# Asset Transfers and Self-Fulfilling Runs on a Diamond-Dybvig Intermediary*

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**Abstract**

We introduce a new mechanism to address self-fulfilling runs on a Diamond-Dybvig intermediary. If a depositor wants to end her relationship with the intermediary early, then she can withdraw goods *or* take ownership of unliquidated assets from the intermediary's balance sheet. We interpret this mechanism as a repo contract or a bankruptcy plan. When the intermediary can cheaply transfer ownership of its assets to depositors, this mechanism can eliminate runs while achieving the first-best outcome. This result highlights the importance of understanding transaction costs and which assets are best held by intermediaries rather than depositors.

## 1 Introduction

Are solvent financial intermediaries vulnerable to self-fulfilling runs? The canonical environment for discussing this question is provided by Diamond and Dybvig (1983) (henceforth DD), who study a financial intermediary characterized by the ability to pool deposits and the requirement to pay withdrawal requests sequentially. They argue that the intermediary may not be able to provide efficient liquidity insurance to depositors without creating the possibility of a run. We show that this result relies on the implicit restriction that the intermediary can transfer goods but not assets to depositors. We formalize an unrestricted version of their environment and show that there exists a simple mechanism that, using asset transfers, eliminates runs and achieves the first-best outcome. This result suggests that the answer to our question depends crucially on whether financial intermediaries can transfer ownership of their assets to depositors. It follows that understanding the impediments to such transfers is critical to understanding self-fulfilling runs. We discuss a transaction cost as well as frictions that make intermediaries more efficient holders of assets than depositors. For the latter explanation, there are many possible frictions. We provide one path forward by showing that idiosyncratic return risk with sufficiently high dispersion and negative skew renders our mechanism ineffective.

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In our mechanism, when each depositor arrives, the intermediary offers them three options: withdrawing goods for immediate consumption, leaving resources with the intermediary until a later period, and immediately taking ownership of an asset from the intermediary's balance sheet. Throughout the rest of the paper, we refer to the third option as an "asset transfer". The essential feature of a self-fulfilling run is that depositors who do not have an imminent consumption need nonetheless choose to withdraw goods not because they are concerned about the quality of the assets on the intermediary's balance sheet, but because they are concerned that so many others are also withdrawing goods that the intermediary will become insolvent. Conceptually, our mechanism eliminates this problem by saying to depositors: if you are worried about a run and don't need to consume immediately, then you can swap your claim on the intermediary for a claim directly on the intermediary's assets. An appealing feature of our mechanism is that, in most cases, depositors don't choose asset transfers in equilibrium. Instead, the asset transfer acts as an off-equilibrium option that allays any fear of patient depositors running and withdrawing goods early.

The asset transfer has many real world interpretations. A literal interpretation is a repurchase agreement (repo contract), in which the intermediary (borrower) gives the depositor (lender) an asset (collateral) that can be kept if the depositor is worried about a run. The key feature is that, like in a repo contract, even if the intermediary goes bankrupt and has outstanding liabilities, the depositor maintains ownership of the asset. Although we do not see these arrangements in the retail banking sector, they are prevalent in the wholesale banking sector, which operates beyond the remit of deposit insurance (see Pozsar, Adrian, Ashcraft, and Boesky (2010) and Pozsar (2014)). Alternatively, the asset transfer can be interpreted as part of a bankruptcy plan that gives depositors the option to progressively disassemble the intermediary if they believe it will become insolvent. This has a flavor of the requirement in the Dodd-Frank Act that intermediaries have a plan for how to dismantle efficiently. In this case, we can think of the transaction cost as a bankruptcy cost. Moreover, this interpretation is appealing because the progressive break up of the intermediary is already inherent to the finite horizon setting of the DD model. In the original DD model, the intermediary is frictionlessly dissolved in the final period; we are simply bringing this process into an earlier period. Finally, we can also interpret asset transfers as giving depositors units of a securitized portfolio or stakes in a mutual fund of the intermediary's assets.

Although, to our knowledge, asset transfers have not been discussed in the DD literature, we believe there is nothing in the original environment preventing their use. In the DD environment, the intermediary has a balance sheet of investments it uses to pay out resources over two periods subject to "banking" restrictions that, as conceptualized by Wallace (1988), are characterized by four essential features: (1) depositors are either *impatient* and only value consumption in the earlier period or *patient* and value consumption in each of the two periods, (2) each depositor's realized patience type is private information, (3) investments cannot be restarted after being converted into goods, and (4) in the earlier period, depositors are isolated and successively visit the intermediary once. The third feature implies that investment generates an asset that allows the holder, in the earlier period, to make an irreversible choice between liquidating then to receive some quantity of the good and liquidating in the final period to receive a higher quantity of the good. We believe the literature has interpreted the third feature as also implying that the intermediary cannot transfer ownership of the asset to a depositor without liquidating. In our baseline model, we make this implicit restriction explicit by introducing a per unit asset transfer cost.

Our mechanism suggests that rather than ignoring asset transfers, we should explore the frictions involved in such transfers to understand why self-fulfilling runs can occur in a DD environment. We believe a likely impediment comes from whether depositors are willing to hold the asset as well as from a literal transfer cost. This focuses our attention on the characteristics of the assets on an intermediary's balance sheet. Traditionally, in the DD environment, the only frictions are on the liability side. Previous literature, such as Diamond (1984) and Williamson (1986), have provided examples in which an intermediary (but not a DD-style intermediary), by aggregating resources, is better able to hold assets than are depositors. We extend the DD setup by introducing idiosyncratic return risk while maintaining that there is no aggregate risk. In our new environment, the intermediary can create a fully diversified portfolio with a deterministic return but depositors can only hold units of the asset that still bear risk. In this setting, our mechanism only eliminates runs when the return distribution has sufficiently low variance or positive skewness. This complements the work of Goldstein and Pauzner (2005) and Huang (2013), who show that shocks to the aggregate return on the intermediary's assets can lead to runs. A noteworthy feature of our model is that if return dispersion increases, then the possibility of a run may emerge even though the aggregate return is unchanged. Under the repo interpretation of the asset transfer, this suggests that an increase in return dispersion could destabilize the repo market. This seems particularly relevant in light of the recent work by Ospina and Uhlig (2018), which shows that the increase in the average loss rate on mortgage backed securities during the crisis was relatively low compared to the increase in the dispersion of loss rates.

We also extend the DD setup by adding aggregate risk without idiosyncratic risk. An important issue created by aggregate risk is that depositors who withdraw early cannot bear the same risk as depositors who withdraw late — their irreversible payments must be made before the aggregate state is fully realized or known by the intermediary. In this case, we argue and demonstrate with a numerical example that the ability of our mechanism to eliminate runs depends on the intermediary's ability to create assets with payoffs that depend on the aggregate state.

Formally, our approach follows a branch of the literature, pioneered by Green and Lin (2000, 2003) and Peck and Shell (2003), which focuses on whether a mechanism designer — interpreted as a competitive financial sector — facing the essential features of the DD "banking" environment can strongly implement the first-best allocation. If only weak implementation can be achieved, then they call the environment "fundamentally unstable" since the intermediary cannot achieve the first-best allocation without introducing suboptimal equilibria such as "bank runs". Technically, our baseline environment is not the same as that of Green and Lin (2000, 2003) and Peck and Shell (2003) because we have an infinite number of agents and no aggregate risk. However, our main point of departure is that we consider an allocation function with the complete set of options available to the intermediary: liquidating units of the asset to provide goods immediately, not liquidating and providing goods in the next period, and transferring ownership of units of the asset to the depositor immediately, *without liquidating*. Previous literature only considers allocation functions with the first two.

Since our mechanism uses all three options, it is an indirect mechanism. In this sense, we are similar to Andolfatto, Nosal, and Sultanum (2014) and Cavalcanti and Monteiro (2016), who argue that other researchers have overlooked the possibility of indirect mechanisms and find ones that strongly implement the first-best in the Peck and Shell (2003) environment. Like their indirect mechanisms, ours includes a third option in addition to the two allowed in the original DD paper that is not chosen in the first-best equilibrium but, if a run were to occur, would dominate withdrawing goods for patient depositors. An important difference is that it is easier to imagine how an intermediary would implement our third option for all possible depositor strategy profiles. In Andolfatto et al. (2014), the intermediary must commit to punishing some depositors who choose the third option even though doing so will not be ex-post optimal and even though depositors may be unable to verify whether their punishments are appropriate. In Cavalcanti and Monteiro (2016), the intermediary uses the third option to extract information, from an arbitrarily small collection of depositors, to work out whether to freeze deposits. On the other hand, in our mechanism, the intermediary is offering an alternative form of payment to depositors, the value of which does not depend on any future actions of depositors or the intermediary. In this sense, our mechanism does not have the same dependence on commitment as do their mechanisms. Another related mechanism that also depends strongly on the intermediary's ability to commit is the direct mechanism in De Nicolo (1996). His intermediary commits to setting aside some resources in period one and commits to a particular distribution of those resources in period two, which, in our baseline environment without aggregate risk, is effectively a traditional withdrawal freeze. Relative to these papers, we believe a significant contribution of our paper is in providing a mechanism that is less dependent on the intermediary's ability to commit.

Green and Lin (2000) suggest that if strong implementation of the first-best is possible in a DD environment, then rather than deciding that self-fulfilling runs only exist if intermediaries behave suboptimally, we should conclude that the environment is incomplete and, to understand runs, consider what additional features would prevent strong implementation. With this in mind, our mechanism, more so than previous indirect mechanisms, suggests a future direction for research into self-fulfilling runs: investigating the difficulties involved in creating and transferring assets.

There is an extensive literature building on the DD environment that does not use a mechanism design framework, some of which is particularly relevant to this paper. In the discussion, we explore the connection between our mechanism and limited commitment, as studied by Ennis and Keister (2009, 2010), and the possibility of allowing depositors to trade amongst themselves, as studied by Jacklin (1987). Jacklin (1987) also introduces an equity contract that, at first glance, may appear similar to our asset transfer. However, while the asset is a claim on goods independent of the intermediary, the equity contract is a claim on the residual holdings of the intermediary. Moreover, unlike the asset, the equity contract is only able to yield different allocations to impatient and patient depositors through trade between them.

