

Inequality and Financial Fragility*

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Abstract. I study how the distribution of wealth influences the government's response to a banking crisis and the fragility of the financial system. Distributional concerns tend to make government guarantees of deposits in a systemic crisis credible for relatively poor agents, but such guarantees may not be credible for wealthier agents. As a result, wealthier agents will have a stronger incentive to panic and, in equilibrium, the institutions in which they invest will be endogenously more likely to experience a run and receive a bailout. Thus, even under a utilitarian policy maker, bailouts tend to benefit the wealthy at the expense of the general public. Moreover, the shape of the wealth distribution affects the level of fragility in the financial system. The recognition of this fact may alter the government's desire to redistribute wealth ex ante.

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1 Introduction

In most of the theoretical literature on banking panics, both deposit insurance and panics are all-or-nothing affairs, in the sense that all deposits are treated equally and a panic affects all banks the same way. However, the financial crises observed in reality do not fit such a simple description. First, panics are often restricted to certain types of institutions or arrangements (i.e. money market mutual funds in the United States in 2008) while others remain effectively insured by the government (commercial banks in the United States in 2008). Second, even within a single institution, some agents may be forced to accept a haircut, whereas others remain protected in full by the government (as in Cyprus in 2008). Third, the written rules of a deposit insurance program might be abandoned in a systemic financial collapse, so that a banking crisis transforms government guarantees from a legal to a political commitment (as in Iceland in 2008 and others).¹

I study a model of financial intermediation in which the government's response to a financial crisis is determined, in part, by distributional concerns. In this setting, agents anticipate differential treatment depending on their wealth level and, in equilibrium, panics are endogenously restricted to certain types of institutions. My model builds on the classic work of Diamond and Dybvig (1983), extended to including heterogeneous endowments (i.e. wealth levels) across agents. In addition, there is a policy maker charged with collecting taxes and with providing a public good. If there is a financial crisis, the policy maker can divert a fraction of the tax revenue in order to bail out banks experiencing runs. As in Keister (2016), the policy maker cannot pre-commit to the details of the bailout package, but chooses bailouts ex post.² Hence, the agents and their

¹A case in point is the Icesave dispute, taking place after the collapse in 2008 of the Icelandic bank Landsbanki. The Court of Justice of the European Free Trade Association States (EFTA) ruled in 2013 that the Icelandic government was not under legal obligation to adhere to its original promise to insure Dutch and UK deposit holders, since doing so would have undermined the stability of the Icelandic banking system.

²Government guarantees here are broadly interpreted to include all forms of fiscal transfers (bailouts) to banks. Ex-post bailouts has been analyzed by, among others, Farhi and Tirole (2012), Keister (2016), Bianchi (2016), Nosal and Ordonez (2016), and Chari and Kehoe (2016). The goal of this assumption is to capture the renegotiation of government guarantees that appears to play a major role in times of systemic banking crises. .

financial intermediaries anticipate that any bailouts will be provided only to the level which is ex-post optimal for the policy maker.

I begin by showing that if bailouts are prohibited, this model does not lead to any novel implications nor give predictions that are consistent with the recent episodes of financial turmoil. If bailouts are allowed, however, a utilitarian policy maker will choose ex post to tilt the bailout package disproportionately towards poorer agents and their financial institutions. As a result, government guarantees of deposits will tend to be credible for relatively poor agents, but may not be credible for the wealthy. The anticipation of this response, in turn, influences agents' behavior and thus determines the form of financial fragility observed in equilibrium. Specifically, wealthier agents will have a stronger incentive to panic and, in equilibrium, the institutions in which they invest are endogenously more likely to experience a run and be bailed out by the government. The resulting pattern, in which small retail depositors do not panic but large agents do and their institutions are endogenously more likely to end up being bailed out, matches well with observations from recent financial crises. Moreover, bailouts, when they occur, will tend to benefit the wealthy and their institutions while, by depressing the level of the public good, imposing a cost on everyone else.

The incentive for an agent to run in this model depends on the size of the bailout his institution would receive in a crisis, which in turn depends on the entire distribution of wealth. In particular, given any initial distribution of wealth, there exists an endogenous cutoff point such that an agent may have an incentive to run only if his wealth level (after any redistribution ex ante) remains above the cutoff. Moving to a new distribution of wealth not only changes the wealth level of some agents, but also changes this cutoff point. I say that such a move increases financial fragility if it increases the measure of agents in this fragile region. I show, both analytically and through numerical examples, how rising inequality can increase financial fragility in some cases and decrease it in others.

This link between inequality and financial fragility changes the desirability of ex-ante redistribution. I show that taking the link into account enhances the government's desire

to redistribute wealth in some cases, but diminishes it in others. In particular, if more redistribution has the effect of increasing the measure of agents in the fragile region of the wealth distribution, the policy maker may choose to scale down the amount of ex-ante redistribution in order to avoid creating excessive fragility. In fact, the link with fragility can prevent a utilitarian government from fully redistributing wealth even if there are zero efficiency costs associated with redistribution or progressive taxation.

