the Metropolitan Area had, in total, mean welfare gains equal to 0.94%, whereas some of the less urban municipalities, which in turn are the poorest, reached welfare gains above 2% of their initial income. Just to serve as reference, low income households in Colombia expend on average 1.13%, 2.04% and 4.79% of their income on pension, health care and education, respectively (DANE, 2015). This illustrates how important are the welfare implications
of utility regulation; price changes in this sector may have huge effects on households’
welfare, especially on the poorest.

The remainder of the paper is organized as follows. Section 1 outlines the complete
endogenous Bayesian modeling strategy, the likelihood formulation as well as our prior
specification, and the deduction of the conditional posterior densities. Section 2 addresses
generalities of the Colombian energy market that are fundamental to the understanding
of our application. Section 3 deals with the microeconomic foundation of the Equiva-
lent Variation welfare measure, its ties to the econometric specification of the system of
equations and derives the measure for the specific case of a logarithmic demand function
taking into consideration a budget constraint for a two-tiered pricing scheme. Section 4
is divided into four subsections. The first of them presents summary statistics for the
data used in the econometric exercise. Secondly, we show specific characteristics of our
econometric specification for the application. The next subsection includes a summary of
our demand equation estimation results with some robustness checks regarding the spatial
structure. Our main findings are in the following subsection, which includes the analysis
of the posterior distribution of the welfare implications and its geographical patterns.
Finally, Section 5, presents our conclusions.

1 Econometric Approach

We propose an endogenous Bayesian approach using simultaneous equations with spatial
random effects, which account for unobserved heterogeneity and spatial dependence, in a
context where there is recursive endogeneity. In particular, we employ an instrumental
variable approach to handle the endogeneity issue. The specification of the model is

\[ y_i = \pi_0 + z'_{1i}\pi_1 + \alpha x_i + u_{1i} + v_i \]  

(1)

where \( y_i \) is the variable of interest that depends on a set of \( K \) exogenous controls \( z_{1i} \),
and an endogenous regressor \( x_i \), such that \( E(x_i u_{1i}) \neq 0 \), and as a consequence omitting this fact generates biased and inconsistent parameter estimates.

In addition, \( u_{1i} \) is an idiosyncratic stochastic shock, and \( v_i \) is a spatial random effect to control for spatial heterogeneity and spatial dependence between cross-sectional units. This dependence emerges due to clusters and/or spillovers effects between neighboring regional units, and allow us to control for unobservable spatial heterogeneity.

Given that we implement an instrumental variable approach to handle endogeneity, we set some exclusion restrictions in the main equation. These are associated with \( K_2 \) instrumental variables \( z_{2i} \) that do not affect \( y_i \) if \( x_i \) is held constant. Then,

\[
x_i = \phi_0 + z'_{1i} \phi_1 + z'_2 \alpha_s + u_{2i}
\]

where \( u_{2i} \) is an idiosyncratic stochastic shock, such that \((u_{1i}, u_{2i})' \sim N(0, \Omega)\), \( \Omega = \{\omega_{ij}\} \). Thus, \( \omega_{12} \) captures the endogeneity of the system (Greenberg, 2008).

At this instance, we should mention that an instrumental variable approach in the Bayesian framework has advantages compared with the Frequentist framework. For instance, Two-Stage Least Squares and Limited Information Maximum Likelihood have some difficulties dealing with weak instruments and small samples (Angrist and Pischke, 2008), whereas the Bayesian approach does not require asymptotic criteria, and works well using weak instruments due to the fact that the likelihood function and its identification are less important for deriving estimates in Bayesian models (Zellner, 1996, Imbens and Rubin, 1997, Zellner, 1998, Crespo-Tenorio and Montgomery, 2013).

One of the most popular spatial econometric alternatives to our specification is a SAR model. For instance, Ohtsuka et al. (2010) use it in a Bayesian econometric framework to model electricity demand in Japan in a spatiotemporal setting without taking into
consideration endogeneity issues. Unfortunately, a SAR specification cannot be used in a
two component disturbances decomposition (Parent and LeSage, 2008), like the one that
we propose. In addition, parameters estimates do not have an easy interpretation in SAR
models due to the presence of the spatial lags (Elhorst, 2014).

We should keep in mind that our final objective is to perform statistical inference re-
lated to complicated non-linear functions of parameter estimates, for instance the Equivalent Variation equations (20) and (21) in Section 3, which can be troublesome using a Frequentist approach. Therefore, this is another argument in favor of using a Bayesian framework to accomplish this task. In particular using Frequentist methods would require estimating equation (1) by means of Instrumental Variables (Rey and Boarnet, 2004, Kele-
jian and Prucha, 2004, Fingleton and Gallo, 2008, Crukker et al., 2013, Liu and Lee, 2013) or Generalized Method of Moments (Fingleton and Gallo, 2008, Crukker et al., 2013), and implementing spatial resampling algorithms (Lahiri and Zhu, 2006) or the Delta method (Casella and Berger, 2002) to find standard errors associated with functions of parameter estimates. These tasks are difficult and tedious, require extra computational effort, and more importantly, are based on asymptotic results. On the other hand, the Bayesian framework allows us to obtain full posterior distributions on all the parameters from equation (1), and using simple probabilistic rules, we obtain the posterior distributions of the functions of parameter estimates without any additional computational effort nor assumptions regarding asymptotic outcomes (Bernardo, 2003).

The likelihood function of the system is:

$$f(y, x|z_1, z_2; \Omega, \pi, \phi, \pi_0, \phi_0, v) = \frac{|\Omega \otimes I_N|^{-1/2}}{(2\pi)^{N/2}} \exp \left\{ -\frac{1}{2} \sum_{i=1}^{N} (y_i - \pi_0 - w_i'\pi - u_{1i} - v_i, x_i - \phi_0 + z_i'\phi - u_{2i})\Omega^{-1} \begin{pmatrix} y_i - \pi_0 - w_i'\pi - u_{1i} - v_i \\ x_i - \phi_0 + z_i'\phi - u_{2i} \end{pmatrix} \right\}$$

(3)

where $w_i' = [z_{1i}', x_i]$, $z_i' = [z_{1i}', z_{2i}']$, $\pi' = [\pi_1', \alpha]$ and $\phi' = [\phi_1', \alpha_s']$. 
Observe that introduction of spatial random effects, \( v_i, i = 1, 2, \ldots, N \), is another argument in favor of the Bayesian approach. In particular, the additional \( N \) parameters cannot be estimated by means of Maximum Likelihood methods due to the limited degrees of freedom (Seya et al., 2012). Thus, we follow a Bayesian hierarchical approach to model the spatial random effects where each unit is associated with a particular \( v_i \), and the conditional distributions of these parameters depend on their neighbors, through a contiguity matrix, and a precision parameter that is drawn from a Gamma distribution.

In particular, we assume that each spatial random effect has as prior distribution an improper (intrinsic) Conditionally Autoregressive (CAR) structure (Besag et al., 1991):

\[
 v_i | v_{i \sim j} \sim \mathcal{N} \left( \sum_{i \sim j} \frac{w_{ij} v_j}{\sum_{i \sim j} w_{ij}}, \frac{\sigma_v^2}{\sum_{i \sim j} w_{ij}} \right)
\]

(4)

where \( v_{i \sim j} \) is a vector composed by the spatial components of stochastic error of the neighbors \( j \) of \( i \) (\( i \sim j \)), and \( w_{ij} \) are the elements of the contiguity matrix which defines the spatial structure of the model. The joint distribution of the improper CAR is \( v \sim \mathcal{N}(\bar{v}, \sigma_v^2(I_N - W_n)^{-1}) \) where \( W_n \) is the contiguity matrix (Wall, 2004).

The contiguity relation is binary, that is, if region \( i \) and \( j \) are neighbors, the element \( i j \) is equal to 1 and 0 in other case, thus the contiguity matrix is symmetric, which is a requirement of the CAR model. By definition, the elements on the main diagonal of the contiguity matrix are set equal to 0. \( \sigma_v^2 \) is a parameter that defines the conditional variance of the spatial process, where the conditional variance must be inversely proportional to the number of neighbors.

