Asymmetric Exchange Rate Pass-through Behavior over the Business Cycle
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Abstract
We estimate a Bayesian threshold vector autoregression (TVAR) to study the relationship between exchange rate pass-through and business cycles in Canada and Mexico. Both the model comparison and the analysis of impulse response functions show strong evidence of a nonlinear relationship and suggest that the exchange rate pass-through is dependent on the business cycle phase. In particular, the pass-through coefficient is higher when the growth rate of output is large and this difference is statistically significant across regimes for both countries. Furthermore, the results show that the degree of pass-through is complete in the case of import prices and that the pass-through coefficient falls along the distribution chain of goods.

Keywords: Exchange rate pass-through, Bayesian Analysis, Asymmetry, Threshold Processes, Vector Autoregression, MCMC Methods.
1 Introduction

In an open economy, fluctuations in the exchange rate are an important source of short-run inflationary pressures. This is especially true in small, open economies because a depreciation of the currency is typically associated with an increase in the domestic price of tradable goods, imported inputs, or final products priced in foreign currency. The effect of these exchange rate fluctuations over the price level is commonly known as the exchange rate pass-through (PT, henceforth).

In this paper, we estimate and test a threshold vector autoregression (TVAR) to make inferences about the behavior of the exchange rate PT in Canada and Mexico for the 2001-2013 period, and the variation of the PT coefficient over the business cycle. Canada and Mexico are small, open economies integrated through the North American Free Trade Agreement (NAFTA), and share similarities with many other developed and emerging countries. Therefore, our results can be extended to the experiences of other economies.

Our empirical model contributes to the existing literature in three main ways. First, we consider a multivariate environment that takes into account possible interdependence of macroeconomic aggregates. For the Mexican case, only Capistrán et al. (2012) and Cortés (2013) have estimated values for the PT coefficient using multivariate (although linear) frameworks. Aleem and Lahiani (2014) estimate a nonlinear VAR and find evidence of asymmetries. However, the change in regimes is driven by inflation dynamics, unlike our focus on business cycle phases. We are not aware of any multivariate studies for Canada. Second, we adopt a Bayesian approach to estimate and test our proposed model. This allows us to formally compare whether our proposed nonlinear model is statistically different when contrasted with a linear VAR. Further, it accounts for coefficient uncertainty and makes it possible to construct generalized impulse responses when the economy evolves from the low output growth state to the high output growth state, which allows us to trace out the dynamics of the PT coefficient. Third, we extend the analysis beyond the consumer price index and consider more disaggregated prices.

Understanding the effect of exchange rates on domestic prices is relevant for the design of monetary policy. Because movements in the exchange rate can significantly influence the dynamics of inflation, policymakers must be able to gauge the size of these effects in order to determine appropriate policy responses. More broadly, the exchange rate can act as a ‘shock absorber’ mechanism for the economy. Accurately determining a low level and persistence of underlying inflationary pressures, for example, would allow a higher degree of independence from exchange rate considerations in the design of monetary policy.

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1A standard way to estimate the pass-through effect is by regressing changes in price indices on variations in nominal exchange rates (Campa and Goldberg 2005; Mihaljek and Klaus 2008). This practice, however, is not necessarily well suited as it implicitly suggests that any movement in the exchange rate is exogenous, while also failing to control for factors, other than the exchange rate, that determine inflation.
policy and inflation targets. Furthermore, PT levels may even influence the measure of inflation central banks should target (Adolfson, 2007; Flamini, 2007).

Recently, a decreasing level of exchange rate PT has been extensively documented. (Goldfajn and Werlang, 2000; Campa and Goldberg, 2005; Ihrig et al., 2006; Burstein et al., 2007; Marazzi and Sheets, 2007; Mihaljek and Klau, 2008; Takhtamanova, 2010) Some studies that focus on a given country, nonetheless, especially emerging economies, have found substantially different PT levels, even when considering similar subsamples. For example, Santaella (2002) finds that the exchange rate pass-through to CPI inflation is 44% over a one year horizon in Mexico, while Capistrán et al. (2012) find that the same coefficient, for the same horizon, is 6%.

In this context, many theoretical models suggest that the PT level is time-varying. While this nonlinear behavior of the PT coefficient has been linked to changing inflationary environments (Taylor, 2000; Devereux and Yetman, 2010) and the size of the exchange rate shock (Smets and Wouters, 2002), we focus on its relationship with the business cycle phase. The size of the PT is intrinsically related to the ability of importers and local producers to transfer their higher costs to final consumers which, in turn, depends on the state of the economy. Intuitively, if the economy is sluggish, even large depreciation shocks may leave prices unchanged as firms will opt to reduce mark-ups in order not to lose market share. By contrast, firms find it optimal to translate increases in costs to final prices in an expansionary economy (Froot and Rogoff, 1995; Rogoff, 1996; Engel, 2000; Goldfajn and Werlang, 2000).

To motivate how the degree of PT can depend on the business cycle phase, we present a theoretical model where the PT coefficient is a nonlinear function of the state of the business cycle, which can be approximated by threshold-type dynamics. While the model is simple, the implications are useful as motivation for the empirical model. The results show that, in general, there is strong evidence in favor of nonlinearity. As shown in section 4, the asymmetry detected by the model comparison also implies a significant asymmetry in the response of inflation to exchange rates. For the case of Mexico, the estimated median PT coefficient in the low-growth regime for CPI, for example, is 4.4% after 12 months and, also, 4.4% after 18 months. The estimated median PT coefficient in the high-growth regime for CPI is 11.4% after 12 months and 16.9% after 18 months, and the responses are significantly different across regimes. In addition, there is also strong evidence in favor of an asymmetric PT coefficient for the imports, producer and goods price indexes.

For the case of Canada, similarly, there is evidence of a nonlinear behavior of the PT coefficient for the consumer, producer, imports and goods price indexes. For example, for CPI, the estimated median

\footnote{Taylor (2000) and Devereux and Yetman (2010) construct theoretical models in which agents keep their prices unchanged in a low-inflation environment, given that exchange rate shocks are perceived as transitory. However, they increase their prices in a high-inflation environment. Similarly, a pronounced depreciation could generate incentives for firms to increase their prices. If the depreciation is relatively small, on the other hand, it is likely that firms decide to keep their prices unchanged, given the existence of menu costs or costs of adjustment (Smets and Wouters, 2002).}
PT coefficient in the low-growth regime is 3.0% after 12 months and 3.4% after 18 months. Meanwhile, the estimated median PT coefficient in the high-growth regime is 7.0% after 12 months and 8.8% after 18 months, and the responses are significantly different across regimes. The evidence is not as strong for PPI and the prices of goods. Overall, the results suggest that the PT coefficient is state-dependent, and that this state-dependence is both statistically and economically relevant.

The rest of the paper is organized in the following way. In the next section, we discuss the background and motivation. Section 3 describes the empirical model and the estimation procedure. The results are presented in the fourth section. We provide some concluding remarks in section 5.

2 Motivation and Background

Different states of the business cycle could alter the effect of exchange rate shocks on inflation because firms are more likely to increase their prices during expansions, when they typically face increasing sales. At the same time, it can be argued that firms reduce production and inventories during recessions. As a consequence, a depreciation of the exchange rate will not affect prices given that inventories can be thought of as production associated with costs incurred in prior to the shock (Campa and Goldberg, 1999) \(^3\) The reader is referred to the appendix for a simple theoretical model motivating the assumption of a PT coefficient that varies across the business cycle, as in the model described in the next section.

Empirically, the effects of the state of the business cycle over PT have received little attention in the literature. There are two exceptions. Goldfajn and Werlang (2000) use a linear panel of 71 countries over the 1980-1998 period and find that the cyclical component of output is an important determinant of the PT. The other exception is the study of Ben Cheikh (2013), who uses a logistic smooth transition autoregression to evaluate nonlinearities in the PT with respect to economic activity in 12 Euro area countries. He finds strong evidence of nonlinearities in 6 out of the 12 countries, with significant differences in the degree of PT between expansions and recessions.

Our paper is, thus, similar in spirit to that of Goldfajn and Werlang (2000). However, we explicitly allow for a possible asymmetric PT behavior over time, while their linear model is not able to determine whether the size of the PT coefficient varies between expansions and recessions. Our model is also closely related to that of Ben Cheikh (2013). But, unlike the univariate model he considers, we allow for possible interdependence between macroeconomic aggregates in a multivariate environment.

