Counterparty Risk and Repo Runs *

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Abstract

I develop a model in which banks finance the purchase of risky assets by borrowing against the assets as collateral. Due to limited liability, banks with impaired balance sheets have incentives to take excessive leverage, exposing their lenders to the risk of counterparty default. I show that lenders' efforts to deter excessive leverage-taking generate a novel bank-run mechanism through which a small shock on banks' balance sheets can generate large decreases in collateralized funding and asset prices. According to the model, a central bank's liquidity backstop is effective in deterring such a run on collateralized funding. In normal times, a countercyclical capital buffer can increase banks' funding stability, whereas a minimum margin requirement can be ineffective in reducing the risk of a market-wide funding squeeze.

Keywords: Financial Intermediation, Collateralized Debts, Repurchase Agreements, Financial Crises, Policy Interventions, Financial Regulations

JEL Classification: G01, G23, G28

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1 Introduction

The fragility in the repurchase agreement (repo) and other short-term collateralized funding markets was a key source of systemic risk during the 2008 financial crisis. Similar to a traditional bank run, the massive withdrawal of repo funding collateralized by non-government securities caused sizable deleveraging-induced fire sales and sudden collapses of highly leveraged financial institutions. Why did lenders run on the repo market? As evidenced by Lehman Brothers' loss of repo funding backed by all types of collateral, lenders' concerns about counterparty default played an important role in generating the funding squeeze in the repo market. In this paper, I develop a theory that explains runs on collateralized debt markets by lenders' concerns about counterparty default.

I build a static model in which banks finance the purchase of risky assets by borrowing against the assets as collateral. Banks borrow from a group of infinitely risk-averse lenders who values the asset less than banks do. Because of this asymmetric valuation, collateralized debts are inherently under-collateralized and lenders suffer losses when their counterparty borrowers default.

In the model, I link the risk of a borrower's default with the borrower's endogenous leverage choice. Due to limited liability, banks with low net worth have incentives to take excessive leverage, through scaling up their asset positions and collateralized borrowing, and become insolvent when their assets lose value. Concerned about counterparty default, lenders set the amount of collateralized funding, that is, the amount of lending per unit of collateral in their collateralized debt contracts, to deter banks from taking excessive leverage.

More interestingly, lenders' efforts to discourage excessive leverage-taking can generate positive feedback between collateralized funding and asset price. On one hand, a decrease in the amount of collateralized funding tightens banks' collateral constraints and lead to a lower asset price. On the other hand, when lenders have serious concerns about the viability of their counterparty borrowers, a lower asset price can make it more profitable for banks to take excessive leverage and thus induce lenders to further decrease the amount of collateralized funding.

Therefore, in times of heightened uncertainty about bank solvency, a small decrease in banks' net worth can trigger the positive feedback, resulting in a significant increase in lenders' concerns about counterparty risk and large decreases in collateralized funding and asset price. The discontinuity of the shock response captures how the initial shock on housing prices during the 2007-2008 financial crisis generated market-wide runs on the repo market and sharp declines in prices of assets that are not directly related to the initial shock. Different from existing theoretical literature that explains such a run either by lenders' concern about collateral value (Brunnermeier and Pedersen, 2009; Dang et al., 2013) or by lenders' strategic withdrawal behaviors (Diamond and Dybvig, 1983; Martin et al., 2014b), my model highlights lenders' concern about counterparty default as a source of fragility in repo and other collateralized debt markets.

The model generates a few testable predictions on repo and other collateralized debt markets. For the repo market, the model predicts the repo margin is increasing in the riskiness of the collateral asset. After controlling for collateral quality, the repo margin is independent of counterparty type for repos backed by liquid assets, that is, assets to which borrowers and lenders attach a similar value, but the repo margin is sensitive to counterparty type for repos backed by illiquid assets, that is, assets that lenders value less than borrowers do. These predictions are broadly consistent with empirical evidence in Gorton and Metrick (2012), Copeland et al. (2014), Krishnamurthy et al. (2014), Auh and Landoni (2016), Baklanova et al. (2017), and Hu et al. (2018) on the cross-sectional variation of repo margins across different collateral and counterparty types. According to the model, repos backed by illiquid assets are under-collateralized, while the ex-post default risk of these repos is zero except when the repo borrower is very close to insolvency. This prediction suggests a bank's credit default swap (CDS) spread may be a poor measure of lenders' concerns about counterparty default in a short-term collateralized funding market.

I use the model to study the effectiveness of ex-post policy interventions related to collateralized funding markets. First, a central bank's provision of a liquidity backstop, such as the Primary Dealer Credit Facility (PDCF) for the repo market, can coordinate lenders' concerns about counterparty default to a lower level and shift the economy out of a crisis. Implementing the liquidity backstop requires the central bank's credibility to lend freely at an amount between crisis and pre-crisis levels. Second, I show that an equity injection to banks is more effective than an asset-purchase program of the same size. Equity injection is more effective in a crisis because it relieves lenders' concerns about counterparty default by making banks better capitalized.

I extend the model in two directions to study its implications for financial regulations. First, I endogenize banks' balance-sheet strength by introducing an illiquid investment opportunity. As banks make more illiquid investments in boom times when the downside risk is limited, banks become less capitalized when the downside risk materializes, rendering the financial market vulnerable to small perturbations. As a result, a financial regulation that restricts banks' risk exposure in boom times, such as the countercyclical capital buffer in Basel III, can increase the stability of the financial market. Second, I confirm the key intuition of the baseline model in a dynamic two-period setting in which banks need to roll over their collateralized debts. In addition, I show that lenders' concerns about counterparty risk are aggravated by the existence of rollover loss due to deleveraging-induced fire sales in bad times. However, a simple minimum margin requirement that restricts banks' ex-ante leverage, such as the minimum haircut floor in Basel III, can be ineffective in eliminating the possibility of a crisis without significantly straining market liquidity in normal times.

Related Literature. The main contribution of the paper is to highlight the role of counterparty risk in collateralized debt markets. The paper primarily contributes to the literature that studies fragility in collateralized debt markets. Brunnermeier and Pedersen (2009) study a model in which borrowers finance their trades through collateralized debts. In their model, the availability of collateralized funding can decrease discontinuously as lenders become increasingly concerned about collateral risk, that is, the future volatility of collateral value. By contrast, my paper shows such a discontinuous decrease can be driven by lenders' concerns about counterparty risk, that is, the risk of counterparty default due to excessive leverage-taking. Martin et al. (2014a,b) study an infinite-horizon Diamond-Dybvig model[1] and derive constraints under which borrowers can withstand lenders' collective withdrawals of funding. The main difference in my paper is that I model runs as lenders setting higher margins in their collateralized lending for fear of counterparty risk. In Kuong (2015), the fragility in collateralized debt markets comes from the feedback between borrowers' ex-ante risk-taking incentives and lenders' ex-post fire sales of collateral assets. In my model, debt default and post-default fire sale need not occur in equilibrium, while lenders' concerns about these adverse events can generate a large tightening of collateralized funding.

Dang et al. (2013) study a model in which the margin/haircut in a repo transaction depends on the information sensitivity of the underlying collateral. When the collateral becomes riskier, the repo margin increases to the point where the lender has no incentive to acquire information about the collateral.[2] Geanakoplos (2010) develops a general equilibrium theory of leverage and asset price. He shows asset prices are more volatile when natural asset buyers leverage their asset purchases through collateralized borrowing. Acharya et al. (2011) show transitory bad news about asset value can significantly reduce the asset's borrowing capacity when banks finance the asset by rolling over short-term collateralized debts. Zhang (2014) demonstrates default can be contagious in the interbank collateralized lending market because the transfer of collateral in a borrower's default worsens the lender's liquidity condition. My paper complements the literature by emphasizing counterparty-risk concern as the source of fragility in collateralized debt markets.

The paper also contributes to the literature on risk-shifting and counterparty risk. In a classical risk-shifting model, such as in Jensen and Meckling (1976) and Acharya and Viswanathan (2011), the risk-shifting opportunity is borrowers' control over the riskiness of their assets. In Diamond and Rajan (2011), the risk-shifting opportunity shows up as borrowers' reluctance to load off illiquid assets due to the expectation of deposit withdrawals and fire-sale losses. In my paper, the risk-shifting problem arises as banks' incentives to build up excessive leverage backed by collateralized borrowing. My paper is related to Bizer and DeMarzo (1992), where borrowers have incentives to take additional debts and exert less effort in managing the assets, and to Admati et al. (2018) and DeMarzo and He (2018), where borrowers find it optimal to leverage up by issuing new debts that dilute existing creditors. The excessive leverage-taking in my model is an out-of-equilibrium concern that shapes the terms of debt contracts and generates large decreases in collateralized funding in response to small shocks.

[1] See Diamond and Dybvig (1983) for the original model of self-fulfilling bank runs. Related papers include Goldstein and Pauzner (2005), He and Xiong (2012), and Gertler and Kiyotaki (2015).

[2] Gorton and Ordonez (2014, 2016) study the macroeconomic implications of the regime shift in the information sensitivity of debt contracts.

The moral hazard of risk-shifting has been applied to study counterparty risk under different contexts. Biais et al. (2016) study the role of counterparty risk in derivatives activity. They show the existence of risk-taking opportunities leads to endogenous counterparty risk and limits the risk-sharing between derivatives counterparties. Bolton and Oehmke (2015) argue that granting automatic-stay exemption to derivative contracts can lead to an increase in derivative-users' credit risk, creating incentives for them to take risky bets. Thompson (2010) studies the effect of counterparty risk on financial insurance contracts. He finds that the existence of excessive counterparty risk due to moral hazard can alleviate information asymmetry between the insurer and the insured. Heider et al. (2015) study the role of counterparty risk in the interbank lending market. They show the information asymmetry about counterparty credit risk can lead to a complete market breakdown. My paper contributes to the literature by modeling the build-up of excessive leverage as the source of endogenous counterparty risk in collateralized funding markets and showing the concerns about counterparty risk can lead to large withdrawals of collateralized funding.

Lastly, my paper is related to the theoretical literature on borrowing constraints of financial institutions. In the model, borrowers are subject to a classic collateral constraint, such as in Kiyotaki and Moore (1997) and Benmelech and Bergman (2012). However, the collateral constraint in my model depends endogenously on lenders' concerns about counterparty risk and is tighter when borrowers are less capitalized. This feature makes the collateral constraint in my paper similar to an equity-issuance constraint in He and Krishnamurthy (2013) and other types of financial constraints that depend on borrowers' net worths, such as in Bernanke and Gertler (1989), Shleifer and Vishny (1997), Gromb and Vayanos (2002), and Gertler and Karadi (2011). My paper contributes to the literature by showing that the tightness of financial constraints can increase discontinuously as lenders become increasingly worried about borrowers' creditworthiness.

In section 2, I discuss relevant contractual and institutional features of repo and repo markets. In section 3, I present the model, study the model's predictions, and discuss relevant empirical evidence. In section 4, I analyze the bank-run mechanism in the model and discuss the robustness of my results. In section 5, I study the model's implications for policy interventions. In section 6, I extend the model to study the model's implications for financial regulations. In section 7, I conclude.

2 Repo and Repo Markets

This section briefly discusses several contractual and institutional features of repo and repo markets. The goal is not to provide a comprehensive reference guide,[3] but to present the key features that motivate this paper's research questions and key modeling assumptions.

[3]See Acharya and Öncü (2010), Adrian et al. (2013), ICMA (2013), Baklanova et al. (2015), and CGFS (2017) for detailed references of repo and repo markets.

Contractual Features of Repos. A repurchase agreement or repo is a sale of a security combined with an agreement to repurchase at a future date and is economically equivalent to a short-term collateralized loan. Repo transactions typically involve over-collateralization. For repos that are driven by the need to borrow cash, the amount of lending in a repo loan is usually smaller than the market value of securities used as collateral.[4] The difference between the two, which measures the extent of over-collateralization, is usually referred to as the repo margin or the haircut of the repo loan. The presence of positive repo margins protects cash lenders against losses when borrowers fail to repay. In addition, repo lenders are further protected by the exemption from automatic stay in bankruptcy. The exemption from automatic stay allows repo lenders to seize and liquidate the collateral immediately upon default without having to go through the bankruptcy procedure.[5] In the US, almost all repo transactions are exempt from automatic stay, including those backed by mortgage loans and mortgaged-related securities since 2005 (Acharya and Öncü, 2014).

Institutional Features of Repo Markets. Repo markets play an important role in facilitating the flow of cash and securities in a modern financial system. According to CGFS (2017), the size of the global repo market as of mid-2016 was around $12 trillion, with more than 70% backed by government bonds. The size of the US repo market was around $3 trillion[6] as of September 2018 and is estimated to have been between $6 trillion and $10 trillion right before the financial crisis.[7]

Dealer banks play a central role in repo markets.[8] In general, dealer banks perform two main functions in repo markets. First, they serve as intermediaries that stand between end repo users and trade "matched-book" repo.[9] Typically, dealer banks first lend cash to end repo borrowers such as hedge funds and pension funds against certain securities as collateral, and then use the securities as collateral to borrow from end repo lenders such as money market mutual funds (MMMFs) and securities lenders. The first transaction is typically a bilateral repo between the borrower and the

[4] Symmetrically, for repos that are driven by the need to borrow certain securities, the amount of lending in a repo loan is usually larger than the market value of securities (Baklanova et al., 2017). These repos are over-collateralized from securities lenders' perspectives. Given this symmetry, throughout this paper I will, without loss of generality, focus on repos that are driven by the need to borrow cash.

[5] In the US, the relevant bankruptcy code provisions are Sections 555, 559, 362(b)(7), 362(o), 546(f), 548(d) of the U.S. Bankruptcy Code. (Roe et al., 2014)

[6] CGFS (2017) estimates the number to be $2.7 trillion from SIFMA repo market fact sheets as of mid-2016. Baklanova et al. (2015) estimates the number to be $3 trillion as of June 2015 from Form FR 2004 data. According to SIFMA repo market fact sheets, the amount of U.S. primary dealer outstanding repo, which takes up about 70% of the total repo volume, has been very stable at $2.2 trillion for the past four years from 2015 to 2018. Therefore, I estimate the total size of the repo market to be around $3 trillion as of September 2018.

[7] Copeland et al. (2014) estimates that the size was $6.1 trillion as of July and August of 2008, whereas Gorton and Metrick (2012) and Singh and Aitken (2010) estimates the amount peaked at $10 trillion, with some double-counting, during pre-crisis period.

[8] In the US, primary dealers' total outstanding repo represents about 70% of total repo activities.

[9] The process is also called repo rehypothecation. Singh and Aitken (2010) show that rehypothecation funding is a large component of the shadow banking system. Infante (2015) and Muley (2015) study the underlying risk in the process of repo rehypothecation.

lender, whereas the second is typically a tri-party repo that involves a clearing bank that provides collateral management and facilitates repo settlement.[10]

Second, dealer banks serve as end borrowers in the repo markets to finance their holding of inventory securities and securitized bonds. I call this activity "net-position" repo, because banks are taking leveraged positions. As an end borrower, a dealer bank can borrow from other dealer banks through bilateral repos[11] or from cash investors in the tri-party repo market. Although a rough calculation shows net-position repo activity takes up less than 25% of primary dealers' total repo activities,[12] King (2008) finds the five largest dealer banks financed about half of their asset positions through repos before the financial crisis. Moreover, two-thirds of these asset positions are risky assets such as equities, corporate bonds, and structured products. Financing these long-term risky assets with short-term repos is a key part of the process of credit and maturity transformation in the US shadow banking system.[13]

To capture financial firms' use of repos to obtain leverage, I study a model in which banks take leveraged positions through collateralized borrowing. In the model, the financial asset serves both as collateral for borrowing and as the investment opportunity. The banks in the model can be interpreted either as dealer banks that finance their inventory assets in the tri-party repo market or as hedge funds that engage in leveraged trading by obtaining funding through bilateral repos.

Counterparty Risk and Excessive Leverage. Empirical studies show significant variation in the repo margin in counterparty type both in normal and crisis times.[14] Clear evidence of this counterparty dependency is the massive withdrawals of repo funding from Bear Stearns and Lehman Brothers before their collapses. For Lehman, the loss of repo funding occurred mainly during its last week.[15] According to Ball (2018), about $20 billion of Lehman's repo financing was cut off between September 10 and 12 in 2008. The loss of repo funding drove Lehman's liquidity to less

[10]Before the financial crisis, the clearing bank also provided intraday credit to repo borrowers. As shown in Copeland et al. (2012) and Martin et al. (2014b), the intraday credit exposure can be a destabilizing force that increases market fragility in crisis times. Since the financial crisis, the New York Fed has conducted the US Tri-Party Repo Infrastructure Reform and reduced clearing banks' intraday credit exposures.