There is spatial literature that favors CAR priors (Banerjee et al., 2004, Parent and LeSage, 2008, Darmofal, 2009, Chakraborty et al., 2013), and other that supports SAR prior specifications (Smith and LeSage, 2004, LeSage et al., 2007, LeSage and Llano, 2013). Our decision to use a CAR prior distribution to model the spatial random effects, instead of the SAR prior, is due to the fact that heteroscedasticity is inherent to the CAR specification, and we achieve a
higher level of heterogeneity (Cressie, 1993). In addition, the CAR specification is a Markovian process in space, that is, spatial heterogeneity is due to local variation, rather than a global spatial pattern, which is present in SAR specification (Anselin, 2003). Our intuition is that unobserved heterogeneity present in our application, which is related to residential electricity consumption in a municipality, is affected by the first order neighbors (see Section 4, Maps (2) and (3) and comments therein). Finally, a CAR prior offers computational convenience because we just need to work with its conditional distributions, avoiding matrix inversion. On the other hand, SAR models do not have full conditional distributions with a convenient form, and this enlarge computational burden (Banerjee et al., 2004).

It is well know that the joint distribution of the CAR process is improper, and although we can obtain a proper CAR process just introducing a single parameter, we work with an improper rather than a proper because the latter includes the spatial autocorrelation parameter that needs to lie in a specific region, usually between -1 and 1. As a consequence, the final solution using MCMC techniques becomes more complicated and computationally expensive, and we would need to use a Gibbs sampler with some steps of the Metropolis-Hastings algorithm (Greenberg, 2008). Additionally, this spatial autocorrelation term also limits the set of spatial patterns that the distribution can replicate and becomes much less intuitive (Banerjee et al., 2004).

We should have in mind that the improper CAR is identified only up to an additive constant, thus to identify the intercepts in our model, it is necessary to add the constraint \( \sum_{i=1}^{N} v_i = 0 \), as a consequence it is necessary to use improper uniform priors for the constant terms (\( \pi_0 \) and \( \phi_0 \)) in both equations.

To complete our Bayesian specification, we set the remaining priors as follow: \( \pi \sim N_{K_1+1}(\underline{\pi}, \underline{\Pi}) \) and \( \phi \sim N_{K_1+K_2}(\underline{\phi}, \underline{\Phi}) \). In our application, we set \( \underline{\pi} = 0_{K_1+1} \), \( \underline{\phi} = 0_{K_1+K_2} \), \( \underline{\Pi} = 1,000I_{K_1+1} \) and \( \underline{\Phi} = 1,000I_{K_1+K_2} \). This implies vague prior information where there is no effect of each control variable on dependent variables.

In addition, we assume a Wishart distribution for \( \Omega^{-1} \), that is \( \Omega^{-1} \sim W_2(\underline{\omega}, \underline{\Omega}) \). In par-
ticular, we set $\omega = 3$ and $\Omega = I_2$, where setting the degrees of freedom to $p + 1$, where $p$ is the dimension of the covariance matrix, the Wishart form reduces to $\pi(\Omega^{-1}) \propto |\Omega^{-1}|^{-(N+1)/2}$, which is a diffuse prior used by Savage that emerges using Jeffrey’s invariance theory (Zellner, 1996). Thus, a priori there is no endogeneity, and the fat-tailed prior will guarantee robustness of outcomes regarding this distribution (Berger, 1985).

To specify the prior distribution of the precision parameter of the CAR component, we must take into account that there are two different sources of stochastic variability in our main equation, $u_{1i}$ and $v_i$. As a consequence, both sets of hyperparameters of the prior distributions of these random effects cannot imply arbitrarily large variability, since these effects would be unidentifiable. We try to identify two random effects using a single observation at each spatial unit. Therefore, we cannot use arbitrarily vague prior distributions in our hierarchical approach. We propose a fair argument to construct the prior distribution of the precision parameter of the CAR component (Banerjee et al., 2004). Specifically, we establish a priori that the proportion of the variability due to spatial effects is 0.5, that is, we set $Var(u_{1i}) = Var(v_i)$. Thus, taking into consideration that $u_{1i} \sim N(0, \omega_{11})$ where $\omega_{11} \sim IG((\omega - 1)/2, 1/2)$ due to our prior assumptions, and $Var(u_{1i}) = \omega_{11} \approx \sigma^2_{u} \sum_{i \sim j} w_{ij} \text{Ave} \approx Var(v_i)$ (Bernardinelli et al., 1995) where $\left(\sum_{i \sim j} w_{ij}\right)^{\text{Ave}}$ is the average number of neighbors, we obtain $1/\sigma^2_{v} \sim \frac{1}{0.7^2 \left(\sum_{i \sim j} w_{ij}\right)^{\text{Ave}}} \mathcal{G}(\omega^{-1}/2, 1/2)$. Moreover, we find in our application that the posterior parameter estimates are robust to changes of hyperparameters of the CAR’s precision (available upon request).

We assume that the prior distributions are independent, that is,

$$
\pi(\Omega, \pi, \phi, \pi_0, \phi_0, v_i, \sigma^2_v) = \pi(\Omega)\pi(\pi)\pi(\phi)\pi(\pi_0)\pi(\phi_0)\pi(v_i|\sigma^2_v)\pi(\sigma^2_v)
$$

(5)
The full conditional posteriors for all parameters are as follow:

\[ \Omega^{-1} | \pi, \phi, \pi_0, \phi_0, v, \sigma_v^2, Data \sim W_2(\bar{\omega}, \bar{\Omega}) \]

\[ \bar{\omega} = \omega + N \]

\[ \bar{\Omega} = \left[ \Omega^{-1} + \sum_{i=1}^{N} \left( \begin{array}{c} y_i - \pi_0 - w_i' \pi - v_i \\ x_i - \phi_0 - z_i' \phi \end{array} \right) \left( y_i - \pi_0 - w_i' \pi - v_i, x_i - \phi_0 - z_i' \phi \right) \right]^{-1} \tag{6} \]

To sample \( \pi \), we use \( f(y_i, x_i | \Theta) = f(y_i | x_i, \Theta)f(x_i | \Theta) \) where \( \Theta = (\Omega, \pi, \phi, \pi_0, \phi_0, v) \). In particular, \( y_i | x_i, \Theta \sim N(\pi_0 + w_i' \pi + v_i + \frac{w_i z_i}{\omega_1^2} (x_i - \phi_0 - z_i' \phi), \psi_{11}) \) where \( \psi_{11} = \omega_1 + \frac{w_i^2}{\omega_2} \). Then,

\[ \pi | \Omega, \phi, \pi_0, \phi_0, v, \sigma_v^2, Data \sim N_{K_1+1}(\bar{\pi}, \bar{\Pi}) \]

\[ \bar{\Pi} = [\psi_{11}^{-1} + W W]^{-1} \]

\[ \bar{\pi} = \bar{\Pi} [\psi_{11}^{-1} \tilde{\pi} + W y_1] \tag{7} \]

where \( W \) is a \( N \times (K_1 + 1) \) matrix whose rows are \( w_i' \) and \( y_1 \) is a \( N \times 1 \) vector whose elements are \( y_i - \frac{w_i z_i}{\omega_2} (x_i - \phi_0 - z_i' \phi) - \pi_0 - v_i \).

We follow the same procedure to deduce the conditional posterior distribution of \( \phi \), that is, we use \( f(y_i, x_i | \Theta) = f(x_i | y_i, \Theta)f(y_i | \Theta) \). In particular, \( x_i | y_i, \Theta \sim N(\phi_0 + z_i' \phi + \frac{w_i z_i}{\omega_1^2} (y_i - \pi_0 - w_i' \pi - v_i), \psi_{22}) \) where \( \psi_{22} = \omega_2 + \frac{w_i^2}{\omega_1} \). Thus,

\[ \phi | \pi, \Omega, \pi_0, \phi_0, v, \sigma_v^2, Data \sim N_{K_1+K_2}(\bar{\phi}, \bar{\Phi}) \]

\[ \bar{\Phi} = [\psi_{22}^{-1} + Z Z]^{-1} \]

\[ \bar{\phi} = \bar{\Phi} [\psi_{22}^{-1} \tilde{\phi} + Z y_2] \tag{8} \]

where \( Z \) is a \( N \times (K_1 + K_2) \) matrix whose rows are \( z_i' \) and \( y_2 \) is a \( N \times 1 \) vector whose elements are \( x_i - \frac{w_i z_i}{\omega_1} (y_i - \pi_0 - w_i' \pi - v_i) - \phi_0 \).