I conclude the analysis by relating the banking arrangements underlying the baseline model to real world financial institutions and regulatory practices. In particular, the banking arrangements in the model can be interpreted as being composed of two sectors – a “commercial” banking sector where all agents are insured and therefore do not run and a “shadow” banking sector where the run risk (and hence bailouts) ends up being concentrated.

The closest paper in the existing literature is Cooper and Kempf (2016), which studies the decision of the government ex post to provide deposit insurance in a version of the Diamond and Dybvig with heterogeneous endowments. They restrict the government to a binary choice of either providing deposit insurance to everyone or abstain from providing deposit insurance altogether. I allow for much more flexible bailout interventions in which the government may choose to impose different haircuts on different agents, as was recently the case in some countries (Iceland and Cyprus). Cooper and Kempf also restrict their analysis to equilibria where runs are zero probability events and, therefore, do not affect ex-ante behavior. The centerpiece of my analysis, in contrast, is the fact that the agents and their financial intermediaries anticipate that a bank run might occur with positive probability and adjust their behavior in response (as in Cooper and Ross, 1998, Peck and Shell, 2003, and others).

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 formulates the strategies, derives allocations, and defines equilibria. Section 4 contains the main results of the analysis. Section 5 applies the model to study the effect of ex-ante redistribution on financial fragility. Section 6 relates the banking arrangements in the model to those observed in reality. The last section concludes.

2 The model

The model is based on that in Keister (2016), which is a version of the Diamond and Dybvig (1983) model augmented to include fiscal policy and a public good.

2.1 The environment

There are three time periods $t = 0, 1, 2$ and a set of agents. The *wealth level* of a given agent is his endowment of private goods and is denoted e . The *c.d.f.* and the *p.d.f.* of the distribution of wealth in the population of agents are denoted $H(e)$ and $h(e)$. The support of the wealth distribution is $[\underline{e}, \bar{e}]$ and mean wealth is normalized to 1. The agents derive utility from the consumption of a private good and the consumption of a public good. In particular, the preferences for agent i with wealth level e are given by

$$u(c_1 + \omega_i^e c_2) + v(g), \quad (1)$$

where c_t denotes his consumption in period t and g denotes the level of the public good, which is provided in period 1. For each wealth level e , the index i runs from 0 to the density of wealth e agents in the population, $h(e)$. The object ω_i^e is a binomial random variable with support $\Omega \equiv \{0, 1\}$. If $\omega_i^e = 0$, this particular agent is *impatient* and values consumption only in period 1. On the other hand, if $\omega_i^e = 1$, this particular agent is *patient* and values consumption equally in periods 1 and 2. Agent i with wealth e learns the realization of ω_i^e privately at the start of period 1. All agents have the same probability of being impatient denoted π . I assume that a law of large numbers holds according to which the mass of impatient agents with wealth level e will be equal to $\pi h(e)$, and therefore, the total mass of impatient agents will be equal to π . I also assume that the functions u and v are of the constant relative risk aversion form, with

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad \text{and} \quad v(g) = \delta \frac{g^{1-\gamma}}{1-\gamma}, \quad (2)$$

where the parameter $\delta > 0$ measures the relative importance of the public good. As in

Diamond and Dybvig (1983) the coefficient of relative risk-aversion γ is assumed to be greater than 1.

There is a single *constant-returns-to-scale* technology, operated at a central location, for transforming the endowments into private consumption in later periods. A unit of good invested in period 0 yields $R > 1$ units in period 2, but only 1 unit in period 1. There is also a linear technology for transforming units of the private good into units of the public good in period 1. Without loss of generality this rate of transformation is assumed to be one-to-one.

In period 0, the agents can pool their endowments into the constant-return-to-scale technology in order to insure against idiosyncratic liquidity risk. Agents are isolated from each other in periods 1 and 2 and no trade can occur between them. As in Ennis and Keister (2010), I assume that agents' order in the withdrawal opportunities is given by their index. Specifically, an agent with wealth e and index i has an opportunity to withdraw before another agent with the same wealth level and index i^* if and only if $i < i^*$. When an agent opportunity to withdraw arrives, he can either visit the central location or wait until the final period to withdraw. Those who decide to withdraw in period 1 must consume immediately what is given to them and return to isolation.³

Wallace (1988, 1990) shows that this environment leads to a *sequential-service* constraint where each payment can depend only on the information available to the intermediation technology at the time of the withdrawal. For each agent, the intermediation technology will observe his wealth level and his choice of whether to withdraw in period 1 or period 2. At the same time, the intermediation technology *does not* observe this agent's consumption preference type ω_i^e or his index i and, therefore, payments cannot depend on this information. Further, I assume that the fraction of wealth e agents who contact the central location during the first π withdrawals is equal to the fraction of wealth e agents in the population. This assumption ensures that the intermediation

³The assumption that agents know their ordering when making withdrawal decisions is for tractability and is based on Green and Lin (2000), Andolfatto *et al.* (2007), and on Ennis and Keister (2010). Similar results obtain under the alternative assumption whereby the agents do not know their ordering as in Diamond and Dybvig (1983), Peck and Shell (2003) and others.

technology cannot make inferences based on the composition of withdrawals.⁴

2.2 The decentralized economy

In the decentralized economy, in addition to agents, there is a set of banks and a single policy maker. I begin with the description of the banks.