This paper is structured as follows. In section 2, the DD environment is set up, our mechanism is defined, and the key results are proven. In section 3, we discuss the connection between our mechanism and commitment as well as the effects of allowing limited depositor interaction. Section 4 concludes.

# 2 The Model

## 2.1 The Environment

Consider the classic DD environment with a sequential service constraint formalized in the manner of Wallace (1988). There are three time periods, $t = 0, 1, 2$, and a continuum of depositors indexed by $i \in [0, 1]$. Each depositor $i$ has preferences given by

$$U(c_{i,1}, c_{i,2}; \theta_i) = \begin{cases} u(c_{i,1}), & \text{if } \theta_i = I \\ u(c_{i,1} + c_{i,2}), & \text{if } \theta_i = P \end{cases}$$

where $c_{i,t}$ represents depositor i's consumption in period $t$ of the good and $\theta_i \in \{I, P\}$ is the depositor's type. If $\theta_i = I$, then depositor $i$ is impatient and only cares about consumption in period 1. If $\theta_i = P$, then depositor $i$ is patient and cares about total consumption across periods 1 and 2. A depositor's type is revealed to her at $t = 1$ and is private information. Denote by $\lambda$ the probability that a depositor is impatient. By the law of large numbers, $\lambda$ is also the fraction of depositors who are impatient, so there is no uncertainty about the aggregate type distribution. The function $u$ is twice differentiable, strictly increasing, strictly concave, and has the property that, for all $c \geq 0$, $-cu''(c)/u'(c) > 1$.

Each depositor is endowed with one unit of the consumption good in period 0. Depositors have access to a constant returns to scale investment technology for transforming the endowment into the consumption good in later periods. An investment in period zero yields a return of $R > 1$ units of the good in period 2 per unit of the good invested. If the project is interrupted in period 1, before completion, it yields 1 unit of the good per unit invested. It is useful to think of this investment as generating a perfectly divisible asset that allows the holder, in period 1, to make an irreversible choice between 1 unit of the good per unit of the asset in period 1 and $R$ units of the good per unit of the asset in period 2. If the holder chooses to receive goods in period 1, then we say that the asset has been liquidated.

There is also an intermediary in which depositors can pool resources to manage liquidity risk. In period 0, endowments are deposited and invested, which generates an intermediary balance sheet in period 1 consisting of a unit measure of the asset described above. In period 1, depositors cannot interact with each other and each depositor contacts the intermediary once. In period 2, depositors can interact with each other and the intermediary freely. At the beginning of period 1, each depositor is allocated a place in line $s \sim U[0, 1]$, where $s$ represents the proportion of depositors ahead of them. When a depositor interacts with an intermediary, all they observe are the options the intermediary offers to them. In particular, they neither observe their own place in line nor the actions of previous depositors. The depositor selects one of the options and the intermediary can make an immediate transfer. The depositor and intermediary do not interact again until the following period. We call these restrictions placed on the intermediary the "sequential service constraint".

The environment described thus far captures all the features from the original DD model. We do not believe this environment precludes the intermediary from transferring ownership of its asset holdings to depositors in period 1. We explore the implications of this possibility. We impose a per unit asset transfer cost of $\epsilon$ units of the consumption good. This implies that if the intermediary transfers 1 unit of the asset, then it must liquidate another $\epsilon$ units of its asset holdings to cover the cost. The most literal interpretation of this cost is a direct transaction cost, but we discuss other interpretations throughout the paper. We view previous papers in the literature as setting $\epsilon = \infty$.

## 2.2 Unconstrained Social Planner

Suppose there is a benevolent social planner who can observe depositor types and directly control allocations. The social planner maximizes depositors' period 0 expected utility subject to the aggregate resource constraint. DD show that the first-best (FB) allocation satisfies:

$$(c_{i,1}, c_{i,2}) = \begin{cases} (c_1^*, 0), & \text{if } \theta_i = I \\ (0, c_2^*), & \text{if } \theta_i = P \end{cases}$$

where $c_1^*$ and $c_2^*$ satisfy the Euler equation $u'(c_1^*) = Ru'(c_2^*)$ and the aggregate resource constraint $(1-\lambda)c_2^* = R(1-\lambda c_1^*)$. Since $R > 1$, it follows that $1 < c_1^* < c_2^* < R$. These inequalities demonstrate that the planner provides insurance for agents' type uncertainty, which can be interpreted as liquidity insurance.

## 2.3 Constrained Social Planner

Following Green and Lin (2000, 2003), we use a mechanism design approach to investigate which outcomes can be achieved by a constrained planner in the environment described in section 2.1. The constrained planner faces the sequential service constraint and an information asymmetry about each depositor's type.

The state space is $\Omega^{[0,1]}$, where $\Omega = \{I, P\} \times [0,1]$ has typical element $\omega = (\theta, s)$, which consists of a depositor's private type $\theta \in \{I, P\}$ and her place in line $s$.[1] Let $\mathcal{A}$ denote the action space for each depositor. The action profile of all depositors is denoted $\mathbf{a} \in \mathcal{A}^{[0,1]}$ and $a_s$ is the action of the depositor at place $s$ in line. An outcome function is a mapping $g : \mathcal{A}^{[0,1]} \times [0,1] \to \mathbb{R}^3_+$. It specifies that if depositors play $\mathbf{a}$, then the depositor at position $s$ gets allocation

$$g(\mathbf{a}, s) = (c_1(a_s, s; \mathbf{a}), c_2(a_s, s; \mathbf{a}), \kappa(a_s, s; \mathbf{a})),$$

where $c_t(a_s, s; \mathbf{a})$ is the units of the consumption good given to the depositor in period $t$ and $\kappa(a_s, s; \mathbf{a})$ is the units of the asset transferred to the depositor in period 1.[2] A mechanism is a pair $\Gamma = (\mathcal{A}, g)$. The constrained social planner is restricted to choose a sequential service feasible mechanism:

**Definition 1 (Sequential Service Feasible Mechanism).** Let $B(s; \mathbf{a})$ denote the measure of the intermediary's holdings of the asset at place $s$ if depositors play $\mathbf{a}$. A mechanism $\Gamma = (\mathcal{A}, g)$ is sequential service feasible if the outcome function satisfies:

1. Budget feasibility: $\int_0^1 c_2(a_s, s; \mathbf{a}) ds \leq RB(1; \mathbf{a})$
2. Sequential service constraint: The period 1 payouts to the depositor at place $s$ in line, $c_1(a_s, s; \mathbf{a})$ and $\kappa(a_s, s; \mathbf{a})$, can only depend on $s$ itself and $\{a_l : l \leq s\}$, the actions of depositors at place up to and including $s$.

This paper uses the Bayes Nash Equilibrium (BNE) concept to discuss implementation. The definition of implementation is standard.[3] The mechanism induces the following game.

[1]For a space $M$, the notation $M^{[0,1]}$ denotes the space of mappings $f : [0,1] \to M$.
[2]These are the only three objects in the economy, so this is as general an outcome space for $g$ as possible.
[3]For example, see Palfrey (1993).

Each depositor chooses a mixed strategy that can depend on whether she is patient, $\theta \in \{I, P\}$, and the options offered to her, but not explicitly on the actions of previous depositors or on her place in line, $s$. In a BNE, depositors play an action $a$ to maximize expected utility

$$\mathbb{E}_{s\in[0,1]}[U(a, s; \mathbf{a})] = \begin{cases} \mathbb{E}[(u(c_1(a, s; \mathbf{a}) + \kappa(a, s; \mathbf{a}))] & \text{if } \theta = I \\ \mathbb{E}[(u(c_1(a, s; \mathbf{a}) + c_2(a, s; \mathbf{a}) + R\kappa(a, s; \mathbf{a}))] & \text{if } \theta = P, \end{cases}$$

where we impose the optimality condition that, in period 1, impatient depositors liquidate their holdings of the asset and patient depositors do not. Observe that a depositor takes $\mathbf{a}$ as given since she has measure zero. We focus on the implementation of social choice functions in which each depositor's allocation depends only on whether she is impatient or patient, i.e. her allocation is given by $h : \{I, P\} \to \mathbb{R}^3_+$.[4] The mechanism $\Gamma$ **weakly implements** the social choice function characterized by $h$ if there exists a BNE of the game induced by $\Gamma$ in which impatient depositors receive $h(I)$ and patient depositors receive $h(P)$. The mechanism $\Gamma$ **strongly implements** the social choice function if, in every BNE of the game induced by $\Gamma$, depositor allocations are given by $h$.

The canonical question in the DD literature can be phrased as: does there exist a sequential service feasible mechanism that strongly implements the first-best allocation? If this is not the case, then we say the environment is "fundamentally unstable" since we cannot achieve the first-best without introducing the possibility of suboptimal equilibria.

## 2.4 Demand Deposit Mechanism

DD and most of the subsequent literature have studied mechanisms with the direct action space $\{W_G, W_0\}$[5], where $W_G$ represents withdrawing goods in period 1 and $W_0$ represents waiting to withdraw goods in period 2.[6] Their original (demand deposit) mechanism $\Gamma_D$ has outcome function

$$\left(c_1^D(a_s, s; \mathbf{a}), c_2^D(a_s, s; \mathbf{a}), \kappa^D(a_s, s; \mathbf{a})\right) = \begin{cases} \begin{bmatrix} (c_1^*,\ 0,\ 0), & \text{if } B(s; \mathbf{a}) > 0 \\ (0,\ 0,\ 0), & \text{if } B(s; \mathbf{a}) = 0 \end{bmatrix}, & \text{if } a_s = W_G \\ (0,\ RB(1; \mathbf{a})/m_0,\ 0), & \text{if } a_s = W_0 \end{cases}$$

where $m_0$ is the fraction of depositors who choose $W_0$ and

$$B(s; \mathbf{a}) = 1 - \int_0^s c_1^D(a_s, s; \mathbf{a})ds$$

is the measure of the intermediary's holdings of the asset remaining at place $s$. DD prove that the mechanism $\Gamma_D$ is sequential service feasible and weakly implements the first-best allocation. More specifically, they show that the game induced by $\Gamma_D$ has a "truth telling" BNE and may also have a "run" BNE. In the truth telling BNE, impatient depositors choose $W_G$, patient depositors choose $W_0$, and the allocation is the first-best. In the run BNE, all

[4]Indeed, we only consider the implementation of the unconstrained first-best allocation, which is of this form.