[11]In the US, interdealer repo transactions with US treasuries are centrally cleared by the Fixed Income Clearing Corporation (FICC) through its General Collateral Financing (GCF) Repo Service. Such a central clearing counterparty (CCP) mechanism plays a larger role in interdealer repo markets in Europe and China. See Mancini et al. (2015) and Chen et al. (2018) for detailed descriptions.

[12]Baklanova et al. (2015) show that primary dealers' net repo amount is about 22% of their total repo activity. Iyer and Macchiavelli (2017b) suggest the fraction of net-position repo activity and the average level of repo margin sum up to 28%.

[13]See Acharya and Öncü (2010), Gorton et al. (2010), Gorton and Metrick (2012), Acharya and Öncü (2014), and Adrian and Ashcraft (2016) for the importance of repo in the shadow banking system.

[14]Dang et al. (2013) and Copeland et al. (2014) present evidence that repo margins depend on counterparty type, after controlling for collateral quality at the asset-class level, in bilateral and tri-party repo markets, repsectively. Hu et al. (2018) show the repo margin depends on counterparty type after controlling for collateral quality at the security level in the tri-party repo market. See section 3.3 for a detailed discussion of this evidence.

[15]Lehman's loss of repo funding can be also seen from the significant decrease in the value of collateral posted in the tri-party repo market shown in Copeland et al. (2014).

than \$1 billion and pushed Lehman to declare bankruptcy because it was unable to meet more repo withdrawals on the coming Monday, September 15.

Lehman's reliance on repos before the crisis is likely to be an important reason it faced the most severe withdrawal of repo funding in the crisis. Although the fraction of Lehman's own assets funded with repos was 46% in mid-2008 and around the average of the top-five US investment banks, Lehman's asset composition was significantly tilted toward non-government securities, such as private-label asset-backed securities (ABS) and corporate bonds (King, 2008; Krishnamurthy et al., 2014). Further suggestive evidence of Lehman's excessive use of repos is that Lehman used the accounting gimmick Repo 105 to move \$50 billion in repo transactions off its balance sheet around financial-reporting days, reducing its reporting net leverage ratio from 17.8 to 16.1 (Valukas, 2010). Since the financial crisis, a key objective of the regulatory reform is to limit excessive leverage in repo and other short-term wholesale funding markets, under the belief that "excessive use of repos can indeed weaken the financial system by facilitating the use of short-term funding and the build-up of leveraged positions backed by collateralised borrowing" (CGFS, 2017).

Motivated by Lehman's over-reliance on repo funding, I model a collateralized lending relation in which the lender is exposed to the default risk of the borrower when the borrower takes excessive leverage through collateralized borrowing. In the model, such a leverage-induced counterparty risk plays little role in good times, but it becomes a primary concern in the lender's lending decision in stress times.

Repo Runs during the Financial Crisis. The collapse of Lehman triggered a massive withdrawal of funding from the repo markets. The run on repo primarily features a large increase in repo margins. Gorton and Metrick (2012) show the average repo margin increased from 20% to 45% for bilateral repos backed by certain non-subprime-related asset classes. Copeland et al. (2014) find the average repo margin increased from 5% to 7% for repos backed by non-government securities in the tri-party repo market.[16]

In addition, the set of eligible collateral in repo markets narrowed during the crisis. Following Lehman's bankruptcy, lenders stopped accepting subprime-related structured products as collateral for bilateral repo lending (Acharya and Öncü, 2014). Meanwhile, MMMFs, important lenders in the tri-party repo market, stopped accepting private-label ABS as collateral (Krishnamurthy et al., 2014). Lastly, the total amount of repo lending decreased significantly, especially for repos against illiquid assets as collateral. From July 2008 to April 2009, the value of ABS and corporate bonds used as tri-party repo collateral declined by 66% and 44%, respectively (Copeland et al., 2014). The

[16]The two-percentage-point increase roughly corresponds to a deleveraging of 30% of existing positions, without considering borrowers posting additional collateral and potential fire-sale loss in deleveraging. As discussed in Copeland et al. (2014) and Martin et al. (2014b), the lack of adjustment in repo margins in the tri-party repo market reflects the fact that lenders simply cut off funding rather than adjust the margin of the repo loans when they have serious concerns about counterparty risk.

total amount of outstanding primary dealer repo dropped from more than \$4.5 trillion in 2008 to less than \$3 trillion in 2009.

In this paper, I explain the large disruption in repo markets by lenders' heightened concerns about counterparty default. I show that a small shock can significantly increase lenders' concerns about counterparty risk, leading to large decreases in collateralized funding and asset price.

3 The Model

Consider an economy with two dates, $t = 1, 2$. The economy is populated with two groups of agents, banks and lenders, of the same size. Banks are risk-neutral. I assume each bank obtains a non-pledgeable charter value $C > 0$ when the bank remains solvent at date 2. When becoming insolvent at date 2, the bank is protected by limited liability but loses the charter value. The charter value captures the bank's continuation payoff due to future trading activities and its avoidance of bankruptcy cost. Without loss of generality, both the risk-free rate and the discount rate in the economy are normalized to zero.

At date 1, each bank owns l units of cash and x units of net illiquid asset. Each unit of net illiquid asset generates no cash flow at date 1, and pays one unit at date 2.[17] Banks differ in their net illiquid asset values. Specifically, a θ fraction of banks have $x = x_H > 0$, whereas the remaining $1-\theta$ fraction of banks have $x = x_L \in (-l, 0)$. I assume a bank's net illiquid asset value is its private information. At date 1, each bank can purchase financial assets in a competitive asset market. I assume for simplicity that financial assets are homogeneous and inelastically supplied at A. If held by a bank, a financial asset pays v_H in the good state, which happens with probability q, and v_L in the bad state, which happens with probability $1-q$, at date 2. Let $\bar{v} = qv_H + (1-q)v_L$ be the expected value of an asset.

To purchase the asset, banks can borrow from a group of lenders. I make two simplifying assumptions about the lenders. First, I assume lenders are infinitely risk averse. This assumption captures the fact that lenders are looking for safe lending opportunities in repo markets. Section 4.3 discusses the robustness of the model's main results under different assumptions about lenders' risk preferences. Second, I assume the aggregate supply of loanable fund is larger than the largest-possible aggregate demand by imposing the following parameter restriction.

Assumption 1: Each lender has more than $v_L(A + \frac{(1-\theta)C}{q(v_H - v_L)})$ units of loanable funds.

Under Assumption 1, lenders face perfect competition, so each lender's profit-maximization problem is equivalent to maximizing her borrower's expected payoff subject to the constraint that the

[17] When $x < 0$, it requires a repayment of $-x$ unit of cash. Think of the net illiquid asset as the value of the bank's illiquid investments (loans) net of the repayment of its existing debt in place.

lender breaks even. Assumption 1 also implies that all lenders are deep-pocketed, in the sense that each lender has enough loanable funds to accommodate the borrowing need of one borrower. Without loss of generality, I assume one bank borrows from one lender only. In section 4.3, I discuss the robustness of the model when a bank has to borrow from multiple lenders.

The key assumption of the model is that lenders only value the financial asset at $u < v_L$ in the bad state at date 2. This assumption is primarily motivated by the fact that some repo lenders have to liquidate the seized collateral immediately due to the existence of institutional restrictions and balance-sheet constraints.[18] Therefore, u captures the short-term liquidation value of the asset in the bad state, which can be significantly less than the asset's fundamental value v_L. The difference between the two captures the illiquidity of the collateral asset, which is exogenous in my main model and can be microfounded by incomplete markets (section 6.2 of the paper) or fire-sale losses (Shleifer and Vishny, 1992).[19]

The financial friction in the model is that contracts are incomplete. Formally, I assume lending contracts can be written only on contingent transfers of the financial asset.[20] Because the only enforceable penalty that a lender can impose on a defaulting borrower is to take a pre-determined amount of financial assets away from the borrower, any lending contract in this environment can be interpreted as a collateralized debt contract.

A collateralized debt contract, denoted by ϕ, specifies three variables $\phi = (d, f, a)$: the amount of lending, the face value, and the amount of assets as collateral. A bank that takes this contract borrows d units of cash at date 1. At date 2, the bank keeps the collateral assets when it repays the face value f.[21] When the bank fails to repay, a units of assets are transferred to the lender. In such a default, I assume the collateralized debt contract is not renegotiated ex post. In the Appendix, I provide two micro-foundations for this no-renegotiation assumption. The first one relies on lenders' preference for immediate liquidation of the collateral, and the second one arises from borrowers' reluctance to strategically renegotiate due to concerns about bank runs.

The timeline of the model is the following. First, each lender is matched with one bank at date 1. Taking the asset price P as given, the lender quotes a set of collateralized debt contracts to a matched bank. Second, taking the asset price and the set of collateralized debt contracts as given,

[18] For example, money market mutual funds (MMMFs), frequent lenders in repo markets, are restricted by Rule 2a-7 to hold assets with significant residual maturity.

[19] In addition, lenders' lower valuation of the collateral also captures the hassle of seizing and liquidating the collateral (Duffie, 2010), the headline risk of lending to troubled banks (Ball, 2018), different beliefs about the asset's future payoff (Geanakoplos, 2010), different valuations of the asset's future collateralizability (Parlatore, 2018), or inferior quality of distressed assets due to adverse selection (Wang, 2018).

[20] See Hart and Moore (1994) and Kiyotaki and Moore (1997) for detailed descriptions the incomplete-contract framework in financial contracting.

[21] This scenario is true under the assumption that lenders always deliver the collateral when payment is made in full. Therefore, the model abstracts from another important source of counterparty risk, which is the risk of cash lenders failing to deliver the collateral at the settlement date. Although much less disastrous, settlement failures can pass through the repo intermediation chain (Iyer and Macchiavelli, 2017c) and cause systemic liquidity squeezes (Fleming and Garbade, 2005).

each bank makes its asset purchase and collateralized borrowing decisions. Then, the asset price is determined in the competitive market. Lastly, assets pay off and debts mature at date 2. Banks repay their lenders subject to limited liability.

3.1 The Equilibrium

Consider a lender's problem when it is matched with a borrower with unknown type. Because the borrower has only two possible types, it is without loss of generality to assume that the lender provides only two collateralized debt contracts in order to attract each type of borrowers. Denote the contract that attracts a high-type bank by $\phi_H = (d_H, f_H, a_H)$ and the contract that attracts a low-type bank by $\phi_L = (d_L, f_L, a_L)$. Because the lender takes the asset price P as given, both the two contracts are functions of P. The model is easily extended to cases where there are more than two types of banks.

For a contract $\phi = (d, f, a)$ to be feasible, it has to satisfy the following conditions. First, the amount of funding must be enough to cover the cost of purchasing the collateral asset, that is, $aP \leq l + d$. Second, the face value of debt must not be lower than the amount of lending, that is, $d \leq f$. Lastly, the face value of debt must be lower than the worst-case value of the collateral asset, that is, $f \leq av_L$. Whenever this condition is violated, the borrower defaults the lender incurs a loss when the asset pays low. Let $\Phi = \{(d, f, a) | aP \leq l + d; d \leq f; f \leq av_L\}$ be the set that contains all feasible collateralized debt contracts. Note that the set depends on the asset price, which is taken as given by the lender.

For a bank with type i, the bank's expected payoff when taking a collateralized debt contract ϕ is $V(x_i, \phi) = q(x_i + l + a(v_H - P) + d - f + C) + (1 - q)(x_i + l + a(v_L - P) + d - f + C)^+$, where $y^+ \equiv y\mathbb{1}(y \geq C)$. The indicator function highlights the possibility that a bank can default and lose the charter value in the bad state. Note because $x_H > 0$, a high-type bank always stays solvent at date 2. In writing banks' expected payoffs, I implicitly assume banks pledge all their assets in the collateralized debt contract. This assumption is without loss of generality. As we will show later, the interest rate of lending in equilibrium is equal to the risk-free rate the bank earns by holding cash. Therefore, any equilibrium of the model will be payoff equivalent to a corresponding equilibrium in which banks pledge all their assets out.

Because lenders face perfect competition under Assumption 1, a lender's objective is to design the contracts, ϕ_H and ϕ_L, to maximize banks' expected payoffs, that is,

$$\max_{\phi_i} V(x_i, \phi_i) \text{ for } i = H, L \tag{1}$$

$$\phi_H, \phi_L \in \Phi \tag{F}$$

$$x_L + l + a_L(v_L - P) + d_L - f_L \geq 0 \text{ or } d_L \leq a_L u \tag{BE}$$

$$V(x_L, \phi_L) \leq V(x_L, \phi_H) \tag{IC}$$

The first constraint is the feasibility constraint. The second constraint is a break-even constraint. Because the lender is infinitely risk-averse, the break-even constraint requires either the low-type bank to stay solvent in the bad state or the amount of lending to be less or equal to the lender's valuation of the collateral. The third constraint, the incentive-compatibility (IC) constraint, makes sure that the low-type bank does not have an incentive to take the equilibrium contract designed for the high-type bank. Note we left out another IC constraint that requires a high-type bank not to take the equilibrium contract designed for the low-type bank, because this constraint will not bind in equilibrium. Definition 1 defines the equilibrium of the model.

Definition 1. *An equilibrium of the model is defined as an asset price P and two collateralized debt contracts $\phi_H = (d_H, f_H, a_H)$ and $\phi_L = (d_L, f_L, a_L)$, such that*

1. *Given the collateral asset price P, each lender quotes ϕ_H and ϕ_L to solve the maximization problem in (1).*
2. *Given P, ϕ_H, and ϕ_L, a bank with type $i = H, L$ chooses the collateralized debt contract ϕ_i and purchases a_i units of assets.*
3. *The collateral asset price P is determined by the market-clearing condition $\theta a_H + (1-\theta) a_L \geq A$, where the strict inequality holds only when $P = \bar{v}$.*

Because lenders take the asset price as given when quoting debt contracts, ϕ_H and ϕ_L are functions of the asset price P. Section 3.2 studies how equilibrium collateralized debt contracts depend on the endogenous asset price. Section 3.3 studies how equilibrium collateralized debt contracts depend on exogenous parameters that capture collateral quality and counterparty type. Section 4 studies the determination of asset price and the novel bank-run mechanism through which small shocks can lead to large decreases of collateralized funding and asset price.

3.2 Collateralized Funding and Asset Price

A lender's problem is to maximize its expected payoff by quoting a set of collateralized debt contracts to banks with different types. Because lenders are infinitely risk averse and face perfect competition, they quote a set of collateralized debt contracts to maximize borrowers' expected payoffs, subject to the constraint that they break even in the worst-case scenario. Proposition 1 states the equilibrium lending contracts between a lender and a borrower.

Proposition 1. *Assume assumption 1 holds. In equilibrium, lenders quote* $\phi_L = (a_L^* D, a_L^* D, a_L^*)$ *and* $\phi_H = (a_H^* D, a_H^* D, a_H^*)$*. Both contracts have the same amount of lending per collateral, which is equal to* $D = \max\{u, \min\{v_L, D^*(P)\}\}$*, where* $D^*(P) \equiv P - \frac{l}{\frac{l+x_L}{P-v_L} + \frac{(1-q)C}{q(v_H-P)}}$*. Banks' asset positions are* $a_H^* = \frac{l}{P-D}$ *and* $a_L^* = \frac{l+x_L}{P-v_L}$ *when* $D > u$ *and* $a_L^* = \frac{l}{P-D}$ *when* $D = u$*.*

The proof of Proposition 1 is in the Appendix. According to Proposition 1, the two equilibrium contracts have zero interest rate and the same amount of lending per collateral. In other words, both high-type and low-type banks borrow the same amount of money against one unit of collateral at zero interest rate. When the amount of lending is equal to the lender's valuation of the collateral $D = u$, the equilibrium debt contract is a pooling contract in which both types of banks purchases the maximum amount of assets and holds no cash at hand. In this case, lenders are fully secured so they provide contracts that allow banks to take maximum leverage regardless of whether banks will default or not. The $D = u$ will be referred to as the full-security condition.

When the full-security condition is violated, that is, $D > u$, the key difference between the two contracts is that $a_L^* < a_H^*$. A high-type bank has $a_H^* = \frac{l}{P-D}$, so it purchases the maximum amount of assets and holds no cash at hand. On the contrary, a low-type bank has $a_L^* = \frac{l+x_L}{P-v_L} < a_H^*$. Instead of spending all money in asset purchase, a low-type bank holds some cash at hand to avoid default in the bad state.