2.2.1 Banks

The intermediation technology is operated by a continuum of banks. In each bank, the fraction of impatient agents will be equal to π . Also, each bank is small and will have no effect on economy-wide outcomes. All banks operate to maximize the expected utilities of their agents at all times. There are no restrictions on the payments a bank is allowed to give other than those imposed by the information structure and the sequential-service constraint. Specifically, banks will adjust the payments they give to their remaining agents whenever new information becomes available. The information available to the banks is the same as the information available to the intermediation technology. Furthermore, as in Ennis and Keister (2010), I assume that banks cannot pre-commit to future payments. Instead the payment given to each agent will be made as a best response to the situation the bank is facing at the time of the withdrawal.⁵

2.2.2 Financial crisis

I follow Cooper and Ross (1998) and others in introducing the possibility of bank runs through a *sunspot state*.⁶ The sunspot state can take on two values, α and β , with respective probabilities $(1 - q, q)$ and is realized at the start of period 1. The state of

⁴For example, those choosing to withdraw in period 1 might be assigned to separate withdrawal lines based on their wealth level. The agents in line e are serviced with speed *proportional* to the fraction of wealth e agents in the population. This corresponds to the “no clock” assumption in Diamond and Dybvig models according to which the speed with which agents arrive to withdraw is not informative about the mass of additional withdrawals that is likely to occur in the future.

⁵As in Ennis and Keister (2009, 2010) this lack of commitment assumption implies that suspension of payments plans as in Diamond and Dybvig (1983) or run-proof contracts as in Cooper and Ross (1998) fails to eliminate runs in this environment since banks will not use them *ex post*. Similarly, priority of claims provisions as analyzed by de Nicolo (1996) will also not work in preventing runs.

⁶I restrict attention solely to runs based on self-fulfilling expectations since doing so allows me to highlight the main implications of the model in a clear and concise way.

the economy in period 1 is thus given by $s \in S = \{\alpha, \beta\}$. The sunspot state has no effect on preferences or technologies, but may serve to coordinate agents' expectations in equilibrium. The agents observe the realization of the sunspot state at the beginning of period 1 before withdrawals begin. Banks do not observe the realization of the sunspot state but instead must infer it based on the flow of withdrawals. A fraction π of the agents in each bank will be impatient and will choose to withdraw in period 1 in both states. At the same time, a given bank will experience a run if and only if a positive mass of its patient agents also chooses to withdraw in period 1. Thus, if withdrawals continue beyond the first π , the bank infers that a run must be underway. In this case the bank can (and will) react to this surge of period 1 withdrawals in choosing the payments it gives to its remaining agents.

2.2.3 Policy maker

There is a single policy maker who is both benevolent and utilitarian. This policy maker can tax the endowments in period 0 and use the tax revenue to provide the public good in period 1. Taxes are set in period 0, before the sunspot state is realized, and therefore taxation cannot be contingent on the sunspot state. Furthermore, taxation is not possible after the initial period.⁷ In the baseline model, I restrict the policy maker to impose the same tax flat rate of $\tau \in [0, 1]$ on all agents. I investigate ex-ante redistribution in Section 5.

If a given bank is experiencing a run, the policy maker can use tax revenue to provide a fiscal transfer (*bailout*) to the bank. The bailout would augment the bank's resources and therefore raise the consumption levels for its remaining agents. At the same time, the opportunity cost of a bailout is that the level of the public good will be lower since

⁷As shown by Wallace (1988), if the policy maker can collect taxes after agents have withdrawn from the bank, it can effectively circumvent the sequential service constraint. It is therefore important that there be some restrictions on the policy maker's ability to tax agents in period 1. Having all taxes collected in period 0 is one way of ensuring that fiscal policy is consistent with the assumptions about sequential service. Boyd et al. (2002), Martin (2006), and Keister (2016) have all introduced taxation in the Diamond-Dybvig model in similar ways. Another approach will be to follow Keister and Narasiman (2016) where taxes are collected in period 1 through a levy on bank deposits. In their setting, a "bailout" is identified as an event where a lower tax is collected from some bank(s). The second approach yields implications for fragility which are largely similar to having all taxes collected in period 0.

some of the tax revenue has been diverted to bail out banks. The policy maker is allowed to provide any bailout payments to banks subject to two restrictions. *First*, a bailout can be given only to a bank that is experiencing a run. *Second*, the bailout to any given bank cannot be so large so as to provide “super insurance”. That is, the agents in a bank that is bailed out cannot receive more consumption than they would have received had there been no run on their bank. These restrictions are necessary to ensure that the policy maker is not using bailouts for pure redistribution, independent of its response to a financial crisis. Finally, as in Keister (2016), the policy maker cannot pre-commit to the bailout policy and will instead choose bailouts ex post, as a best response to the situation at hand, and subject to the restrictions outlined above.