[5]Technically, DD allow depositors to withdraw a fraction in period 1. However, this is equivalent to the setup here because we have a continuum of depositors and allow for mixed strategies. $\{W_G, W_0\}$ can be thought of as a direct action space because it has the same dimension as the type space.

[6]Notable exceptions are Andolfatto et al. (2014) and Cavalcanti and Monteiro (2016), which look at indirect mechanisms in environments with aggregate uncertainty.

depositors choose $W_G$ in period 1 and the intermediary runs out of resources before the end of the line (i.e., there exists an $\bar{s} < 1$ such that $B(\bar{s}; \mathbf{a}) = 0$ and so all depositors in places $s \geq \bar{s}$ receive no goods in either period).

## 2.5 Asset Transfer Mechanism

We define an Asset Transfer Mechanism $\Gamma_K$ that makes use of the intermediary's ability to transfer ownership of units of the asset. Let the action space be $\{W_0, W_G, W_K\}$, where the actions $W_0$ and $W_G$ have the same interpretations as before and the new action $W_K$ transfers ownership of $\kappa$ units of the intermediary's holdings of the asset to the depositor (without liquidating those units of the asset). The outcome function is defined by:

$$\left(c_1^K(a_s, s; \mathbf{a}), c_2^K(a_s, s; \mathbf{a}), \kappa^K(a_s, s; \mathbf{a})\right) = \begin{cases} \begin{bmatrix} (c_1^*,\ 0,\ 0), & \text{if } B(s; \mathbf{a}) > 0 \\ (0,\ 0,\ 0), & \text{if } B(s; \mathbf{a}) = 0 \end{bmatrix}, & \text{if } a_s = W_G \\ \begin{bmatrix} (0,\ 0,\ \kappa), & \text{if } B(s; \mathbf{a}) > 0 \\ (0,\ 0,\ 0), & \text{if } B(s; \mathbf{a}) = 0 \end{bmatrix}, & \text{if } a_s = W_K \\ (0,\ RB(1; \mathbf{a})/m_0,\ 0), & \text{if } a_s = W_0 \end{cases}$$

where $\kappa$ is a single number[7] and, if we let $\epsilon \geq 0$ be the per unit cost of transferring the asset, then

$$B(s; \mathbf{a}) = 1 - \int_0^s c_1^K(a_s, s; \mathbf{a}) ds - (1 + \epsilon) \int_0^s \kappa^K(a_s, s; \mathbf{a}) ds$$

is the measure of the intermediary's holdings of the asset left at place $s$. By construction, $\Gamma_K$ is sequential service feasible.

**Theorem 1.** *$\Gamma_K$ can strongly implement the first-best allocation if and only if $\epsilon \in [0, c_2^*/c_1^* - 1)$.*

*Proof.* Necessity: We show the contrapositive that if $\epsilon \geq c_2^*/c_1^* - 1$, then $\Gamma_K$ cannot strongly implement the FB. If $\epsilon \geq c_2^*/c_1^* - 1$, then $(c_1^*/R, c_2^*/(R(1+\epsilon))]$ is empty, so it must be that either $\kappa \leq c_1^*/R$ or $\kappa > c_2^*/(R(1+\epsilon))$. If $\kappa \leq c_1^*/R$, then $\Gamma_K$ does not strongly implement the FB. In this case, both impatient and patient depositors weakly prefer to choose $W_G$ over $W_K$. Hence, as in the game induced by $\Gamma_D$, there exists a BNE in which all depositors choose $W_G$ and the FB is not achieved.

Next, we show that if $\kappa > c_2^*/(R(1+\epsilon))$, then $\Gamma_K$ does not strongly implement the FB. In this case, there exists a non-FB BNE in which impatient and patient depositors choose whichever of $W_K$ and $W_G$ gives them higher utility (and mix in any proportion if they are indifferent) and the intermediary runs out of resources strictly before the end of period 1. To see this, suppose depositors mix only between $W_K$ and $W_G$ and let $\alpha$ and $\beta$ be the probabilities with which impatient and patient depositors choose $W_G$, respectively. For convenience, define

$$\tilde{B}(s; \mathbf{a}) = 1 - \int_0^s c_1^* \mathbb{1}_{\{a_s = W_G\}} ds - (1 + \epsilon) \int_0^s \kappa \mathbb{1}_{\{a_s = W_K\}} ds,$$

[7] In Appendix A, we prove that this restriction does not affect whether the first-best is strongly implementable.

which is the measure of the intermediary's holdings of the asset that theoretically would be left at place $s$ (possibly negative) if the intermediary were to pay out $c_1^*$ units of the consumption good to each depositor who chooses $W_G$ and $\kappa$ units of the asset to each depositor who chooses $W_K$, regardless of the value of $B(s;\mathbf{a})$. If $\tilde{B}(s;\mathbf{a}) \geq 0$, then $B(s;\mathbf{a}) = \tilde{B}(s;\mathbf{a})$. If $\tilde{B}(s;\mathbf{a}) < 0$, then $B(s;\mathbf{a}) = 0$. Now, we will show that, given $\alpha$ and $\beta$, there exists an $s_0 \in (0,1)$ such that $\tilde{B}(s_0;\mathbf{a}) = 0$. First, $\tilde{B}(0;\mathbf{a}) = 1$. Moreover,

$$\tilde{B}(1;\mathbf{a}) = 1 - \lambda(\alpha c_1^* + (1-\alpha)(1+\epsilon)\kappa) - (1-\lambda)(\beta c_1^* + (1-\beta)(1+\epsilon)\kappa).$$

We can only have $\alpha < 1$ if impatient depositors weakly prefer to choose $W_K$ over $W_G$, which requires that $\kappa \geq c_1^*$, which implies that $(1+\epsilon)\kappa \geq c_1^*$. It follows that either $\alpha = 1$ or $\alpha < 1$ and $(1+\epsilon)\kappa \geq c_1^*$. Hence,

$$\alpha c_1^* + (1-\alpha)(1+\epsilon)\kappa \geq c_1^*.$$

Furthermore, both $c_1^*$ and $(1+\epsilon)\kappa$ are strictly greater than $c_2^*/R$. As such,

$$\beta c_1^* + (1-\beta)(1+\epsilon)\kappa > c_2^*/R.$$

Bringing the two inequalities together yields

$$\begin{aligned}\tilde{B}(1;\mathbf{a}) &< 1 - \lambda c_1^* - (1-\lambda)c_2^*/R \\ &= 1 - \lambda c_1^* - (1 - \lambda c_1^*) \\ &= 0,\end{aligned}$$

where the second line follows from the fact that optimality of the FB implies that the resource constraint, $(1-\lambda)c_2^* \leq R(1-\lambda c_1^*)$, binds. It then follows from the continuity of $\tilde{B}(s;\mathbf{a})$ in $s$ that there exists an $s_0 \in (0,1)$ such that $\tilde{B}(s_0;\mathbf{a}) = 0$ and so $B(s_0;\mathbf{a}) = 0$. As such, the intermediary has a measure 0 of the asset remaining at the end of period 1, i.e. $B(1;\mathbf{a}) = 0$, and any depositor choosing $W_0$ gets 0 units of the consumption good in period 2. Therefore, all depositors weakly prefer to choose one of $W_G$ and $W_K$, which implies that impatient and patient depositors choosing whichever of $W_G$ and $W_K$ gives them higher utility forms a BNE. This BNE does not achieve the FB because a positive measure of depositors with sufficiently large $s$ ($s > s_0$) receive 0 units of the consumption good in both periods 1 and 2.

This argument also shows that $(1+\epsilon)\kappa < 1$ is not sufficient for $\Gamma_K$ to strongly implement the FB. If $(1+\epsilon)\kappa \in (c_2^*/R, 1)$ and all depositors choose $W_K$, then the intermediary will not run out of resources before the end of period 1. However, in this case, since $c_1^* > 1$, all impatient depositors will choose $W_G$. Then, the intermediary will not have sufficient resources to provide $\kappa$ (and certainly not $c_1^*$) to each patient depositor because

$$(1-\lambda)(1+\epsilon)\kappa + \lambda c_1^* > (1-\lambda)c_2^*/R + \lambda c_1^* = 1.$$

Sufficiency: We show that if $\epsilon < c_2^*/c_1^* - 1$, then there exists a choice of $\kappa$ such that $\Gamma_K$ strongly implements the FB. If $\epsilon < c_2^*/c_1^* - 1$, then $(c_1^*/R, c_2^*/(R(1+\epsilon))]$ is non-empty. Let $\kappa = c_2^*/(R(1+\epsilon))$. Since $\kappa \leq c_2^*/R < c_1^*$, impatient depositors have a strictly dominant strategy: $W_G$. Moreover, since $c_2^*/(R(1+\epsilon)) > c_1^*/R$, we have that $R\kappa > c_1^*$ and patient depositors strictly prefer to choose $W_K$ over $W_G$. It follows that impatient depositors choose $W_G$ and patient depositors mix between $W_K$ and $W_0$. Suppose patient depositors choose $W_K$ with probability $\beta$. Then, the measure of the intermediary's holdings of the asset at the end of period 1 is

$$\begin{aligned} B(1;\mathbf{a}) &= 1 - \lambda c_1^* - (1-\lambda)\beta(1+\epsilon)\kappa \\ &= 1 - \lambda c_1^* - (1-\lambda)\beta\frac{c_2^*}{R} \\ &= (1-\lambda)(1-\beta)\frac{c_2^*}{R}, \end{aligned}$$

where the last equality follows from the resource constraint and optimality of the FB. As such, depositors who choose $W_0$ receive

$$\frac{B(1;\mathbf{a})}{(1-\lambda)(1-\beta)} = c_2^*$$

units of the consumption good in period 2. If $\epsilon > 0$, then $\kappa = c_2^*/(R(1+\epsilon)) < c_2^*/R$ and, for any $\beta \in [0,1]$, $W_0$ is a strictly dominant strategy for patient depositors. In this case, the unique BNE of the game induced by $\Gamma_K$ consists of impatient depositors choosing $W_G$ and patient depositors choosing $W_0$ and yields the FB allocation. Alternatively, if $\epsilon = 0$, then $\kappa = c_2^*/R$ and, for all $\beta \in [0,1]$, patient depositors are indifferent between choosing $W_K$ and $W_0$. As such, for all $\beta \in [0,1]$, there exists a BNE in which impatient depositors choose $W_G$ and patient depositors mix over $W_K$ and $W_0$, choosing $W_K$ with probability $\beta$. These are all the BNE and each yields the FB allocation.