When $D > u$, the key variable in lenders' lending decision D, which is the amount of lending per unit of collateral, is set to make the lending risk free and is determined by two conditions. Note D will also be referred to as the amount of collateralized funding and the tightness of banks' collateral constraints. First, D must satisfy the no-recourse condition $D \leq v_L$. This condition arises from the feasibility constraint and sets the amount of lending per collateral to be lower or equal to the asset's worst-case value. If this condition is violated, borrowers have incentives not to buy back the collateral the at debt repayment date.

More importantly, D must satisfy the no-default condition $D \leq D^*(P)$. This condition arises from the IC constraint in (1). Intuitively, the lender wants to induce the low-type bank not to take the equilibrium contract designed for high-type banks. Without the IC constraint, the low-type bank has an incentive to accept the high-type bank's debt contract, because it allows a larger asset position or, equivalently, higher leverage. A higher leverage generates more profit in the upside, while shifting a part of the default cost to lenders in the downside. Because lenders are averse to the default risk, they restricts the amount of collateralized funding in the high-type bank's debt contract, through imposing the IC constraint, in order to deter low-type banks from taking that contract.[22]

[22]In Proposition1, lenders also restrict the amount of collateralized funding in the low-type banks' debt contract.

Intuitively, lenders set the amount of lending per collateral to deter banks from taking excessive leverage and becoming insolvent in the bad state. Whenever $D > u$, excessive leverage-taking and counterparty default do not arise in equilibrium. However, lenders' concerns about these adverse events are what determine the amount of collateralized funding in equilibrium. The intuition here is similar to a classical moral hazard problem, such as in Holmstrom and Tirole (1997), with the twist that the moral hazard to deter in my model is the borrower's excessive leverage choice.

Assumption 2: Each lender's valuation of a financial asset satisfies $u < \frac{qx_L v_H + (ql+(1-q)C)v_L}{qx_L + (ql+(1-q)C)}$.

Assumption 2 is a sufficient condition for $u < \min_{P \in [v_L, \bar{v}]} D^*(P)$, which says that lenders' valuation of the collateral is strictly lower than the maximum amount of lending implied by the no-default condition for all possible market-clearing prices. As a result, $D > u$ always holds if Assumption 2 holds. A collateral asset that satisfies Assumption 2 is interpreted as being illiquid, because lenders' valuation of the collateral asset is much lower than borrowers' valuation.

For a liquid asset, that is, an asset to which u is very close to v_L, the model predicts that the amount of lending against the asset is close to the asset's short-term liquidation value. By contrast, for an illiquid asset, that is, an asset that satisfies Assumption 2, the model predicts that the amount of lending against the asset is higher than the short-term liquidation value of the collateral. In other words, collateralized debts that are backed by illiquid assets are default free but under-collateralized. The prediction is consistent with empirical evidence in the bilateral repo market. Baklanova et al. (2017) find that more than 80% of bilateral repos that are backed by treasuries are associated with repo margins high enough to protect lenders against future price changes, whereas only 22% of bilateral repos backed by equities have the same feature.[23]

Because of this under-collateralization feature, a lender's lending decision depends not only on the collateral's worst-state payoff, but also on the borrower's perceived creditworthiness, which itself depends on the collateral asset price and other model primitives. To study the effect of the collateral asset price on collateralized funding, I first look at $m^*(P) \equiv P - D^*(P)$, the minimum margin that satisfies the no-default condition:

$$m^*(P) = \frac{l}{\underbrace{\frac{l + x_L}{P - v_L}}_{\text{fundamental-risk}} + \underbrace{\frac{(1-q)C}{q(v_H - P)}}_{\text{risk-taking}}} \tag{2}$$

This restriction is to avoid low-type banks taking excessive leverage through borrowing from other lenders. See the Appendix for a detailed discussion of proper off-equilibrium beliefs and actions that lead to this result.

[23] In their sample, all equity repos are associated with negative repo margins, reflecting cash borrowers' (securities lenders') worry about not getting the collateral back. To the extent that securities borrowers have an incentive to establish excessive leveraged short positions by selling the borrowed securities, the model is suitable to analyze these cases, with the twist that the counterparty risk now arises from cash lenders' (securities borrowers') leverage choices.

Here the margin of a collateralized debt is defined as the dollar-amount difference between the collateral asset price and the amount of lending per collateral.[24] In equation 2, a decrease in the asset price affects the margin $m^*(P)$ through two channels. First, a fundamental-risk channel exists in which a decrease in asset price reduces the bank's potential loss in purchasing the risky asset. As a result, the lender becomes less concerned about counterparty risk and sets a lower margin or, equivalently, increases the amount of collateralized funding. Second, a risk-taking channel exists in which a decrease in asset price increases the bank's potential gain in purchasing the risky asset. As a result, the borrower has a high incentive to take excessive leverage, making the lender more concerned about counterparty risk. In response, the lender sets a higher margin or, equivalently, reduces the amount of collateralized funding.

The presence of the two channels makes $m^*(P)$ a non-monotonic function in asset price. Specifically, it is increasing in asset price when the asset price is low and the fundamental-risk channel dominates, but decreasing in asset price when the asset price is high and the risk-taking channel dominates. Proposition 2 summarizes the property of the equilibrium amount of lending D.

Proposition 2. *Under Assumption 1 and 2, each lender sets the amount of lending per unit of collateral at* $D = \min\{v_L, D^*(P)\}$ *and* $D > u$. *Further, it has the following properties*

1. *When* $P \in [P_1, \bar{v}]$, $D = v_L$. *This region is non-empty when* $x_L + C > 0$.
2. *When* $P \in [v_L, P_1]$, $D = D^*(P)$. *Further, there exists a threshold value* $P_2 \in (v_L, P_1)$ *such that* $\frac{\partial D}{\partial P} > 0$ *when* $P \in [P_2, P_1)$, *and* $\frac{\partial D}{\partial P} < 0$ *when* $P \in [v_L, P_2)$.

The threshold values in Proposition 2 are given by $P_1 = \frac{(1-q)Cv_L - qx_L v_H}{(1-q)C - qx_L}$ and $\frac{\partial D^*(P)}{\partial P}|_{P=P_2} = 0$. Figure 1 plots the amount of lending per collateral (D, in the right y-axis) and the corresponding margin (m, in the left y-axis) as functions of asset price on $[v_L, \bar{v}]$. When the asset price is above the threshold value P_1, the amount of lending per collateral is simply v_L, the asset's payoff in the bad state. Intuitively, when the asset price is relatively high, the borrower has few incentives to take excessive leverage. As a result, the lender sets the amount of lending per collateral at the highest possible value given by the no-recourse condition.

When the asset price is below the threshold value P_1, the amount of collateralized funding is $D^*(P)$, determined by the no-default condition, and the margin is $m^*(P) = P - D^*(P)$. In this region, borrowers have incentives to take excessive leverage were they given v_L units of funding per collateral. So lenders reduce the amount of lending in order to incentivize borrowers to refrain from taking excessive leverage and to always repay. As discussed in equation 2, both the amount of lending and the margin of the collateralized debt are non-monotonic functions of asset price.

[24] The model's main results will be qualitatively the same when the repo margin is instead defined as $\frac{P-D}{P}$, the percentage difference between the collateral asset price and the amount of lending.

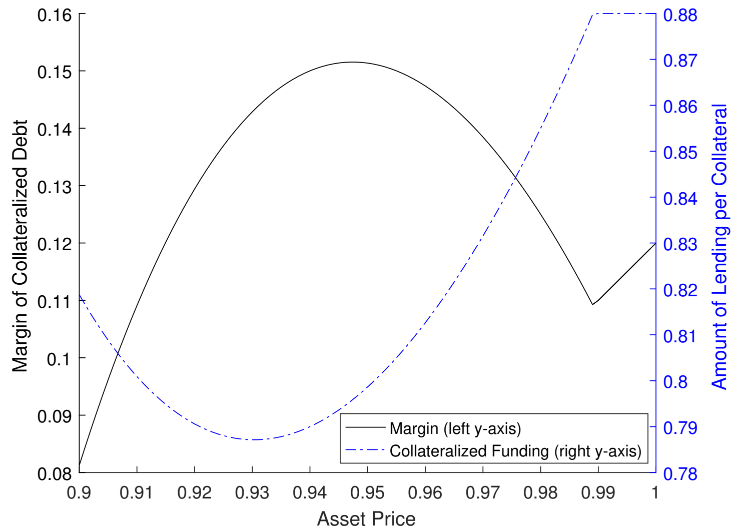

Figure 1: The figure plots the margin of the collateralized debt (solid black) and the amount of lending per collateral (dash-dot blue) as functions of the collateral asset price. Parameter values are $v_H = 1.03$, $v_L = 0.88$, $q = 0.8$, $\bar{v} = 1$, $l = 0.5$, $x_H = 0.1$, $x_L = -0.4$, $\theta = 0.8$, $u = 0.78$, $A = 3.3$, and $C = 0.6$.

3.3 Collateralized Funding and Counterparty Type

Keeping the collateral asset price fixed, lenders' provision of collateralized funding depends on exogenous parameters that capture the quality of the collateral and the creditworthiness of the borrower. In this subsection, I analyze how the margin of the collateralized debt depends on these parameters and discuss relevant empirical evidence in repo markets.

Collateral Quality

To study the effect of collateral risk, I write the asset's payoff as $v_H = v_H(\delta) \equiv v_H^0 + \frac{(1-q)\delta}{q}$ in the good state and $v_L = v_L(\delta) \equiv v_L^0 - \delta$ in the bad state. δ captures the riskiness of the collateral asset. In the model, collateral assets that have the same riskiness can differ in their liquidity, which is captured by u, namely, lenders' valuation of the collateral.

Corollary 3.1. *The margin of a collateralized debt is weakly increasing in* δ *and weakly decreasing in* u*, that is,* $\frac{\partial m}{\partial \delta} \geq 0$ *and* $\frac{\partial m}{\partial u} \leq 0$.

The model predicts that the margin of a collateralized debt is higher when the underlying collateral asset is riskier and less liquid. The margin is increasing in the riskiness of the collateral asset for two reasons. First, because of the no-recourse condition, the amount of lending per collateral is capped by the worst-state value of the collateral asset v_L. Second, the increase in the riskiness of the asset makes it more profitable for the borrower to take excessive leverage, inducing lenders to set a higher margin. The margin is higher when the collateral asset is more illiquid, because lenders are concerned about counterparty risk when lending against illiquid assets as collateral.

This prediction is consistent with the variation of repo margins in collateral quality. Auh and Landoni (2016) find that bilateral repos backed by securitized bonds are associated with lower repo margins when the underlying collateral assets have higher ratings and higher seniority among all tranches of the same collateral. Gorton and Metrick (2012) show that a proxy for the expected price volatility of a given asset class can explain the variation of the average margin of bilateral repos backed by assets in that asset class. For the tri-party repo market, the average margin of repo loans backed by non-government securities, such as corporate bonds, equities, and private-label ABS, is consistently higher than that of repo loans backed by treasuries and agency securities.[25] Moreover, Copeland et al. (2014) and Krishnamurthy et al. (2014) show the difference in repo margins between repos backed by non-government securities and those backed by government securities widened during the crisis.

Interestingly, Krishnamurthy et al. (2014) show that MMMFs stopped accepting private-label ABS as collateral after Lehman defaulted, whereas empirical evidence in Copeland et al. (2014) suggests other lenders in the tri-party repo market continued to lend against private-label ABS, albeit at a higher repo margin, during the same time period. This difference can be explained by assuming MMMFs value the collateral asset less than other lenders do. In reality, MMMFs are subject to higher liquidation cost due to institutional restrictions in holding assets with significant residual maturity. Besides, the lower valuation also captures the fact that MMMFs are concerned about large redemption requests after the bad publicity of lending to a failing borrower. As a result of Corollary 3.1, repo margins set by MMMFs can be strictly higher than those set by other lenders.

Counterparty Creditworthiness

Both x_L and C capture the borrower's creditworthiness. A decrease in x_L means the bank is perceived as likely to be more poorly-capitalized, while a decrease in C means a bank has a lower charter value.[26] Lenders perceive a bank that has a lower x_L or C less creditworthy borrower because it has a higher incentive to take excessive leverage.

Corollary 3.2. *The margin of a collateralized debt is weakly decreasing in x_L, the low-type bank's net illiquid asset value, and C, the bank's charter value; that is, $\frac{\partial m}{\partial x_L} \leq 0$ and $\frac{\partial m}{\partial C} \leq 0$. Further, the inequality is strict when Assumption 2 holds and $x_L < -\frac{(1-q)C}{q}\frac{P-v_L}{v_H-P}$.*

[25] See Copeland et al. (2014) and Krishnamurthy et al. (2014) for evidence during the 2007-2008 financial crisis, and Hu et al. (2018) for evidence after the crisis.

[26] A decrease in C can be interpreted as the hedge fund clients of a troubled dealer bank shifting their cash and securities to better capitalized banks. See Duffie (2010) for a detailed discussion.

The model suggests the margin is decreasing in the creditworthiness of the borrower when the underlying collateral asset is illiquid and the borrower is perceived as likely to be poorly capitalized. In these cases, lenders set a higher margin to reduce the provision of collateralized funding and to deter borrowers from taking excessive leverage.

Consistent with this prediction, empirical evidence in the tri-party repo market shows the repo margin depends on counterparty type. Copeland et al. (2014) find the identity of the borrower is an important factor that determines the repo margin in the tri-party repo market during the financial crisis. Hu et al. (2018) show that repo margins set by MMMFs after the financial crisis depend on counterparty type for repos backed by equities and high-yield corporate bonds, but not for those backed by US treasuries and investment-grade corporate bonds.

However, both papers find that a repo borrower's credit risk, measured by its five-year CDS spread, has little power in explaining the variation in the repo margin and repo funding. Hu et al. (2018) show that after controlling for borrower fixed effects, an increase in the borrower's CDS spread has negligible effects on the margin of a repo loan. Copeland et al. (2014) find a similar result for repos backed by government securities. For repos backed by non-government securities, they show the effect of the borrower's CDS spread on the repo margin is highly non-linear and economically significant only for a very large increase in the borrower's CDS spread. Moreover, they find the effect of a borrower's CDS spread on the borrower's total repo funding, measured by the amount of collateral posted by the borrower, is statistically insignificant. These pieces of evidence do not necessarily contradict the model's prediction, because a borrower's five-year CDS spread may be a poor measure of the borrower's creditworthiness from a repo lender's perspective. Because most repos are overnight, lenders should care more about the borrower's exposure to short-term risk factors, which are not captured by the CDS spread.

Iyer and Macchiavelli (2017a) measure lenders' exposure to counterparty risk by the ratio of a borrower's net positions in non-government securities to the sum of its long and short positions. They find the change of a dealer bank's secured funding[27] is not negatively correlated with the bank's net-position ratio until a month prior to Lehman's bankruptcy. During that last month, the negative correlation was present for both secured funding backed by non-government securities and those backed by US treasuries, and was more significant when the borrower was Lehman.[28] These empirical findings are broadly consistent with the prediction in Corollary 3.2, except for the finding that repo funding backed by government securities also became sensitive to counterparty risk during

[27]The change of secured funding corresponds to the amount of funds received in agreements to deliver securities to counterparties from Columns 9 through 12 in Form C of FR2004 Primary Government Securities Dealers Reports. These securities-out activities are mainly repos, securities lending, and securities delivered to a derivative counterparty for margin calls.

[28]This result is also consistent with Krishnamurthy et al. (2014), who show a dealer bank's change in repo funding from prior to Bear's collapse to after Lehman's bankruptcy is negatively correlated with the bank's reliance on non-government repo collateral prior to Bear's collapse.

the crisis. The contraction of a dealer bank's repo funding backed by safe assets can be instead explained by the dealer bank's clients pulling out these assets from the bank for fear of counterparty risk (Duffie, 2010) or the dealer bank selling safe assets in order to raise liquidity (Copeland et al., 2014). These channels are outside the current model and are interesting topics for future research.

Bear and Lehman: Ineligible Collateral

Clear evidence of the repo margin being counterparty dependent is the observation that Bear Stearns and Lehman Brothers lost funding in the tri-party repo market before their collapses during the financial crisis. To study runs on these risky borrowers, I introduce a borrower named Lehman into the model. Let x_{Lehman} be lenders' expectation of Lehman's lowest-possible date-2 net illiquid asset value.

Corollary 3.3. *The amount of lending per collateral to Lehman is equal to u, that is, lenders' valuation of the collateral, when Lehman's initial net worth* $x_{Lehman}+l < \max\{0, (P - v_L)(\frac{l}{P} - \frac{(1-q)C}{q(v_H-P)})\}$.