Regarding the posterior distribution of the constant term \( \pi_0 \), using as prior an improper
uniform distribution and given $f(y_i, x_i|\Theta) = f(y_i|x_i, \Theta)f(x_i|\Theta)$, we obtain

$$\pi_0|\phi, \pi, \Omega, \phi_0, v, \sigma^2_v, Data \sim N(\bar{\pi}_0, \psi_{11}/N)$$

$$\bar{\pi}_0 = \frac{1}{N} \sum_{i=1}^{N} \left\{ y_i - \frac{\omega_{12}}{\omega_{22}} (x_i - \phi_0 - z_i^t \phi) - w_i^t \pi - v_i \right\}$$

In a similar way, using as prior an improper uniform distribution for $\phi_0$, and the fact that $f(y_i, x_i|\Theta) = f(x_i|y_i, \Theta)f(y_i|\Theta)$, we obtain

$$\phi_0|\pi_0, \phi, \pi, \Omega, v, \sigma^2_v, Data \sim N(\bar{\phi}_0, \psi_{22}/N)$$

$$\bar{\phi}_0 = \frac{1}{N} \sum_{i=1}^{N} \left\{ x_i - \frac{\omega_{12}}{\omega_{11}} (y_i - \pi_0 - w_i^t \pi - v_i) - z_i^t \phi \right\}$$

As we mentioned, we just need to use the conditional prior distribution to obtain the posterior distribution of the spatial random effects. In particular, using the fact that $f(y_i, x_i|\Theta) = f(y_i|x_i, \Theta)f(x_i|\Theta)$, we find that

$$v_i|v_{-i}, \phi_0, \pi_0, \phi, \pi, \Omega, \sigma^2_v, Data \sim N(\bar{\xi}_i, \bar{\eta}_i)$$

$$\bar{\eta}_i = \left[ \psi^{-1}_{11} + \left( \frac{\sigma^2_v}{w_{i+}} \right)^{-1} \right]^{-1}$$

$$\bar{\xi}_i = \bar{\eta}_i \left[ \frac{\sigma^2_v}{w_{i+}} \bar{v}_i \right]^{-1} \left( \sum_{i\sim j} \frac{w_{ij}}{w_{i+}} \bar{v}_j \right) + \psi_{11}^{-1} v_i^0$$

where $v_{-i}$ is the set of spatial random effects excluding region $i$, $w_{i+} = \sum_{i\sim j} w_{ij}$ and $v_i^0 = y_i - \frac{\omega_{12}}{\omega_{22}} (x_i - \phi_0 - z_i^t \phi) - \pi_0 - w_i^t \pi$. To identify $\pi_0$, we must add the constraint $\sum_{i=1}^{N} v_i = 0$. Therefore, this constraint must be imposed numerically by recentering each $v$ vector around its own mean following each Gibbs iteration.
Finally,

\[ \frac{1}{\sigma_v^2} | v, \phi_0, \pi_0, \phi, \pi, \Omega, Data \sim \mathcal{G}(\bar{\alpha}, \bar{\beta}) \]

\[ \bar{\alpha} = \frac{\omega - 1}{2} + \frac{1}{2} \]

\[ \bar{\beta} = \frac{1}{2} + \frac{w_i+}{2} \left( v_i - \sum_{i \sim j} \left( \frac{w_{ij}}{w_i+} \right) v_j \right)^2 \]  

(12)

We generate samples from the distributions 6 to 12 using the Gibbs sampler algorithm (Geman and Geman, 1984).

2 The Colombian energy market

To better understand our application and the microeconomic foundation of it, there are some characteristics of the Colombian electricity market that must be taken into consideration. In particular, we must explain the price changes, and thus, their welfare implications. First, the Colombian law segments its population into socioeconomic strata. This segmentation is defined as “an instrument that allows a municipality or district to classify its population in distinct groups or strata with similar social and economic characteristics.” (Bushnell and Hudson, 1996). This classification was actually initiated to establish cross-subsidies that would help the lower socioeconomic classes to pay for utilities such as electricity. Housing characteristics are the main criteria used for classifying the population into six strata in total: one represents lower-low, two is low, three is upper-low, four is medium, five is medium-high and six is high. Second, the Colombian energy regulator establishes a subsistence electricity consumption that is subsidized for strata one, two and three. The regulator determines the maximum subsidy percentage, and each municipality defines its own measure within this limit. In addition, the subsistence consumption level depends on whether the altitude of the municipality exceeds one thousand meters above sea level or not, due to weather conditions that may affect electricity consumption. Municipalities located near sea level have higher temperatures, and as a consequence they present a higher electricity consumption. Specifically, the subsistence consumption is 173 kWh a month.
per household for the municipalities below this threshold and 130 kWh for the ones above it.\footnote{Resolution 0355 by the Mining/Energy Planning Unit (UPME).}

Third, the Colombian energy regulator stipulates that each electricity company must have the same reference tariff in all its market, which involves many municipalities. And fourth, there are basically four components to establish the reference electricity tariff for each company: electricity generation, transport at country level, distribution at market level and commercialization. As a consequence of this regulation framework, we should have in mind that, although there is just one reference tariff for each electricity company, there are different average electricity prices among consumers of different strata and municipalities.

The acquisition of EADE by EPM implied a tariff unification process that generated welfare effects, especially for the households belonging to stratum one. These households’ electricity consumption is approximately 5% of its income in the province of Antioquia, whereas this share is less than 1% for stratum six. In particular, there is EPM, whose market was characterized as an urban region with high population density, and on the other hand is EADE, whose market was a rural area with low population density. Under the Colombian electricity regulation framework, \textit{ceteris paribus}, these market structure differences imply a higher reference tariff in the latter company compared with the former. This is because of the third and four components of the reference tariff: distribution at market level and commercialization. Thus, the acquisition of EADE by EPM implied that the stratum one electricity consumers of the former company experienced a huge decrease in the electricity charge, while the consumers of the latter company faced a slight increase.\footnote{Regulation stipulates that strata one and two cannot have a tariff increase higher than inflation rate.} As a consequence, these changes generated considerable welfare impacts on the poorest inhabitants of the province of Antioquia who live in the rural areas.

## 3 Microeconomic Foundation: Equivalent Variation

We apply our proposed methodology to analyze the welfare changes arising from tariff unification in the municipalities of Antioquia using an Equivalent Variation (EV) approach. The Equivalent Variation measures the “amount that the consumer would be indifferent about accepting in
lieu of the price change; that is, it is the change in her wealth that would be equivalent to
the price change in terms of its welfare impact” (Mas-Colell et al., 1995). The EV presents
several advantages over other welfare measures used in applied economic work, such as the
Compensating Variation (CV) and Consumer Surplus (CS). In particular, Chipman and Moore
(1980) and Mas-Colell et al. (1995) show that the EV is the appropriate measure to correctly
order different pricing policies in welfare analysis. The CV orders alternatives correctly only
when consumers exhibit homothetic preferences and income remains unchanged. However, in
our empirical application tariff unification translates into implied subsidies for some consumers,
and therefore, changes in income. Another argument in favor of the EV is that, by definition, it
is an ex-post measure of welfare based on the Hicksian demand function. It accounts for income
effects associated with price changes that are ignored by the Marshallian demand function on
which the CS is based on. Furthermore, Hausman (1981) showed that it was possible to derive
EV as a product of observable Marshallian demand functions. His method can be applied to
the case of linear budget constraints, and both Reiss and White (2006) and Ruijs (2009) extend
it to the case of budget constraints generated by block-pricing systems using linear demand
functions.