As an aside, the above arguments show that if $\Gamma_K$ strongly implements the FB, then the FB is also the unique outcome that survives the iterated deletion of strictly dominated strategies. □

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To understand this proof, it is helpful to recall why a run can occur in the demand deposit mechanism. The first-best allocation is exactly budget feasible if the impatient depositors choose $W_G$ and the patient depositors choose $W_0$. In this case, which we call "truth telling", the intermediary provides insurance by liquidating $c_1^* > 1$ units of the asset for each impatient depositor and keeping the remaining assets $1 - \lambda c_1^*$ in order to provide $c_2^* < R$ units of the good to each patient depositor in period 2. Since $c_1^* > 1$, if a sufficiently large fraction of patient depositors choose $W_G$, then the intermediary's holdings of the asset are depleted before all depositors have been served and any depositors who choose $W_0$ receive nothing. Thus, it is not "incentive compatible" for a patient depositor to choose $W_0$ if sufficiently many other patient depositors choose $W_G$.

Now consider the Asset Transfer Mechanism when $\epsilon$ is sufficiently small so that the intermediary can choose a $\kappa$ that satisfies $c_1^*/R < \kappa = c_2^*/(R(1+\epsilon))$. For any depositor strategy profile, all impatient depositors strictly prefer to choose $W_G$ over $W_K$ or $W_0$ and all patient depositors strictly prefer to choose $W_K$ over $W_G$. Hence, the only possible incentive compatible deviation from truth telling is for patient depositors to choose $W_K$. Since $(1+\epsilon)\kappa < c_2^*/R$, if a positive measure of patient depositors choose $W_K$, then, unlike if they were to choose $W_G$, the intermediary is left, at the end of period 1, with *more* units of the asset per patient depositor who chose $W_0$. In this sense, $W_K$ is an off-equilibrium path option that, by strictly dominating $W_G$ for patient depositors, ensures that it is always "incentive compatible" for patient depositors to choose $W_0$. As such, the indirect mechanism $\Gamma_K$ is able to strongly implement the first-best while the direct mechanism $\Gamma_D$ cannot.[8] Finally, when $\epsilon = 0$ and $\kappa = c_2^*/R$, then $W_K$ can be played in equilibrium and a direct mechanism is able to strongly implement the first-best. If the asset transfer is costless, then, in period 1, the intermediary can conclude its relationships with *all* depositors by giving units of the good to impatient depositors and units of the asset to patient depositors.[9]

In this setting, it is natural to interpret the Asset Transfer Mechanism as a bankruptcy plan that allows depositors to progressively dismantle the intermediary if they believe it will become insolvent. In this case, we view the transfer cost as a bankruptcy cost. We also interpret the mechanism as offering depositors repo contracts. To see this more clearly, suppose the intermediary executes the mechanism by doing the following. In period 0, each depositor keeps $\kappa$ units of the asset and gives $1-\kappa$ units to the intermediary. In period 1, if a depositor chooses $W_G$ or $W_0$, then the intermediary repurchases the depositor's holdings of the asset using period 1 goods or the promise of period 2 goods, respectively. If a depositor chooses $W_K$, then she ends the repurchase agreement and walks away with her $\kappa$ units of the asset. Under this interpretation of the mechanism, the asset transfer cost is a penalty imposed on depositors who walk away from the repurchase agreement.

## 2.6 Idiosyncratic Return Risk

A clear economic implication of Theorem 1 is that we need to understand the impediments to transferring assets from intermediaries to depositors in order to understand why runs can occur in a DD environment. This was captured in the previous section by introducing a transfer cost, which was interpreted as a direct transaction cost, a bankruptcy cost, or a penalty for breaking repo contracts. In this section, we abstract from such costs and instead focus on why depositors might have a disadvantage in holding the asset relative to a DD intermediary whose only ability is aggregation. This is particularly relevant for the repo contract interpretation, in which the penalty cost may be hard to motivate. A prominent example in the literature is delegated monitoring (see Diamond (1984) and Williamson (1986)), in which, through aggregation, the intermediary can exploit economies of scale to more efficiently monitor investments. Indeed, if we introduce delegated monitoring into the model of the previous section, the asset transfer cost is endogenized as the additional monitoring cost depositors must pay, relative to the intermediary, when holding the asset.

We further explore the difficulties depositors face when holding the asset by introducing idiosyncratic return risk but no aggregate risk. While we expand on the original DD environment, we maintain that the intermediary functions as an insurance provider with

[8] Note that, in this context, the revelation principle does not imply that the restriction to direct mechanisms is without loss of generality. As discussed in Palfrey (1993), it is possible that an indirect mechanism strongly implements the first-best whereas the direct mechanism only weakly implements the first-best.

[9] The intermediary has two options, transferring the asset immediately and transferring the good immediately, one of which is relatively more valuable to patient depositors, the other of which is relatively more valuable to impatient depositors, and neither of which has a value dependent on the actions of other depositors. With these two options, the intermediary can separate impatient and patient depositors into their FB allocations without creating the possibility of a run.

only the ability to pool resources. As such, a depositor in autarky faces return risk, but the intermediary may not if it holds a diversified portfolio. In principle, the intermediary could use its diversified portfolio to generate "non-risky" units of the asset, but, in practice, we restrict the intermediary's ability to create units of the asset that are as diversified as its entire portfolio. Thus, even a diversified intermediary can only transfer "risky" units of the asset during period 1. We formalize this environment below.

Set $\epsilon = 0$. Suppose depositors have the same preferences as in section 2.1 but impose the additional restriction that $\lim_{c \to 0} u(c) = -\infty$. Suppose depositors have access to the following investment technology for transforming the endowment into the consumption good in later periods. An agent chooses the number of projects in which to invest and the amount of their endowment to invest in each project. Each investment of $x$ units of the endowment yields a distinct project that generates $x$ units of the consumption good if interrupted in period 1 and $Rx$ units of the consumption good if held until period 2, where $R$ is random and i.i.d. across projects:

$$R = \begin{cases} R_H, & \text{w.p. } q \\ R_L, & \text{w.p. } 1-q \end{cases}$$

and the expected return is $\bar{R} = qR_H + (1-q)R_L$. We impose that each investment must be of size greater than or equal to $\bar{x} > 0$. This stylized restriction captures that agents may face an increasingly large average cost as they invest in increasingly small projects. The required minimum size makes our investment technology similar to the one in Diamond (1984) and Williamson (1986), although it plays a different role in their papers in which it justifies costly monitoring. Here, the important result is that depositors can only invest in finitely many projects, leaving them exposed to not only liquidity preference risk but also idiosyncratic return risk. An intermediary, by contrast, can invest in uncountably many projects and so will create a fully diversified portfolio with non-random period 2 return $\bar{R}$.

We restrict the intermediary by imposing that each project and the ownership of each project are perfectly indivisible; all the intermediary can do with each project is liquidate, transfer, or hold it until period 2 *in its entirety.* In particular, the intermediary can transfer projects to a depositor, but cannot create a new, diversified, and transferable asset consisting of fractions of projects. Thus, even if the intermediary fully diversifies its portfolio, it can only transfer assets to a depositor that bear the same idiosyncratic risks that the depositor would have faced had she invested on her own. We interpret this setup as allowing the intermediary to provide insurance by pooling resources but limiting the intermediary's ability to securitize its portfolio.[10]

We explore the ability of the Asset Transfer Mechanism to strongly implement the first-best in this setting. For simplicity, we suppose parameters are such that it is sufficient to consider the case in which the intermediary offers, as its asset transfer, to transfer one project of size $\kappa$ to each depositor and, when choosing $\kappa$, the constraint $\kappa \geq \bar{x}$ is not binding.[11] Hence, the intermediary has a measure $1/\kappa$ of distinct investments, each of size

[10] We consider the case in which the intermediary effectively cannot perform any securitization, but we believe the results are qualitatively unchanged so long as the intermediary cannot perfectly securitize its portfolio.

[11] More specifically, we suppose parameters are such that the following hold. If the intermediary can strongly implement the first-best by offering, as its asset transfer, a single project of size $\kappa$, then $\bar{x} \leq \kappa$. If the intermediary cannot strongly implement the first-best with any such asset transfer, then $\bar{x}$ is sufficiently large so that offering to transfer two or more projects to any depositors wouldn't allow for strong implementation

$\kappa$.

Define $\eta \equiv R_H/R_L$. Observe that the environment in the first part of this paper is the special case with $\eta = 1$. As such, given $\epsilon = 0$, we know that if $\eta = 1$, then $\Gamma_K$ can strongly implement the FB. For $\eta > 1$, we prove the following theorem, which says that, for fixed preferences and a fixed average return, the first-best can only be strongly implemented if the dispersion of idiosyncratic returns is sufficiently low.

**Theorem 2.** *If $\eta = R_H/R_L$ is varied while $R_H$ and $R_L$ adjust so that $\bar{R}$ and $q$ are held constant, then there exists a $\bar{\eta} \in (1, \infty)$ such that $\Gamma_K$ can strongly implement the FB (with an appropriate choice of $\kappa$) if and only if $\eta < \bar{\eta}$.*

See Appendix B for the proof. The intuition for the theorem is the following. Patient depositors accepting the asset transfer now face idiosyncratic return risk. As return dispersion increases, the intermediary must offer a higher $\kappa$ to maintain patient depositors' preference for the risky asset transfer over the risk free withdrawal of goods. Eventually, the required $\kappa$ is so large that a non-first-best equilibrium emerges in which all patient depositors take the asset transfer. In this equilibrium, the intermediary's holdings of the asset are depleted before all depositors have been served in period 1, leaving nothing for any depositors who choose $W_0$ and for some of the depositors who choose $W_G$ or $W_K$.