Intuitively, whenever lenders expect a positive probability of the borrower (Lehman) being sufficiently close to insolvency, their lending will always be exposed to positive counterparty risk. As a result, infinitely risk-averse lenders will only lend an amount equal to their valuation of the collateral. In the presence of headline risk and market illiquidity, u can be very small and close to zero for illiquid assets. When $u = 0$, lenders effectively refuse to lend to Lehman.

The $u = 0$ scenario in the model is consistent with market participants' perception that in the tri-party repo market, "a margin may not be sufficient for an investor if it has serious concerns about the viability of its counterparty" (Valukas, 2010). Indeed, as discussed in section 2, repo lenders pulled out funding massively from Bear and Lehman right before their collapses, even against collateral assets that were relatively safe. Paul Friedman, senior managing director of Bear Stearns, recalled that "repo market lenders declined to roll over or renew repo loans, even when the loans were supported by high-quality collateral such as agency securities." (Friedman, 2010). For Lehman, Copeland et al. (2014) show the amount of collateral posted by Lehman in the tri-party repo market decreased dramatically for all collateral types during its last week.[29]

If interpreted directly, the model says that a bank's loss of collateralized funding is just a consequence, but not the cause, of its near-insolvency. However, the cause of its near-insolvency, which is the bank's low net worth, is exogenous in the model. So the baseline model is silent on whether the loss of collateralized funding contributes to a bank's insolvency. In reality, a bank's low net worth can be either due to a decrease in the value of its illiquid investment (see section 6.1 for the

[29] Again, a large decrease in repo quantity should be read as evidence of a repo run with cautions. More anecdotal evidence of Lehman's loss of repo funding can be found in Ball (2018).

extension), which is orthogonal to its loss of funding, or due to fire-sale losses in deleveraging (see section 6.2 for the extension), which is caused by its loss of funding. During the financial crisis, both forces had contributed to the collapse of Bear and Lehman.

Repo Rate and Repo Maturity

Because of the assumption that lenders are infinitely risk averse, the model predicts the interest rate of the collateralized debt is always equal to the risk-free interest rate. Krishnamurthy et al. (2014) show that repo rates set by MMMFs in the tri-party repo market were close to the Fed fund rate before the financial crisis. Even in times when both margins and rates of repos backed by non-government securities spiked following Lehman's default, the repo rate recovered to a few basis points above the risk-free rate relatively quickly, while the repo margin continued to stay at a high level and repo funding kept falling for an extensive period of time. Auh and Landoni (2016) find that bilateral repos backed by riskier collateral are associated with higher repo rate and repo margin. According to their findings, the difference in repo rates is a few basis points, whereas the difference in repo margins can be as large as 15%. These facts suggest repo lenders primarily choose to reduce their risk exposure through quantity adjustment, rather than to ask for more risk compensation through price adjustment, when lending to riskier borrowers against riskier collateral. In section 4.4, I argue these empirical facts can be rationalized in a model where lenders are assumed to be highly, but less than infinitely, risk averse.

The model does not have a direct implication about the effect of maturity on the margin of collateralized debt. Indirectly, a collateralized debt with shorter maturity can be viewed as one backed by a safer asset, because the expected volatility of the collateral asset price is lower at a shorter horizon. Therefore, the model predicts a lender can reduce their concerns about counterparty risk by shortening the maturity of the collateralized debt. Krishnamurthy et al. (2014) shows evidence of such maturity compression in the tri-party repo market in the time series during the crisis. On the contrary, Auh and Landoni (2016) find that bilateral repos backed by riskier collateral are associated with longer maturity for periods before the crisis. Future study is needed to better understand the determination of repo maturity under different trading mechanisms and in different market environments.

4 The Model: Repo Runs

I have shown that lenders' provision of collateralized funding depends on the price of the collateral asset through lenders' concerns about counterparty default. In this section, I show the price of the collateral asset is determined by the tightness of collateralized funding. Under certain conditions, the model generates a positive feedback between collateralized funding and asset price, resulting in multiple equilibria. To see this, first note that the equilibrium can be summarized by the following two equations

$$D = \max\{u, \min\{v_L, D^*(P)\}\} \tag{CC}$$

$$\theta\frac{l}{P-D} + (1-\theta)\frac{l+x_L}{P-v_L} = A \tag{MC}$$

The first equation is the collateral constraint equation from Proposition 1. Intuitively, it shows how the amount of collateralized funding depends on the price of the collateral asset through lenders' concerns about counterparty default. The second equation is the market clearing equation in an interior equilibrium. Note that there can be a corner equilibrium in which $P = \bar{v}$ and the MC equation holds with strict inequality $>$. Intuitively, this equation shows how the asset price depends on the amount of collateralized funding through market clearing. In an interior equilibrium, the asset price is determined by cash-in-the-market pricing in the spirit of Allen and Gale (1994). Proposition 3 shows the economy can have multiple equilibria.

Proposition 3. *Assume Assumptions 1 and 2 hold. There exist two threshold values x_1 and x_2, such that $-l \leq x_1 \leq x_2 < 0$, and*

1. *When $x_L \in (x_2, 0)$, the economy has a unique equilibrium with $D = v_L$.*
2. *When $x_L \in (x_1, x_2)$, the economy has three equilibria with distinct levels of collateralized funding D. Two of the three equilibria are stable against small perturbations, whereas the other one is unstable.*
3. *When $x_L \in (-l, x_1)$, the economy has a unique equilibrium with $D < v_L$.*
4. *All three regions are non-empty, that is, $-l < x_1 < x_2 < 0$, when the supply of financial assets satisfies $A \in [A_1, A_2]$.*

Proof of Proposition 3 and the threshold values (A_1, A_2, x_1, x_2) are given in the Appendix. I define a run on collateralized debt (repo) as the decrease in collateralized (repo) funding when the economy shifts from one stable equilibrium to the other. In the next two subsections, I show such a run can be driven by self-fulfilling beliefs or small negative shocks on collateral quality and borrower creditworthiness in a comparative static sense. In section 6.2, I extend the model to a two-period setting to study the dynamic implication of the run mechanism.

Belief-Driven Repo Runs

Figure 2 plots the aggregate asset demand and supply as functions of the asset price when the model has multiple equilibria. In Figure 2, the aggregate asset demand is increasing in asset price within a certain region. In this region, a decrease in asset price leads to an increase in the margin of collateralized debts as the risk-taking channel dominates the fundamental-risk channel. A higher margin tightens banks' collateral constraints and leads to a reduction in aggregate asset demand.

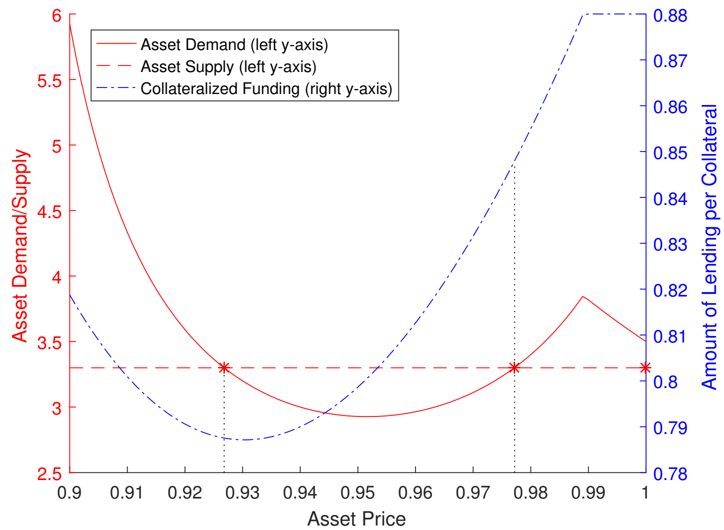

Figure 2: The figure plots the asset demand (in solid blue), the asset supply (in dash blue), and the amount of lending per collateral (in dash-dot red) as functions of the collateral asset price. The figure also pinpoints the three equilibria of the model. Three equilibria exist: the bad stable equilibrium A with $(P_A, D_A) = (0.9268, 0.7875)$, the unstable equilibrium B with $(P_B, D_B) = (0.9772, 0.8480)$, and the good stable equilibrium C with $(P_C, D_C) = (1, 0.88)$

Because of this upward-sloping asset demand, the model can have multiple equilibria. According to Proposition 3, the model has the two stable equilibria when $x_L \in (x_1, x_2)$, that is, when borrowers are perceived as likely to be poorly capitalized. Among the two stable equilibria, the bad equilibrium features significant concerns about counterparty risk and low levels of collateralized funding and asset price, whereas the good equilibrium features few concerns about counterparty risk and high levels of collateralized funding and asset price.

The source of equilibrium multiplicity is the feedback effect between asset price and collateralized funding. A decrease in collateralized funding leads to a decrease in asset price through tightening high-type banks' collateral constraint. However, a decrease in asset price can result in a further decrease in collateralized funding, because the decrease in asset price increases low-type banks' incentives for excessive leverage and raises lenders' concerns about counterparty risk. When low-type banks have low net worth, this positive feedback effect can be large enough to create an upward-sloping asset demand curve, leading to the existence of multiple equilibria.

Whenever multiple equilibria exist, the economy can shift from the good to the bad equilibrium due to self-fulfilling concerns about counterparty default. In other words, lenders' heightened concerns about counterparty default can rationalize themselves, generating a large decrease in collateralized funding and asset price. Different from the traditional bank-run model in Diamond and Dybvig (1983), the strategic complementarity that leads to self-fulfilling equilibria exists not among lenders of the same borrower, but among lenders in the entire lending market through the endogenous determination of the collateral asset price.

Fundamental-Driven Repo Runs

According to Proposition 3, a small decrease in x_L can make the economy shift from having only one equilibrium with a high level of collateralized funding to having multiple equilibria. Assuming the economy always coordinates to the equilibrium associated with the lowest amount of collateralized funding, a small decrease in x_L, which measures the creditworthiness of borrowers, can lead to a shift from the good to the bad equilibrium. In other words, the run on collateralized debt can be driven by a small fundamental shock.[30]

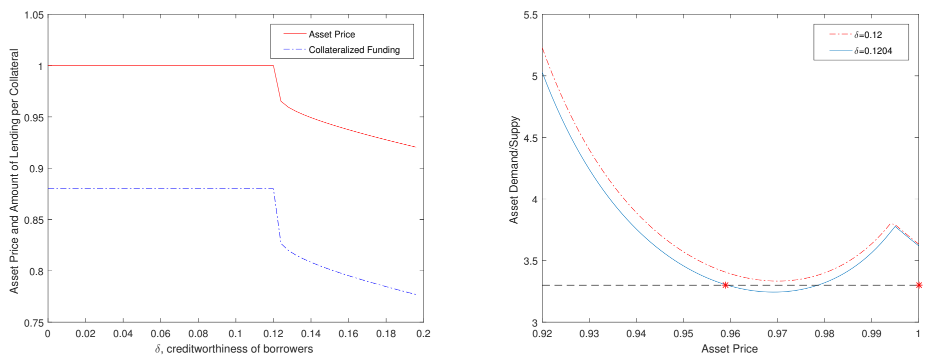

Figure 3: This graph plots the equilibrium asset price and collateralized funding (the amount of lending per collateral) as functions of δ. Here, I consider $x_L = -0.2 - \delta$, where $\delta \in [0, 0.2]$.

Consider a decrease in low-type banks' net illiquid asset value from x_L to $x_L - \delta$. Figure 3 plots the equilibrium asset price and collateralized funding as functions of δ. As shown in the left panel, when the net illiquid asset value of low-type banks decreases below a threshold value $\delta^* = 0.12$, both the asset price and the amount of lending per collateral decrease discontinuously. The right panel of Figure 3 shows the discontinuous decrease happens because a small decrease in x_L at the threshold gives rise to the existence of the bad equilibrium, resulting in a shift from the good to the bad equilibrium. Therefore, a small decrease in borrowers' creditworthiness can also make lenders more concerned about counterparty risk, generating large decreases in collateralized funding and asset price.

The run mechanism in the model sheds light on the runs in repo markets following the collapse

[30] Similarly, one can show that a small increase in the riskiness of the collateral asset can also lead to such a fundamental-driven repo run.

of Lehman Brothers in the financial crisis. The model predicts that a small negative shock on the borrowers' perceived creditworthiness (a decrease in x_L) or a sunspot shock can significantly increase lenders' concerns about counterparty risk, leading to a market-wide decrease in repo funding backed by illiquid assets and a sharp decline in asset prices. This prediction is consistent with evidence of a repo run during the 2007-2008 financial crisis. For example, Gorton and Metrick (2012) show Lehman's bankruptcy triggered a sharp increase in the LIB-OIS spread, a measure of counterparty risk in the market, which is followed by large increases in margins of bilateral repos that are backed by non-subprime securitized bonds and in spreads of these bonds.

My paper provides a formal analysis of how concerns about counterparty risk can intensify and lead to run-like behaviors in repo markets. In a related speech, Bernanke (2008) argues that "even repo markets could be severely disrupted when investors believe they might need to sell the underlying collateral in illiquid markets. Such forced asset sales can set up a particularly adverse dynamic, in which further substantial price declines fan investor concerns about counterparty credit risk, which then feed back in the form of intensifying funding pressures." Note the mechanism in my model does not work through actual default and post-default fire sales. In my model, lenders' concerns about these adverse scenarios are what generate the sudden tightening of collateralized funding and substantial declines in asset prices.

Discussion

This subsection justifies the key assumptions of the model and discusses the robustness of the main results with respect to a few simplifying assumptions.

Asymmetric information. The repo-run mechanism in the paper is driven by two key assumptions. First is the assumption that lenders are uninformed of their counterparty's creditworthiness. Under this assumption, lenders in equilibrium provide the same amount of lending per unit of collateral to both high-type and low-type banks. If borrowers' types are common knowledge, lenders can lend a higher amount to good-type banks that have little default risk. In that case, the aggregate demand function will always be downward sloping in asset price, because a change in bad-type banks' incentives for excessive leverage will no longer tighten the collateral constraint of good-type banks. Asymmetric information about the balance-sheet strength of the counterparty is relevant for collateralized debt markets, because lenders typically have less knowledge of borrowers' balance-sheet strength than borrowers do.

Difference in collateral valuation. Another key assumption is that lenders value the collateral less than borrowers do. Because of asymmetric valuation, lenders who lend an amount larger than their valuation of the collateral are concerned about counterparty risk. If lenders and borrowers have the same valuation over the asset in the bad state, that is, $u = v_L$, the amount of lending against a collateral asset will be $D = v_L$, which is independent of borrowers' incentives for excessive leverage. Consequently, the repo-run mechanism in the paper is more relevant for repos that are backed by illiquid assets, including corporate bonds, equities, and private-label ABS. The list can expand to relatively liquid assets such as long-term treasuries and agency MBS, because liquidation of these assets can also generate sizable fire-sale loss during stress times.[31]

Infinite risk aversion. The model's main results are robust when lenders are less than infinitely risk averse. When lenders are less than infinitely risk averse, the equilibrium debt contract that attracts low-type banks may have positive interest rate as low-type banks choose to take excessive leverage and default in the bad state. Mathematically, the break-even constraint in (1) will be changed to take into account borrowers' default risk and lenders' risk preference. However, the equilibrium debt contract that attracts high-type banks still has zero interest rate, and the amount of lending per collateral in that contract still depends on the IC constraint. As a result, whenever low-type banks become less capitalized, high-type banks will face a tighter collateral constraint as lenders reduce the amount of lending per collateral to deter low-type banks from choosing the high-type bank's debt contract. Therefore, the model can still generate multiple equilibria. Somehow differently, a shift from the good to the bad equilibrium will be associated with a large increase in the interest rate of the contract that attracts low-type banks. As discussed in section 3.3, this result is consistent with evidence in both bilateral and tri-party repo markets during the 2008 financial crisis.

One-borrower-one-lender assumption. In the model, one borrower borrows only from one lender in equilibrium. Therefore, a lender fully internalizes the effect of her lending decision on her borrower's portfolio choice. The one-borrower-one-lender assumption assumes away the coordination-failure problem among lenders of a single borrower, which is the key mechanism in Diamond-Dybvig-style bank-run models. My model shows multiple equilibria can still arise as a result of strategic complementarity in margin-setting decisions of all lenders in the market.