To build our application, we consider the two-good case in which a representative agent
consumes a good \( x \), say electricity, and an aggregate good as a numeraire \( (x_a) \). We note that,
for our application, the representative agent assumption is not as restrictive as it may seem. In
particular, given that we work with stratum one population at the municipality level, a fairly
homogeneous group within each polygon, the assumption that agents with similar preferences can
be aggregated into a single agent per municipality is not unthinkable. This could be thought of
as a case of dispersion in preferences and income where, although individuals may present erratic
utility functions, the aggregate demand for the commodities of interest are well-behaved (Trockel,
1987). In addition, representative agent models dominate microfounded macroeconomics due
to its simplicity and tractability (Acemoglu, 2008). One final argument is the impossibility of
obtaining data at the micro-level to correct for the bias raised by agent heterogeneity. Therefore,
although we are aware of the disadvantages of the representative agent (Kirman, 1992, Reiss
and White, 2006), we will continue to work under this assumption.
Throughout this paper we will indicate a situation before or after tariff unification with subscript 0 or 1, respectively. Subsistence consumption will be denoted by $\bar{x}$. This subsistence consumption divides demand into two possible tiers, denoted by superscript 1 when the consumer demands a quantity less than $\bar{x}$ and 2 when it is greater than $\bar{x}$. As shown in Varian (1992), EV can be represented in terms of the expenditure function as

$$EV(p_0, p_1, y_0) = e(p_0, u_1) - e(p_1, u_1) = e(p_0, u_1) - y_0$$

(13)

with $p_i = (p_1^i, p_2^i, 1)$, $i = 0, 1$ and $e(p_0, u_1)$ the expenditure needed to reach the utility level $u_1$ when facing prices $p_0$. Call $x_1$ the new demand at prices $p_1$ and income $e(p_1, u_1) = y_0$. Tangency between $u_1$ and the budget curve characterized by $p_0$ and income $e(p_0, u_1)$ is referred to as virtual consumption ($x_e$, see Figure 1). As in Ruijs (2009) and You and Lim (2013), we assume that new consumption $x_1$ and virtual consumption $x_e$ fall on the same tier. Therefore, utility level $u_1$ and expenditures $e(p_0, u_1)$ are such that (i) $x_1, x_e < \bar{x}$ or (ii) $x_1, x_e > \bar{x}$. For the first case:

$$u_1 = V((p_1^1, 1), y_0)$$
with $V((p_1^1, 1), y_0) = \max\{u(x) : p_1^1 x_1 + x_a \leq y_0\}$, the indirect utility function. At this utility level, $e(p_0, u_1) = e(p_1^1, u_1)$ and Equivalent Variation can be written as

$$EV(p_0, p_1, y_0) = e(p_0^1, u_1) - y_0$$

(14)

For the second case, when $x_1$ is in the second tier, the utility level associated with this consumption is

$$u_1 = V((p_1^2, 1), y_0 + (p_1^2 - p_1^1)\bar{x})$$

A consumer pays $p_1^1$ for consumption up to $\bar{x}$ and $p_1^2$ after, up to $x_1$, which means consumers save $(p_1^2 - p_1^1)\bar{x}$. Expenditures $e(p_0, u_1)$ are also subject to change since an amount of $(p_0^2 - p_0^1)\bar{x}$ less is needed to acquire $x_e$ and therefore, $e(p_0, u_1) = e(p_0^2, u_1) - (p_0^2 - p_0^1)\bar{x}$. In this context, $e(p_0^2, u_1)$ is the virtual expenditure under the assumption of a flat rate of $p_0^2$ throughout the whole system, and $(p_0^2 - p_0^1)\bar{x}$ is an implicit subsidy perceived by consumers in the second block. Under these conditions, the Equivalent Variation turns out to be

$$EV(p_0, p_1, y_0) = e(p_0^2, u_1) - (p_0^2 - p_0^1)\bar{x} - y_0$$

(15)

In this paper we ignore the possibility that $x_e$ is located exactly at the kink $\bar{x}$ since this is not the case for any of the municipalities we analyze.\(^3\) Given that any alternative represents a convex budget set and that we estimate a quasi-concave demand function, we can focus all of our attention to the two cases stated above (Hausman, 1985, Moffitt, 1990).\(^4\)

Hausman (1981) developed a method that relates the econometric specification of a Marshallian demand function to the definitions stated above. He starts with Roy’s identity:

$$x(p, y) = -\frac{\partial V(p, y)/\partial p}{\partial V(p, y)/\partial y}$$

(16)

\(^3\)See Ruijs (2009) and You and Lim (2013) for EV under this condition.

\(^4\)A kinked budget constraint represents a convex budget set when $p^2 \geq p^1$. 
For our case, \( x(p, y) \) is specified as an exponential function

\[
x(p, y) = p^\alpha y^{\delta_1} e^{z' \delta}
\]  

(17)

with \( \alpha, \delta_1 \) and \( \delta \), the price elasticity of demand, income elasticity of demand and vector of coefficients for covariates \( z \), respectively. The next step is to solve the differential equation implicit in (16), by using the separation of variables technique, to find

\[
V(p, y) = c = -e^{z' \delta} \frac{p^{1+\alpha}}{1 + \alpha} + \frac{y^{1-\delta_1}}{1 - \delta_1}
\]  

(18)

Setting the constant of integration \( c = \bar{u} \) and solving for \( y \), results in the expenditure function in terms of \( p \) and \( \bar{u} \)

\[
e(p, \bar{u}) = \left[ \frac{1 - \delta_1}{1 + \alpha} \left( \bar{u} + e^{z' \delta} \frac{p^{1+\alpha}}{1 + \alpha} \right) \right]^{\frac{1}{1-\delta_1}}
\]  

(19)

Now all we need to do is plug equations (18) and (19) into (14) and (15) using the appropriate prices and income to find expressions for the Equivalent Variation in terms of the parameters of our proposed model. For case (i) with \( u_1 = V(p_1, y_0) \)

\[
u_1 = -e^{z' \delta} \frac{p_1^{1(1+\alpha)}}{1 + \alpha} + \frac{y_0^{1-\delta_1}}{1 - \delta_1}
\]

\[
e(p_0, u_1) = \left[ \frac{1 - \delta_1}{1 + \alpha} \left( p_0^{1(1+\alpha)} - p_1^{1(1+\alpha)} \right) e^{z' \delta} + y_0^{1-\delta_1} \right]^{\frac{1}{1-\delta_1}}
\]

and the Equivalent Variation

\[
EV(p_0, p_1, y_0) = \left[ \frac{1 - \delta_1}{1 + \alpha} \left( p_0^{1(1+\alpha)} - p_1^{1(1+\alpha)} \right) e^{z' \delta} + y_0^{1-\delta_1} \right]^{\frac{1}{1-\delta_1}} - y_0
\]  

(20)

For case (ii) with \( u_1 = V(p_2, y_0 + (p_2 - p_1)\bar{x}) \)

\[
u_1 = -e^{z' \delta} \frac{p_2^{1(1+\alpha)}}{1 + \alpha} + \frac{(y_0 + (p_2 - p_1)\bar{x})^{1-\delta_1}}{1 - \delta_1}
\]

\[
e(p_0, u_1) = \left[ \frac{1 - \delta_1}{1 + \alpha} \left( p_0^{2(1+\alpha)} - p_1^{2(1+\alpha)} \right) e^{z' \delta} + (y_0 + (p_2 - p_1)\bar{x})^{1-\delta_1} \right]^{\frac{1}{1-\delta_1}}
\]
and the Equivalent Variation

\[ EV(p_0, p_1, y_0) = \left[ \frac{1 - \delta_1}{1 + \alpha} \left( p_0^{2(1+\alpha)} - p_1^{2(1+\alpha)} \right) e^{\bar{z} \delta} + \left( y_0 + (p_2^1 - p_1^1) \bar{x} \right)^{1-\delta_1} \right]^{\frac{1}{1-\delta_1}} - \]

(21)

We can see from equations (20) and (21) that a price decrease of an inelastic normal good produces welfare gains that can be quantified through a positive Equivalent Variation. Equation (21) takes into consideration that the subsidy has effects on the expenditure function as well as in the agent’s income.

4 Results

4.1 Data

Data was collected for the average individual of stratum one in all the 125 municipalities of the department of Antioquia (Colombia) in 2005. Table (A.1) in the Appendix, lists all the variables, their measurement units and sources. Stratum level income was constructed from regional participation in total production of the province and population (Ramírez and Londoño, 2009). We standardized both electricity and substitute good prices to US$/kWh by taking their calorific power into account. For municipalities in the Metropolitan Area, the substitute good was natural gas. For the other municipalities, it was liquefied petroleum gas due to absence of natural gas. In addition, we have to mention that by construction, the average price is affected by electricity consumption because average electricity tariff is a weighted average between tariffs in the first and second tiers where the weights depend on the observed and subsistence consumption levels. This generates the endogeneity problem in our application.