Now that idiosyncratic return risk is present, it may seem reasonable to expect that some depositors, after they are offered asset transfers, would receive signals about those assets' returns. In appendix C, we show that if these signals are perfectly revealing and not observed by the intermediary, then the Asset Transfer Mechanism can only strongly implement the first-best if the dispersion of idiosyncratic returns is sufficiently low or the proportion of high return projects is sufficiently large.

### 2.7 Aggregate Risk

Preliminary version available upon request.

## 3 Discussion

In this section, we compare the Asset Transfer Mechanism to two other mechanisms frequently discussed in the literature: the withdrawal freeze mechanism introduced by DD and the equity contract introduced by Jacklin (1987). We argue that the Asset Transfer Mechanism does not require the same level of commitment as does the deposit freeze mechanism and may be more compatible with a competitive asset market than is an equity contract.

Under the withdrawal freeze mechanism introduced by DD, the intermediary only pays out goods in period 1 if it has sufficient holdings of the asset to pay out $c_2^*$ units of the good in period 2 to each of a measure $(1 - \lambda)$ of depositors. This ensures that a patient depositor strictly prefers $W_0$ regardless of the actions of other depositors and so there is no run equilibrium. If a benevolent planner has full commitment, then the withdrawal freeze strongly implements the first-best. However, Ennis and Keister (2009, 2010) provide an example in which if a sufficiently large fraction of patient depositors choose $W_G$, then a benevolent planner will want to abandon the withdrawal freeze and make such large

of the first-best.

payments to unserved impatient depositors in period 1 that the period 2 payments to patient depositors who chose $W_0$ will fall below $c_1^*$. Hence, if the benevolent planner cannot commit to freezing withdrawals, then whether a patient depositor strictly prefers $W_0$ over $W_G$ again depends on the actions of other depositors and the run equilibrium reemerges. By contrast, under the Asset Transfer Mechanism with $\kappa > c_1^*/R$, a patient depositor always strictly prefers $W_K$ over $W_G$ regardless of the actions of other depositors *and* regardless of whether the intermediary later abandons the mechanism. Taking the asset transfer, like withdrawing goods immediately, ends a depositor's relationship with the intermediary and gives the depositor something whose value does not depend on the actions of others. In this sense, our mechanism works without the form of commitment required by the withdrawal freeze.

In the environments discussed so far, depositors have been completely isolated during period 1 and so have been unable to trade with each other. We now consider what happens when we partially relax the sequential service constraint and potentially allow some markets to form. Wallace (1988) argues that the sequential service constraint consists of both depositor isolation and the restriction that impatient depositors only get utility from consuming the good immediately after contacting the intermediary. If we relax the isolation restriction but maintain the immediate consumption restriction, then the only trade that could ever occur is between depositors who choose $W_K$ but don't liquidate the asset and depositors who choose $W_0$ leaving them with claims on the intermediary's period 2 holdings. Opening markets to allow for such trades to occur would have no effect because only patient depositors would choose $W_0$ or choose $W_K$ and not liquidate and so there can be no gains from trade.

If we relax the immediate consumption restriction as well as the depositor isolation restriction, then depositors can store and potentially trade the good, the asset, and claims on the intermediary's period 2 holdings at the end of period 1; such claims are given to depositors who choose $W_0$. This is the environment studied by Jacklin (1987), who shows that if preferences are generalized so that both types of depositors have strictly positive marginal utilities of consumption in both periods and if depositors are allowed to trade claims on the intermediary's period 2 holdings for the good at the end of period 1, then the first-best can only be even weakly implemented if it is also achieved in autarky.[12] This suggests that the intermediary should prevent the trading of claims on period 2 goods. The intermediary might be able to do this by refusing to honor any claims on its period 2 resources that have changed ownership. However, it is harder to see how the intermediary could restrict trade in units of the asset after it has distributed them to depositors since such depositors need no longer have any interactions with the intermediary. Nonetheless, permitting trade in the asset is less problematic than allowing trade in the claims on the intermediary's period 2 holdings. Suppose the asset transfer is chosen so that the Asset Transfer Mechanism strongly implements the first-best in our baseline environment. If the sequential service constraint is relaxed, depositors can trade units of the asset – but

[12] Jacklin also introduces an equity contract that, at first glance, may appear similar to the asset introduced here. However, while the asset is a claim on goods independent of the intermediary, the equity contract is a claim on the residual holdings of the intermediary in period 2. In fact, after the equity contract pays out dividends in period 1, it looks identical to the claim received by a depositor who chose $W_0$. Moreover, unlike the asset, the equity contract is only able to yield different allocations to impatient and patient depositors through trade between them at the end of period 1.

not claims on the intermediary's period 2 holdings – for the good at the end of period 1, and preferences are generalized as in Jacklin (1987), then the intermediary still weakly implements the first-best. Yet, whether the intermediary strongly implements the first-best is ambiguous.[13]

Although it is an intriguing question whether the first-best can still be strongly implemented in this setting, we do not believe we have the correct environment to study the broader question of whether limited trade in the asset amongst depositors disrupts the Asset Transfer Mechanism. Further exploration requires an environment in which some form of a sequential service constraint is present – so that the asset transfer is still required to eliminate the classic run equilibrium – alongside some form of inter-depositor trade in the asset. We leave this for future work.

# 4 Conclusion

We showed that a benevolent intermediary facing the "classic" DD environment could choose a simple mechanism that strongly implements the first-best outcome. This mechanism involves offering asset transfers that are sufficiently large to prevent patient depositors from withdrawing goods early but sufficiently small that the intermediary never runs out of resources from transferring assets. We interpret this result as saying that the "classic" DD environment is incomplete. We suggest that to understand self-fulfilling runs greater attention needs to be given to the assets on intermediaries' balance sheets, the frictions involved in transferring them, and whether depositors are willing to hold them.

There are many possible directions this line of research could take. In addition to a direct asset transfer cost, we provided a simple example in which we added idiosyncratic return risk and found that the first-best outcome is only strongly implementable if the idiosyncratic return distribution has sufficiently low dispersion or sufficiently positive skew. A noteworthy feature of this model is that if return dispersion increases, then the possibility of a run may emerge even though the aggregate return on the intermediary's assets is unchanged. These implications could be tested empirically in future work.

[13] Once we allow trade at the end of period 1, incentive compatibility depends on the market values of the allocations promised to each depositor action. In the first-best equilibrium, the market value of the good in period 2 implies that selling the asset and liquidating the asset yield the same amount of the period 1 good. As such, opening the market does not change an individual depositor's optimization problem in the first-best equilibrium. However, the market values associated with the depositor actions depend on the strategy profile of all depositors. If positive measures of patient depositors choose $W_G$ and $W_K$, then there will be a price (strictly greater than 1 unit of the good per unit of the asset) at which those who chose $W_G$ buy the asset from those who chose $W_K$. Impatient depositors who chose $W_G$ and patient depositors who chose $W_0$ do not participate in this market because, at the margin, they are indifferent between the period 1 good and the asset and they do not have any of the asset to sell. As supply and demand dictate, as more patient depositors choose $W_G$, the price rises and as more patient depositors choose $W_K$, the price falls. Hence, there is a ratio of patient depositors who choose $W_G$ to patient depositors who choose $W_K$ at which the price is such that the market values of choosing $W_G$ and of choosing $W_K$ are the same. If and only if this ratio is sufficiently large, then there is a suboptimal run equilibrium in which some patient depositors choose $W_G$, the rest choose $W_K$, the intermediary runs out of resources before the end of period 1, and patient depositors are indifferent between $W_G$ and $W_K$ and prefer either to $W_0$.

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# A Proof of Corollary 1

**Corollary 1.** *Define a mechanism $\Gamma$ that offers depositors three options: $W_G$, $W_0$, and $W_K$. Suppose that if depositors choose $W_G$ or $W_0$, then they get the same payoff as they would in the demand deposit mechanism (with an appropriate definition of $B(s;\mathbf{a})$). If depositors choose $W_K$, then they only receive units of the asset. If there does not exist a constant $\kappa$ such that $\Gamma_K$ strongly implements the first-best, then $\Gamma$ does not strongly implement the first-best.*

*Proof.* Suppose that $\Gamma_K$ cannot strongly implement the first-best allocation. It follows from Theorem 1 that $1+\epsilon \geq c_2^*/c_1^*$. Suppose impatient depositors choose $W_G$ and patient depositors choose whichever they prefer of $W_G$ and $W_K$, breaking ties in favor of $W_G$. In this equilibrium, suppose a fraction $\beta$ of depositors are offered a $\kappa > c_1^*/R$. Define $\bar{\kappa}$ to be the average value of $\kappa$ offered to such depositors (or 0 if $\beta = 0$). It follows that $(1+\epsilon)\bar{\kappa} > c_2^*/R$. Hence,

$$\tilde{B}(1;\mathbf{a}) = 1 - \lambda c_1^* - (1-\lambda)(1-\beta)c_1^* - (1-\lambda)\beta(1+\epsilon)\bar{\kappa}$$

is strictly less than 0 since $\lambda c_1^* + (1-\lambda)c_2^*/R = 1$ and $c_1^* > c_2^*/R$. As such, the specified strategy profile is indeed optimal. The equilibrium does not achieve the first-best allocation since a positive measure of depositors receive no goods in either period. □

# B Proof of Theorem 2

Recall that, throughout this proof, $\bar{R}$ is fixed, which implies that $c_1^*$ and $c_2^*$ are fixed as well. If $\eta = 1$, then we are in the model without idiosyncratic return risk and with $\epsilon = 0$. As such, Theorem 1 shows that $\Gamma_K$ can strongly implement the FB. For the rest of the proof, we suppose that $\eta > 1$.

Proof of Theorem 2. Suppose depositors are unsophisticated. First, we show that if $\kappa > c_2^*/\bar{R}$, then $\Gamma_K$ does not strongly implement the FB. Suppose that patient and impatient depositors choose whichever of $W_G$ and $W_K$ gives them higher utility (and mix in any proportion if they are indifferent). We can show that this forms a BNE by using an analogous argument as in the proof of Theorem 1 (for the case in which $\kappa > c_2^*/(R(1+\epsilon))$) to demonstrate that the bank runs out of resources strictly before the end of period 1. Then, any depositor playing $W_0$ gets 0 units of consumption in period 2, confirming that it is indeed optimal for impatient and patient depositors to choose whichever of $W_G$ and $W_K$ yields higher utility. This BNE does not implement the FB since depositors arriving after the place in line at which the bank runs out of resources recieve nothing.