\(^3\)This type of business cycle dependence could be correlated with other factors that might explain a time-varying PT coefficient. Dvir (2007), for example, argues that large importers arising due to the globalization process have contributed to the decline in PT because they have significant market power. Meanwhile, the nonlinear effects of exchange rate shocks on inflation can also be related to the literature on sticky prices and menu costs. For a given size of the menu cost, firms will choose a higher frequency of price adjustments (Ball and Romer, 1990; Ball and Mankiw, 1994; Lucas and Cooley, 2007; Alvarez and Lippi, 2013; Alvarez et al., 2014). Therefore, the higher the frequency of the price change, the higher the PT coefficient (Devereux and Yetman, 2002, 2010; Burnstein et al., 2005, 2007).
We focus on Mexico and Canada for three main reasons. First, trade integration could explain the reduction in the degree of PT over time. Thus, our analysis focuses on countries that are integrated through the North American Free Trade Agreement (NAFTA). Second, they are both small open economies, where the interrelated large economy is clearly identified and the volume of trade is sizable for the entire duration of our sample. Third, the recent behavior of inflation dynamics in Canada and Mexico shares many similarities with that of other small, open economies. Therefore, our analysis could be easily extended to the experiences of other similar economies. The post-2001 sample, in turn, is motivated by the large number of disruptions that generated uncertainty and instability in global markets during the 1990s (e.g., the 1995 Mexican crisis, specifically, and the global crises in 1997 and 1998). Moreover, Mexico introduced an inflation targeting (IT) regime in 2001 (and, Canada, in 1991). By focusing on the post-inflation targeting period, we can study potential asymmetries in an environment of stable inflation, thus avoiding structural breaks in inflation dynamics that could drive changes in regimes.

Several studies have analyzed the effect of exchange rate fluctuations on the price level for the Mexican case (Conesa, 1998; González, 1998; Goldfajn and Werlang, 2000; Garcés, 2001; Hausmann et al., 2001; Santaella, 2002; Baqueiro et al., 2004; Capistrán et al., 2012; Cortés, 2013). However, most of these studies use a univariate setting, and none of them have introduced possible nonlinearities in the estimation of PT. One exception is the study of Aleem and Lahiani (2014), who estimate a nonlinear VAR to examine the responses of domestic prices to exchange rate shocks. However, they focus on possible asymmetries in the behavior of the PT coefficient associated with changes in inflation, unlike our focus on output growth as the threshold variable. Considering the variability of the results obtained thus far, the empirical evidence is not conclusive and the estimates of PT are not uniform.

The analysis of the PT for Canada is, on the other hand, more limited. Gagnon and Ihrig (2004) and Bailliu and Fujii (2004) estimate a linear panel data model or cross-country linear regressions and find that the PT coefficient in Canada has declined. Neither of them formally tests nonlinearities or considers a multivariate environment. From a theoretical perspective, Bouakez and Rebei (2008) and Murchison (2009) estimate DSGE models calibrated to the Canadian economy over two sub-samples, before and after the adoption of the IT regime, and find that PT declined after 1991. However, they focus on the effects of the inflationary environment over the PT coefficient, the reduction of PT is associated with a structural break, and they make an assumption that exchange rate shocks are eventually matched up by a proportionate increase in prices, so that PT is complete in the long-run by construction.

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4For example, many developed small, open economies like Canada (e.g., New Zealand, Sweden, among others) and many emerging, small, open economies like Mexico (e.g., Chile, Brazil, among others) have adopted inflation targeting regimes. As a consequence, they have experienced periods of low-stable inflation in the aftermath of the monetary regime change.
Since discrepancies in the estimates of the PT coefficient can arise because of a time-varying behavior, studying any possible asymmetries in the behavior of the PT value is relevant for several reasons. First, it contributes to a better understanding of the factors that affect the value of the PT, and the aggregate process of price determination. In particular, an asymmetric PT could imply varying effects of monetary policy actions over the central bank’s inflation target. Second, a possible nonlinear behavior in the relationship between exchange rates and prices could affect inflation forecasts. This, in turn, could lead to wrong inferences and, consequently, inadequate policies. Third, a number of Central Banks, both in Mexico and many other emerging markets, have recently expressed a concern about the raise in the Federal Funds rate (FFR), the appreciation of the U.S. over the value of their currencies and, consequently, their inflation objectives. Lastly, the estimates of the PT coefficients from VAR models are frequently used to calibrate behavioral parameters in theoretical DSGE models for small open economies (see, for example, Faruqee, 2006). If there is significant nonlinearity in the empirical relationship between exchange rates and inflation, models that assume a stable relationship may not describe the economy accurately.

3 Model and Estimation

In order to quantify the magnitude of the PT generated by shocks to the exchange rate, this section first introduces a linear vector autoregression (VAR) that serves as a benchmark model and explains the PT coefficient in terms of elasticities of cumulative impulse-response functions. In the second subsection, we extend the model to allow for nonlinear behavior.

3.1 A benchmark model: Linear VAR

The linear VAR that serves as a benchmark model is given by

$$Y_t = \Phi_0 + \Phi(L) Y_{t-1} + \Psi(L) X_t + \varepsilon_t$$

(1)

where $Y_t$ is a vector containing a measure of economic activity, the domestic interest rate, the exchange rate (peso-to-dollar and Canadian dollar to U.S. Dollar, for Mexico and Canada, respectively) and a measure of prices; $X_t$ is a vector of exogenous variables that contains a measure of U.S. economic activity, a measure of the international interest rate, a measure of foreign prices, and an index of international commodities prices; $\Phi_0$ is a vector of constants; and $\Phi(L)$ and $\Psi(L)$ are polynomial matrices in the lag operator, $L$. The autoregressive polynomial matrix, $\Phi(L)$, is assumed to have roots that are
strictly outside the unit circle. The shocks $\varepsilon_t$ are assumed to follow an i.i.d. Gaussian process with mean zero and variance $\Omega$. The VAR framework provides a direct way to obtain the dynamic response of prices to exchange rate innovations by using impulse-response functions. To make the interpretation of the results straightforward, we evaluate the effect of exchange rate shocks on prices by means of elasticities from accumulated changes. These can be interpreted as changes in prices, expressed in percentage points, when the domestic currency depreciates by 1 percentage point. In this context, the PT coefficient $\kappa$ periods after an exchange rate shock, $\kappa_\tau$, can be calculated using the accumulated impulse-response functions as a starting point:

$$
\kappa_\tau = \frac{\sum_{j=0}^{\tau} \frac{\partial \pi_{t+j}}{\partial \varepsilon_t} \overline{\Delta \varepsilon_t}}{\sum_{j=0}^{\tau} \frac{\partial \Delta \varepsilon_t}{\partial \varepsilon_t}}
$$

The numerator in equation (2) measures the accumulated effect of an exchange rate depreciation over the inflation rate up until period $\tau$. It is important to note that, given the multivariate environment of the VAR framework, the response of inflation will reflect, in addition to the reaction to the structural innovation, the endogenous response to the exchange rate. That is, the response of inflation up until period $\tau$ is generated both by the initial shock, and by the subsequent depreciation following the initial shock. This could overestimate the actual value of the PT, especially within a nonlinear environment. Hence, the denominator in equation (2) considers the accumulated and endogenous response of the exchange rate to the initial shock. Following the literature, the structural shocks are identified using a Cholesky decomposition, and the foreign block is assumed to be completely exogenous. The Cholesky decomposition corresponds to the assumption that real variables respond to nominal variables with a lag, and that higher frequency movements in exchange rates are driven by asset market disturbances and not by goods market disturbances.

A linear VAR is a popular method for analyzing the effects of exchange rate shocks on prices because they have the advantage of being parsimonious, and the impulse responses are easy to construct and to interpret [see, for example, Cortés (2013), for a study that focuses on Mexico, or Faruqee (2006), for a study that focuses on the Euro area.] This parsimony, however, comes at a price. In particular, the response of any of the variables in the system to any shock in the linear VAR framework does not depend on the history of the shock. For example, the response of inflation to exchange rate shocks would, necessarily, be the same during recessions and during expansions. Hence, the process given in equation (1), as well as the dynamic response of prices to exchange rate shocks, are unable to capture any potential asymmetries or state-dependent behavior of the pass-through coefficient.