2.3 Timeline

The sequence of events is depicted in Figure 1. In period 0, the policy maker sets the tax rate τ and agents deposit their after-tax endowment with a bank. At the start of period 1, each agent observes his consumption type (impatient or patient) and the realization of the sunspot state. Withdrawals then begin. The banks and the policy maker do not observe the state and must infer it based on withdrawals. A fraction π of the agents in each bank will be impatient and will choose to withdraw in period 1 regardless of the state. As a result, payments during the initial π withdrawals cannot be made contingent on the state. If withdrawals within a bank stop once they reach π , then this bank is not experiencing a run and there will be only impatient agents left within the bank. On the other hand, if withdrawals within a bank continue beyond the first π , then this bank is experiencing a run. As a result, a fraction of the remaining agents within the bank will be impatient and the bank can potentially receive a bailout from the policy maker. After receiving a bailout (if any), each bank makes payments to those agents still arriving to withdraw in period 1. After all bailouts have been made, the remaining tax revenue will be used to provide the public good. In period 2 each bank divides the matured value of its remaining resources evenly among its remaining agents.

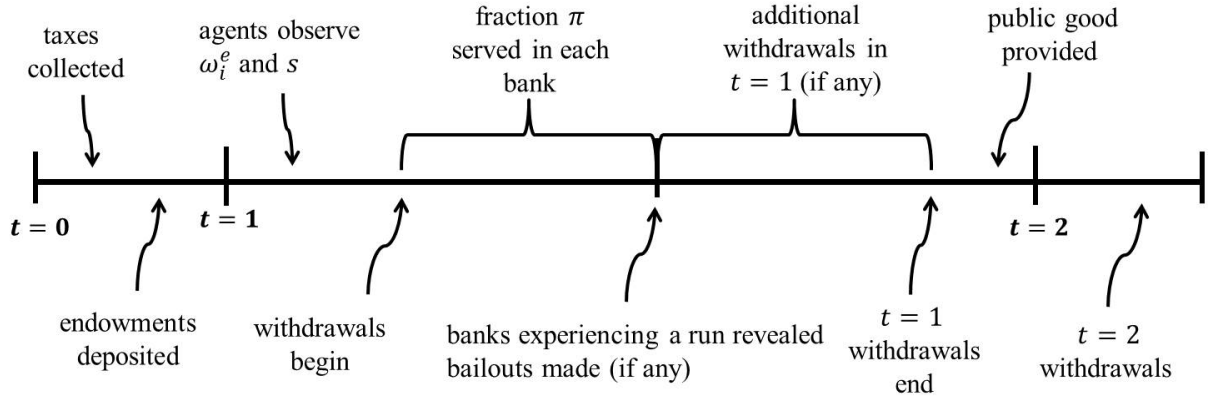


Figure 1: Timeline.

3 Strategies, best responses, and equilibrium

In the baseline version of the model, I restrict attention to financial arrangements such that all agents within any given bank have equal wealth levels and hold a deposit of equal amount. That is, all agents in bank e have wealth level e and have deposited their entire after-tax wealth. This assumption may appear to be restrictive at first, not least because this is not the way most financial institutions operate in reality. Nonetheless, once we derive the equilibrium outcomes under this special financial arrangement, it will become clear that the main results hold under more general financial arrangements where agents with different wealth levels can be part of the same bank. The results from this extension of the model will be presented in Section 6. Going through the analysis first under the baseline scenario where agents are separated by wealth levels simplifies the presentation considerably and allows me to highlight the key mechanisms in the clearest possible way.

3.1 Strategies

A withdrawal strategy for an agent i with wealth e is a mapping from his consumption preference type $\Omega = \{0, 1\}$ and the realization of the sunspot state $S = \{\alpha, \beta\}$ to a choice of whether to withdraw in period 1 or period 2. That is,

$$y_i^e : \Omega \times S \rightarrow \{0, 1\}, \quad (3)$$

where $y_i^e = 0$ corresponds to withdrawing in period 1 and $y_i^e = 1$ corresponds to withdrawing in period 2. Let y^e be the withdrawal profile for all wealth e agents and let $y = \{y^e\}$ be the withdrawal profile for all agents in all banks. I will focus on symmetric equilibria in which all agents within the same bank follow the same strategy and on equilibria where all patient agents in each bank choose to withdraw in period 2 when the state is α .