Next, as in the proof of Theorem 1, we want to find a lower bound on $\kappa$ such that patient depositors strictly prefer to play $W_K$ over $W_G$. Define $U(\kappa)$ to be the expected utility a patient depositor receives from choosing $W_K$:

$$U(\kappa) \equiv qu(R_H\kappa) + (1-q)u(R_L\kappa).$$

Observe that, given risk-aversion and $\eta > 1$, $U(\kappa) < u(\bar{R}\kappa)$. If $\kappa \leq U^{-1}(u(c_1^*))$, then patient depositors weakly prefer to play $W_G$ over $W_K$ and, as in the game induced by $\Gamma_D$, there exists a BNE in which all depositors play $W_G$ and the FB is not achieved.

Using the results shown thus far, we can see that if $\left(U^{-1}(u(c_1^*)), c_2^*/\bar{R}\right]$ is empty, then $\Gamma_K$ cannot strongly implement the FB. On the other hand, suppose $\left(U^{-1}(u(c_1^*)), c_2^*/\bar{R}\right]$ is non-empty. We show that, in this case, $\Gamma_K$ can strongly implement the FB with $\kappa = c_2^*/\bar{R}$. Let $\kappa = c_2^*/\bar{R}$. Then, since $U(\kappa) > u(c_1^*)$, we have that, for patient depositors, $W_K$ strictly dominates $W_G$. Also, since $c_2^*/\bar{R} < c_1^*$, we have that, for impatient depositors, $W_G$ strictly dominates $W_K$ and $W_0$. As such, in any BNE, impatient depositors choose $W_G$ and patient depositors mix between $W_K$ and $W_0$. If patient depositors play $W_K$ with probability $\beta$, then the bank assets remaining at the end of period 1 are

$$\begin{aligned} B(1;\mathbf{a}) &= 1 - \lambda c_1^* - (1-\lambda)\beta\kappa \\ &= 1 - \lambda c_1^* - (1-\lambda)\beta\frac{c_2^*}{\bar{R}} \\ &= (1-\lambda)(1-\beta)\frac{c_2^*}{\bar{R}}. \end{aligned}$$

In that case, depositors who play $W_0$ receive

$$\bar{R}\frac{B(1;\mathbf{a})}{(1-\lambda)(1-\beta)} = c_2^*$$

units of consumption in period 2 with certainty. Since $U\left(c_2^*/\bar{R}\right) < u(c_2^*)$, it follows that patient depositors strictly prefer to play $W_0$ over $W_K$. Hence, the unique BNE consists of impatient depositors choosing $W_G$ and patient depositors choosing $W_0$, yielding the FB allocation.

We have shown that $\Gamma_K$ can strongly implement the FB if and only if $U\left(c_2^*/\bar{R}\right) > u(c_1^*)$. We now show that there exists an $\bar{\eta} \in (1, \infty)$ such that $U\left(c_2^*/\bar{R}\right) > u(c_1^*)$ if and only if $\eta < \bar{\eta}$. First, observe that as $\eta$ goes to infinity, $R_L$ converges to 0 and $U\left(c_2^*/\bar{R}\right)$ diverges to negative infinity (which is strictly less than $u(c_1^*)$). Also, at $\eta = 1$, $U\left(c_2^*/\bar{R}\right) = u(c_2^*)$, which is strictly greater than $u(c_1^*)$. Next, we show that $U\left(c_2^*/\bar{R}\right)$ is strictly decreasing in $\eta$. Since $R_L = \bar{R}/(q\eta + 1 - q)$ and $R_H = (\bar{R}\eta)/(q\eta + 1 - q)$, the expected utility a patient depositor receives from an asset transfer of $c_2^*/\bar{R}$ is

$$\begin{aligned} U\left(\frac{c_2^*}{\bar{R}}\right) &= qu\left(R_H\frac{c_2^*}{\bar{R}}\right) + (1-q)u\left(R_L\frac{c_2^*}{\bar{R}}\right) \\ &= qu\left(\frac{\eta c_2^*}{q\eta + 1 - q}\right) + (1-q)u\left(\frac{c_2^*}{q\eta+1-q}\right). \end{aligned}$$

The derivative of this expected utility with respect to $\eta$ is

$$\frac{\partial U\left(\frac{c_2^*}{\bar{R}}\right)}{\partial \eta} = \left[u'\left(\frac{\eta c_2^*}{q\eta+1-q}\right) - u'\left(\frac{c_2^*}{q\eta+1-q}\right)\right]\frac{q(1-q)c_2^*}{(q\eta+1-q)^2},$$

which is strictly negative because $\eta > 1$ and $u'$ is strictly decreasing. Therefore, since $U(c_2^*/\bar{R})$ is continuous in $\eta$, it follows that there exists the desired $\bar{\eta}$.

□

# C Sophisticated Depositors

In this appendix, we consider the case in which each depositor is "sophisticated" and, after viewing the choices offered by the intermediary but before choosing their action, can observe the returns on the projects they would receive if they chose $W_K$. The intermediary still cannot observe the returns on individual projects, but uses Bayes' rule to learn about the returns on particular projects by observing the actions of depositors.

We prove the following theorem, which says that if all depositors are sophisticated, for fixed preferences and a fixed average return, the first-best can only be strongly implemented if the dispersion of idiosyncratic returns is sufficiently low or the proportion of high return projects is sufficiently large.

**Theorem 3.** *Suppose all depositors are "sophisticated". If $\eta = R_H/R_L$ is varied while $R_H$ and $R_L$ adjust so that $\bar{R}$ and $q$ are held constant, then $\Gamma_K$ can strongly implement the FB (with an appropriate choice of $\kappa$) if $\eta < c_2^*/c_1^*$. If $\eta \geq c_2^*/c_1^*$ and $q$ is varied while $R_H$ and $R_L$ adjust so that $\bar{R}$ and $\eta$ are held constant, then there exists a $\bar{q} \in (0,1)$ such that $\Gamma_K$ can strongly implement the FB (with an appropriate choice of $\kappa$) if and only if $q > \bar{q}$.*

The intuition for the theorem is the following. Patient depositors no longer face idiosyncratic return risk. Instead, there are two types of patient depositors: H-types, who observe a return $R = R_H$, and L-types, who observe a return $R = R_L$. If return dispersion is low, then the intermediary can offer a $\kappa$ that is large enough so that both types of patient depositors prefer the asset transfer over the early withdrawal of goods and small enough so that even if all patient depositors take the asset transfer, the intermediary's asset holdings are not depleted before the end of period 1. If return dispersion is high, then any $\kappa$ that is large enough so that L-types prefer the asset transfer over the early withdrawal of goods is so large that H-types get more than $c_2^*$ units of consumption in period 2 from taking the asset transfer, implying that the FB is not achieved. However, even when return dispersion is high, if $q$ is sufficiently large, then the FB can be achieved by choosing a $\kappa$ that is only large enough for H-types to prefer the asset transfer to the early withdrawal of goods and small enough for the intermediary not to run out of resources if all L-types withdraw goods early and all H-types take the asset transfer.

Before beginning the proof, we introduce some additional concepts. In the proof of Theorem 2, neither the intermediary nor the depositors observed the return on any units of the asset so the intermediary had no preference over which "protocol" they used for selecting assets for liquidation. Now, we consider the case in which depositors are sophisticated and observe the return on the $\kappa$ units of the asset offered. The intermediary then learns about the return of particular subsets of its asset holdings by observing the actions of depositors. Hence, when the intermediary offers $\kappa$ units of the asset to a depositor or liquidates units of the asset in response to a depositor choosing $W_G$, it may have a non-trivial choice of the average quality of the units.

In order to discuss this environment, we need to make precise how the intermediary's asset portfolio ends up being structured. Initially, the intermediary's holdings of the asset is a collection of zero measure sets, each of which contains $\kappa$ units of the asset that are either all high quality or all low quality. After some fraction have contacted the intermediary, the intermediary's holdings of the asset are split into two collections of zero measure sets. The first collection consists of the sets of the units of the asset that have not yet been offered to any depositors. We call these sets the **unviewed sets**. Since no depositor has seen the units of the asset in an unviewed set, the intermediary knows by the law of large numbers that, with probability $q$, the units are high quality. The other collection consists of the sets of the units of the asset that particular depositors have viewed and then rejected. We call these sets the **rejected sets**. The intermediary's belief that all the units in a particular rejected set are high quality is a function of the place in line of the depositor that rejected those units of the asset, whether the depositor chose $W_G$ or $W_0$, and the strategy profile of depositors.

Given the structure of the intermediary's asset portfolio, a **protocol** describes, for each place in line, which set of $\kappa$ units of the asset the intermediary offers to the depositor and, if the depositor chooses $W_G$, from which sets the intermediary liquidates. In line with the assumption that a depositor choosing $W_K$ must receive $\kappa$ units of the asset all of the same quality, we impose that if the intermediary liquidates fractions of the units of the asset in particular sets, then it can hold or liquidate the remaining units, but it cannot combine the remaining units into $\kappa$ units to offer to a future depositor.[14] Throughout the proof, we will

[14] In this case, we have setup the mechanism design problem informally but if we want to map back into

refer to the following protocol as the **standard protocol**. When the intermediary selects the $\kappa$ units of the asset to offer to a depositor, it offers an unviewed set and, if no unviewed sets remain, it offers a rejected set. If a depositor chooses $W_G$, then the intermediary liquidates as much as possible of the $c_1^*$ units of the asset from the $\kappa$ units of the asset offered to the depositor. Any further goods come from liquidating unviewed sets or – if no unviewed sets remain – from rejected sets, leaving at most one partially liquidated set.[15]

The standard protocol is useful because it best protects the intermediary from the type of run that may occur when depositors are sophisticated. As we show more formally below, when dispersion is low, the intermediary can strongly implement the FB under any protocol. But, when dispersion is high, to prevent patient $H$-type depositors from getting more than they're due in the FB, the intermediary must choose a $\kappa$ that is so low that patient $L$-type depositors no longer prefer to choose $W_K$ over $W_G$. Then, the possibility of a run in which impatient and patient $L$-type depositors, but not patient $H$-type depositors, choose $W_G$ emerges.