5 Similarly, the response of inflation to exchange rate shocks in a linear VAR does not depend on the sign or the size of the shock.
3.2 Introducing Asymmetry: A Threshold VAR Model

Threshold vector autoregressions (TVAR) are natural extensions of linear VAR processes. A TVAR model provides a simple and direct way to capture nonlinearities, such as regime-switching, as well as asymmetries in the response of prices to exchange rate innovations, by allowing for the possibility that the dynamics of the entire system depends on an observed threshold variable. We build a two-regime TVAR model that extends the linear model by allowing the relationship between the variables to vary over time depending on the state of the business cycle. The TVAR process can be described according to:

\[ Y_t = \left[ \Phi_0^1 + \Phi_1^1 (L) Y_{t-1} + \Psi_1^1 (L) X_t \right] + \left[ \Phi_0^2 + \Phi_2^1 (L) Y_{t-1} + \Psi_2^1 (L) X_t \right] I(q_{t-d} \leq c) + \varepsilon_t \]

(3)

where \( Y_t \) is defined as in equation (3); \( \Phi_0^1 \) and \( \Phi_0^2 \) are vectors of constants; \( \Phi_1^1, \Phi_2^1, \Psi_1^1 \) and \( \Psi_2^1 \) are lag polynomial matrices; \( \varepsilon_t \) is the vector of disturbances with mean zero and covariance matrix \( \Omega \); \( q_{t-d} \) is the threshold variable, which determines the prevailing regime of the system; \( c \) is the threshold parameter at which the regime switching occurs; and \( I(.) \) is an indicator function that equals 1 when \( q_{t-d} \leq c \) and zero otherwise. The integer \( d \) is the delay lag with which the threshold variable determines the change in regimes. This model splits a time series process endogenously into different regimes. For the threshold variable, we consider the lagged growth of the industrial production index, as a proxy for the state of the business cycle.

3.3 Data

For the measure of domestic economic activity, we use the IGAE Global indicator for economic activity for Mexico and the index of industrial production for Canada. We use the index for U.S. industrial production as the measure of the foreign business cycle both for Mexico and Canada. For the case of domestic prices, we consider the consumer price index (CPI), the producer price index (PPI), imports prices, intermediates prices and goods prices. The foreign price level is measured using the U.S. CPI. We use the 91-day CETES interest rate as a measure of the domestic interest rate for Mexico, the discount rate for Canada, and the Federal Funds rate (FFR) as a measure of the foreign interest rate. We use the monthly average of the Peso-to-Dollar and the Canadian-to-US dollar exchange rates. The global indicator for economic activity and the Mexican prices were obtained from INEGI (Instituto Nacional de Estadística y Geografía). The domestic interest rate and the exchange rate were obtained from the Banco de México website. All other series were obtained from the St. Louis Federal Reserve...
database (FRED). The recession index used in the subsequent sections is the OECD peak-to-trough index. We use the IMF commodity price index as a measure of commodity prices. In order to facilitate the computation of the PT coefficient, all data series are converted to logarithms and differenced to ensure stationarity, except interest rates, which are only differenced. The use of first differences also facilitates the comparison of results with the benchmark, linear case.

3.4 Estimation

Because both the VAR and the threshold VAR models are highly parameterized, we make inferences about the threshold and the coefficients using Bayesian methods. In particular, we use a multi-block Metropolis-Hastings (MH) algorithm to sample from the marginal posterior distributions of the parameters and to calculate the marginal likelihoods for models. The advantages of using a Bayesian approach in this setting are twofold. First, it allows us to capture the uncertainty about the parameter values when constructing the impulse response functions. Second, despite the presence of the nuisance parameter \( c \) in the nonlinear model, comparing the linear to the nonlinear model and examining the presence of nonlinear effects is straightforward. The reader is referred to the appendix for the technical details.

To provide an accurate approximation of the target posterior distribution of the parameters, we follow the standard approach in the applied literature and use a tailored multivariate Student-\( t \) distribution as the proposal distribution. Our prior for \( \Phi \) is a normal distribution, truncated to ensure stationarity. The prior for \( \Omega \) is an inverse-Wishart, and the prior for \( c \) is a Student-\( t \) distribution centered around the maximum likelihood estimate, with 5 degrees of freedom (relatively uninformative, but proper prior), truncated over \([c_l, c_h]\) where \( c_l \) and \( c_h \) are the highest and the lowest observed values of the vector of ordered thresholds. The full technical details of the posterior sampler and the priors are relegated to the appendix.

There are two ways to evaluate whether there is evidence of nonlinearity in this framework. First, we can evaluate if the model exhibits nonlinearity by comparing the posterior likelihoods of different models using a Bayesian information criterion (BIC) or marginal likelihood comparison. Second, we can make inferences about the asymmetry of the impulse responses and the PT coefficient directly. To compare whether there is any evidence of nonlinearity, we first estimate the threshold VAR model using the MH algorithm and then compare the expected posterior likelihood to that of the restricted linear version of the VAR model. Because the model has a large number of parameters, the marginal likelihood could be affected by the choice of priors. Campolieti, Gefang, and Koop (2014) suggest using

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6 Following the standard approach in the literature, \( c \) is assumed to be restricted to a bounded set \( C = [c_l, c_h] \)

7 Using a uniform prior for \( c \) leads to very similar posterior estimates.
the BIC as an alternative for the marginal likelihood comparison, as the former is not sensitive to the choice of priors. We compare the linear model to the nonlinear model by means of a BIC constructed using the expected posterior likelihood.

Rejecting nonlinearity in this way necessarily implies that at least one of the impulse responses to at least one of the structural shocks is different across regimes, but the degree of this asymmetry and its dynamic impact on the PT coefficient can only be evaluated by looking at the impulse response functions themselves. Two sets of impulse responses are constructed. First, we compare the impulse response functions (IRFs) in the fixed low-growth state, when the economy is assumed to remain below the estimated threshold permanently, and the responses in the high-growth regime, when the economy is assumed to remain above the estimated threshold permanently. It is important to note that the entire distribution of IRFs is evaluated to obtain these fixed responses. While these assumptions are somewhat restrictive, fixed-state impulse responses are routinely reported in the TVAR literature because they can be used as a useful benchmark to test for evidence of nonlinearity. In the second set of impulse responses, we relax the assumption that the economy remains in one state forever, and consider dynamic responses to an exchange rate shock when the economy is allowed to evolve endogenously across regimes.

Following Koop, Pesaran and Potter (1996) and Teräsvirta and Yang (2014a, 2014b), we consider simulation-based IRFs for the nonlinear model in order to measure the responses when the threshold variable is allowed to respond endogenously. When we allow the system to evolve and switch between regimes, the impulse responses depend on the initial state, and are defined as the change in the conditional expectation of $Y_{t+k}$ as a result of a shock at time $t$:

$$IRF[\text{shock}_t, F_{t-1}] = E[Y_{t+k}|\text{shock}_t, F_{t-1}] - E[Y_{t+k}|F_{t-1}]$$

(4)

where $F_{t-1}$ is the information set at time $t-1$. Calculating the IRFs requires specifying the nature of the shock and the initial conditions $F_{t-1}$. Then, the conditional expectations $E[Y_{t+k}|\text{shock}_t, F_{t-1}]$ and $E[Y_{t+k}|F_{t-1}]$ are computed by simulating the model.

We consider an orthogonal exogenous shock identified from the TVAR model rather than a forecast error from the reduced-form VAR, as considered in Koop, Pesaran and Potter (1996). Because threshold models imply that the predicted responses from the model to a given shock depend on a particular history, we average over all histories when $q_{t-d} < \hat{c}$ to get the average response when the switching variable is below the threshold, and over all histories when $q_{t-d} > \hat{c}$ to get the average response when

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8It is important to note, however, that the results obtained using marginal likelihood comparisons also favored the nonlinear model in all cases.
the switching variable is above the threshold. The reader is referred to the appendix for details on the computation of the simulation-based IRFs.

To evaluate whether the difference between the responses across regimes is significant, we compare the posterior distributions for the PT coefficients. This approach is rather general because it allows for threshold effects, endogenously evolving regimes, and parameter uncertainty.

4 Results

4.1 Model Comparison and Estimated Thresholds

Table 1 below reports the logarithms of the expected posterior likelihoods, as well as the changes in the BIC, for linear VAR and TVAR models of different price indices for Mexico and Canada. The switching variable is the lagged growth of industrial production and the estimated highest posterior density (HPD) and median thresholds for each TVAR model, $\hat{c}$, are also provided.