In principle, the strategy of a bank specifies how much it will pay each agent at the time of the withdrawal. Thus, under sequential service, the payments in bank e can be summarized by the function $x^e : [0, 1] \rightarrow \mathbb{R}_+$, where $x^e(\mu)$ is the payment given to the μ th agent who arrives to withdraw from bank e in period 1. We can, however, use the structure of the model to simplify the strategy space of a bank as follows. First, note that bank e anticipates that a fraction π of its agents will be impatient and will choose to withdraw in period 1 in each state. As these withdrawals take place, the bank is unable to make any inference about the sunspot state. Since agents are risk averse, the bank will choose to give the same payment to each of the first π agents to withdraw. Denote this payment as c_1^e . I show below that a bank's choice of c_1^e can be used to determine the payments it will make to the remaining fraction $1 - \pi$ of its agents as well. We can, therefore, express the strategy of bank e as the choice of a single number c_1^e . Let c_1 be the specification of the early payments chosen by all banks.

The policy maker chooses a bailout payment b^e (in per-agent terms) for each bank e . Let b denote the specification of bailout payments to all banks. A strategy for the policy maker is then a choice of a bailout specification b .

3.2 Allocations and welfare

For given tax rate τ and a triple (c_1^e, b^e, y^e) , we can compute the entire consumption allocation for the agents in bank e . If the state is α , only impatient agents will choose to withdraw in period 1. As a result, withdrawals stop after the first π , there will be no bailouts, and the payment given by the bank to each of its patient agents in period 2 will be

$$c_{2\alpha}^e = \frac{R((1-\tau)e - \pi c_1^e)}{1-\pi}. \quad (4)$$

In other words, the bank's remaining $1 - \pi$ agents evenly share the matured value of the initial deposits minus the payments made to the first π agents. On the other hand, if the state is β , we can use the strategy profile of the agents in bank e to compute the additional mass of period-1 withdrawals (i.e. after the first π) that will take place in any bank e , denoted $\pi_\beta^e \in [0, 1 - \pi]$. Thus, if there is no run on bank e we have $\pi_\beta^e = 0$. On the other hand, if there is a partial (or full) run on bank e when the state is β we have $\pi_\beta^e \in (0, 1 - \pi]$. The remaining (per-agent) quantity resources in bank e , including a bailout payment of $b^e \geq 0$, will be equal to

$$\psi_\beta^e \equiv (1 - \tau)e - \pi c_1^e + b^e. \quad (5)$$

The bank will choose to give a common payment of $c_{1\beta}^e$ to each of the additional π_β^e agents arriving to withdraw in period 1 and a common payment of $c_{2\beta}^e$ to all agents withdrawing in period 2. These payments will be chosen to maximize the sum of expected utilities for the remaining agents in bank e . That is,

$$\max_{\{c_{1\beta}^e, c_{2\beta}^e\}} \{\pi_\beta^e u(c_{1\beta}^e) + (1 - \pi - \pi_\beta^e)u(c_{2\beta}^e)\}, \quad (6)$$

subject to the resource constraint

$$\pi_\beta^e c_{1\beta}^e + (1 - \pi - \pi_\beta^e) \frac{c_{2\beta}^e}{R} \leq \psi_\beta^e. \quad (7)$$

The solution of the program in (6) is characterized by the following first order condition

$$u'(c_{1\beta}^e) = R u'(c_{2\beta}^e). \quad (8)$$

From (7) and (8), it follows that the consumption levels for the remaining agents in the bank, $c_{1\beta}^e$ and $c_{2\beta}^e$, are increasing functions of ψ_β^e and decreasing functions of π_β^e . The consumption allocation for the agents in bank e is thus summarized in the following

vector,

$$(c_1^e, c_{2\alpha}^e, c_{1\beta}^e, c_{2\beta}^e), \quad (9)$$

where $c_{2\alpha}^e$ is characterized in (4), and $c_{1\beta}^e, c_{2\beta}^e$ are characterized as the solution to (7) - (8). The level of the public good in each state is given by

$$g_\alpha = \tau \quad \text{and} \quad g_\beta = \tau - \int_e^{\bar{e}} b^e dH(e). \quad (10)$$

For given tax rate τ and a triple (c_1, b, y) we can therefore compute the consumption allocation in each bank in each state, as given in (9), and the level of the public good g_s in each state, as given in (10). As a result, we can completely characterize the consumption allocation over the private and the public good for each agent in each bank. Welfare for wealth e agents (expressed in per-agent term) is equal to the the sum of expected utilities for the agents in bank e , that is,

$$W^e(c_1^e, b^e, y^e) = \left\{ \begin{array}{l} \pi u(c_1^e) + (1 - q) [(1 - \pi)u(c_{2\alpha}^e) + v(g_\alpha)] \\ + q [\pi_\beta^e u(c_{1\beta}^e) + (1 - \pi - \pi_\beta^e)u(c_{2\beta}^e) + v(g_\beta)] \end{array} \right\}. \quad (11)$$