This equilibrium multiplicity result in my model is robust in an alternative specification where each borrower has to borrow from multiple lenders. Interestingly, the existence of multiple lenders introduces an additional layer of fragility as a result of coordination failure among lenders of the same borrower. This additional layer of fragility comes from the fact that lenders of the same borrower do not fully internalize the effect of their lending decisions on the borrower's leverage choice. Consequently, the bad equilibrium that is stable in the main model becomes unstable against small perturbations in the presence of multiple lenders. The intuition is the following. In this bad equilibrium, a low-type bank is indifferent between taking excessive leverage and not doing

[31] One proxy for a collateral asset's secondary-market illiquidity, as in Begalle et al. (2016), is the expected time to liquidate a certain amount of the asset without a large price impact.

so. Expecting a tiny amount of lenders to deviate from the equilibrium by providing a little more funding, all lenders expect the borrower to take excessive leverage and become insolvent in the bad state. As a result, all lenders reduce the amount of lending to their valuation of the collateral to avoid losses in default.

This alternative specification is particularly relevant for the tri-party repo market in which large dealer banks borrow from multiple cash investors. Copeland et al. (2014) finds the top-five dealers of that clearing bank borrow from an average of 53 investors. My paper suggests banks that borrow from multiple lenders in the tri-party repo market can be subject to higher run risks due to the strategic uncertainty among multiple lenders of the same borrower.

5 Implications for Policy Interventions

This section considers the model's implications about ex-post government interventions related to repo markets. I now interpret borrowers as investment banks and collateralized debts as tri-party repos that these banks use to finance their leverage positions. Consider the economy in the bad equilibrium associated with a low level of collateralized funding.

To compare the effects of different intervention programs, I focus on studying their effects on the aggregate asset demand for two reasons. First, in the model, a higher aggregate asset demand makes the bad equilibrium less likely to exist. So a policy that is more effective in raising asset demand is more likely to shift the economy out of the bad equilibrium. Second, to the extent that extremely low asset prices in a market turmoil can depress real economic activity and lead to lower welfare, which is outside of the current model, the government will lean toward policies that are more effective in raising asset demand.

Liquidity backstop. Let's first consider programs in which central banks provide repo lending to dealer banks. I have in mind the Primary Dealer Credit Facility (PDCF), which was established in March 2008 to provide funding, in the form of collateralized loans, to primary dealers when such funding may not be readily available elsewhere. During the last week of Lehman, the set of eligible collateral in PDCF was expanded from investment-grade securities to nearly all types of instruments that can be pledged in the tri-party repo market. The expansion of eligible collaterals effectively made the PDCF the liquidity backstop or the lender of last resort in the tri-party repo market.

In the model, consider the economy to be at the bad equilibrium (Equilibrium A in Figure 2) after a belief-driven repo run. I ask how a central bank can move the equilibrium to the good one through providing collateralized funding to borrowers. If the central bank is lending at the current market level $D = 0.7875$, it does not help the economy. Indeed, the problem of the bad equilibrium is not a lack of liquidity per se, but heightened concerns about counterparty risk.

I find the government can shift the equilibrium to the good one by promising to lend freely at $D = 0.8480 + \epsilon$ per unit of collateral, where 0.8480 is the amount of lending per collateral in the unstable equilibrium B and ϵ is an infinitely small positive amount. When the central bank promises to lend freely at a little more than $D_B = 0.8480$ per unit of collateral, borrowers expect the asset price to increase above the unstable equilibrium level $P_B = 0.9772$. The increase in asset price reduces borrowers' incentives for excessive leverage and relieves lenders' concerns about counterparty risk. Consequently, the policy will drive the economy to the good equilibrium where the asset price is $P_C = 1$, and private lenders are lending $D_C = 0.88$ per collateral, an amount larger than the central bank's liquidity provision.

After the PDCF was put into use on 2008, evidence suggests PDCF was lending to primary dealers against risky collateral assets at a repo margin smaller than the current market level. For example, according to Ball (2018), the repo margin for speculative-grade corporate bonds on September 12 was 15% in the market and only 6.5% for Morgan Stanley and Goldman Sachs to borrow in the PDCF. My model provides a reason why such a choice of repo margin in the PDCF can shift the financial system out of a crisis. Interestingly, PDCF required a repo margin of 16.7% from Lehman against the collateral in the same category on that day, suggesting the central bank's lending is also subject to counterparty-risk concerns.

To coordinate banks to move to the good equilibrium, the central bank must have the ability to lend freely at an amount larger than the current market level. However, once the policy works, the equilibrium shifts to the good one in which lenders provide more funding than the central bank does. As a result, none of the banks are actually lending from the central bank. In other words, the liquidity backstop works as an insurance that coordinates lenders' concerns about counterparty risk to a lower level.[32] Nevertheless, if the central bank does not have the ability, it may fail to shift the economy to the good equilibrium. Worse still, the central bank can then be making risky loans as thinly capitalized banks borrow from the central bank to take excessive leverage.

In sum, a central bank can forestall a systemic run on the repo market by being the lender of last resort (LOLR). The concept of LOLR goes back to Thornton (1802) and Bagehot (1873).[33] Bagehot suggests that in a liquidity crisis, a central bank should lend freely, at a high interest rate relative to the pre-crisis period, to any borrower with good collateral and value the collateral at between panic and pre-panic prices. Similarly, my model shows that by lending against collateral assets at an amount between crisis and normal levels, central banks can relieve lenders' concerns about counterparty risk and shift the economy out of a crisis. Freixas et al. (2004) study the optimal form of LOLR policy in a model where the role of LOLR is to provide liquidity to illiquid banks when illiquidity shocks cannot be distinguished from solvency ones. By contrast, my model focuses

[32]Copeland et al. (2014) make a similar point. According to my model, the economics of a liquidity backstop for repos is very similar to that of deposit insurance for retail deposits. The existence of deposit insurance can forestall a panic-driven bank run without any actual insurance payment to depositors.

[33]See Tucker (2014) for detailed reviews of literature on LOLR.

on the role of LOLR in reducing lenders' concerns about counterparty risk.

Equity injection and asset purchase. During the 2007-08 financial crisis, equity injection and asset purchase were used by governments to stabilize the financial system. In the US, the Department of Treasury injected capital into banks through the Troubled Asset Relief Program (TARP), and the Federal Reserve purchased large amounts of non-government securities through multiple rounds of quantitative easing (QE). I use the model to compare the effect of equity injection and asset purchase. Specifically, consider that a government can spend m units of cash freely either to purchase the financial asset directly or to purchase equity claims of borrowers. For simplicity, I assume the government cannot suffer a loss in conducting these two policies.

For asset purchase, I consider the following two scenarios: (i) The government purchases the asset and uses it as collateral to borrow in the private lending market, or (ii) the government purchases the asset and uses it as collateral to borrow at the lowest possible repo margin, which corresponds to $D = v_L$ amount of lending per unit of collateral.

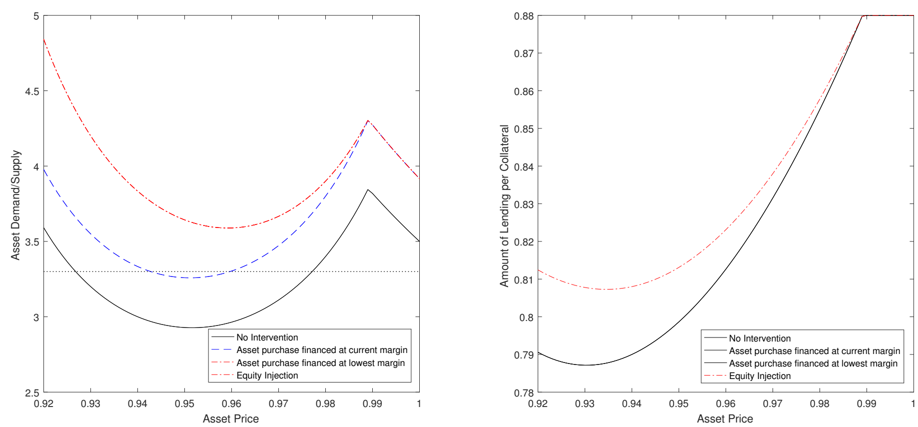

Figure 4: This left panel shows the asset-demand curve when no intervention occurs (black solid), when purchasing assets financed at the current margin (blue dash), when purchasing assets financed at the lowest margin (red dash-dot), and when injecting equity into banks (red dash-dot). This right panel shows the amount of collateralized funding when no intervention occurs (black solid), when purchasing assets financed at the current margin (black solid), when purchasing assets financed at the lowest margin (black solid), and when injecting equity into banks (red dash-dot). The total spending of asset purchase or equity injection is fixed at $m = 0.05$.

Figure 4 shows the asset demand increases in both scenarios, and that the increase in asset demand is larger when the government can use the asset as collateral to borrow at the lowest-possible margin. This is because a lower margin allows the central bank to take more leveraged positions. A government's direct asset purchase does not change the amount of collateralized funding as a function of the collateral asset price, because borrowers' creditworthiness is unchanged when the asset price is fixed.

For equity injection, I assume the government is uninformed of borrowers' types and thus equally purchases equity claims of all banks. In Figure 4, the equity injection shifts both the asset-demand curve and the collateral-constraint curve upwards. Intuitively, injecting capital into banks increases asset demand through two channels: a direct purchase channel in which injecting equity gives all banks more cash to buy assets at the current margin, and a collateral-constraint channel in which injecting equity makes banks safer. Note the collateral-constraint channel relieves lenders' concerns about counterparty risk and leads to an increase in the amount of lending per collateral.

I formally show that when the economy is in a bad equilibrium associated with a low level of collateralized funding, that is, $D < v_L$, the government injecting equity into all banks is more effective in raising asset demand than purchasing the asset financed at the market-prevailing margin. The effect of the direct purchase channel of equity injection alone, $(\frac{\theta}{P-D} + \frac{1-\theta}{P-v_L})m$, is larger than $\frac{m}{P-D}$, the effect of purchasing the asset financed at the market-prevailing margin. Further, equity injection also leads to a lower margin through the collateral-constraint channel. Combining the effects of the two channels, I find that equity injection is as effective as purchasing the asset financed at the lowest-possible margin.

He and Krishnamurthy (2013) show that equity injection is more effective than asset purchase and borrowing subsidy in increasing asset demand in a dynamic model in which marginal asset buyers face an equity capital constraint. In Philippon and Schnabl (2013), equity injection dominates other common forms of intervention, such as asset purchases and debt guarantees, because it reduces opportunistic participation by banks that do not actually need the intervention. My paper argues in favor of equity injection for a different reason. I show equity injection is more effective because it relieves lenders' concerns about counterparty risk and relaxes banks' collateral constraints.

Other Policies. I briefly discuss other policies related to repo markets. During the financial crisis, the US Treasury introduced the Temporary Guarantee Program that offered a guarantee to MMMFs in exchange for an insurance premium. The debt guarantee successfully stopped large-scale runs on MMMFs. A side effect is that it reduces the headline risk of MMMFs' lending to risky counterparties in repo markets. Therefore, I interpret this policy as an increase in u, lenders' valuation of the collateral. In the model, an increase in u can increase lenders' provision of repo lending and raise asset demand. Moreover, an increase in u makes the asset demand less upward-sloping and the financial system less vulnerable to repo runs by setting a higher floor value for the amount of repo lending.

During the financial crisis, the US Treasury and FDIC created the Legacy Loan Program that tapped TARP money to purchase troubled and illiquid loans and other assets from insured banks and thrifts. If the model is interpreted directly, the government purchasing these low-quality loans from banks' balance sheets at fair prices has no effect on the economy, because these bad illiquid loans in the model are known to be bad for sure. In an alternative specification in which low-quality loans can become worse at date 2, removing these low-quality assets that entail large downside risk from banks' balance sheets can be effective in raising asset prices, because the removal makes banks better capitalized in the worst-state scenario and relieves lenders' concerns about counterparty risk. In a different setting, Tirole (2012) shows the optimal intervention mechanism features the government cleaning up low-quality assets to reduce adverse selection in the private market.

Since the financial crisis, considerable debates have occurred over the role of automatic-stay exemptions in repo markets.[34] In the Appendix, I show the exemption from automatic stay is one reason lenders are willing to borrow more than their valuation of the collateral in an incomplete-contract environment. If repos are subject to automatic stay, lenders will reduce the amount of lending for fear of strategic renegotiation. The reduction in repo funding can lead to a lower asset price, but it also makes the repo contracts better collateralized and less subject to runs due to counterparty-risk concerns.[35] A quantitative analysis of this tradeoff is beyond the scope of this paper but an interesting topic for future research.

6 Implications for Financial Regulations

This section extends the baseline model to study its implications for financial regulations. I extend the model to a two-period setting by introducing a pre-period (from date 0 to date 1) and an aggregate shock that realizes at date 1. In section 6.1, banks at date 0 choose their exposure to an illiquid investment that affects their net illiquid asset value at date 1. In section 6.2, banks at date 0 purchase the financial asset backed by collateralized funding and roll over collateralized debts and rebalance asset positions at date 1. In this section, I focus on presenting the key results of these two extension models, while leaving definitions, parameter restrictions, and proofs to the Appendix.

6.1 Illiquid Investment and Financial Fragility

In this section, I endogenize banks' date-1 net illiquid asset value by allowing banks to choose between holding cash and investing in an illiquid loan at date 0. At date 0, each bank is endowed with 1 unit of cash and $\eta < 1$ units of debt that matures at date 2. Each bank can choose to invest $\rho \in [0, 1]$ units of cash in making an illiquid loan. I focus on a symmetric equilibrium in which banks choose the same ρ. Each unit of initial investment yields no liquidation value at date 1, and

[34]Duffie and Skeel (2012) discuss benefits and costs of the exemption from automatic stay for repos and other qualified financial contracts. Roe et al. (2014) argue that repos backed by illiquid assets should not be exempt from automatic stay, because the exemptions can facilitate panic selling and exacerbate an ongoing crisis. Gorton et al. (2010) propose reforms that restrict the exemption from automatic stay to repos associated with qualified borrowers, against approved collaterals, and regulated by a minimum haircut requirement.

[35]However, repos that are subject to automatic stay are not immune to runs. Theoretical works, such as Infante (2013), Kuong (2015), and Ma (2017), show that lenders' valuation of the collateral can vary endogenously with collateral quality and counterparty characteristics through post-default fire sales.

only pays off at date 2.

At date 1, each bank's illiquid loans receive a publicly observable aggregate shock and a privately observable idiosyncratic shock. With probability α, the aggregate state is good and all illiquid loans yield $X_H > 1$ for sure. With probability $1-\alpha$, the aggregate state is bad, and each bank is privately informed that its loan quality is either $X_H > 1$, which happens with probability θ, or $X_L < 1$, which happens with probability $1-\theta$, at date 2. Banks' idiosyncratic shocks are independent of each other. Assume this investment opportunity has positive NPV.

Once the payoff uncertainty of illiquid loans realizes at date 1, the model becomes the same as the baseline model. Specifically, banks can use $1-\rho$ units of cash to purchase financial assets and partially finance the purchases through collateralized debt. At date 2, all assets pay off and banks repay subject to limited liability.

To simplify, I focus on cases in which asset price and collateralized funding in the good aggregate state $j = g$ are at their highest levels, $P_g = \bar{v}$ and $D_g = v_L$. The corresponding parameter restriction is given in the Appendix. In the bad aggregate state, asset price and collateralized funding are given by

$$D_b = \min\{v_L, D_b^*(P)\} \tag{CC'}$$

$$\theta\frac{l(\rho)}{P_b - D_b} + (1-\theta)\frac{l(\rho) + x_L(\rho)}{P_b - v_L} = A \tag{MC'}$$

where $D_b^*(P_b) \equiv P_b - \frac{l(\rho)}{\frac{l(\rho)+x_L(\rho)}{P_b - v_L} + \frac{(1-q)C}{q(v_H - P_b)}}$. The asset-market equilibrium in the bad aggregate state is almost the same as that in the main model, except that now banks' holding of cash $l(\rho) = 1-\rho$ and net illiquid asset value $x_i(\rho) = \rho X_i - \eta$, where $i = H, L$, depend endogenously on banks' date-0 investment decisions. Because banks at date 0 choose ρ to maximize their expected utility, ρ is pinned down by the following first-order condition:

$$(\alpha + (1-\alpha)\theta)(X_H - 1) = (1-\alpha)(\theta\frac{\bar{v} - P_b}{P_b - D_b} + (1-\theta)\frac{\bar{v} - v_L}{P_b - v_L}(1 - X_L)) \tag{FOC}$$

In the Appendix, I prove this extended model has a unique equilibrium under a few regulatory conditions. Here is the intuition for why introducing the investment decision makes the equilibrium more likely to be unique. Expecting a lower asset price at date 1 in the bad aggregate state, banks reduce their investment in illiquid loans at date 0 in order to take positions in the asset market. As a result, the decrease in asset price is more likely to lead to a higher asset demand at date 1.