Table 1: Model Comparison: Log expected posterior likelihood values and estimated thresholds

<table>
<thead>
<tr>
<th>Price index</th>
<th>Linear VAR</th>
<th>TVAR</th>
<th>$\Delta$BIC</th>
<th>HPD $\hat{c}$</th>
<th>90% Cred. interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>-807.52</td>
<td>-757.88</td>
<td>-27.28</td>
<td>0.06</td>
<td>[-0.25, 0.40]</td>
</tr>
<tr>
<td>PPI</td>
<td>-813.48</td>
<td>-772.62</td>
<td>-9.72</td>
<td>-0.83</td>
<td>[-1.03, 0.47]</td>
</tr>
<tr>
<td>Imports</td>
<td>-919.93</td>
<td>-867.01</td>
<td>-33.84</td>
<td>0.49</td>
<td>[-0.76, 1.02]</td>
</tr>
<tr>
<td>Intermediates</td>
<td>-494.67</td>
<td>-460.98</td>
<td>4.62</td>
<td>-1.06</td>
<td>[-2.75, 1.60]</td>
</tr>
<tr>
<td>Goods</td>
<td>-861.23</td>
<td>-825.18</td>
<td>-0.10</td>
<td>-0.70</td>
<td>[-0.87, 0.32]</td>
</tr>
<tr>
<td>Canada</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>-722.96</td>
<td>-677.84</td>
<td>-18.24</td>
<td>-2.50</td>
<td>[-5.70, 0.03]</td>
</tr>
<tr>
<td>PPI</td>
<td>-809.36</td>
<td>-747.31</td>
<td>-52.10</td>
<td>-0.002</td>
<td>[-1.50, 0.93]</td>
</tr>
<tr>
<td>Imports</td>
<td>-788.79</td>
<td>-731.18</td>
<td>-43.22</td>
<td>-2.21</td>
<td>[-5.24, 0.44]</td>
</tr>
<tr>
<td>Intermediates</td>
<td>-825.03</td>
<td>-774.15</td>
<td>-29.76</td>
<td>0.08</td>
<td>[-4.47, 0.84]</td>
</tr>
<tr>
<td>Goods</td>
<td>-722.16</td>
<td>-702.74</td>
<td>-66.84</td>
<td>-0.45</td>
<td>[-5.25, 1.19]</td>
</tr>
</tbody>
</table>

This table reports the HPD, their 90% credibility intervals and the values of the logarithms of the expected posterior likelihoods and changes in BIC of the linear and nonlinear models associated with each price index considered. The sample is 2001-2013 for all cases. Due to data availability, the model that considers the price of intermediate goods in Mexico is estimated for the 2004-2013 period.

The results from table 1 show a strong support in favor of the nonlinear model. The analysis of the log expected posterior likelihoods suggests that the TVAR nonlinear models provide a better description of the data for all prices considered, both for Mexico and for Canada. $\Delta$BIC is computed as the BIC of the benchmark model minus the BIC of the TVAR model. Hence, a large, negative value associated
with $\Delta BIC$ evidences a nonlinear behavior of the data. The analysis of the values of $\Delta BIC$ implies that there is evidence of nonlinearities for disaggregated prices in Canada. The evidence is also strong for CPI, PPI and imports prices in Mexico, but not as strong in the case of goods prices. Based on the analysis of the $\Delta BIC$, there is no evidence of asymmetries for the prices of intermediates in Mexico. The results for the case of intermediate prices in Mexico should be interpreted carefully, nonetheless. Due to data availability, the estimation of the models only included information for the 2005-2013 period.

Figure 1 plots the switching variables, the estimated modes (highest expected posterior likelihood) for the threshold parameters and the 90% credibility intervals for Mexican and Canadian data when evaluating the pass-through from exchange rate to the consumer price index. For the case of Mexico, the HPD estimated threshold is 0.06, suggesting that the PT coefficient changes when lagged output growth rises or falls relative to this threshold parameter. Notice that the timing of the observations that fall into the low growth regime correspond closely to OECD-dated recession periods (shaded in gray). For the case of Canada, the HPD estimated threshold is -2.50 and the low-growth regime closely matches OECD-dated recessions. In both cases, the threshold variables remain in the high-growth regime for the majority of the sample, switching to the low-growth regime only during periods of deep recessions.

A natural question in this context is whether the regime switching and nonlinear behavior of the PT coefficient may be driven by changes in trend inflation that are correlated with the business cycle, thus potentially incorrectly identified in the baseline model as nonlinearity drive by the business cycle. To address this issue, we estimate versions of the models where cyclical inflation replaces headline

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9Campolieti, Gefang and Koop (2014) suggest a value of -2 as evidence of significant model differences, and a value of -10 as sufficiently large to favor the nonlinear model.
inflation as a robustness check. By considering the deviations from trend in the inflation rates, the model estimates are innocuous to change in long-run inflation patterns that could produce changes in regimes. Table 2 reports the log estimated posterior likelihood values for linear VAR and TVAR models, using cyclical inflation instead of headline inflation, for the cases of CPI and PPI. All other variables, including the switching variable, are identical to those used in the main model. The HPD estimated threshold parameters are also reported.

Table 2: Model Comparison: Log expected posterior likelihood values and estimated thresholds using cyclical inflation

<table>
<thead>
<tr>
<th>Price index</th>
<th>Mexico</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VAR</td>
<td>TVAR*</td>
</tr>
<tr>
<td>CPI</td>
<td>-804.92</td>
<td>-744.30</td>
</tr>
<tr>
<td>PPI</td>
<td>-805.91</td>
<td>-761.65</td>
</tr>
</tbody>
</table>

*The nonlinear model is also preferred by the BIC analysis.

This table reports HPD estimated thresholds and the values of the logarithms of the marginal likelihood of the linear and nonlinear models when the measure of inflation is cyclical CPI inflation and cyclical PPI inflation. All other variables, including the threshold variable, are identical to those used in the main model. The sample is 2001-2013.

The results shown in table 2 are in line with those reported in table 1. The HPD estimated threshold parameters are not very different from those estimated in the models where observed inflation is used. The model comparison shows that the evidence for asymmetry, although slightly reduced in the case of Canadian PPI, is still strong. Furthermore, economic activity exhibited several switches between the low- and high-growth regimes, including in the period prior to 2007, which shows that the change in regimes not driven only by the Great Recession period. This robustness check suggests that inflation dynamics do not drive the regime-switching in the main model. After accounting for possible changes in the long-run behavior of inflation, the regime-switching driven by output growth is still highly significant.

4.2 Impulse Response Analysis

The model comparison detailed in the previous subsection rejected the linear VAR in favor of the nonlinear model, indicating that at least one of the coefficients in the matrices Φ₀, Φ₁, Ψ is non-zero.

Following the convention in the TVAR literature, we report 85% and 95% credibility levels for the simulation-based impulse-response functions (IRFs).

---

9 To obtain cyclical inflation, a trend was fitted using the Hodrick-Prescott filter.
11 The robustness check results from all other prices are similar to the results for CPI and PPI and are available from the authors upon request.
12 The asymmetric credibility intervals for the estimated threshold are in line with the previous discussion. In the case of Mexico, the PT is low in periods that are identified as recessions, and high during expansions. For Canada, the PT is low, except during rapid expansions.
13 Posterior distributions of the coefficient matrices Φ₀, Φ₁, Ψ are available from the authors upon request.
Even though the estimated coefficients are different across regimes, it is important to evaluate these responses over time, given the nonlinear nature of the model. In this kind of setting, there are two potential sources of variability in the impulse responses. First, parameter and estimation uncertainty are present, as in any econometric model. Second, impulse response functions (IRFs) exhibit variability due to history dependence for the evolving system. Even if the VAR parameters and the threshold parameter are fixed, the impulse responses will depend on the initial state. If the economy starts below the threshold, but close to it, the response may be different than the case when the economy starts far from the threshold, because the economy may switch to the high state sooner.

For this reason, we estimate two sets simulation-based IRFs as described in subsection 3.4. In the first set, the economy is assumed to remain in a given regime permanently, and we look at the entire posterior distribution of the responses for the fixed low state and for the fixed high state. By considering the entire posterior distributions of the impulse responses and differences between those two distributions, we can both evaluate the role of parameter uncertainty, and determine whether the responses are significantly different across states. When the economy is endogenously allowed to evolve from one regime to another, however, the impulse responses and the PT coefficient are highly nonlinear functions of the parameters. A small difference in the parameters across regimes might result in large differences in impulse responses. Therefore, we also estimate dynamic responses when the economy is allowed to evolve endogenously across regimes. In this case, we fix the parameter values at the HPD estimates, and we evaluate the responses for all periods that are in the low-growth state and in the high-growth state.

To evaluate whether the nonlinearity is statistically significant, as well as economically significant, we directly compare the impulse responses for the low growth and high growth regimes. Following the literature, all price responses are constructed as responses to a 1% depreciation of the domestic currency at the time of the shock. The results for Mexico and Canada are described in the following two subsections.