We present descriptive statistics in Table (1). Average annual electricity consumption is 234.87 kWh with a standard deviation of 117.81 kWh. Prices for electricity and the substitute good averaged 6.10 and 3.00 cents a kWh, respectively. Additionally, average annual per capita income was US$397 exhibiting a standard deviation of US$95.24. The maximum values for both electricity consumption and average annual income were located at the municipality of Envigado,
which is located in the Metropolitan Area. The lowest value for consumption was found at Vigía del Fuerte, a municipality that faces many adversities due to its geographical location. El Bagre was found to have the lowest per capita income. Regarding electricity and substitute prices, the maximum values were located at Enterrios and Chigorodó respectively. On the other hand, minimum values were located in San José de la Montaña and Venecia. Approximately 29% of the municipalities in the province Antioquia are located less than 1000 meters above sea level, the average urbanization rate is 45.8%, and 77.4% of the municipalities used to be covered by EADE prior to its acquisition by EPM.

### Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption (kWh)</td>
<td>234.874</td>
<td>117.811</td>
<td>26.595</td>
<td>588.937</td>
</tr>
<tr>
<td>Electricity Price (US$)</td>
<td>0.061</td>
<td>0.024</td>
<td>0.027</td>
<td>0.240</td>
</tr>
<tr>
<td>Income (US$)</td>
<td>397.085</td>
<td>95.242</td>
<td>230.514</td>
<td>619.227</td>
</tr>
<tr>
<td>Substitute Price (US$)</td>
<td>0.030</td>
<td>0.006</td>
<td>0.016</td>
<td>0.056</td>
</tr>
<tr>
<td>Sea level</td>
<td>29.032%</td>
<td>45.575%</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Urbanization</td>
<td>45.876%</td>
<td>19.917%</td>
<td>10.700%</td>
<td>98.247%</td>
</tr>
<tr>
<td>Coverage (EADE)</td>
<td>77.419%</td>
<td>41.981%</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

*Source: Author’s calculations*

We can observe in Map (2) the geographical distribution of the electricity consumption. In particular, average consumption of electricity tends to be higher in regions that are located less than one thousand meters above sea level. Consumption is also exceptionally high in the Metropolitan Area of Antioquia (South-Central region), where most of the population and economic activity of the province are focused.

We can see in Map (3) that most of the spatial autocorrelation is due to local clusters. This obeys to unobserved social, cultural, economical and geographical restrictions, like limited and bad roads between municipalities or constrained budgets that poor households face in these municipalities. Avoiding a global spatial effect regarding electricity consumption for the inhabitants of the province appears to be the most natural approach as is provided by the CAR specification.
**Figure 2:** Average Annual Electricity Consumption per Household (kWh): Province of Antioquia (Colombia) in 2005, Stratum One

Source: Empresas Públicas de Medellín

**Figure 3:** Local Moran’s I test p-values of Average Annual Electricity Consumption per Household (kWh): Province of Antioquia (Colombia) in 2005, Stratum One

Source: Authors’ calculations
4.2 Model Specification

We need to estimate the electricity demand function to perform statistical inference of the Equivalent Variation. However, it is necessary to take into account the endogeneity issue between price and consumption to avoid biased and inconsistent parameter estimates. We use as instrument a dummy variable that is equal to 1, if the municipality was serviced by EADE, and 0 otherwise. The argument behind this instrument is that the national electricity regulation generates restrictions that imply that the only effect of the electricity supplier on average consumption in each municipality is just through price. However, regional market reference tariff, and as a consequence average electricity price of low strata in each municipality, is drastically affected by each supplier.

The structural specification of our system is

\[
\ln x_i = \delta_0 + z'_i \delta + \alpha \ln p_i + u_{1i} + v_i \quad (22)
\]

\[
\ln p_i = \phi_0 + z'_i \phi + \alpha_s z_{2i} + u_{2i} \quad (23)
\]

where \(x_i\) and \(p_i\) are electricity consumption and price, \(z'_i = (\ln y_i, \ln p_{s,i}, alt_i, \ln urb_i)\) is a vector of exogenous covariates that affects the system (income, substitute price, sea level dummy and urbanization rate) and \(z_{2i} = EADE_i\) is our instrument. Additionally, \(\delta_0, \delta' = (\delta_1, \delta_2, \delta_3, \delta_4)\), \(\phi_0, \phi' = (\phi_1, \phi_2, \phi_3, \phi_4)\), \(\alpha\) and \(\alpha_s\) are parameters to estimate. Finally, \(u_{1i}\) and \(u_{2i}\) are the idiosyncratic error terms associated with the demand and price of each municipality, and \(v_i\) are spatial random effects to control for spatial heterogeneity and spatial dependence between cross-sectional units that is present in our application (see Map (3)). These emerge due to clusters and/or spillovers effects between neighboring municipalities, and allow us to control for unobservable spatial heterogeneity. Omitting this last component can cause loss of good statistical properties of estimators (Anselin, 1988).

Using just one instrument allows us to exactly identify the structural parameters from a reduced model. As a consequence, the model can be estimated using the reduced form equations.
that result from replacing (23) into (22)

\[
\ln x_i = \pi_0 + \pi_1 \ln y_i + \pi_2 \ln p_i + \pi_3 \text{alt}_i + \pi_4 \ln \text{urb}_i + \pi_5 EADE_i + \mu_{1i} + v_i
\]

\[
\ln p_i = \phi_0 + \phi_1 \ln y_i + \phi_2 \ln p_i + \phi_3 \text{alt}_i + \phi_4 \ln \text{urb}_i + \alpha s EADE_i + \mu_{2i}
\]

where \(\mu_{1i} = u_{1i} + \alpha u_{2i}\) and \(\mu_{2i} = u_{2i}\), such that \((\mu_{1i}, \mu_{2i})' \sim N(0, \Omega)\),

\[
\Omega = \begin{bmatrix}
\sigma_1^2 + \alpha^2 \sigma_2^2 + 2\alpha \sigma_{12} & \sigma_{12} + \alpha \sigma_2^2 \\
\sigma_{12} + \alpha \sigma_2^2 & \sigma_2^2
\end{bmatrix} = \begin{bmatrix}
\omega_{11} & \omega_{12} \\
\omega_{12} & \omega_{22}
\end{bmatrix}
\]

The structural parameters can be recovered from the following equations:

\[
\begin{align*}
\delta_0 &= \pi_0 - \phi_0 \alpha \\
\delta_1 &= \pi_1 - \phi_1 \alpha \\
\delta_2 &= \pi_2 - \phi_2 \alpha \\
\delta_3 &= \pi_3 - \phi_3 \alpha \\
\delta_4 &= \pi_4 - \phi_4 \alpha \\
\alpha &= \frac{\pi_5}{\alpha s}
\end{align*}
\]

Setting \(z_i' = (\ln y_i, \ln p_i, \text{alt}_i, \ln \text{urb}_i, EADE_i)\), \(\pi' = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)\), \(\phi' = (\phi_1, \phi_2, \phi_3, \phi_4, \alpha s)\) and \(\nu' = (v_1, v_2, \ldots, v_n)\), and taking into consideration that the determinant of the Jacobian matrix is 1, the likelihood function of the system is:

\[
f(\ln x, \ln p|z; \Omega, \pi, \phi, \pi_0, \phi_0, \nu) = \frac{|\Omega|^{-N/2}}{(2\pi)^{N/2}} \times 
\exp \left\{ -\frac{1}{2} \sum_{i=1}^{N} (\ln x_i - z_i' \pi - \pi_0 - v_i, \ln p_i - z_i' \phi - \phi_0 - v_i) \Omega^{-1} (\ln x_i - z_i' \pi - \pi_0 - v_i, \ln p_i - z_i' \phi - \phi_0 - v_i) \right\}
\]

We can estimate our endogenous Bayesian model with spatial random effects using this likelihood, the prior distributions shown in Section (1), and the conditional posterior distributions developed therein, where we must take into consideration that \(w_i' = \mathbf{z}_i' = (\ln y_i, \ln p_i, \text{alt}_i, \ln \text{urb}_i, EADE_i)\).