Interestingly, in this unique equilibrium of the extended model, once the investment decision has been made at date 0, the asset market at date 1 in the bad aggregate state can be in an unstable equilibrium, with the equilibrium concept defined in the baseline model. In this unstable equilibrium, a small negative perturbation of asset demand can lead to large decreases in asset price and collateralized funding. Figure 5 shows the date-1 bad-state asset market is in such an unstable equilibrium when the probability of the good aggregate state increases above a threshold value.

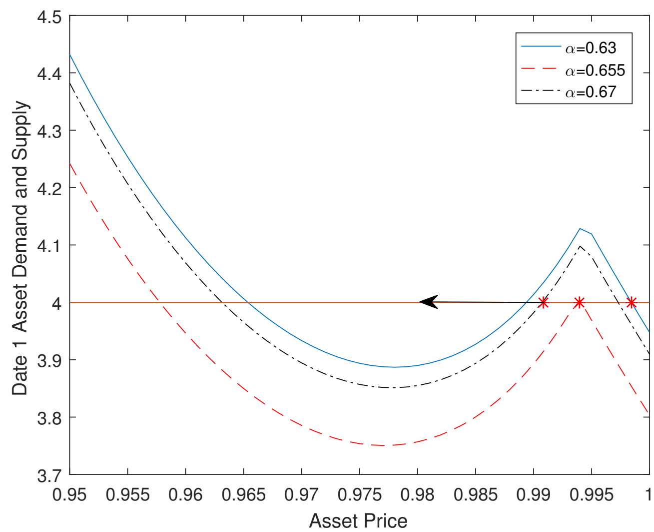

Figure 5: The blue solid line is the date-1 bad-state asset demand when $\alpha = 0.63$, the red dash line is the date-1 bad-state asset demand when $\alpha = \alpha^* = 0.655$, and the black dash-dot line is the date-1 bad-state asset demand when $\alpha = 0.67$. When α increases above the threshold value α^*, the date-1 bad-state asset market equilibrium is unstable. The shift of the asset-demand curve is non-monotonic in α, but the equilibrium date-1 bad-state asset price is decreasing in α. Parameter values (that are new or different from those in Figure 1) are $X_H = 1.1$, $X_L = 0.3$, $\eta = 0.45$, $l = 1$, $C = 0.4$, $\theta = 0.7$ and $A = 4$.

The model characterizes fragility in the financial market in boom times. In the model, boom times are defined as cases when the probability of a bad aggregate shock at date 1 is low. In boom times, banks invest heavily in illiquid loans that are exposed to aggregate risk. When the good aggregate state realizes, everything works out perfectly: The illiquid loans pay off high, the financial asset is priced high, and the margin of collateralized debts is low. However, when the economy is hit by a negative aggregate shock, some banks suffer large losses on their illiquid loans. Due to the overhang of these poorly capitalized banks, the asset market can be unstable against small perturbations and thus becomes inherently fragile.

The build-up of financial fragility in boom times provides a rationale for countercyclical financial regulations. A regulation that limits banks' investment of illiquid loans in boom times, such as the countercyclical capital buffer in Basel III, can make the financial system less fragile when hit by negative aggregate shocks. Although such a regulation can make the financial system more robust against small perturbations, it is ineffective in preventing a financial crisis caused by self-fulfilling concerns about counterparty risk. When such a self-fulfilling crisis occurs, the economy must rely on ex-post policy interventions, such as the ones analyzed in section 5, to deter large decreases in asset price and collateralized funding.

6.2 Rollover Loss and Margin Requirement

In this section, I extend the baseline model to a dynamic two-period setting where banks have to roll over their collateralized debts at the interim date. At date 0, each bank has l units of cash and x units of net illiquid asset. Similar to the model in section 6.1, a bank's net illiquid asset value depends on an aggregate shock and an idiosyncratic shock, and both of them realize at date 1. With probability β, the aggregate state is good, and all banks' net illiquid asset values are x_H. With probability $1-\beta$, the aggregate state is bad, and each bank is privately informed that his net illiquid asset value is either $x_H > 0$, which happens with probability θ, or $x_L < 0$, which happens with probability $1-\theta$, at date 2. Banks' idiosyncratic shocks are independent of each other. Unlike in section 6.1, I assume the two possible net illiquid asset values are exogenous.

Banks can trade the financial asset both at date 0 and date 1. The financial asset is inelastically supplied at A. At date 2, the asset pays v_H with probability q and v_L with probability $1-q$ if held by a bank, and pays $u < v_L$ if held by a lender. At date 0, banks purchase assets and finance the purchases through collateralized borrowing. Because banks are identical at date 0, we focus on symmetric equilibrium in which banks hold the same amount of assets at date 0. At date 1, shocks on banks' net illiquid asset values realize. Then banks rebalance their asset positions and roll over their collateralized debts. I assume only banks have access to the asset market at date 1. In the Appendix, I show this incomplete market assumption leads to asymmetric valuation of the asset at date 1. At date 2, all assets pay off and banks repay debt subject to limited liability.

To simplify, I focus on cases in which asset price and collateralized funding in the good aggregate state $j = g$ are at their highest levels, $P_g = \bar{v}$ and $D_g = v_L$. The correponding parameter restriction is given in the Appendix. In the bad aggregate state, asset price and collateralized funding are given by

$$D_b = \min\{v_L, D^*(P_b)\} \tag{CC1}$$

$$\theta\frac{l_b}{P_b - D_b} + (1-\theta)\frac{l_b + x_L}{P_b - v_L} = A \tag{MC1}$$

where $D^*(P_b) \equiv P_b - \frac{l}{\frac{l_b+x_L}{P_b-v_L}+\frac{(1-q)C/q}{v_H-P_b}}$. The above two equations characterize an interior equilibrium of the asset market. In addition, a corner equilibrium can exist when $P_b = \bar{v}$ and equation (MC1) holds with inequality. The asset-market equilibrium in the bad aggregate state is almost the same as that in the main model, except that now banks' holding of cash (liquid asset value) $l_b = l + A(P_b - P_0)$ depends endogenously on rollover loss, which is determined by asset prices at date 0 and at date 1 in the bad aggregate state.

In general, the asset price at date 0 depends both on banks' reservation price, taking into account the potential illiquidity at date 1, and on their collateral constraints, which depend on lenders' concerns about counterparty risk. In the Appendix, I derive the formula for D_0, the amount of collateralized funding at date 0. To simplify the analysis, I focus on cases in which collateral constraints do not bind at date 0 and banks are indifferent between asset purchase and cash holding at date 0. The asset price at date 0 is determined by

$$P_0 = \frac{\beta\bar{v} + (1-\beta)(\theta R_1(P_b, D_b)) + (1-\theta)R_2(P_b))P_b}{\beta + (1-\beta)(\theta R_1(P_b, D_b)) + (1-\theta)R_2(P_b))} \tag{MC0}$$

where $R_1(P_b, D_b) = \frac{\bar{v}-D_b}{P_b-D_b}$ and $R_2(P_b) = \frac{\bar{v}-v_L}{P_b-v_L}$. Figure 6 shows the equilibrium asset price at date 0 and at the bad state in date 1. In the Appendix, I show the extended model can have multiple equilibria. In the good equilibrium, asset prices at date 0 and date 1 are equal to the fundamental value $P_0 = P_b = P_g = \bar{v}$. In this equilibrium, banks face no rollover loss at date 1 in either state, and the asset market is liquid even in the bad aggregate state. Lenders have few concerns about counterparty risk, so they set the highest possible amount of collateralized funding $D_b = D_g = v_L$ at date 1.

In the bad equilibrium, asset prices satisfy $P_b < P_0 < P_g = \bar{v}$. When the good aggregate state realizes at date 1, banks are well capitalized and lenders have few concerns about counterparty default. As a result, lenders set the highest possible amount of collateralized funding $D_g = v_L$, pushing up asset price from P_0 to P_g. Due to the increase in asset price, banks realize capital gains and do not need to deleverage. However, when the bad aggregate state realizes at date 1, some banks become poorly capitalized. Moreover, these banks suffer rollover losses because they need to sell a fraction of their assets at a fire-sale price P_b to obtain their charter values at date 2. Both the exogenous shock and the endogenous deleveraging make lenders concerned about counterparty default. As a result, lenders set a lower amount of collateralized funding $D_b < v_L$ to deter excessive leverage-taking. Due to the tightening of banks' collateral constraints, the asset price decreases from P_0 to P_b. As low-type banks deleverage, high-type banks acquire more assets and benefit from the illiquidity of the asset market. The asset price at date 0 is below the fundamental, that is, $P_0 < \bar{v}$, because of the expectation of future market illiquidity caused by lenders' concerns about counterparty risk.

Here I make three interesting observations about the bad equilibrium of the model. First, we show in the Appendix that D_0, the amount of funding per collateral at date 0, is negatively correlated with P_b, which depends on lenders' concerns about counterparty risk at date 1. Intuitively, the concerns about counterparty risk tomorrow can have an effect on the amount of collateralized funding today through determining the collateral asset price tomorrow. Second, we assume a symmetric equilibrium in which every bank holds the same amount of assets at date 0. When banks make asymmetric asset-holding decisions, banks that hold more assets at date 0 are subject to higher rollover loss and face a more severe funding squeeze in the bad state. This result suggests monitoring whether a funding squeeze happens disproportionately to banks that hold larger positions on risky assets can be useful in gauging the stress in a collateralized funding market. Third, we assume the economy has only two aggregate states, with the fraction of low-type banks being either 0 or β. When the economy has a continuum of aggregate states that differ in the value of β, any equilibrium of the model has the feature that only when the aggregate shock is worse than a certain threshold does the economy generate sizable deleveraging-induced fire sales and large decreases in collateralized funding and asset price. This result echoes the repo-run mechanism in the main model.

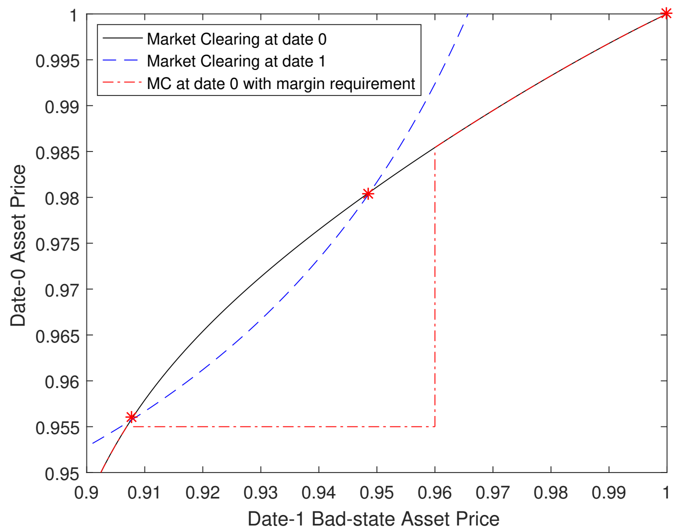

Figure 6: The blue dash line is the market-clearing condition at date-1 in the bad state. The black solid line is the market-clearing condition at date 0 without a minimum margin requirement. The red dash-dot line is the market-clearing condition at date 0 with a minimum margin requirement of 17.5% when the asset price is between 0.955 and 0.985. Imposing such a price-contingent requirement leads to a unique equilibrium with $P_0 = 1$. Parameter values (that are new or different from those in Figure 1) are $\beta = 0.7$, $x_L = -0.3$, and $A = 3$.

I show in the Appendix that the economy has multiple equilibria when the probability of being in a good aggregate state is larger than a threshold value β^*. Intuitively, when the probability of the good aggregate state is high, the date-0 asset price is high. Then banks are subject to higher rollover loss at date 1 in the bad aggregate state. A higher rollover loss makes low-type banks less capitalized, raising lenders' concerns about counterparty default. Therefore, the model is more likely to have multiple equilibria when the probability of a negative aggregate shock is lower.

Because the existence of multiple equilibria is partly driven by rollover loss in deleveraging, a financial regulation that limits banks' leverage at date 0 can be effective in eliminating the bad equilibrium in the model. Specifically, I ask whether a government can eliminate the bad equilibrium by imposing a minimum margin requirement at date 0.[36] To be in line with the minimum haircut

[36] A minimum margin requirement at date 1 is ineffective. Although such a requirement lowers low-type banks' incentives to take excessive leverage, it also reduces high-type banks' ability to obtain leverage. As a result, the asset price in the bad state at date 1 is lower and the rollover loss is higher when imposing such a requirement at date 1.

floor, let's now define the margin of a collateralized debt as $\frac{P-D}{P}$, the percentage difference between asset price and collateralized funding. The government can require all margins to be at least $y\%$. When such a minimum margin requirement binds at date 0, the market-clearing condition at date 0 is instead determined by cash-in-the-cash pricing $P_0 = \frac{l}{y\%A}$.

According to the model, a minimum margin requirement that is conditional on the aggregate state of the economy β and the date-0 asset price can be effective in eliminating the bad equilibria. First, because multiple equilibria arise when β is high, the model suggests the minimum margin requirement should be imposed only in boom times. Figure 6 draws the multiple equilibria in such a scenario. More importantly, such a requirement should bind only when the date-0 asset price is low. Such a binding constraint effectively reduces banks' rollover loss at date 1 when agents expect a bad equilibrium associated with lower asset prices. Lower rollover loss relieves lenders' concerns about counterparty risk in the bad state at date 1 and pushes up asset prices, rendering the expectation of a bad equilibrium irrational. In other words, the expectation of a tighter margin requirement in times when asset prices are low eliminates any equilibrium associated with low asset prices.

However, a minimum margin requirement that is unconditional on asset price, such as the minimum haircut floor in Basel III, is ineffective in eliminating the bad equilibrium. Note that a binding minimum margin requirement sets a price ceiling in the asset market at date 0. As a result, this requirement only serves to strain market liquidity in the good equilibrium without eliminating the bad equilibrium. Moreover, such a requirement may encourage regulatory arbitrage as banks have incentives to find ways to borrow with other forms of debts that are not subject to the requirement. In sum, a simple minimum margin requirement can be ineffective in reducing rollover loss and fire sale in stress times without unduly straining market liquidity in normal times.

7 Conclusion

Short-term collateralized debts are important sources of funding for financial institutions in a modern financial system. Unlike retail deposits that are protected by the government provision of deposit insurance, collateralized debts, such as repurchase agreements, derive their safety from the private provision of financial assets as collateral. In normal times, the provision of collateral provides lenders with a sense of security and allows financial institutions to obtain high leverage through collateralized borrowing. However, to the extent that lenders are reluctant to seize and liquidate the collateral, repos and other collateralized debts are inherently under-collateralized and subject to the risk of counterparty default. During times of market stress, lenders can become increasingly concerned about counterparty solvency and significantly reduce their collateralized lending.

My paper proposes a repo-run mechanism in which small shocks on banks' balance sheets can significantly increase lenders' concerns about counterparty risk and lead to large decreases in collateralized funding and asset prices. One potential weakness of the mechanism is that it is based on a static analysis of multiple equilibria. A full dynamic analysis is needed to capture the run dynamics of the proposed mechanism. Section 6.2 of the paper provides one step in this direction. In a related paper (Wang, 2018), I show a small deterioration of asset quality can lead to prolonged periods of credit crunch and market freeze when the asset market suffers from asymmetric information.

My paper generates new policy insights about interventions and regulations related to collateralized funding markets. My paper suggests a liquidity backstop can shift the economy out of a crisis and that the government should inject equity into banks instead of directly purchasing assets. Further, a countercyclical capital buffer can increase banks' funding stability in bad times, whereas a simple minimum margin requirement can be ineffective in reducing the risk of funding squeeze. As pointed by CGFS (2017), post-crisis regulatory reforms increase the stability of repo markets while raising the cost for various repo users. Although my paper contains qualitative insights about this tradeoff, future works are needed to quantify the optimal form of regulations in repo and other collateralized funding markets under different market scenarios.

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8 Appendix

8.1 No Renegotiations in Default

This subsection discusses the assumption that collateralized debt contracts in the model are not subject to renegotiations ex post. Specifically, I provide two possible micro-foundations for this no-renegotiation assumption. The first micro-foundation is specific to repo contracts. Because repos are exempt from automatic stay, a repo lender can immediately seize the collateral assets when her counterparty borrower fails to repay. Imagine the borrower decides not to repay and start a renegotiation at date 2. Then the collateral is automatically transferred to the lender. The lender cannot hold the collateral and has to sell it either to the market at the liquidation price u or to the borrower in a renegotiation. Note the borrower's attempt to renegotiate down the debt repayment is equivalent to buying back the collateral at a price lower than the original face value per unit collateral.