Total welfare in the economy is measured by the *equal-weighted* sum of all agents' expected utilities. That is,

$$W(c_1, b, y) = \int_e^{\bar{e}} W^e(c_1^e, b^e, y^e) dH(e), \quad (12)$$

3.3 Feasible bailouts

The policy maker can freely choose the bailout payment b^e for each bank subject to the following restrictions. First, the payment must be non-negative. As in Keister (2016), this assumption prevents the policy maker from collecting additional tax revenue at $t = 1$. Instead, the fiscal capacity of the public sector must be set in period 0, before the sunspot state is revealed. Second, the bailout payment cannot be set so high that agents in the bank consume more than they would have consumed had there been no

run. Because my focus is on policy reactions to a crisis, I want to separate *ex ante* redistribution, which occurs in normal times, from *ex post* redistribution during a crisis.⁸ To formalize these restrictions, note that for each bank e , the choice of c_1^e implies a value of $c_{2\alpha}^e$. Furthermore, a given triple (c_1^e, b^e, y^e) implies values for $c_{1\beta}^e$ and $c_{2\beta}^e$. The bailout policy is restricted so that, after the bank is bailed out, the payments given in period 1 cannot exceed c_1^e and the payments given in period 2 cannot exceed $c_{2\alpha}^e$. That is, the allocation in bank e must satisfy $c_{1\beta}^e \leq c_1^e$ and $c_{2\beta}^e \leq c_{2\alpha}^e$.⁹

Formally, the policy maker's strategy set includes any non-negative bailout payment for each bank. However, in order to prevent super-insurance, I model the payoff to the policy maker as taking a prohibitively low value (i.e. $-\infty$) whenever some bank(s) receive super-insurance. Therefore, the equilibrium bailout specification would always satisfy the no-super insurance restriction.

3.4 Partial bank runs

The agents in bank e will be said to follow the *no-run strategy profile* in which each agent withdraws in period 1 when impatient, regardless of the sunspot state. There is always an equilibrium in this model in which no bank run occurs at any bank, no bailouts are made, and the first-best allocation of resources obtains. As is standard in the literature on Diamond-Dybvig models, my interest is in whether or not the banking system is *fragile* in the sense that there exist other equilibria in which some agents run on their bank. To look for such equilibria, we could in principle start from an arbitrary withdrawal profile for all agents, derive the best response of the banks and the policy maker, and then check whether the agents in each bank are best responding with their strategy profile. Notice, however, that the analysis can be simplified by observing that the first order condition in (8) implies that the remaining payments in state β satisfy $c_{1\beta}^e < c_{2\beta}^e$ for each e . That is, all patient agents whose opportunity to withdraw arrives

⁸For now, I assume that the distribution of wealth in the initial period reflects the outcome of any ex-ante redistribution policy. In Section 5, I return to this issue and explicitly introduce a choice of ex ante redistribution policy into the model.

⁹Alternatively one can restrict the bailouts so that the agents in banks with a bailout do not receive more than the *face value* of the deposit that is $c_{1\beta}^e \leq c_1^e$ and $c_{2\beta}^e \leq c_1^e$. Both approaches yield similar results.

after the first π will choose to withdraw in period 2. As a result, in equilibrium a run on any bank is necessarily partial and will not continue beyond the first π withdrawals. In other words, a banking panic in this model consists of a wave of withdrawals from some bank(s), followed by a policy reaction that halts the run. Note that a panic will lead to excessive liquidation of long-term investment and a misallocation of resources. As result, even after the run has ended, a bank's remaining creditors will suffer losses and receive less from the bank because of the run.¹⁰

In view of this discussion, I consider the following *partial run strategy profile* for the agents in bank e ,

$$y_i^e(\omega_i^e, \alpha) = \omega_i^e \quad \text{for all } i$$

$$y_i^e(\omega_i^e, \beta) = \begin{cases} 0 \\ \omega_i^e \end{cases} \quad \begin{cases} \text{if } i \leq \pi h(e) \\ \text{if } i > \pi h(e) \end{cases}. \quad (13)$$

According to this strategy profile, an impatient agent with wealth level e always chooses to withdraw in period 1. A patient agent with wealth level e , on the other hand, chooses to withdraw in period 1 if and only if the state is β and his order in the withdrawal opportunities is among the first fraction π within his bank. Otherwise, this patient agent withdraws in period 2. Thus, any withdrawals after the first π will be made only by the impatient agents and the mass of additional period-1 withdrawals from a bank whose agents follow the strategy in (13) will be equal to $\pi_\beta^e = (1 - \pi)\pi$. Recall that during the first fraction π of withdrawals, the bank is unable to infer whether a run is under way. According to the partial run strategy profile in (13), when patient, an agent will run only if he can withdraw *before* his bank infers that a run is underway.¹¹ The

¹⁰Partial run strategy profiles as in (13) were introduced by Ennis and Keister (2010), who showed that in settings where banks are able to react by changing payments when withdrawal demand is high, and agents know their position in the withdrawal order, an equilibrium bank run will be necessarily partial and restricted to agents that can withdraw before banks infer that a run is underway.