### 4.3 Estimation Results

Since the solution for the model depends on the election of the contiguity matrix, we will test our specification under 3 different matrices. The first one uses road lengths between each
municipality, regards two regions to be neighbors if the roads connecting them are less than 300 kilometers long, which ensures that each region has at least one neighbor. The second is based on the queen criterion, where two regions are considered as neighbors if they share at least a single border point. The third one enforces the rook criterion, where regions are considered neighbors if they share more than one border point.

We estimate each of our models using Markov chain Monte Carlo techniques (MCMC, see Robert and Casella, 2004, for details). In particular, we use the Gibbs sampling algorithm due to having all the conditional posterior distributions (Geman and Geman, 1984). After running the chains for 10 million iterations, we discard the first 5 million and draw an observation every 500 iterations to get an effective sample size of 10,000. This procedure allows us to keep autocorrelation to a minimum and ensure convergence of the chains (See Table (B.1) in Appendix subsection B).

The correlation of the instrument to the logarithm of price is approximately -0.46, its variability is very low due to being a dummy variable, a standard deviation equal to 0.42, and its 90% Probability Highest Density interval in the price equation is (−0.48, −0.22) with a mean and median equal to -0.35. However, even if this instrument were weak, the Bayesian approach works well in this context using proper priors due to the fact that the likelihood function and its identification are less important for deriving estimates in Bayesian models (Zellner, 1996, Imbens and Rubin, 1997, Zellner, 1998, Crespo-Tenorio and Montgomery, 2013).

We present in Table (2) the main outcomes of our estimation exercises using different contiguity criteria to check robustness regarding this issue. And as we can see, all contiguity criteria present similar results. In particular, we show the posterior mean and median, which minimize the quadratic and absolute value loss functions under a decision theory framework, respectively. In addition, to describe the inferential content of the posterior distributions of the parameters, there are the 90% Highest Probability Density credible intervals for each parameter of interest.

---

5 All the simulations exercises and posterior analysis were performed using R (R Core Team, 2014).

6 As this instrument is a Bernoulli random variable, its maximum standard deviation is equal to 0.5. This would be a limitation of using dummy variables as instruments in a Frequentist approach.
Finally, to test if microeconomic restrictions are compatible with observed data, we calculate the odds ratio in favor of the null hypothesis $H_0 : \theta \in (0, \infty)$ versus $H_1 : \theta \in (-\infty, 0]$ using 0.5 as prior probabilities for each of these hypotheses. This procedure is consistent with a symmetric loss function, for instance a zero–one loss function (Berger, 1985, Zellner, 1996). Testing microeconomic restrictions is very important in this setting because our main objective is to perform statistical inference regarding Equivalent Variation, and so, there are some implicit restrictions placed on the parameter estimates. Thus, we follow a statistical decision theory framework, where an action regarding the domain of the posterior densities must be made. These actions are based on prior and sample information.

Regarding endogeneity in our application, we find that posterior median estimates of $\omega_{12}$ are approximately -0.06 using different contiguity criteria, and the Highest Probability Intervals at 90% of credibility are $(-0.100, -0.022)$, $(-0.088, -0.043)$ and $(-0.096, -0.019)$ using roads, queen and rook contiguity criteria, respectively. This evidence suggests that there is endogeneity between electricity consumption and price.

Given that we obtain robust outcomes regarding contiguity criteria, we discuss the results associated with the road length criterion. This criterion better illustrates the connectivity between municipalities in a province that is characterized by irregular geographical conditions and bad roads. Thus, when we observe the posterior mean and median, we see that all those point estimates have the expected signs. Electricity behaves as both an ordinary and normal good, given the negative price-demand elasticity and positive income-demand elasticity, respectively. For instance, the average as well as the median price demand elasticity is -0.88, that is, an increase of 1% in electricity price implies a reduction of 0.88% in consumption. In addition, the average and median income elasticity is approximately 0.30, which implies that a 1% income increase means a 0.30% increase in electricity consumption. Regarding the Highest Probability Density credible intervals of these parameters, we have that these are $(-1.45, -0.28)$ and $(-0.05, 0.64)$ for the price elasticity and income elasticity, respectively. In addition, we calculate the inverse odds ratio in favor of the null hypothesis $H_0 : \alpha \in (-\infty, 0]$ to check the microeconomic restriction of a negative price elasticity. This is equal to 0.014 that means an odds ratio
Table 2: Summary of structural parameter posterior estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Median</th>
<th>90% HPD Interval</th>
<th>$R_{01} = \frac{P(\theta \in (0, \infty))}{P(\theta \in (-\infty, 0))}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Road Length Contiguity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.913</td>
<td>1.940</td>
<td>-1.539 5.393</td>
<td>4.663</td>
</tr>
<tr>
<td>Price</td>
<td>-0.886</td>
<td>-0.882</td>
<td>-1.449 -0.278</td>
<td>0.014</td>
</tr>
<tr>
<td>Income</td>
<td>0.301</td>
<td>0.297</td>
<td>-0.054 0.636</td>
<td>11.920</td>
</tr>
<tr>
<td>Subs. Price</td>
<td>0.123</td>
<td>0.120</td>
<td>-0.215 0.449</td>
<td>2.560</td>
</tr>
<tr>
<td>Altitude</td>
<td>0.139</td>
<td>0.137</td>
<td>-0.041 0.304</td>
<td>9.235</td>
</tr>
<tr>
<td>Urbanization</td>
<td>0.571</td>
<td>0.566</td>
<td>0.410 0.724</td>
<td>908.090</td>
</tr>
<tr>
<td><strong>Queen Contiguity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.873</td>
<td>1.961</td>
<td>-2.081 5.704</td>
<td>4.038</td>
</tr>
<tr>
<td>Price</td>
<td>-0.877</td>
<td>-0.876</td>
<td>-1.570 -0.200</td>
<td>0.027</td>
</tr>
<tr>
<td>Income</td>
<td>0.308</td>
<td>0.298</td>
<td>-0.068 0.701</td>
<td>9.941</td>
</tr>
<tr>
<td>Subs. Price</td>
<td>0.117</td>
<td>0.117</td>
<td>-0.276 0.488</td>
<td>2.331</td>
</tr>
<tr>
<td>Altitude</td>
<td>0.130</td>
<td>0.135</td>
<td>-0.097 0.391</td>
<td>5.826</td>
</tr>
<tr>
<td>Urbanization</td>
<td>0.575</td>
<td>0.566</td>
<td>0.375 0.751</td>
<td>139.840</td>
</tr>
<tr>
<td><strong>Rook Contiguity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.956</td>
<td>1.965</td>
<td>-2.126 5.662</td>
<td>4.061</td>
</tr>
<tr>
<td>Price</td>
<td>-0.818</td>
<td>-0.876</td>
<td>-1.567 -0.189</td>
<td>0.027</td>
</tr>
<tr>
<td>Income</td>
<td>0.297</td>
<td>0.298</td>
<td>-0.095 0.674</td>
<td>9.834</td>
</tr>
<tr>
<td>Subs. Price</td>
<td>0.084</td>
<td>0.117</td>
<td>-0.283 0.482</td>
<td>2.328</td>
</tr>
<tr>
<td>Altitude</td>
<td>0.178</td>
<td>0.135</td>
<td>-0.104 0.384</td>
<td>5.775</td>
</tr>
<tr>
<td>Urbanization</td>
<td>0.575</td>
<td>0.565</td>
<td>0.377 0.756</td>
<td>124.000</td>
</tr>
</tbody>
</table>

*Source: Author’s calculations*

supporting $H_0$ equal to 71.42, which implies that $\log_{10}(R_{01}) = 1.85$. Thus, we have very strong evidence for $H_0$ following Jeffreys’ guidelines (Greenberg, 2008). Regarding the null hypothesis of a positive income elasticity, $H_0 : \delta_1 \in (0, \infty)$, which is suggested by most of the literature about electricity demand (Hsiao and Mountain, 1985, Dergiades and Tsoulfidis, 2008), we have $\log_{10}(R_{01}) = 1.07$ indicating strong evidence for $H_0$.