4.2.1 Results for Mexico

Figure 2 shows the simulation-based responses of Mexican prices to a 1% depreciation shock of the exchange rate, computed according to equation (2). In addition to the response of CPI inflation to exchange rate shocks, we also considered the exchange rate pass-through to other disaggregated prices. Although many items in the CPI are produced domestically, their manufacturing may require imported inputs that are priced in foreign currency. In this way, the pass-through effect may manifest itself through a distribution chain of goods (i.e., import prices respond more strongly and rapidly to exchange
rate shocks than produce or consumer prices). An increase in the exchange rate will lead to higher prices of imported inputs, which importers transfer to local producers. As their production costs increase, local producers will then charge higher prices to consumers. Because distribution costs and nominal rigidities buffer the impact of a currency depreciation as it moves forward in the distribution chain, it is important to evaluate the effects of exchange rate shocks in these disaggregated prices. The disaggregated prices considered are the producer price index (PPI), imports, intermediates and goods prices.

The left column of figure 2 depicts quantiles of the posterior distribution for the estimated pass-through coefficients under the assumption of fixed regimes. The median PT coefficient in the low-growth regime is shown in a solid line with vertical line marks. 85% and 95% credibility bands for the PT coefficient for each horizon in the low-growth regime are plotted in dashed lines. The median PT coefficient in the high-growth regime is shown in a solid green line with square marks. The middle column of figure 2 shows quantiles of the posterior distribution of the difference in PT coefficients across regimes. The median posterior difference is depicted in a solid line. 85% and 95% credibility sets are shown in long and short dashed lines, respectively. The right column of figure 2 reports median PT coefficients when the economy starts in the low-growth regime (solid line) and when the economy starts in the high-growth regime (solid line with square marks), but is otherwise allowed to evolve endogenously across regimes. All lines in the low-growth regimes are depicted in green, while those in the high-growth regime are shown in red. All IRFs are obtained for a horizon of 18 months.

In the first row of figure 2, corresponding to the exchange rate pass-through to CPI inflation, the median PT for the high-growth regime is significantly higher than the estimated PT for the low-growth regime after the fourth month when regimes are fixed. The median PT coefficients after 12 months and after 18 months are 4.4% and 11.4% in both cases, in the low growth-regime, and 11.4% and 16.9%, respectively, in the high-growth regime. When considering the 85% credibility intervals, the PT coefficient in the high-growth regime is significantly higher than in the low-growth regime 6 months after the exchange rate shock. In the case of the 95% credibility intervals, the former is significantly higher than the latter 11 months after the shock. The evidence from the posterior distributions of the difference in PT coefficients across regimes further illustrates this result. The median posterior difference is above zero at all horizons, reaching a difference of 7.2 percentage points after 12 months and peaking at a difference of 12.7 percentage points after 18 months. This difference is significant at the 95% credibility level (except in the first few months after the shock). When the economy is allowed to endogenously evolve across regimes, the results support these findings. The PT coefficients after 12 months and after 18 months are 4.3% and 5.2%, respectively, when the economy starts in the low-growth regime, and 8.4% and 17.6%, respectively, when the economy starts in the high-growth regime.
Figure 2: Mexico: Impulse Response functions for all price indices

The figure shows the response of different prices to a 1% exchange rate depreciation shock. The left column shows PT coefficients with fixed regimes. The middle column shows quantiles of the posterior distribution of the difference in PT coefficient across regimes. The right column shows the median PT coefficients in low growth and high growth regimes for evolving states. The responses for CPI, PPI, imports (the horizontal line is at 1 in this case), intermediates and goods are shown in the first, second, third, fourth and fifth rows, respectively.
The results of the second row of figure 2 correspond to the exchange rate pass-through to PPI inflation and are, in general, similar to the results for the case of CPI inflation. When the economy is assumed to remain in a fixed regime, the median PT for the high-growth regime is 3.6% after 12 months, and also after 18 months, while the PT coefficient in the high-growth regime is 10.8% after 12 months and 15.2% after 18 months. The PT coefficient is significantly higher in the high-growth regimes 5 months after the shock, when considering 85% credibility bands, and 9 months after the shocks, when considering 95% credibility bands. The results form the posterior of the PT difference, similarly, show that the median difference is 6.9 and 11.1 percentage points after 12 months and 18 months, respectively, and that it is significantly above zero even at the 95% credibility level, with the exception of the first few months. When the regimes evolve endogenously, the PT coefficient is 10.7% and 11.7% after 12 months and 18 months, respectively, when the economy starts in the low-growth regime. Similarly, when the economy starts in the high-growth regime, the PT coefficient is 26.0% and 35.6% after 12 months and 18 months, respectively.

PT coefficients and differences across regimes for imports prices are depicted in the third row of figure 2. As it is to be expected, the exchange rate pass-through to imports inflation is the highest of all prices considered. When the state of the economy is fixed permanently after the exchange rate shock, the median PT coefficient is complete in the sense that it is 100% (or very close to it) at all horizons in the high-growth regime. In the low-growth regime, the median PT coefficient is 88.8% and 85.7% after 12 months and 18 months, respectively. The median PT coefficient is significantly higher in the high-growth regime 3 months after the shock at the 85% credibility level, and at all horizons at the 95% credibility level. The analysis of the posterior distribution of the PT difference strongly supports the evidence for asymmetric PT coefficients in low- and high-growth regimes. The median PT difference is 13.1 and 17.0 percentage points after 12 months and 18 months, respectively, and such difference is statistically significant, even at the 95% credibility level, at all horizons. When the economy evolves across regimes, however, the asymmetry is less clear. The PT coefficient when the economy starts in the low-growth regime is 90.3% and 87.2% after 12 months and 18 months, respectively. When the economy starts in the high-growth regime, the PT coefficient is 91.7% and 89.0% after 12 months and 18 months, respectively.

Intermediate prices are shown in the fourth row of figure 2. In the case of fixed regimes, the median PT coefficient is higher in the high-growth regime after 6 months. In the low-growth regimes, the median PT coefficient is 7.0% and 7.2% after 12 and 18 months, respectively. In the high-growth regime, the median PT coefficient is 12.5% and 17.1% after 12 and 18 months, respectively. However, the PT coefficient in the high-growth regime does not seem to be statistically different, neither at the
85% nor the 95% credibility levels. This is also evident in the analysis of the posterior distribution of the PT difference. The median PT difference is 4.8 and 8.3 percentage points after 12 and 18 months, respectively, but it is not statistically different from zero at any of the credibility levels considered. These results are consistent with the PT coefficients estimated as the economy evolves across regimes. The median PT coefficient 12 months after the exchange rate shock is 17.4%, when the economy starts in the low-growth regime, and 17.5%, when the economy starts in the high-growth regime. Such coefficient 18 months after the shocks is 21.1%, when the economy starts in the low-growth regime, and 21.2%, when the economy starts in the high-growth regime. As explained in the previous subsection, however, the results for intermediate prices in Mexico should be analyzed carefully, as the sample size is smaller due to data availability.

The results for the case of goods prices, reported in the last row of figure 2, are mixed. When the responses are fixed, the median PT coefficient is higher in the high-growth regime, but only 10 months (at the 85% credibility level) or 12 months (at the 95% credibility level) after the initial shock. Indeed, the posterior distribution of the PT difference further illustrates this finding. The median PT difference, for example, is 7.3 percentage points after 12 months and 31.9 percentage points after 18 months and this difference across regimes is significantly different from zero after 12 months at all credibility levels. When the economy endogenously evolves across regimes, the median PT coefficient 12 months after the shock is 23.6% and 29.2% when the economy starts in the low- and high-growth regimes, respectively. The median PT coefficient 18 months after the shock is 29.1% and 57.4% when the economy starts in the low- and high-growth regimes, respectively.

Notice that, as expected, the PT coefficient falls along the distribution chain of goods, from being complete in the case of the price of imports, to a lower response in the case of producer prices, to an even lower response in the case of consumer prices. Intuitively, the reduction of the PT coefficient in the distribution chain responds to the fact that the fraction of tradable goods, more vulnerable to exchange rate fluctuations, is smaller in the last stages of that distribution chain. Price indexes introduce different costs, such as transaction and transportation costs, which reduces the transfer of higher costs to final prices (Burnstein et al., 2005, 2007). In general, the analysis of the IRFs provides evidence that the nonlinearity detected by the model comparison and by the comparison of the expected posterior distributions is not only statistically, but also economically significant.