¹¹The fact that a run necessarily halts after π withdrawals depends on my assumption that there are only two sunspot states. Ennis and Keister (2010) show that, with a richer sunspot variable, there can be equilibria in which runs occur in multiple waves, with a policy reaction following each wave. Restricting attention to single-wave runs allows me to simplify notation and focus more clearly on how the distribution of wealth shapes financial fragility. Extending the analysis here to study multiple-wave runs is left for future research.

partial run strategy profile for the agents in bank e is consistent with equilibrium if and only if the following *incentive to run* condition holds,

$$c_1^e \geq c_{2\beta}^e. \quad (14)$$

In state β , an agent with wealth e and an opportunity to be among the first π to withdraw will receive c_1^e if he withdraws in period 1 and $c_{2\beta}^e$ if he withdraws in period 2. Condition (14) ensures that joining the run by withdrawing in period 1 is a best response.

3.5 Optimal early payments

In this and the next section, I characterize banks' and the policy maker's best responses to the withdrawal strategies of the agents. I consider equilibria in which agents in each bank all play either the no-run strategy profile or the partial run strategy profile in (14). Given my assumptions, this approach is without loss of generality since, as described above, any bank run in this model is necessarily partial and will halt after a fraction π of a bank's agents have withdrawn.

In this setting some banks might experience a run in state β , while others not, depending on the withdrawal profile of their agents. Each bank e chooses its early payment c_1^e to maximize the sum of its agents expected utilities in (11), taking as given the pair (b^e, y^e) , the level of the public good in each state g_s , and recognizing that the remaining payments within the bank (i.e. $c_{2\alpha}^e$, $c_{1\beta}^e$ and $c_{2\beta}^e$) will be chosen to satisfy (4), (7) and (8). The optimal choice of early payments in any bank e will satisfy the following first order condition,

$$u'(c_1^e) = R[(1 - q)u'(c_{2\alpha}^e) + qu'(c_{2\beta}^e)]. \quad (15)$$

Bank e sets its early payment to equalize the marginal utility of the agents withdrawing before the state is known to the expected marginal utility of the agents withdrawing after the state is known.¹² Furthermore, if the agents in any bank e follow the no-run

¹²Note that as the probability of the run state q converges to zero then the condition in (15) converges to the standard equation characterizing optimal risk sharing between impatient and patient agents

strategy profile, then $b^e = 0$. In this case, using the utility function in (2), we obtain the solution in closed form,

$$c_1^e = \frac{(1-\tau)e}{\pi+(1-\pi)R^{(1-\gamma)/\gamma}} \quad \text{and} \quad c_{2s}^e = R^{1/\gamma} \frac{(1-\tau)e}{\pi+(1-\pi)R^{(1-\gamma)/\gamma}}. \quad (16)$$

The above implies $c_1^e < c_{2s}^e$ for $s \in \{\alpha, \beta\}$, and therefore, the agents in any bank e would always best respond with the no-run strategy profile.

3.6 Optimal bailout policy

The policy maker infers the state s after the initial π withdrawals. If the state is α there are no runs, no bailouts will be made, and all tax revenue is used to provide the public good. On the other hand, if the state is β the existing tax revenue is allocated between the public good and bailouts to those banks experiencing runs, subject to the restriction that super-insurance is not allowed. Recall that the consumption levels for the remaining agents in bank e , i.e. $c_{1\beta}^e$ and $c_{2\beta}^e$, is increasing in the bailout received by the bank. The policy maker chooses a bailout specification to maximize the expression for total welfare in (12). The optimal bailout specification is characterized as follows: each bank either receives the maximally allowed (per-agent) bailout $b^e = \bar{b}^e$ or an amount which ensures that the marginal utility for the remaining agents within the bank is equal to the marginal utility from the public good, that is,

$$u'(c_{1\beta}^e) = Ru'(c_{2\beta}^e) = v' \left(\tau - \int_e^{\bar{e}} b^e dH(e) \right). \quad (17)$$

Thus, the bailout given to any bank experiencing a run would raise the consumption level of its remaining agents until their marginal utilities are equalized to the marginal utility from the public good or until the bailout to this particular bank reaches the upper limit imposed by the no super-insurance restriction.¹³

in Diamond-Dybvig models.

¹³The bailout policy I study here differs from that in Keister (2016) in that the policy maker does not observe a bank's choice of c_1^e before choosing the bailout payment b^e . Of course, the policy maker will correctly anticipate this choice in equilibrium and will choose b^e accordingly. However, if a bank were to deviate to a higher value of c_1^e , the policy maker would not observe this choice in my model and, therefore, would not reward the bank with a larger bailout. In this way, the incentive distortion

3.7 Definition of equilibrium

In this section, first, I define the equilibrium of the withdrawal game within a bank, holding as fixed the actions of banks and the policy maker. Second, I define the equilibrium of the overall game where the agents, the banks and the policy maker all react to each other. In order to set the stage for the first definition, note that the pair (c_1^e, b^e) defines a *withdrawal game* for the agents in bank e . The payoffs in this game are the consumption levels described in (9). In particular, the payments received by agent i with wealth level e will be completely determined by the triple (c_1^e, b^e, y^e) , the realization of the sunspot state s , and the realization of his consumption type ω_i^e . The (indirect) expected utility for this individual can be defined as follows,

$$v_i^e(c_1^e, b^e, y^e) = \mathbb{E}[u(c_{1i}^e + \omega_i^e c_{2i}^e) + v(g)], \quad (18)$$

where \mathbb{E} represents the expectation over ω_i^e and s . In the above expression the level of the public good in each state will be independent of the outcome within bank e and will therefore be treated as fixed both by the bank and by its agents. Hence, we can use the withdrawal profile for the agents in bank e together with the consumption allocation in (9) to compute the payoff for each agent within the bank and also to determine whether this agent is best responding with his strategy. Now we are ready to state the first definition.