**Lemma 1.** *Suppose $\eta \geq c_2^*/c_1^*$ and $\kappa \in (c_1^*/R_H, c_2^*/R_H]$. Suppose impatient and patient $L$-type depositors choose $W_G$ and patient $H$-type depositors choose $W_K$. If $\tilde{B}(1;\mathbf{a}) < 0$ under the standard protocol, then $\Gamma_K$ cannot strongly implement the FB under any protocol.*

*Proof.* Since $R_H > \bar{R}$ and $c_1^* > c_2^*/\bar{R}$, it follows that $\kappa < c_1^*$. As such, choosing $W_G$ is a strictly dominant strategy for impatient depositors. Moreover, since $\eta \geq c_2^*/c_1^*$, we can see that $R_L\kappa \leq c_1^*$. Hence, patient $L$-type depositors weakly prefer to choose $W_G$ over $W_K$. Finally, since $R_H\kappa > c_1^*$, patient $H$-type depositors strictly prefer to choose $W_K$ over $W_G$. Now, suppose impatient and patient $L$-type depositors choose $W_G$ and patient $H$-type depositors choose $W_K$. If, under whichever protocol the intermediary is using, $\tilde{B}(1;\mathbf{a}) < 0$ (recall that $\tilde{B}(1;\mathbf{a})$, as defined in the proof of Theorem 1, is the measure of units of the asset (of any quality) that theoretically would remain at the end of period 1, possibly negative, if the intermediary were to pay out $c_1^*$ units of the good to each depositor who chooses $W_G$ and $\kappa$ units of the asset to each depositor who chooses $W_K$), then any depositor who chooses $W_0$ receives 0 units of the good in both periods, which confirms that the specified strategy profile forms a BNE. This BNE does not yield the FB because patient $L$-type depositors receive $c_1^* < c_2^*$ units of the good. The remainder of the proof shows that if depositors choosing the specified strategy profile leads to $\tilde{B}(1;\mathbf{a}) < 0$ under the standard protocol, then it leads to $\tilde{B}(1;\mathbf{a}) < 0$ under any other protocol as well.

Suppose depositors choose the specified strategy profile and the intermediary uses the standard protocol. If an impatient or patient $L$-type depositor chooses $W_G$, then, since $\kappa < c_1^*$, the intermediary liquidates all $\kappa$ units of the asset that were offered. If a patient

a canonical mechanism design formulation, then we can think of the protocol as specifying an outcome function that describes both payouts and from which part of the intermediary's holdings the payouts come.

[15]Leaving a partially liquidated set is necessary if $c_1^*$ is not a multiple of $\kappa$, which is generically the case.

$H$-type depositor chooses $W_K$, then the intermediary transfers all $\kappa$ units of the asset that were offered. It follows that when the intermediary selects $\kappa$ units of the asset to offer a depositor (if the intermediary has a positive measure of the asset), then the intermediary offers an unviewed set.

Now, consider an arbitrary protocol and suppose depositors still choose the strategy profile described above. Suppose, for a particular depositor, the probability that the $\kappa$ units of the asset offered are high quality is $\tilde{q}$. Given the depositors' strategy profile, if the depositor chooses $W_G$, then, using Bayes' rule, the probability that the units of the asset rejected are high quality is

$$\frac{\tilde{q}\lambda}{\tilde{q}\lambda + (1-\tilde{q})\,\lambda + (1-\tilde{q})\,(1-\lambda)} = \frac{\lambda}{\lambda + (1-\tilde{q})\,(1-\lambda)}\tilde{q},$$

which is strictly less than $\tilde{q}$ (as long as $\tilde{q} < 1$), the probability that the units of the asset offered were high quality. It follows that, for any rejected set, the intermediary believes that the units of the asset in that set are high quality with probability strictly less than $q$. As such, under any protocol, since $\kappa < c_1^*$,

$$\tilde{B}(1;\mathbf{a}) \leq 1 - \lambda c_1^* - (1-\lambda)((1-q)c_1^* + q\kappa),$$

which is the value of $\tilde{B}(1;\mathbf{a})$ under the standard protocol. Hence, if $\tilde{B}(1;\mathbf{a}) < 0$ under the standard protocol, then $\tilde{B}(1;\mathbf{a}) < 0$ under any protocol. □

We now prove Theorem 3.

Proof of Theorem 3. Consider the case in which depositors are sophisticated. First, we show that a sufficient condition for there to exist a protocol such that $\Gamma_K$ can strongly implement the FB is that $\eta < c_2^*/c_1^*$. We do this by showing that $\eta < c_2^*/c_1^*$ is a sufficient condition under the standard protocol described above. The argument is similar to the one used in the proof of Theorem 1. Since $\eta < c_2^*/c_1^*$, we know that $c_2^*/R_H > c_1^*/R_L$. As such, if $\kappa = c_2^*/R_H$, then, since $R_L < R_H$, we have that $\kappa > c_1^*/R_L$ and $\kappa > c_1^*/R_H$. It follows that all patient depositors strictly prefer to choose $W_K$ over $W_G$. Moreover, since $R_H > \bar{R}$ and $c_1^* > c_2^*/\bar{R}$, it follows that $\kappa < c_1^*$ and choosing $W_G$ is a strictly dominant strategy for all impatient depositors. Hence, we can restrict our attention to strategy profiles in which impatient depositors choose $W_G$ and patient depositors mix between $W_K$ and $W_0$. Also, since $\kappa < c_1^*$, if a depositor chooses $W_G$, then all the $\kappa$ units of the asset that had been offered to the depositor are liquidated – and no rejected set is created. Then, at place $s$, the measure of units of the asset in unviewed sets is $1 - \lambda s c_1^* - (1-\lambda)s c_2^*/R_H$, which is strictly greater than 0 since $R_H > \bar{R}$ and $\lambda c_1^* + (1-\lambda)c_2^*/\bar{R} = 1$. It follows that the intermediary always selects the $\kappa$ units of the asset to offer to a depositor from the unviewed sets. As such, a fraction $q$ of depositors are H-types and a fraction $1-q$ are L-types. Suppose that patient L-types choose $W_K$ with probability $\alpha$ and patient H-types choose $W_K$ with probability $\beta$. Define $B_L(1;\mathbf{a})$ and $B_H(1;\mathbf{a})$ to be the measures of low and high quality units of the asset, respectively, left at the end of period 1 in both unviewed and rejected sets. Then,

$$B_L(1;\mathbf{a}) = 1-q-\lambda\left((1-q)\kappa + (1-q)\left(c_1^*-\kappa\right)\right) - (1-\lambda)(1-q)\alpha\kappa$$

and

$$B_H(1;\mathbf{a}) = q-\lambda\left(q\kappa + q\left(c_1^*-\kappa\right)\right) - (1-\lambda)q\beta\kappa,$$

where the terms for impatient depositors capture that the first units liquidated are the $\kappa$ units offered and the remaining $c_1^*-\kappa$ units liquidated are randomly chosen from unviewed sets. The expressions for $B_L(1;\mathbf{a})$ and $B_H(1;\mathbf{a})$ are strictly greater than 0 since $\kappa = c_2^*/R_H$, $R_H > \bar{R}$, and $\lambda c_1^* + (1-\lambda)c_2^*/\bar{R} = 1$. Therefore, the expressions are indeed the correct expressions for $B_L(1;\mathbf{a})$ and $B_H(1;\mathbf{a})$. A depositor who chooses $W_0$ receives

$$C \equiv \frac{R_L B_L(1;\mathbf{a}) + R_H B_H(1;\mathbf{a})}{1-\lambda-(1-\lambda)((1-q)\alpha+q\beta)}$$

units of the good in period 2. Since $C$ is strictly increasing in $\alpha$, weakly increasing in $\beta$, and equal to $c_2^*$ if $\alpha=\beta=0$, it follows that, for all $\alpha$ and $\beta$, patient L-type depositors strictly prefer to choose $W_0$ over $W_K$. Hence, in any BNE, $\alpha=0$. Then, for any $\beta$, $C=c_2^*$ and patient H-type depositors are indifferent between $W_0$ and $W_K$. Therefore, there are a continuum of BNE in all of which impatient depositors choose $W_G$, patient L-type depositors choose $W_0$, and patient H-type depositors mix between $W_K$ and $W_0$. In all BNE, the FB is achieved. If $\kappa \in (c_1^*/R_L, c_2^*/R_H)$, then the same argument shows that there is an unique BNE. In this BNE, impatient depositors choose $W_G$, patient depositors choose $W_0$, and the equilibrium allocation is the FB.

For the remainder of the proof, suppose $\eta \geq c_2^*/c_1^*$. We now find a $\bar{q}$ such that there exists a protocol under which $\Gamma_K$ can strongly implement the FB if and only if $q > \bar{q}$. We will proceeds as follows. We first develop a series of necessary conditions for there to exist a protocol under which $\Gamma_K$ can strongly implement the FB. Next, we show that, for any protocol, those necessary conditions cannot be satisfied if $q \leq \bar{q}$. Finally, we show that when $q > \bar{q}$, under the standard protocol, $\Gamma_K$ can strongly implement the FB.

First, a necessary condition for $\Gamma_K$ to strongly implement the FB is that $\kappa \in (c_1^*/R_H, c_2^*/R_H]$. If $\kappa \leq c_1^*/R_H$, then, since $R_L < R_H$, we have that both H-type and L-type patient depositors weakly prefer to choose $W_G$ over $W_K$. It follows that, for any protocol, the classic suboptimal run BNE in which all depositors choose $W_G$ exists. If $\kappa > c_2^*/R_H$, then, regardless of the protocol, patient H-type depositors can always achieve strictly more utility than they receive in the FB by choosing $W_K$ and consuming $R_H\kappa > c_2^*$ units of consumption in the final period. Thus, the FB is not even weakly implemented.