Regarding cross elasticity with the substitute good, although this is positive in average, a
1% increase in substitute price implies a 0.12% increase in electricity demand, there is weak evidence for $H_0: \delta_2 \in (0, \infty)$ due to the fact that $\log_{10}(R_{01}) = 0.40$. Probably, this is because of the lack of electricity substitutes in rural areas, or the fact that demand for electricity is derived for most household appliances which cannot function with anything but electricity. For this parameter, we observe a HPD credible interval between -0.21 and 0.45. The mean altitude semi-elasticity is equal to 0.14, which means that municipalities located at lower altitude demand approximately 14% more electricity, *ceteris paribus*. In this case, we have $\log_{10}(R_{01}) = 0.97$, that is substantial evidence for $H_0: \delta_3 \in (0, \infty)$. Finally, there is the urbanization rate that has a strong positive effect on electricity consumption as one would expect, $\log_{10}(R_{01}) = 2.96$ for $H_0: \delta_4 \in (0, \infty)$, that means decisive support for $H_0$. The median and mean urbanization rate elasticity is approximately 0.57 with a Highest Probability Density credible interval equal to (0.41, 0.72).

Despite that our prior assumption regarding the participation of the spatial effects on electricity consumption variability is 50%, we find that the posterior proportion is 13%. This outcome is robust to many hyperparameter combinations of the prior distribution of the precision parameter of the CAR component (available upon request).

Finally, we compute several diagnostics to assess convergence and stationarity of the chains. In particular, we compute the method due to Heidelberger and Welch (1983), the mean difference test proposed by Geweke (1992) and the diagnostic from Raftery et al. (1992). We show that in general all chains under different contiguity criteria achieve convergence and stationarity in the Table (B.1) in Appendix subsection B.

### 4.4 Welfare Implications

The tariff unification procedure brought forth by the acquisition of EADE by EPM, created tier price variations that depended on whether the municipality was part of the Metropolitan Area or not. In particular, by the end of this process, $p_1^1$ and $p_2^2$ changed according to the values in Table (3) with respect to their pre-unification values. We expect to see that the municipalities which consume below the subsistence consumption and are not a part of the Metropolitan Area
have the largest welfare gains, followed by those that are not part of the Metropolitan Area and have average consumptions higher than the subsistence consumption. The welfare effects in the municipalities that belong to the Metropolitan Area are not clear and will depend on whether they consume above subsistence consumption or not, and how much of their consumption is above this threshold, amongst other factors (Ramírez and Londoño, 2009). Here, we note that subsistence consumption is measured in kilowatts/hour a month per household. Therefore, in order to make it comparable with our measure of income, we work with an annual per capita consumption for each altitude.\footnote{The original levels were multiplied by 12 to obtain an annual measure and then divided by an average of 4.04 people per household in stratum one to get our variable of interest.}

Table 3: Tariff variations due to unification

<table>
<thead>
<tr>
<th>Location</th>
<th>$p_1^1$</th>
<th>$p_1^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metropolitan Area</td>
<td>-0.33%</td>
<td>8.12%</td>
</tr>
<tr>
<td>Rest</td>
<td>-17.53%</td>
<td>-0.95%</td>
</tr>
</tbody>
</table>

Source: Author’s calculations

To compute the posterior distribution of the Equivalent Variation, we follow the guidelines of the Bayes theorem, and renormalize the unrestricted posterior distribution of each parameter according to the outcomes of the microeconomic restrictions in Table (2) where statistical evidence suggests accomplishment of these restrictions (Berger, 1985, Bernardo, 2003). This allows us to obtain sensible results, based on a statistical decision theory framework, regarding the Equivalent Variation, which is calculated for each municipality at each observation of these new chains, through expressions (20) and (21). This procedure leaves an effective sample size of 5,410 with which to do the computations.

Table (4) lists the mean, median and 90\% Highest Probability Density interval for the total, Metropolitan Area and rest of Antioquia as a share of original income $y_0$. As can be observed, the median Equivalent Variation in the whole province is approximately 0.63\%, and its standard error is approximately 0.67\%. This welfare gain is lower in the Metropolitan Area (0.14\%) and greater in the rest of the province (0.65\%). The 90\% HPD interval is equal to (0.0\%, 1.9\%).
and the posteriors tend to be skewed to the right as the mean is greater than the median. For stratum one households, this impact can be very substantial, especially for those who are located in regions other than the Metropolitan Area of Antioquia.

Table 4: Equivalent Variation as share of income by Total, Metropolitan Area and Rest

<table>
<thead>
<tr>
<th>Road Length Contiguity</th>
<th>Equivalent Variation</th>
<th>Mean</th>
<th>Median</th>
<th>90% HPD Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>Lower</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M. Area</td>
<td>0.126%</td>
<td>0.141%</td>
<td>0.006%</td>
<td>0.209%</td>
</tr>
<tr>
<td>Rest</td>
<td>0.940%</td>
<td>0.655%</td>
<td>0.257%</td>
<td>2.006%</td>
</tr>
<tr>
<td>Total</td>
<td>0.874%</td>
<td>0.630%</td>
<td>0.005%</td>
<td>1.913%</td>
</tr>
</tbody>
</table>

| Queen Contiguity       | Equivalent Variation | Mean  | Median | 90% HPD Interval |
|------------------------|                      | Mean  | Median | Lower           |
|                        |                      | Upper |        |                 |
| M. Area                | 0.127%               | 0.141% | 0.005% | 0.212%          |
| Rest                   | 0.937%               | 0.653% | 0.256% | 2.003%          |
| Total                  | 0.872%               | 0.628% | 0.004% | 1.907%          |

| Rook Contiguity        | Equivalent Variation | Mean  | Median | 90% HPD Interval |
|------------------------|                      | Mean  | Median | Lower           |
|                        |                      | Upper |        |                 |
| M. Area                | 0.127%               | 0.141% | 0.005% | 0.212%          |
| Rest                   | 0.936%               | 0.653% | 0.257% | 2.000%          |
| Total                  | 0.871%               | 0.627% | 0.005% | 1.905%          |

*Source: Author’s calculations*

Table (5) shows the same summary statistics as before for a few representative municipalities. Each of these four municipalities presents a different price variation according to their location and consumption levels. Medellín, the capital of Antioquia, located in the Metropolitan Area, is characterized by high consumption, high urbanization levels and high altitude. Its average and median welfare gains are approximately 0.14%. Barbosa, a member of the Metropolitan Area but with low consumption, presents the lowest of all average welfare gains along with Girardota (0.00%), since they received the lowest price reduction and do not receive any implicit subsidy from their consumption; all located in tier 1 on average. San José de la Montaña is a special case, exhibiting higher than subsistence consumption but being located outside of the Metropolitan Area. The welfare increase for this municipality is greater than for those of the
Metropolitan Area (0.31%). Santa Fe de Antioquia, whose welfare gain is on average 2.29%, is part of the largest group of municipalities: those with lower consumptions located outside of the Metropolitan Area. These municipalities tend to have large welfare gains but they vary greatly depending on their initial price levels, urbanization rates and altitude. Caucasia, part of the same group as Santa Fe de Antioquia, had the largest average increase in welfare with 3.30% as a share of income.

Table 5: Equivalent Variation as share of income for representative municipalities

<table>
<thead>
<tr>
<th>Municipality</th>
<th>Road Length Contiguity</th>
<th>Queen Contiguity</th>
<th>Rook Contiguity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>90% HPD Interval</td>
</tr>
<tr>
<td></td>
<td>Lower</td>
<td>Upper</td>
<td></td>
</tr>
<tr>
<td>Medellín</td>
<td>0.137%</td>
<td>0.139%</td>
<td>0.081% 0.194%</td>
</tr>
<tr>
<td>Barbosa</td>
<td>0.008%</td>
<td>0.008%</td>
<td>0.006% 0.009%</td>
</tr>
<tr>
<td>San José de la Montaña</td>
<td>0.313%</td>
<td>0.313%</td>
<td>0.307% 0.319%</td>
</tr>
<tr>
<td>Santa Fe de Antioquia</td>
<td>2.297%</td>
<td>2.297%</td>
<td>1.956% 2.619%</td>
</tr>
</tbody>
</table>

Source: Author’s calculations

Finally, Map (4) presents the median Equivalent Variation as a share of income in the province. The spatial distribution is low around the metropolitan area and high on the more rural areas that received the greater improvement and benefits from the tariff unification.
5 Concluding Remarks

In this paper, we introduced spatial random effects into an endogenous Bayesian framework with simultaneous equations and deduced the complete conditional posterior distributions. Thus, we were able to draw observations from the model using a Gibbs sampler algorithm. This approach allows to deal with three shortcomings at a time, that would prove quite difficult to manage simultaneously with a Frequentist approach. First, it permits to account for endogeneity issues using weak instruments in our estimation procedure. Second, we can perform statistical inference of a complicated non-linear function of the parameters estimates in our application. Third, it allows to control the non-observable heterogeneity and spatial autocorrelation present in our cross-sectional data.