In this example, let's assume the bank's valuation of the asset arises from selling it to an end-investor. Specifically, at date 2, lender finds an end-investor who is willing to pay v_H for each unit of an asset with probability q, and pay v_L with probability $1 - q$. When the borrower repays the debt, I assume the borrower is able to sell the collateral asset first and uses the proceeds to repay his debt. One way to realize this transaction is the unwind mechanism in the US tri-party repo market. However, when the borrower decides not to repay and start a renegotiation, the collateral is transferred to the lender, and the borrower is no longer able to apply his specific trading skill on behalf of the lender.

As a result, to buy back the collateral asset in the renegotiation, the borrower needs to have enough cash at date 2. I assume raising cash takes time and cannot be completed at date 2. Then the only way the borrower can buy back the collateral asset before the lender dumps it to the market at date 2 is to hold enough cash at date 1. I show that in the equilibrium of the model, neither the high-type bank nor the low-type bank has enough cash to buy back the collateral under the parameter restriction that $-x_L \leq \frac{u}{q(v_H - v_L)}(l + x_L)$. In other words, ex-post renegotiation is not possible in equilibrium because banks do not have sufficient cash in advance to buy back the collateral.

The second micro-foundation is that borrowers may refrain from strategically renegotiating their collateralized debts due to concerns about bank runs. This interpretation applies to all types of collateralized debt contracts, especially to ones associated with highly leveraged financial institutions as borrowers. Let's interpret a bank's net illiquid asset as the bank's illiquid asset net of the amount of deposits the bank needs to repay. Further assume the bank has many retail depositors, who can demand payment at any time subject to a first-come-first-served constraint. Moreover, I split the date 2 into two sub-dates: date 2.1 and date 2.2. At date 2.1, the payoff uncertainty realizes and the borrower can renegotiate with the lender. At date 2.2, all assets pay off and both the collateralized debt and the deposits mature.

From a depositor's point of view, there are two reasons why the bank starts a renegotiation with his collateralized debt holder (the lender in the model). First, the bank is renegotiating strategically to lower the repayment of the debt. In that case, the bank is solvent and depositors can get their money back, assuming they do not coordinate to run. Second, the bank becomes insolvent and is trying to renegotiate down the debt repayment in order to stay solvent. I assume each depositor assign a non-zero probability to the second case, and a non-zero probability to the event that renegotiation breaks down.

As a result, depositors assign a non-zero probability to the event that the bank defaults. Assuming the liquidation value of the illiquid asset at date 2.1 is less than the amount of deposits the bank owes, depositors strictly prefer to run the bank by withdrawing their deposits at date 2.1. To the extent that such a deposit run drives the bank's profit to zero, the bank will not strategically renegotiate with its collateralized debt holder at date 2.1. This interpretation is similar to Diamond and Rajan (2001), in the sense that the threat of run commits the borrower not to renegotiate. Somehow differently, the deposit run in my interpretation is triggered by depositors inferring a bank's potential insolvency from the bank renegotiating with other debt holders.

8.2 Proof of Proposition 1

I first show that without loss of generality, we can assume that each borrower only borrows from one lender in any equilibrium of the lending market. Assume in an equilibrium, a borrower borrows from two lenders with the following two contracts: (D_1, F_1, A_1) and (D_2, F_2, A_2), where the three variables are the total amount of lending, the face value, and the amount of collateral in this repo loan. Because this is an equilibrium, lenders at least break even with both of the two contracts. As a result, one of the two lenders can just propose a new contract $(D_1 + D_2, F_1 + F_2, A_1 + A_2)$, which is feasible under Assumption 1 and makes the lender break even and the borrower indifferent.

Therefore, we can focus on the lending relation between a borrower and a lender. I first show the lender quotes two repo contracts to screen borrowers of different types. Then, I pin down the exact terms of the two contracts by specifying a no-deviation condition under a proper off-equilibrium belief about repo contracts in the lending market. Last, I show the two contracts have the same amount of lending per collateral and zero interest rate.

As a standard result of screening in an asymmetric information setup, the lender quotes two repo contracts: (D_1, F_1, A_1) to attract a low-type bank and (D_2, F_2, A_2) to attract a high-type bank. Let's first assume the borrower only borrows from one of the two contracts. A borrower that takes a contract $j = 1, 2$ purchases A_j units of assets and uses all of the assets as collateral. Later I check whether banks have incentives to take more debt contracts.

The expected payoff of a bank with type $i = H, L$ taking the contract (D_j, F_j, A_j) is $V(x_i, D_j, F_j, A_j) = q(x + l + A_j(v_H - P) + D_j - F_j + C) + (1-q)(x + l + A_j(v_L - P) + D_j - F_j + C)^+$. Given perfect competition, the lender's problem is to maximize the utility of each borrower $\max V(x_L, D_1, F_1, A_1)$ and $\max V(x_H, D_2, F_2, A_2)$, subject to the following constraint:

$$V(x_L, D_1, F_1, A_1) \geq V(x_L, D_2, F_2, A_2) \tag{IC}$$

$$D_j \leq F_j \leq v_L \quad \text{for} \quad j = 1, 2 \tag{F1}$$

$$A_j P \leq l + D_j \quad \text{for} \quad j = 1, 2 \tag{F2}$$

The first is the incentive compatibility (IC) constraint that a low-type bank does not want to choose the contract designed for a high-type bank. The high-type bank's IC condition is neglected because it will not bind in equilibrium. The second and he third are the feasibility constraint discussed in section 3.1.

For a low-type bank, first note $F_1 = D_1$. Otherwise, the low-type bank's expected payoff is not maximized. Next, the amount of collateral must satisfy $A_1 = \frac{x_L + l}{P - v_L}$, which is the maximized amount of asset positions a low-type bank can hold without being insolvent in the bad state. Any amount lower than that does not maximize the low-type's expected payoff. Then, given the feasibility constraint, the optimal contract can be written as (D_1, D_1, A_1), and $D_1 \in [A_1 P - l, A_1 v_L]$.

For a high-type bank, again we have $F_2 = D_2$. To show this result, let's prove by way of contradiction. Assume $F_2 > D_2$. Let $F_2' = F_2 - \Delta$ and $A_2' = A_2 - \frac{\Delta}{v_H - P}$. This change keeps the IC constraint the same, relaxes the feasibility constraint, and increases the expected payoff of a high-type bank by $1 - \frac{\bar{v} - P}{v_H - P} > 0$. Therefore, $F_2 > D_2$ contradicts the payoff-maximization objective. So we must have $F_2 = D_2$ in equilibrium.

As a result, A_2 is determined by the IC constraint, that is, $A_2 = A_1 + \frac{(1-q)C}{q(v_H - P)}$. According to the feasibility constraint, the high-type bank's contract is given by (D_2, D_2, A_2), where $D_2 \in [A_2 P - l, A_2 v_L]$.

The above contracts are optimal if banks can commit not to take additional collateralized debt contracts. However, such commitment is not enforceable by assumption. Therefore, any equilibrium contract must provide borrowers with no incentive to seek for additional leveraged position.

As a result, I show $D_2 = A_2 P - l$. Because the high-type bank's contract makes a low-type bank just indifferent between staying safe and taking excessive risks. If $D_2 > A_2 P - l$, the low-type bank will choose the high-type bank's contract and use the extra cash to purchase additional assets, even if no other lenders are willing to provide collateralized funding. As a result, the lender will be exposed the risk of counterparty default. Therefore, the original lender will only set $D_2 = A_2 P - l$. Let $D^* \equiv \frac{D_2}{A_2}$.

Now the only undetermined element is D_1, which is within the range of $[A_1 P - l, A_1 v_L]$. However, a low-type bank is indifferent between any contract with $D_1 \in [A_1 P - l, A_1 v_L]$. Therefore, these contracts are payoff equivalent, as long as they provide the low-type bank with no incentive to seek for additional collateralized debt contracts.

To pin down the exact contract used in equilibrium, I need to specify off-equilibrium beliefs. Here, I assume that if a low-type bank believes he can borrow at D^* per unit of collateral from other lenders in the lending market. Given this off-equilibrium belief, I show $D_1 = A_1 D^*$. If $D_1 > A_1 D^*$, the low-type bank has an incentive to seek for additional funding and take excessive leverage. If $D_1 < A_1 D^*$, the low-type bank may switch to another lender, because he can obtain more funding from other lenders.[37]

Given this off-equilibrium belief, the pair of equilibrium contracts is uniquely determined by $(A_1 D^*, A_1 D^*, A_1)$ and $(A_2 D^*, A_2 D^*, A_2)$. Given this equilibrium contract, we can verify that it is indeed rational for other lenders to lend D^* per unit of collateral to a deviating borrower, and that borrowers have no incentive to deviate in equilibrium.

The above analysis neglects the no-recourse condition and the full-security condition. But the analysis still applies when the two constraints are added back. As a result, the amount of lending per collateral is given by $D = \max\{u, \min\{v_L, D^*\}\}$.

8.3 Proof of Proposition 3

Let's first derive the condition that leads to the existence of a good equilibrium with $D = v_L$. In this case, the amount of collateralized funding is determined by the no-recourse condition, so $v_L \geq D^*(P)$ or, equivalently, $P \geq P_1 = \frac{\frac{(1-q)Cv_L}{q} - x_L v_H}{\frac{(1-q)C}{q} - x_L}$. The aggregate asset demand can be written as $D_{agg}(P) = \frac{l+(1-\theta)x_L}{P - v_L}$. Because $D'_{agg}(P) > 0$, the good equilibrium exists when $D_{agg}(P_1) \geq A$, because a good equilibrium can have a price that satisfies either $D_{agg}(P) = A$ or $P = \bar{v}$ and $D_{agg}(\bar{v}) > A$. This condition can be written as $\frac{(t - x_L)(l+(1-\theta)x_L)}{-x_L(v_H - v_L)} \geq A$. Note the LHS is increasing in x_L. Lemma 1 summarizes the existence of the good equilibrium.

Lemma 3.1 *Let $x_1 < 0$ be the unique solution to $\frac{(\frac{(1-q)C}{q} - x_1)(l+(1-\theta)x_1)}{-x_1(v_H - v_L)} = A$. A good equilibrium with $D = v_L$ exists when $x > x_1$. Further, the threshold value is in the interior $x_1 \in (-l, 0)$ if and only if $A < A_1 \equiv \frac{(\frac{(1-q)C}{q} + l)\theta}{v_H - v_L}$.*

Let's then look at the existence of a bad equilibrium that is associated with $D < v_L$. In this case, the amount of lending is determined by the no-default condition, so $v_L \leq D^*(P)$ or, equivalently, $P \leq P_1$. The asset demand function is now $D_{agg}(P) = \frac{l+x_L}{P - v_L} + \frac{\theta(1-q)C}{q(v_H - P)}$. Note the asset-demand is a bell-shape function such that $D'_{agg} < 0$ when $P < P_3 \equiv \frac{v_H s + v_L}{s+1}$, where $s = \sqrt{\frac{q(l+x_L)}{\theta(1-q)C}}$, and $D'_{agg} > 0$.

[37]The latter will be strictly true when imposing the tiebreaker rule that borrowers earn a small interest rate on their cash holdings.

when $P > P_3$. Given these properties, it is easy to see that when $D_{agg}(P_1) \leq A$, the model has only one bad equilibrium. When $D_{agg}(P_1) > A$, the model can have 2 or 1 or 0 bad equilibrium, depending on the comparison between $D_{agg}(P_3)$ and A. Lemma 2 summarizes the existence of the bad equilibrium(s).

Lemma 3.2 *When $x \leq x_1$, the model has only one bad equilibrium. There exists a $x_2 < 0$ such that the model has two bad equilibria when $x \in (x_1, x_2)$, one bad equilibrium when $x = x_2$, and no bad equilibria when $x > x_2$. Further, $x_2 > x_1$ if and only if $A > A_2 \equiv \frac{(\sqrt{l+x_1}+\sqrt{t\theta})^2}{v_H - v_L}$. x_2 is given by $(\sqrt{A(v_H - v_L)} - \sqrt{\frac{(1-q)C}{q}\theta})^2 - l$.*

Note that $A_2 > A_1$ holds when $(\frac{(1-q)C}{q} + l)\theta > (\sqrt{l + x_1} + \sqrt{\frac{(1-q)C}{q}\theta})^2$. Proposition 3 is then just a combination of Lemma 3.1 and 3.2. Q.E.D.

8.4 The Model in Section 6.1

In this subsection, I provide a detailed analysis of the extension model in section 6.1. Banks are risk neutral and obtain a charter value C whenever they are solvent at date 2. At date 0, each bank is endowed with 1 unit of cash and $\eta < 1$ units of debt in place. Each bank can choose to invest $\rho \in [0, 1]$ units of cash in a risky and illiquid loan. I focus on a symmetric equilibrium where banks choose the same ρ. Each unit of initial investment yields no liquidation value at date 1, and only pays off at date 2. For simplicity, assume loans cannot be used as collateral.

At date 1, each bank's illiquid loan receives a publicly observable aggregate shock and a privately observable idiosyncratic shock. With probability α, the aggregate state is good and all illiquid loans yield $X_H > 1$. With probability $1-\alpha$, the aggregate state is bad, and each bank is privately informed that its loan quality is either $X_H > 1$, which happens with probability θ, or $X_L < 1$, which happens with probability θ, at date 2. Banks' idiosyncratic shocks are independent of each other. Assume this investment opportunity is positive NPV, that is, $(\alpha + (1-\alpha)\theta)X_H + (1-\alpha)(1-\theta)X_L > 1$.

At date 1, once the payoff uncertainty of illiquid loans realizes, banks can use $1-\rho$ units of cash to purchase a financial asset and partially finance the purchase through collateralized borrowing. The asset market and lending market are the same as those in the baseline model. The financial asset is inelastically supplied at A, pays v_H with probability q and v_L with probability $1-q$ at date 2, and can be used as collateral. Each cash lender is assumed to have sufficient loanable funds (Assumption 1 in the main model), so that lenders face perfect competition. As a result of Proposition 1, lenders quote lending contracts with the amount of lending per collateral at $D_j = \max\{u, \min\{v_L, D_j^*(P_j)\}\}$, where $D_j^*(P_j) \equiv P_j - \frac{l(\rho)}{\frac{l(\rho)+x_i(\rho)}{P_j - v_L} + \frac{(1-q)C}{q(v_H - P_j)}}$, $l(\rho) = 1 - \rho$, and $x_i(\rho) = \rho X_i - \eta$. Note that i is the worst type in aggregate state j, so $i = H$ when $j = g$ and $i = L$ when $j = b$.

A symmetric equilibrium of the extended model is defined as $(\rho, P_g, D_g, P_b, D_b)$, such that (1) in the aggregate state, $j = g, b$, D_j satisfies the collateral constraint given by $D_j = \max\{u, \min\{v_L, D_j^*(P)\}\}$, (2) the asset price P_j is determined by market-clearing conditions at date 1 in state $j = g, b$, and (3) taking all future prices as given, banks choose ρ to maximize their expected payoffs at date 0.

To focus on the most interesting case, I impose assumptions 6.1.1 and 6.1.2, which are given at the end of this subsection, so that $x_H(\rho) > 0$ and $x_L(\rho) < 0$ in equilibrium. In the good aggregate state $j = g$, all banks have positive net illiquid asset values, so lenders are not concerned about counterparty risk. The amount of lending per collateral is just $D_g = v_L$, given by the no-recourse condition. The aggregate asset demand is $\frac{l(\rho)}{P_g - v_L}$. I impose assumption 6.1.3 so that $\frac{l(\rho)}{\bar{v} - v_L} > A$ in equilibrium. As a result, the asset price in the good aggregate state is $P_g = \bar{v}$.

In the bad aggregate state, a θ fraction of them have a high net illiquid asset value $x_H(\rho) = \rho X_H - \eta$, and a $1 - \theta$ fraction of them have a low value $x_L(\rho) = \rho X_L - \eta$. I assume $u < \frac{-q\eta v_H + (ql + (1-q)C)v_L}{-q\eta + (ql + (1-q)C)}$ (a variant of Assumption 2 in the main model). Given this assumption, the equilibrium in the bad aggregate state can be characterized by equation CC' and MC', which are given in section 6.1. Note the market-clearing equation must hold with equality; otherwise, banks will invest all their cash to profitable illiquid investment. On the contrary, the collateral constraint may hold with inequality $\leq$, because the amount of collateralized funding is capped at v_L.