4.2.2 Canada

Figure 3 reports simulation-based IRFs for Canada. The PT coefficients are constructed according to equation (2), where the exchange rate shock is equal to a 1% depreciation of the domestic currency. The
rows and columns are similar to those in figure 2 for the case of Mexico. In general, the estimated PT coefficients for Canada are lower and the evidence of asymmetry is not as strong as that for the case of Mexico. Regarding the exchange rate pass-through to CPI inflation, the median PT coefficient is higher in the high-growth regime at all horizons. After 12 months, the median PT is 3.0% in the low-growth regime and 7.0% in the high-growth regime. Meanwhile, the median PT coefficient, after 18 months, is 3.4% and 4.8% in the low- and high-growth regimes, respectively. At the 85% credibility level, the median PT coefficient in the high-growth regime is significantly higher than that in the low-growth regime 2 months after the exchange rate shock. However, it is not significantly higher at the 95% credibility level. This is also shown in the results from the posterior distribution of the PT difference across regimes. The median difference is 4.2 and 6.0 percentage points 12 months and 18 months after the shock. The difference is significantly different from zero at the 75% credibility level, but not at any other credibility levels considered. When the economy is allowed to evolve across regimes endogenously, the median PT coefficients are not very different. 12 months after the exchange rate shock, the median PT is 5.9% and 6.3% when the economy starts in the low- and high-growth regimes, respectively. After 18 months, they become 6.0% and 6.5%, respectively.

The results for the pass-through from exchange rates to PPI inflation are reported in the second row of figure 3. The results for the case of the fixed-regime responses are similar to those for CPI inflation. The median PT coefficient is marginally higher in the high-growth regime: in the low-growth regime, it is 16.8% and 18.9% after 12 months and 18 months, respectively. In the high-growth regime, it is 18.4% and 20.5% after 12 months and 18 months, respectively. At the 85% credibility level, the PT coefficient is significantly higher only in the first 6 months after the exchange rate shock. It is not significantly higher at the 95% credibility level. The median difference is 2.6 and 2.8 percentage points after 12 months and 18 months, but the difference is only significant at the 75% credibility level. When the economy evolves across regimes, nonetheless, the median PT coefficients are substantially different. When the economy starts in the low-growth regime, the median PT coefficient is 15.8% and 18.3% 12 months and 18 months after the initial shock, respectively. When the economy starts in the high-growth regime, the median PT coefficient is 28.7% and 30.6% after 12 months and 18 months, respectively.

With respect to the exchange rate pass-through to imports prices, the results are similar to those for Mexico. When the economy is permanently fixed in a given regime, the median PT coefficient in the high-growth regime is complete (or very close to being complete) at all horizons. The median PT coefficient is significantly smaller in the low-growth regimes 2 months after the initial shock at all horizons. When analyzing the posterior distribution of the PT difference across regimes, the median PT difference is 16.6 and 17.8 percentage points after 12 months and 18 months, respectively. This
The figure shows the response of different prices to a 1% exchange rate depreciation shock. The left column shows PT coefficients with fixed regimes. The middle column shows quantiles of the posterior distribution of the difference in PT coefficient across regimes. The right column shows the median PT coefficients in low growth and high growth regimes for evolving states. The responses for CPI, PPI, imports (the horizontal line is at 1 in this case), intermediates and goods are shown in the first, second, third, fourth and fifth rows, respectively.
difference is significant at all horizons and at all credibility levels considered. This difference across regimes persists even when the economy is allowed to evolve across regimes, as depicted in figure 3.

Dynamically, there is no evidence that the response of intermediate prices to exchange rate shocks persists more than a few months. Under the assumption of fixed states, the median high-growth PT coefficient is higher only 8 months after the initial shock, but this higher degree of pass-through is not significant at either the 85% nor the 95% credibility levels. The analysis of the posterior distribution of the PT difference provides further evidence of this finding: the median PT difference is only marginally higher than zero, but not significantly different from zero at any credibility level considered. When the economy evolves across regimes, the median PT coefficient is 18.0% and 20.1% after 12 months and 18 months of the initial shock and when the economy starts in the low-growth regime. Meanwhile, these responses are 26.8% and 28.8%, respectively, when the economy starts in the high-growth regime.

Finally, the responses of goods prices to exchange rate shocks are depicted in the last row of figure 3. As in the Mexican case, the evidence in favor of nonlinearity for the price of goods is mixed. When the responses are fixed, the high-growth median PT coefficient is higher 4 months after the initial shock. At the 85% credibility level, the high-growth median PT coefficient is significantly higher 8 months after the initial shock, while at the 95% credibility level, it is significantly higher only 12 months after the initial shock. In terms of the analysis of the posterior distribution of the PT difference across regimes, the median PT difference is 5.6 and 6.4 percentage points after 12 months and 18 months, respectively. However, this difference is statistically different from zero after 8 months at the 75% credibility level and after 10 months at the 85% credibility level, although it’s not statistically different from zero at the 90% or 95% credibility levels. When the economy starts in the low-growth regime and then evolves across regimes, the median PT coefficient is 29.9% and 30.7% after 12 months and 18 months, respectively. Meanwhile, when the economy starts in the high-growth regime and then evolves across states, the median PT coefficient is 32.1% and 32.8% after 12 months and 18 months, respectively.

Consistent with the results for the case of Mexico, the degree of PT falls along the distribution chain of goods. Overall, these results suggest that firms respond to exchange rate shocks and that, to some extent, they have more market power in the earlier stages of the distribution chain of goods. Further, the asymmetric behavior of the PT coefficient suggests that the evolution of economic conditions between low-growth and high-growth regimes allows firms to perceive exchange rate shocks as transitory during low-growth periods and partially absorb the costs associated with a depreciation of the currency.
5 Concluding Remarks

We estimate a TVAR process to study the behavior of the exchange rate pass-through to different price indices in Mexico and Canada. Strong empirical evidence is presented in favor of nonlinearities in the behavior of the exchange rate pass-through coefficient. In particular, when output growth is high, the pass-through of exchange rates to prices is high. This effect is smaller, and much less persistent, when output growth is low. The evidence shown is consistent with the results from theoretical models of optimizing importing firms, which suggests that the degree of exchange rate pass-through is higher during periods of economic expansion since firms can increase their mark-ups without losing much market power.

Large reductions in inflation in the aftermath of the introduction of inflation targeting regimes do not completely eliminate high PT episodes in Mexico, which shows that the asymmetric behavior of the PT coefficients found is driven by evolving business cycle phases. This evidence of nonlinearity has important implications for policy design and evaluation. When the PT coefficient is high, the Central Bank needs to coordinate inflation targeting and exchange rate policies. Since estimates of pass-through coefficients obtained from linear VAR models are frequently used to calibrate DSGE models for small open economies, accounting for nonlinearity that depends on the business cycle, and not just on inflation, is important when constructing theoretical models.

We also find that the pass-through is complete in the case of the price of imports, and that its value falls along the distribution chain (the PT coefficient is smaller for producer prices and even lower for consumer prices). Finally, the result that the pass-through of exchange rate fluctuations into different price indices depends on the business cycle phase indicates that the behavior of the margins of domestic producers will continue to change according to the growth level of output. To the extent that the so-called ‘Great Moderation’ period seems to have come to an end, the variations in those margins may increase in coming years. Examining this issue is a promising topic for future research.

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References


### Table 3: Pass-through coefficients under fixed and evolving regimes for selected horizons - NOT FOR PUBLICATION

<table>
<thead>
<tr>
<th>Price index</th>
<th>Mexico</th>
<th></th>
<th></th>
<th>Canada</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Low growth regime</td>
<td>High growth regime</td>
<td></td>
<td>Low growth regime</td>
<td>High growth regime</td>
</tr>
<tr>
<td></td>
<td>12 months</td>
<td>18 months</td>
<td>12 months</td>
<td>18 months</td>
<td>12 months</td>
<td>18 months</td>
</tr>
<tr>
<td>Fixed-state responses</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer price index</td>
<td>0.044</td>
<td>0.044</td>
<td>0.114</td>
<td>0.169</td>
<td>0.030</td>
<td>0.034</td>
</tr>
<tr>
<td>Producer price index</td>
<td>0.036</td>
<td>0.036</td>
<td>0.108</td>
<td>0.152</td>
<td>0.168</td>
<td>0.180</td>
</tr>
<tr>
<td>Imports</td>
<td>0.888</td>
<td>0.857</td>
<td>0.907</td>
<td>0.906</td>
<td>0.880</td>
<td>0.896</td>
</tr>
<tr>
<td>Intermediates</td>
<td>0.076</td>
<td>0.073</td>
<td>0.135</td>
<td>0.171</td>
<td>0.166</td>
<td>0.178</td>
</tr>
<tr>
<td>Goods</td>
<td>0.232</td>
<td>0.2250</td>
<td>0.296</td>
<td>0.568</td>
<td>0.258</td>
<td>0.266</td>
</tr>
<tr>
<td>Evolving responses</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer price index</td>
<td>0.044</td>
<td>0.052</td>
<td>0.084</td>
<td>0.176</td>
<td>0.059</td>
<td>0.060</td>
</tr>
<tr>
<td>Producer price index</td>
<td>0.107</td>
<td>0.117</td>
<td>0.200</td>
<td>0.356</td>
<td>0.158</td>
<td>0.183</td>
</tr>
<tr>
<td>Imports</td>
<td>0.903</td>
<td>0.872</td>
<td>0.917</td>
<td>0.880</td>
<td>0.972</td>
<td>1.042</td>
</tr>
<tr>
<td>Intermediates</td>
<td>0.174</td>
<td>0.211</td>
<td>0.175</td>
<td>0.212</td>
<td>0.180</td>
<td>0.201</td>
</tr>
<tr>
<td>Goods</td>
<td>0.236</td>
<td>0.294</td>
<td>0.292</td>
<td>0.574</td>
<td>0.299</td>
<td>0.307</td>
</tr>
</tbody>
</table>