It now follows from the Lemma that a further necessary condition for $\Gamma_K$, under any protocol, to strongly implement the FB is that, under the standard protocol, if impatient and patient $L$-type depositors choose $W_G$ and patient $H$-type depositors choose $W_K$, then $\tilde{B}(1;\mathbf{a}) \geq 0$. We find a $\bar{q}$ such that there exists a $\kappa \in (c_1^*/R_H, c_2^*/R_H]$ for which this holds if and only if $q > \bar{q}$. Suppose $\kappa \in (c_1^*/R_H, c_2^*/R_H]$, depositors choose the specified strategy profile, and the intermediary uses the standard protocol. It follows from the definition of the standard protocol that the intermediary only offers depositors units of the asset that have not been offered to previous depositors and, when an impatient or patient L-type depositor chooses $W_G$, all $\kappa$ units of the asset that were offered are liquidated. As such,

$$\tilde{B}(1;\mathbf{a}) = 1 - (\lambda + (1-\lambda)(1-q))c_1^* - (1-\lambda)q\kappa,$$

which is strictly decreasing in $\kappa$. If $\kappa = c_1^*/R_H$, then using $R_H = \left(\bar{R}\eta\right)/(q\eta + 1 - q)$ yields

$$\tilde{B}(1;\mathbf{a}) = 1 - c_1^* + (1-\lambda)c_1^* q - (1-\lambda)c_1^* \frac{q + (\eta-1)q^2}{\bar{R}\eta},$$

which is strictly concave in $q$, is negative when $q = 0$, and is positive when $q = 1$. Hence, there exists a $\bar{q} \in (0,1)$ such that if $\kappa = c_1^*/R_H$, then $\tilde{B}(1;\mathbf{a}) = 0$ when $q = \bar{q}$ and $\tilde{B}(1;\mathbf{a}) > 0$ when $q \in (\bar{q}, 1]$. Moreover, $\bar{q}$ is the lesser of the two real solutions to the quadratic equation

$$\frac{(1-\lambda)(\eta-1)c_1^*}{\bar{R}\eta} q^2 - \frac{(1-\lambda)\left(\bar{R}\eta - 1\right)c_1^*}{\bar{R}\eta} + c_1^* - 1 = 0,$$

i.e.

$$\bar{q} = \frac{\left(\bar{R}\eta - 1\right) - \sqrt{\left(\bar{R}\eta - 1\right)^2 - 4\frac{(\eta-1)(c_1^*-1)\bar{R}\eta}{(1-\lambda)c_1^*}}}{2(\eta-1)}.$$

It follows that if $q \leq \bar{q}$, then, for any $\kappa > c_1^*/R_H$, we have that $\tilde{B}(1;\mathbf{a}) < 0$. On the other hand, it $q > \bar{q}$, then there exists a $\kappa \in (c_1^*/R_H, c_2^*/R_H]$ such that $\tilde{B}(1;\mathbf{a}) > 0$.

Finally, we show that if $q > \bar{q}$, then, using the standard protocol, $\Gamma_K$ can strongly implement the FB. Suppose the intermediary uses the standard protocol and $q > \bar{q}$. The argument in the previous paragraph shows that we can choose a $\kappa \in (c_1^*/R_H, c_2^*/R_H]$ such that if impatient and patient L-type depositors choose $W_G$ and patient H-type depositors choose $W_K$, then $\tilde{B}(1,\mathbf{a}) > 0$. If $\kappa = c_2^*/R_H$ and $q = 1$, then if impatient and patient L-type depositors choose $W_G$ and patient H-type depositors choose $W_K$, then

$$\tilde{B}(1;\mathbf{a}) = 1 - \lambda c_1^* - (1-\lambda)\frac{c_2^*}{R_H},$$

which is strictly less than 0 since $c_2^* = \bar{R}(1 - \lambda c_1^*)/(1 - \lambda)$ and $R_H > \bar{R}$. It follows that the desired $\kappa$ is strictly less than $c_2^*/R_H$. Then, since $\eta \geq c_2^*/c_1^*$, we know that $\kappa < c_1^*/R_L$, which implies that patient L-type depositors strictly prefer to choose $W_G$ over $W_K$. Hence, recalling that $\kappa \in (c_1^*/R_H, c_2^*/R_H]$ implies that impatient depositors strictly prefer to choose $W_G$ over $W_K$ or $W_0$ and that patient H-type depositors strictly prefer to choose $W_K$ over $W_G$, we can, without loss of generality, restrict our attention to strategy profiles in which impatient depositors choose $W_G$, patient L-type depositors mix between $W_G$ and $W_0$, choosing the former with probability $\alpha$, and patient H-type depositors mix between $W_K$ and $W_0$, choosing the former with probability $\beta$. Moreover, since $B(1; \mathbf{a}) > 0$ when $\alpha$ and $\beta$ are equal to 1, we know that, for any $\alpha$ and $\beta$, the intermediary always has a positive measure of the asset that has not been offered to previous depositors. It follows that the intermediary only offers depositors units of the asset that have not been offered to previous depositors and, when an impatient or patient L-type depositor chooses $W_G$, all $\kappa$ units of the asset that were offered are liquidated.

We must know both the total measure of the asset remaining with the intermediary at the end of period 1 as well as the fraction that is high quality to compute the units of the good received in period 2 by a depositor who chose $W_0$. If $\alpha \neq \beta$, then the fraction of the asset remaining with the intermediary that is high quality at the end of period 1 will not be equal to $q$. With this in mind, we define

$$B_L(\alpha, \beta) = 1 - q - \lambda(1 - q)c_1^* - (1 - \lambda)(1 - q)\alpha(\kappa + (1 - q)(c_1^* - \kappa))$$

and

$$B_H(\alpha, \beta) = q - \lambda q c_1^* - (1 - \lambda)(1 - q)\alpha q(c_1^* - \kappa) - (1 - \lambda)q\beta\kappa$$

as the measures of the low and high quality asset, respectively, that are left with the intermediary at the end of period 1. Each is decreasing in $\alpha$ and $\beta$ and if $\alpha = \beta$, then $B_H(\alpha, \beta)/B_L(\alpha, \beta) = q/(1 - q)$, the initial ratio of the measures of the high and low quality asset. We know, given $q > \bar{q}$ and from our choice of $\kappa$, that $B_L(1, 1) + B_H(1, 1) > 0$. Hence, both $B_L(1, 1)$ and $B_H(1, 1)$ must be strictly positive, which implies that for all $\alpha, \beta \in [0, 1]$, $B_L(\alpha, \beta)$ and $B_H(\alpha, \beta)$ are strictly positive. Then, the units of consumption received in period 2 by a depositor who chose $W_0$ is

$$C(\alpha, \beta) = \frac{R_L B_L(\alpha, \beta) + R_H B_H(\alpha, \beta)}{(1 - \lambda)(1 - \alpha(1 - q) - \beta q)}.$$

We show that for all $\alpha, \beta \in [0, 1]$, $C(\alpha, \beta) > R_H\kappa > c_1^*$, which implies that choosing $W_0$ is a strictly dominant strategy for both patient L-type and patient H-type depositors. It then follows that the unique BNE consists of impatient depositors choosing $W_G$ and patient depositors choosing $W_0$, which achieves the FB allocation. The partial derivative of $C(\alpha, \beta)$

with respect to $\alpha$ is

$$\frac{-R_L(1-\lambda)(1-q)(q\kappa+(1-q)c_1^*)-R_H(1-\lambda)(1-q)q(c_1^*-\kappa)+(1-\lambda)(1-q)C(\alpha,\beta)}{(1-\lambda)(1-\alpha(1-q)-\beta q)},$$

which is positive if and only if the numerator is positive. Multiplying the numerator by the denominator of $C(\alpha, \beta)$ and dividing by $(1-\lambda)(1-q)$ yields

$$-[R_L(q\kappa+(1-q)c_1^*)+R_H q(c_1^*-\kappa)](1-\lambda)(1-\alpha(1-q)-\beta q)+R_L B_L(\alpha,\beta)+R_H B_H(\alpha,\beta),$$

which again is positive if and only if $\partial C(\alpha, \beta)/\partial\alpha$ is positive. Ignoring the terms not dependent on $\alpha$ in the above expression leaves

$$[R_L(q\kappa+(1-q)c_1^*)+R_H q(c_1^*-\kappa)](1-\lambda)(1-q)\alpha \\ -[R_L(1-\lambda)(1-q)(q\kappa+(1-q)c_1^*)+R_H(1-\lambda)(1-q)q(c_1^*-\kappa)]\alpha,$$

which is equal to 0. It follows that whether $\partial C(\alpha, \beta)/\partial\alpha$ is positive is not dependent on $\alpha$, which implies that, given $\beta$, $C(\alpha, \beta)$ is monotone in $\alpha$. Hence, given $\beta$, $C(\alpha, \beta) > R_H\kappa$ if $C(0, \beta) > R_H\kappa$ and $C(1, \beta) > R_H\kappa$. Next, the partial derivative of $C(\alpha, \beta)$ with respect to $\beta$ is

$$\frac{\partial C(\alpha,\beta)}{\partial\beta}=\frac{-R_H(1-\lambda)q\kappa+(1-\lambda)qC(\alpha,\beta)}{(1-\lambda)(1-\alpha(1-q)-\beta q)},$$

which is strictly positive if and only if $C(\alpha, \beta) > R_H\kappa$. As such, given $\alpha$, if there exists a $\hat{\beta} \in [0, 1]$ such that $C\left(\alpha, \hat{\beta}\right) > R_H\kappa$, then for all $\beta \in [0, 1]$, $C(\alpha, \beta) > R_H\kappa$. We know that $C(0, 0) = c_2^* > R_H\kappa$ and we know that the limit of $C(1, \beta)$ as $\beta$ goes to 1 is $+\infty$, so we can see that, for all $\beta \in [0, 1]$, $C(0, \beta)$ and $C(1, \beta)$ are both strictly greater than $R_H\kappa$. It follows that for all $\alpha, \beta \in [0, 1]$, $C(\alpha, \beta) > R_H\kappa$, completing the proof.

Intuitively, given that an asset transfer that appeals to both L-type and H-type patient depositors is unavailable, the possibility of a run equilibrium in which L-type patient depositors choose $W_G$ and H-type patient depositors choose $W_K$ emerges. In this equilibrium, L-type patient depositors are depleting the intermediary's holdings of the asset more than in the FB equilibrium. As such, the best protocol for preventing this run equilibrium is one that minimizes the measure of L-type patient depositors, which is one that liquidates as much of the low quality asset as possible by always liquidating the units of the asset offered to a depositor when that depositor chooses $W_G$. This is the only protocol that the intermediary must ever use to strongly implement the FB when strong implementation is possible. □