To pin down the equilibrium, consider banks' choice of ρ at date 0. A bank maximizes its expected utility $\max_{\rho \in [0,1]} \alpha U(x_H(\rho); P_g, D_g) + (1 - \alpha)(\theta U(x_H(\rho); P_b, D_b) + \theta U(x_L(\rho); P_b, D_b))$, where the U function is the maximized utility of banks given in equation 1. The first-order condition is given by the FOC equation.

Note that ρ is not in the FOC equation, because the problem is linear in ρ. When a bank deviates by investing a little more on the illiquid investment, I assume lenders are aware of the fact that some borrower made a deviation, but they do not know the identity of the borrower. Given such an off-equilibrium assumption, banks have no incentives to deviate from the equilibrium characterized by the FOC condition.

First, consider the existence of an interior equilibrium in which $D_b < v_L$. In this case, equations CC' and FOC hold with equality and the equation MC' binds at the no-default condition. As a result, the asset price is determined by

$$\frac{(\alpha + (1 - \alpha)\theta)}{1 - \alpha} \frac{X_H - 1}{1 - X_L} = \theta \frac{(\bar{v} - P_b)(A(v_H - P_b) + (1 - \theta)\frac{(1-q)C}{q})}{(P_b - v_L)(A(v_H - P_b) - \theta\frac{(1-q)C}{q}) + \eta - X_L} + (1 - \theta)\frac{\bar{v} - v_L}{P_b - v_L}$$

The RHS of the above equation is decreasing in asset price P_b whenever $\eta \geq X_L$, which is assumption 6.1.2. As a result, at most one interior equilibrium exists. The equilibrium asset price is uniquely determined by the above equation, denoted by P^*, which is indeed an interior equilibrium when the corresponding $D(P^*)$ is less than v_L. Whenever $D(P^*) > v_L$, the economy has a corner equilibrium with $(D_b, P_b, \rho) = (v_L, P^*, \rho^*)$, where (P^*, ρ_c) are given by plugging $D_b = v_L$ into the

CC' and FOC equations. $P^* = \min\{v_L, P^{**}\}$, where $P^{**} \in [v_L, \bar{v}]$ is the unique solution that satisfies $-\frac{\eta - X_L}{P - v_L} + \frac{(1-q)(1+(1-\theta)X_L)C}{q(v_H - P)} = A$, and $\rho^* = 1 - (A + (1-\theta)\frac{(1-q)C}{q(v_H - P^*)})(P^* - v_L)$. In the corner equilibrium, the collateral-constraint equation holds with inequality "$\leq$".

Given this equilibrium uniqueness, I next establish the main result about how the stability of equilibrium in the asset market varies with the probability of the good aggregate state, α. I define the cutting-edge case when $\alpha = \alpha^*$ to be the threshold value that lets the unique equilibrium be just in the interior, that is , $D_b = v_L$, and the no-default condition in equation MC', equation CC', and, equation FOC all hold with equality. Let $S(\alpha) \equiv \frac{\frac{(\alpha+(1-\alpha)\theta)(X_H - 1)}{1-\alpha} - \theta}{\theta + (1-\theta)(1-X_L)}$. Then, $\alpha^* = S^{-1}(\frac{\bar{v} - v_L}{P^* - v_L})$.

I am interested in the stability of the equilibrium at the date-1 asset market after the investment decision has been made at date 0. The concept of equilibrium is defined in the main model where the investment decision is treated as an exogenous parameter. When α is below α^*, the economy has a unique corner equilibrium with $D_b = v_L$. As a result, the aggregate demand function is $D_{agg}(P_b) = \theta\frac{l(\rho)}{P_b - v_L} + (1-\theta)\frac{l(\rho) + x_L(\rho)}{P_b - v_L}$. When ρ is fixed, the aggregate demand is decreasing in asset price.

However, when α is above α^*, the economy has a unique interior equilibrium with $D_b < v_L$. As a result, the aggregate demand function is $D_{agg}(P_b) = \theta\frac{l(\rho)}{P_b - D_b} + (1-\theta)\frac{l(\rho) + x_L(\rho)}{P_b - v_L}$, and can be rewritten by plugging in the collateral-constraint equation CC' as, $D_{agg}(P_b) = \frac{l(\rho) + x_L(\rho)}{P_b - v_L} + \frac{\theta(1-q)C}{q(v_H - P_b)}$. Around the neighborhood of α^*, the aggregate demand is increasing in asset price when $\frac{dD_{agg}}{dP_b}|_{P_b = P^*} > 0$, which corresponds to assumption 6.1.4 given below.

Therefore, when α is around the neighborhood of α^* and increases above the threshold value, the aggregate asset-demand curve switches from being downward-sloping to being upward-sloping, once banks have made their investment decision at date 0. As can be seen in Figure 5, the upward-sloping demand makes the asset market vulnerable to small perturbations, which can drive the economy to a worse equilibrium with significantly lower repo funding and asset price.

The above-mentioned assumptions are
Assumption 6.1.1: $X_H > \frac{\eta}{\rho^*}$.
Assumption 6.1.2: $X_L < \eta$.
Assumption 6.1.3: $\bar{v} - v_L < \frac{1-\rho^*}{A}$.
Assumption 6.1.4: $\frac{v_H - P^*}{P^* - v_L} > \sqrt{\frac{\theta(1-q)C}{q(1-\rho^* + \rho^* X_L - \eta)}}$.

The first three assumptions are just simplifying assumptions that drive us to the most interesting cases. Assumption 6.1.4 is the key assumption that leads to the build-up of financial fragility in boom times. This assumption is more likely to hold when the illiquid loans are more risky, that is, when X_L is low.

8.5 The Model in Section 6.2

This section provides a detailed analysis of the extension model in section 6.2. Again, banks are risk neutral and obtain a charter value C whenever they are solvent at date 2. Each bank has l units of cash and x units of net illiquid asset. A bank's net illiquid asset value depends on an aggregate shock and an idiosyncratic shock, and both of them realize at date 1. With probability β, the aggregate state is good, and all banks' net illiquid asset values are x_H. With probability $1-\beta$, the aggregate state is bad, and each bank is privately informed that his net illiquid asset value is either $x = x_H > 0$, which happens with probability θ, or $x = x_L < 0$, which happens with probability $1-\theta$, at date 2. Banks' idiosyncratic shocks are independent of each other.

At both date 0 and date 1, banks can trade financial assets in a competitive market. The financial asset is inelastically supplied at A, and its date 2 value is v_H with probability q and v_L with probability $1-q$. For simplicity, assume the payoff uncertainty of the financial asset is independent of the aggregate net-worth shock and is revealed at date 2. At date 0, banks can borrow with the purchased assets as collateral. I assume all collateralized debt contracts are one period, so they have to be rolled over at date 1. At date 1, shocks on banks' net illiquid asset values realize. Then, the asset market reopens. Banks rebalance their asset positions and roll over their repos. At date 2, all assets pay off and banks repay debt subject to limited liability.

Banks are the only asset buyers in this model, and they are identical at date 0. Therefore, I focus on a symmetric equilibrium in which each bank ends up holding A units of the financial asset at date 0. At date 1, each bank's net worth is the sum of its initial cash l, its rollover gain/loss $A(P_j - P_0)$, and its net illiquid asset value x_i. P_j is the asset price at date 1 in the aggregate state $j = g, b$, P_0 is the asset price at date 0, and $i = H, L$. Let $l(P_j, P_0) = l + A(P_j - P_0)$ be the liquid part of a bank's net worth that can be used in purchasing assets.

I assume each cash lender has sufficient loanable funds (Assumption 1 in the main model), so that lenders face perfect competition. As a result of Proposition 1, lenders at date 1 quote lending contracts whose amount of lending per collateral is $D_j = \max\{u, \min\{v_L, D_j^*(P_j)\}\}$, where $D_j^*(P_j) \equiv P_j - \frac{l(P_j,P_0)}{\frac{l(P_j,P_0)+x_i}{P_j - v_L} + \frac{(1-q)C}{q(v_H - P_j)}}$ and $l(P_j, P_0) = l + A(P_j - P_0)$. Here i is the worst type in aggregate state j, so $i = H$ when $j = g$ and $i = L$ when $j = b$.

Importantly, I assume lenders have no access to the date-1 asset market. This assumption can be motivated by lenders' lack of skills to trade with other borrowers in the asset market. To formalize this idea in a more structured way, I assume two asset markets exist and open sequentially within date 1. At date 1.1, borrowers but not lenders can trade in a competitive asset market, which is called the pre-default asset market. At date 1.2, both borrowers and lenders who seize the collateral assets can trade in another competitive asset market, which is called the post-default asset market.

To simplify the analysis, I focus on the equilibrium in which no debt default and no trade in the post-default asset market occur. In such an equilibrium, borrowers all participate in the pre-default asset market. When the price in that market is $P_{pre} = \bar{v}$, the banking sector has additional liquidity to absorb assets in the post-default market. So the price in the post-default market, although no trade occurs there, will be $P_{post} = \bar{v}$.

However, when the price in the pre-default market is $P_{pre} < \bar{v}$, the banking sector is no longer willing to absorb assets at a high price. High-type banks run out of cash by spending it all in purchasing assets in the pre-default market. Low-type banks hold some cash at hand due to solvency concerns, but are unwilling to give up cash for additional asset positions. As a result, the price in the post-default market will be given by $P_{post} = v_L$, which is a price that makes purchasing the asset riskless. The difference between the pre-default and post-default prices are driven by the assumption of market incompleteness. Note the asset price in the pre-default market is the single price of the asset for banks, whereas the asset price in the post-default market is lenders' valuation of the asset.

A symmetric equilibrium of the extended model is defined as $(P_0, P_g, D_g, P_b, D_b)$, such that (1) in the aggregate state $j = g, b$, D_j satisfies the collateral constraint given by $D_j = \max\{u, \min\{v_L, D_j^*(P)\}\}$, (2) the asset price P_j is determined by the market-clearing condition at date 1 in state $j = g, b$, and (3) the date-0 asset price P_0 is determined by the market-clearing condition at date 0.

When the aggregate state is good, all banks have positive net illiquid asset values, so they will always be solvent at date 2. Consequently, lenders set the amount of collateralized funding at $D_g = v_L$ because they are not concerned about counterparty risk. Given the high level of collateralized funding, the date-1 asset demand is $\frac{l+A(v_L-P_0)}{P_g-v_L} + A$. To simplify the analysis, I impose assumption 6.2.1 to make sure the market in good times clears at the highest-possible price $P_g = \bar{v}$.

When the aggregate state is bad, some banks are thinly capitalized, but lenders are uninformed of the types of their counterparty. Similar to the baseline model, an interior equilibrium of the asset market and the lending market at date 1 satisfies equations CC1 and MC1. Note that a corner equilibrium can exist when $MC1$ holds with inequality "$\geq$" at the price of $P_b = \bar{v}$. The only difference between the baseline model and the extended model is that the amount of cash a bank can use to participate in the asset market depends endogenously on its rollover loss, which is proportional to the change in asset price from date 0 to date 1.

The asset price at date 0 is determined by a market-clearing condition. Note that lenders at date 0 set the amount of lending D_0 so that no banks want to take excessive leverage, through borrowing from other lenders, and become insolvent when hit by both a negative aggregate and a negative idiosyncratic shock. The no-default condition that determines D_0 can be written as

$$D_0 = \max\{v_L, \min\{P_b, D_0^*(P_0)\}\}$$

where $D_0^*(P_0) = P_0 - \frac{l}{A+\frac{(1-\beta)C'}{\beta(v_H-P_0)}}$, and $C' = \theta(l + A(P_b - P_0))\frac{\bar{v}-D_b}{P_b-D_b} + (1-\theta)((l + x_L + A(P_b -$

$P_0))\frac{\bar{v}-v_L}{P_b-v_L}+C)$. The determination of D_0 is very similar to that of D_1 in the main model. It must satisfy the no-recourse condition, in this case, P_1, the asset's endogenous worst-state price at date 1. It can either satisfy the full-security condition or the no-default condition. In the former, lenders' valuation of the collateral is v_L as argued before. In the latter, the amount of lending must provide borrowers with incentives not to take excessive leverage. Note a bank's charter value C' at date 1 depends on both its exogenous charter value at date 2 and its endogenous trading gain at date 1.

Under assumption 6.2.1, the amount of collateralized funding at date 0 is sufficient for banks to purchase A units of financial assets at their expected value. As a result, banks' collateral constraints are loose at date 0, and the asset price at date 0 is given by the indifference condition in equation MC0.

First, consider a corner equilibrium in which $D_b = v_L$. To simplify the analysis, I impose assumption 6.2.2. As a result, a corner equilibrium always exists where $P_b = P_0 = \bar{v}$. Second, consider an interior equilibrium in which $D_b < v_L$. As a result, the no-default condition of equation MC1 binds. I can rewrite the problem with a system of two equations and two unknowns (P_0, P_b) such that

$$P_0 = P_b + \frac{\beta(\bar{v}-P_b)}{\beta+(1-\beta)(\theta\frac{\bar{v}-D_b}{P_b-D_b}+(1-\theta)\frac{\bar{v}-v_L}{P_b-v_L})}$$

$$P_0 = v_L + \frac{1}{A}(\frac{\theta(1-q)C}{q}\frac{P_b-v_L}{v_H-P_b}+x_L+l)$$

where $D_b = P_b - \frac{l}{\frac{l+A(P_b-P_0)+x_L}{P_b-v_L}+\frac{(1-q)C/q}{v_H-P_b}}$. The first equation is a variant of equation MC1 and the second is a variant of equation MC0.

Denote the first equation by $P_0 = f_1(P_b)$ and the second by $P_0 = f_2(P_b)$. We have $f_1(v_L) = v_L$, $f_1(\bar{v}) = \bar{v}$, $f_2(v_L) = v_L + \frac{l+x_L}{A} > v_L$, $f_2(\bar{v}) = v_L + \frac{\frac{\theta(1-q)C}{q}+l+x_L}{A}$ Under assumption 6.2.2, $f_2(\bar{v}) > \bar{v}$. Further, I can show $f_1' > 0$, $f_1'' < 0$, $f_2' > 0$, and $f_2'' > 0$, when $P_b \in [v_L, \bar{v}]$.

Given the shapes of these two functions, there is a unique corner equilibrium with high asset prices when β is below a threshold value β^* and that there are one corner equilibrium and two interior equilibria when β is larger than the threshold.[38] As argued in section 6.2, a high probability of the good aggregate state opens the possibility of a bad equilibrium, resulting in large decreases in asset price and collateralized funding when the economy is hit by a negative aggregate shock.

The above-mentioned assumptions are

Assumption 6.2.1: $\bar{v} - v_L < \frac{l}{A}$.

Assumption 6.2.2: $\bar{v} - v_L < \frac{l+\theta C+x_L}{A}$.

[38] When Assumption 6.2.2 fails, there could be another threshold $\beta^{**} > \beta^*$ above which the economy will have only one equilibrium in which $P_b < P_0 < \bar{v}$.

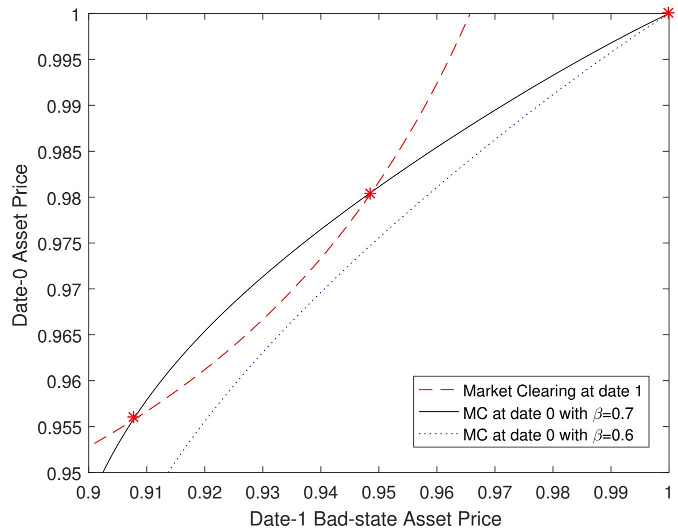

Figure 7: The red dash line is the market-clearing condition at date 0. The black solid line is the market-clearing condition at date 1 in the bad state when $\beta = 0.7$, and the blue dotted line is the market-clearing condition at date 1 in the bad state when $\beta = 0.6$. Note multiple equilibria exist when $\beta = 0.7$, but only one equilibrium exists when $\beta = 0.6$. Parameter values (that are new or different from those in Figure 1) are $x_L = -0.3$ and $A = 3$.

Assumption 6.2.1 is a simplifying assumption that drives us to the most interesting cases. When Assumption 6.2.2 holds, the economy always has a good equilibrium with high asset prices.