This table reports responses of inflation to a one percent depreciation shock, derived from the generalized impulse response functions described in Appendix C. The upper part of the table reports responses when output growth is fixed to a given regime. The bottom part of the table reports responses when output growth is allowed to evolve across regimes. The selected horizons correspond to the PT coefficient 12 and 18 months after the exchange rate shock.
### Table 4: Posterior differences of PT coefficients across regimes - NOT FOR PUBLICATION

<table>
<thead>
<tr>
<th>Price index</th>
<th>Mexico 12 months</th>
<th>Mexico 18 months</th>
<th>Canada 12 months</th>
<th>Canada 18 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer price index</td>
<td>0.072</td>
<td>0.127</td>
<td>0.042</td>
<td>0.050</td>
</tr>
<tr>
<td>Producer price index</td>
<td>0.000</td>
<td>0.111</td>
<td>0.025</td>
<td>0.028</td>
</tr>
<tr>
<td>Imports</td>
<td>0.131</td>
<td>0.170</td>
<td>0.166</td>
<td>0.178</td>
</tr>
<tr>
<td>Intermediates</td>
<td>0.048</td>
<td>0.083</td>
<td>0.020</td>
<td>0.024</td>
</tr>
<tr>
<td>Goods</td>
<td>0.073</td>
<td>0.319</td>
<td>0.056</td>
<td>0.064</td>
</tr>
</tbody>
</table>

This table reports the median of the posterior distributions of the difference in PT coefficients across regimes, directly obtained from the Metropolis-Hastings algorithm described in Appendix B. The selected horizons correspond to the PT coefficient 12 and 18 months after the exchange rate shock.
Appendix A: Theoretical Motivation

In this section, we briefly present a simple theoretical model whose results are used only to motivate an exchange-rate pass-through that depends non-linearly on economic conditions. The model builds on Devereux and Yetman (2010). There is a continuum of monopolistically competitive importing firms that purchase differentiated goods from abroad indexed by \( i \in [0, 1] \). A representative domestic final producer buys all the imported intermediate goods to produce output. The demand function is given by

\[
C_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\theta} Y_t \tag{A-1}
\]

where \( \theta \) is interpreted as the elasticity of demand for each individual good, \( P_t \) is the price index and \( P_{i,t} \) is the price associated with firm \( i \) at time \( t \). All intermediate goods are imported at the foreign currency price, \( P_f^t \), which importers take as given. The firm's profits are given by

\[
\Pi_{i,t} = C_{i,t}(P_{i,t} - \tau S_t P_f^t) \tag{A-2}
\]

with \( S_t \) being the exchange rate and, \( \tau \), a transportation cost. The optimal price is, thus:

\[
\hat{P}_{i,t} = \hat{\theta} \tau S_t P_f^t \tag{A-3}
\]

where \( \hat{\theta} = \frac{\theta}{1 - \theta} \) is the mark-up and \( \tau S_t P_f^t \) represents the marginal cost. That is, the producer with market power sets the price as a mark-up over the marginal cost, where the magnitude of the mark-up is determined by the elasticity of demand. Because the firm's market power increases during more favorable economic conditions, the firm is able to increase its price above the marginal cost during expansions, but not during recessions. With probability \( z \in (0, 1] \), the economy is in a recessionary period. Therefore, the firm's price can be re-expressed in the following way:

\[
\hat{P}_{i,t} = \hat{\theta}(Y_t) \hat{P}_{i,t} \tag{A-4}
\]

where \( \hat{P}_t \) is the price set during recessionary times and

\[
\hat{\theta}(Y_t) = \begin{cases} 
\frac{\theta}{\theta - 1}, & Y_t > 0 \\
1, & \text{otherwise}
\end{cases} \tag{A-5}
\]
with $\theta > 1$. In expansionary times, the importing firms will set the mark-up optimally. However, in recessionary times, firms decide not to vary the prices and absorb exchange rate fluctuations to avoid losing market power. Therefore, when the economy is in a recession, importing firms simply set their price to $P_{t,i} = S_{t-1}P_{t-1}^{f}$. In this way, the aggregate price index at time $t$, is then:

$$P_t = (1 - z)\hat{P}_{t,i} + z\hat{P}_{t,i}$$

$$= (1 - z)(\hat{\theta}(Y_t)S_t P_t^{f}) + z(S_{t-1}P_{t-1}^{f})$$

(A-5)

Since we can define the inflation rate $\pi_t$ as the log difference in the price level, then $\ln \hat{P}_t = \mu_t + s_t + p_t^{f}$, where $\mu_t = \ln(\hat{\theta}(Y_t))$, and lower cases denote logs. It follows that:

$$p_t = (1 - z)(\mu_t + s_t + p_t^{f}) + z(s_{t-1} + p_{t-1}^{f})$$

$$= (\mu_t + s_t + p_t^{f}) - z\mu_t - z\Delta(s_t + p_t^{f})$$

(A-6)

Hence, the inflation dynamics can be described according to:

$$\pi_t = \ln P_t - \ln P_{t-1}$$

$$= (\mu_t + s_t + p_t^{f}) - z\mu_t - z\Delta(s_t + p_t^{f}) - (\mu_{t-1} + s_{t-1} + p_{t-1}^{f}) + z\mu_{t-1} + z\Delta(s_{t-1} + p_{t-1}^{f})$$

$$= (1 - z)\Delta(s_t + p_t^{f}) + (1 - z)\Delta\mu_t - z\Delta(s_{t-1} + p_{t-1}^{f})$$

(A-7)

Following Devereux and Yetman (2010), we define the ERPT as the first derivative of $\pi_t$ with respect to $(s_t + p_t^{f})$. Hence:

$$\frac{\partial \pi_t}{\partial (s_t + p_t^{f})} = (1 - z)$$

(A-8)

In this way, the ERPT depends on $z$. In recessions, when $z = 1$, ERPT is zero. In expansions, it is complete.\footnote{However, note that, empirically, $z$ can depend on output growth, monetary policy actions, the global economic environment, among other variables.}
Appendix B: Bayesian Estimation and Derivation of Posterior Distributions

In this appendix, we provide details for the estimation procedure for both, the linear VAR and the threshold VAR processes. We also discuss the priors.

Linear Model

For convenience, we reproduce the linear model given in equation (1) here:

\[ Y_t = \Phi_0 + \Phi(L)Y_{t-1} + \Psi(L)X_t + \varepsilon_t \]  

(B-1)

where \( \Phi_0 \) is a vector of constants and \( \Phi(L) \), \( \Psi(L) \) are polynomial matrices in the lag operator, \( L \). The autoregressive polynomial matrix, \( \Phi(L) \), is assumed to have roots that are strictly outside the unit circle. The shocks \( \varepsilon_t \) are assumed to follow an i.i.d. Gaussian process with variance-covariance matrix, \( \Omega \).