Using these features of the Bayesian framework to our advantage, we estimate the Equivalent Variation welfare measure, as a share of mean income, that stemmed from a process of electricity tariff unification in the province of Antioquia (Colombia), with data at the municipality level. We estimate a demand function for electricity and find the average price, income, substitute
and urbanization rate demand elasticities to be -0.88, 0.30, 0.12 and 0.57, respectively. The semi-elasticity associated with a dummy of the altitude of municipalities was approximately 0.13. Using this information as input, we find the Equivalent Variation for the total of the province to be 0.87% in average and 0.63% in median. When taking into account the welfare gains of those municipalities of the Metropolitan Area, these amount only to 0.13%. However, the municipalities that are not part of the Metropolitan Area, gained in average 0.94%, while the less urban and poorest municipalities, increased their welfare well above 2% of the initial income. Comparing these figures with the the amount that low income households expend in pension (1.13%), health care (2.04%) and education (4.79%) illustrates the huge effect of electricity regulation on the welfare of the poor.
References


Appendix

A Variable Definitions

Table A.1: Variables definitions and sources

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption ((x))</td>
<td>Average annual electricity consumption per household in kilowatts hour ((\text{kWh}))</td>
<td>EPM\textsuperscript{a}</td>
</tr>
<tr>
<td>Price ((p))</td>
<td>Average annual electricity price in US$ by kilowatt hour ((\text{US$/kWh}))</td>
<td>EPM</td>
</tr>
<tr>
<td>Income ((y))</td>
<td>Average annual per capita income in US$</td>
<td>Authors’ calculations</td>
</tr>
<tr>
<td>Substitute price ((p^*))</td>
<td>Average annual price of the substitute good in US$ by kilowatt hour ((\text{US$/kWh}))</td>
<td>CREG\textsuperscript{b}</td>
</tr>
<tr>
<td>Urbanization ((\text{urb}))</td>
<td>Ratio of urban to total population</td>
<td>DANE\textsuperscript{c}</td>
</tr>
<tr>
<td>Altitude ((\text{alt}))</td>
<td>Dummy variable taking on 1 when the municipality is located less than 1000m above sea level</td>
<td>Anuario Estadístico de Antioquia\textsuperscript{d}</td>
</tr>
<tr>
<td>Coverage ((EADE))</td>
<td>Dummy variable taking on 1 when municipality used to be covered by EADE and 0 otherwise</td>
<td>SUI\textsuperscript{e}</td>
</tr>
</tbody>
</table>

Notes: \textsuperscript{a} Empresas Públicas de Medellín, \textsuperscript{b} Comisión de Regulación de Energía y Gas, \textsuperscript{c} Departamento Administrativo Nacional de Estadística, \textsuperscript{d} Antioquia’s Statistical Yearbook compiled by the Government of Antioquia, \textsuperscript{e} Sistema Único de Información

B Diagnostics

The method due to Heidelberger and Welch (1983) is divided into two tests used to assess convergence: a stationarity test and a half-width test. For the first part, the null hypothesis considers the chain of estimated values to come from a stationary distribution. In particular, we contrast this hypothesis using the Crámer–von-Mises statistic. If the chain does not pass the stationarity test, the first 10% is discarded and the statistic is calculated again. The algorithm stops whenever the chain passes the test or 50% of the observations have been discarded without passing. The second part of the diagnostic calculates half the width of the \((1 - \alpha)\%\) credible
interval around the mean using the portion of the chain remaining from the previous step. If
the ratio between the half-width and the mean is lower than some small value, usually of 0.1,
the chain passes the test and there is evidence that the mean is being estimated with sufficient
accuracy. As can be observed, all of the chains for the structural parameters pass the test when
using the road length criterion. So, these values support to the conclusion that parameter chains
have been run long enough to reach convergence. The second diagnostic, due to Geweke (1992),
is based on a test of equality of the means from the first and last portions of a Markov chain
(usually 10% and 50% respectively). If the sampled values came from a stationary distribution,
the two means should be close to equal and the z-score statistic should remain below the critical
values. Each parameter appears to have statistically equal means in the first and last parts of
the chain, when comparing the statistics to a 10% or even 5% critical value. Finally, the table
contains the dependence factor included in the diagnostic due to Raftery et al. (1992). This
diagnostic attempts to provide a researcher with an appropriate number of burn-in and total
iterations to estimate a certain quantile, with given tolerance and probability from a pilot chain.
However, it also outputs an estimate of the dependence factor, that is, the extent to which
autocorrelation inflates the sample size necessary to estimate the quantile of interest accurately.
Values above 5 can indicate strong autocorrelation due poor choice of starting value or high
posterior correlations. It can be observed that the dependence factor for each parameter remains
below 5 and the sample size is well above the minimum of 2,638 observations recommended by
the diagnostic.
Table B.1: Stationarity and Convergence diagnostics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Road Length Contiguity</th>
<th>Queen Contiguity</th>
<th>Rook Contiguity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heidelberger (1st Part/p-value)</td>
<td>Heidelberger (2nd Part)</td>
<td>Geweke</td>
</tr>
<tr>
<td>Constant</td>
<td>0.887</td>
<td>0.064</td>
<td>0.758</td>
</tr>
<tr>
<td>Price</td>
<td>0.923</td>
<td>-0.047</td>
<td>0.820</td>
</tr>
<tr>
<td>Income</td>
<td>0.414</td>
<td>0.035</td>
<td>-0.740</td>
</tr>
<tr>
<td>Subs. Price</td>
<td>0.909</td>
<td>0.067</td>
<td>-0.311</td>
</tr>
<tr>
<td>Altitude</td>
<td>0.581</td>
<td>0.052</td>
<td>-0.515</td>
</tr>
<tr>
<td>Urbanization</td>
<td>0.871</td>
<td>0.024</td>
<td>-0.407</td>
</tr>
<tr>
<td></td>
<td>Heidelberger (1st Part/p-value)</td>
<td>Heidelberger (2nd Part)</td>
<td>Geweke</td>
</tr>
<tr>
<td>Constant</td>
<td>0.830</td>
<td>0.085</td>
<td>-0.079</td>
</tr>
<tr>
<td>Price</td>
<td>0.142</td>
<td>-0.054</td>
<td>-0.238</td>
</tr>
<tr>
<td>Income</td>
<td>0.578</td>
<td>0.037</td>
<td>-0.399</td>
</tr>
<tr>
<td>Subs. Price</td>
<td>0.530</td>
<td>0.111</td>
<td>-0.450</td>
</tr>
<tr>
<td>Altitude</td>
<td>0.266</td>
<td>0.216</td>
<td>1.083</td>
</tr>
<tr>
<td>Urbanization</td>
<td>0.126</td>
<td>0.013</td>
<td>0.550</td>
</tr>
<tr>
<td></td>
<td>Heidelberger (1st Part/p-value)</td>
<td>Heidelberger (2nd Part)</td>
<td>Geweke</td>
</tr>
<tr>
<td>Constant</td>
<td>0.604</td>
<td>0.151</td>
<td>-0.930</td>
</tr>
<tr>
<td>Price</td>
<td>0.280</td>
<td>-0.155</td>
<td>-0.967</td>
</tr>
<tr>
<td>Income</td>
<td>0.226</td>
<td>0.095</td>
<td>0.866</td>
</tr>
<tr>
<td>Subs. Price</td>
<td>0.634</td>
<td>0.455</td>
<td>-0.745</td>
</tr>
<tr>
<td>Altitude</td>
<td>0.935</td>
<td>0.356</td>
<td>0.541</td>
</tr>
<tr>
<td>Urbanization</td>
<td>0.894</td>
<td>0.034</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Notes: a Null hypothesis is stationarity of the chain, b Half-width to mean ratio, c Mean difference test z-score, d Dependence factor