Let \( \Phi = \text{vec}(\Phi_0) | \text{vec}(\Phi_1) | \ldots | \text{vec}(\Psi) \) and define \( W_t^* = [1 \ Y_{t-1} \ X_t] \). Thus, we can rewrite the VAR model as a seemingly unrelated regressions (SUR) model with heteroskedastic errors:

\[ Z_t = \Phi W_t + \varepsilon_t \]  

(B-2)

where

\[
W_t = \begin{bmatrix}
W_t^* & 0 & 0 & 0 \\
0 & W_t^* & 0 & 0 \\
0 & 0 & W_t^* & 0 \\
0 & 0 & 0 & W_t^*
\end{bmatrix}.
\]

In this case, the likelihood function, given the parameters, can be described according to:

\[
f(y, \Phi, \Omega) = \Pi_{t=p+1}^{T} (2\pi)^{-2.5} |\Omega|^{-0.5} \exp\{-0.5 * (y_t - w_t \Phi)' \Omega^{-1} (y_t - w_t \Phi)\} \]  

(B-3)

where \( y_t \) and \( w_t \) are elements of \( Y_t \) and \( W_t \), respectively. If we assume that the prior for \( \Phi \) is truncated...
Gaussian distribution with mean 0 and variance-covariance matrix $B_0$, it is straightforward to show that $\Phi|\Omega, y$ follows a Gaussian distribution with mean

$$\mu = B^{-1} \left( \sum_{t=p+1}^{T} x_t x_t' (\Omega)^{-1} x_t \right).$$

and variance

$$B = (B_0^{-1} + \sum_{t=p+1}^{T} w_t' (\Omega)^{-1} w_t)^{-1}$$

Similarly, it is not difficult to show that the posterior for $\Omega$ follows an inverse Wishart distribution. Since both the posterior for the VAR coefficients and the posterior for the variance-covariance matrices are standard distributions, the linear model can be estimated using a two-block Gibbs algorithm.

**Nonlinear Models**

Notice that if the parameter $c$ is known, the nonlinear model is equivalent to a SUR model. If we assume that the prior distribution for $\Phi$ is a multivariate Gaussian distribution, conditional on $\Omega, c$, the posterior distribution for $\Phi$ is normal, so we can sample $(\Phi^i|c^{i-1}, \Omega^i)$ directly, using a Gibbs step (the derivation of the posterior is identical to the derivation for the linear case). Similarly, conditional on $c$ and $\Phi$, the posterior of $\Omega$ is inverse Wishart distribution, so we can also sample from the posterior directly using a Gibbs step.

The posterior for $c$ in the TVAR case is nonstandard, so a Metropolis-Hastings step is used to sample from those distributions. Following the standard literature, the proposal density follows a Student-$t$ distribution with 5 degrees of freedom. We obtain the mode for the proposal distribution for the first draw through concentrated ML estimation and use a grid-search procedure to elicit a good guess for the posterior mode of the threshold parameter, $c$. Because $\varepsilon_t$ is assumed to be Gaussian, the ML estimators can be obtained by using least squares estimation. For this maximization, $c$ is restricted to a bounded set $C = [c_l, c_h]$ that covered the middle 90% of all observed values of the switching variable.

Conditional on $c$, the model is linear in $\Phi$ and $\Omega$. Estimating the linear model by splitting the sample into two subsamples yields the conditional estimators $\hat{\Phi}$ and $\hat{\Omega}$. The estimated threshold value (conditional on the delay lag $d$) can be identified uniquely as

$$\hat{c}_{mle} = \arg\max_{c \in C} L(\Phi, \Omega|c)$$

$^{15}$The Gaussian distribution is truncated to ensure that the autoregressive parameters are stationary.
where $L(.)$ is the likelihood function. We then use the estimated value for $\hat{c}$ to construct the proposal for the first draw of the MH algorithm. Given a sufficiently large burn-in sample, the initial value of $\hat{c}$ does not affect the Bayesian estimates. It provides, however, a plausible starting value for the mode and it enables us to easily compare the Bayesian mode with the ML estimate. It is important to note that the grid search makes it infeasible to obtain the variance of the estimate of $c$.

To address this issue, we follow Lo and Morley (2015) to obtain a measure of the curvature of the posterior with respect to $c$. In particular, we invert the likelihood ratio (LR) statistics for the threshold parameter, based on the assumption that the parameter estimates are normally distributed and the LR statistics for testing whether a single parameter is equal to a given value follows a $\chi^2(1)$ distribution. We then use the 95% confidence interval for the LR statistics to obtain an implied standard error for $c$. In doing so, we do not attempt to perform frequentist inferences. This method simply provides a way to obtain a plausible value for the curvature that enables us to tailor the proposal density to ensure a slightly faster convergence. Notice that this approach is only an approximation, and it is not asymptotically accurate for obtaining standard errors of the ML estimate for $c$. It is, however, much faster than bootstrapping and inverting the LR test in order to obtain the scale for the first draw of the MH algorithm. Since this approximation affects only the first draw, it will not affect the estimates of the parameters for a large enough burn-in sample.

For the $i^{th}$ iteration, the transition density for $c$ follows a Student-t distribution with mean equal to $c$ and variance equal to $\kappa \hat{\sigma}_c^2$, where $\hat{\sigma}_c$ is obtained by inverting the LR test. The parameter $\kappa$ is a scaling factor for the proposal density, adjusted to attain an acceptance rate for the MH algorithm between 20% and 60%.

We use a large burn-in sample of 10,000 draws and make inference based on additional 20,000 MH iterations. The results presented and discussed in the text were obtained using the following priors:

Table 5: Prior Distributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Mean</th>
<th>Variance / scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi$</td>
<td>Multivariate Gaussian</td>
<td>0</td>
<td>$10 \times I_k$</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Inverse Wishart</td>
<td>$I_4$</td>
<td>$16 \times I_4$</td>
</tr>
<tr>
<td>$c$</td>
<td>Student-$t$, 5 d.f.*</td>
<td>$c_{MLE}$</td>
<td>$\hat{\sigma}_c^2$</td>
</tr>
</tbody>
</table>

*truncated at $[c_{0.15}, c_{0.85}]$.

This table reports the prior distributions for the Bayesian estimation detailed in Appendix B.
Appendix C: Simulation-Based Impulse Response Functions

The procedure for computing the generalized impulse response functions (IRFs) follows Koop, Pesaran and Potter (1996), with the modification of considering an orthogonal structural shock. The generalized impulse response is defined as the effect of a one-time shock on the forecasted level of variables in the model, and the response is compared against a baseline ‘no shock’ scenario:

\[ IRF_y(k, \text{shock}_t, F_{t-1}) = E[Y_{t+k} | \text{shock}_t, F_{t-1}] - E[Y_{t+k} | F_{t-1}] \]  

where \( k \) is the forecasting horizon, \( F_{t-1} \) denotes the initial values of the variables in the model, and \( \text{shock}_t \) is a one-time shock to the exchange rate. The impulse response is then computed by simulating the model. The positive shock to the exchange rate is normalized to be equal to 1 percent (at the time the shock occurs). Thus, for \( \Theta = \{ \Phi, c, \Omega \} \), the response \( IRF_y \) for a given draw \( i \), \( \Theta^{(i)} \), of the MH algorithm is generated using the following steps:

1. Pick a history \( F_{t-1} \). The history is the actual value of the lagged endogenous variable at a particular date.

2. Pick a sequence of 4-dimensional forecast errors \( \varepsilon_{t+k}, k = 0, 1, ..., 18 \). The forecast errors are simulated assuming an independent Gaussian process with mean zero and variance-covariance matrix equal to \( \Omega^{(i)} \).

3. Using \( \Psi_{t-1} \) and \( \varepsilon_{t+k} \), simulate the evolution of \( Y_{t+k} \) over \( l+1 \) periods. Denote the resulting path \( Y_{t+k}(\varepsilon_{t+k}, F_{t-1}) \) for \( k = 0, 1, ..., l \).

4. Using the Cholesky decomposition of \( \Omega_t \) to orthogonalize the shocks, solve for the exchange rate shock at time \( t \), replace it with a shock equal to 1 percent, and reconstruct the implied vector of forecast errors. Denote the implied vector of forecast errors as \( \varepsilon_{t+k}^{\text{shock}_t} \), the sequence of forecast errors as \( \varepsilon_{t+k}^{\text{shock}_t} \), and the resulting simulated evolution of \( Y_{t+k} \) over \( l+1 \) periods as \( Y_{t+k}(\varepsilon_{t+k}^{\text{shock}_t}, F_{t-1}) \) for \( k = 0, 1, ..., l \).

5. Construct a draw of a sequence of impulse responses as \( Y_{t+k}(\varepsilon_{t+k}^{\text{shock}_t}, F_{t-1}) - Y_{t+k}(\varepsilon_{t+k}, F_{t-1}) \) for \( k = 0, 1, ..., l \). Then, calculate the PT coefficient according to equation \( \text{[2]} \).

6. Repeat steps 2 to 5 for \( B \) times, with \( B = 1,000 \), and average the sequences of responses to obtain a consistent estimate of the impulse response function conditional on the history.

7. To obtain the average response for a subset of histories, repeat steps 1-6 for a the subset of histories of interest, and report the response and the PT coefficient averaged over all histories.