Definition 1. For given tax rate τ and a pair (c_1^e, b^e) , an *equilibrium of the withdrawal*

game in bank e is a profile of withdrawal strategies y^{*e} such that $y_i^{*e} \in \operatorname{argmax}_{y_i^e \in \{0,1\}} v_i^e(c_1^e, b^e, (y_{-i}^{*e}, y_i^e))$ for all $i \in [0, h(e)]$.

Let $\hat{Y}^e(c_1^e, b^e)$ be the set of symmetric pure strategy equilibrium strategy profiles for

that arises in Keister (2016) does not arise here and there is no need to study macro-prudential policies that aim to correct this distortion. The approach I take leads to the same outcome as that in Keister (2016) when the optimal macro-prudential policy is in place, while being notationally much simpler.

the agents in bank e . The following definition of equilibrium in the overall banking game is based on Ennis and Keister (2010).

Definition 2. For given tax rate τ , an *equilibrium without commitment* is a triple (c_1^*, b^*, y^*) such that,

- (i) $y^{e*} \in \hat{Y}^e(c_1^{e*}, b^{e*})$ for each $e \in [\underline{e}, \bar{e}]$,
- (ii) $c_1^{e*} \in \operatorname{argmax}_{c_1^e} W^e(c_1^e, b^{e*}, y^{e*})$ for each $e \in [\underline{e}, \bar{e}]$,
- (iii) $b^* \in \operatorname{argmax}_b W(c_1^*, b, y^*)$.

According to (i), for a given early payment c_1^{e*} and bailout b^{e*} , the profile of agents' strategies in bank e is an equilibrium of the withdrawal game in that bank. According to (ii), bank e chooses its early payment optimally taking as given the strategy profile of its agents y^{e*} and the bailout payment of the policy maker. Note that conditions (i) and (ii) define equilibrium *within a bank* holding the specification of bailouts across banks fixed. Finally, according to (iii) the policy maker chooses bailouts optimally taking as given early payments in all banks and the strategy profile for all agents in all banks.

Observe that if the banks and the policy maker had commitment power, then their plans would be set once and for all - before the agents have made their withdrawal decisions - and no change can be made later. In this environment, however, both the banks and the policy maker cannot irrevocably set their plans before the agents choose their strategies. Instead the payment given to each agent and the bailout given to each bank will be finally determined only when it is actually made. This no-commitment approach to defining equilibrium follows Ennis and Keister (2010) in that both the rescheduling of payments that ensues in banking crisis and the bailouts made by the policy maker are not set in advance but instead end up being determined as a best response to the events unfolding during the crisis.

3.8 The optimal tax rate

Each possible choice of the tax rate τ generates a game in which banks, agents, and the policy maker will play an equilibrium as described in Definition 2. . This game may, of course, have multiple equilibria. To choose an optimal tax rate, the policy maker must have some belief about which equilibrium will be played for each choice of τ .¹⁴ I define the *maximally fragile equilibrium* to be the equilibrium in which the highest fraction of agents in the population follow the partial run strategy. In choosing the tax rate in period 0, I assume the policy maker has the most “pessimistic” belief in the sense the he anticipates that the maximally fragile equilibrium associated with each choice of τ would obtain.

4 How the distribution of wealth affects fragility

In this section, I present the main results of the analysis. The focus will be on characterizing the type of runs that can occur in equilibrium and connecting them to the bailout intervention of the policy maker. First, and as a motivating example, I show that removing bailouts leads to an economy where the initial distribution of wealth is not linked to financial fragility. Second, returning to the economy where bailouts are allowed, I show that financial fragility can be linked to the distribution of wealth through the bailout policy. Third, both analytically and through numerical examples, I examine how varying the initial distribution of wealth affects financial fragility.

Before proceeding, I formally define the notion of fragility used for the remainder of the paper. First, an *economy* is a collection of parameter values (i.e. R , π , q , γ and δ) together with a c.d.f. for the initial distribution of wealth H . The definition of fragility is the following.

Definition 3. Given an economy, the financial system is *fragile for agents with wealth level e* if and only if there exist an equilibrium where this agent type follows the partial

¹⁴See Ennis and Keister (2006) for a general discussion of how optimal policy problems can be formulated in models with multiple equilibria.

run strategy profile in (13).

If, on the other hand, one cannot construct an equilibrium where agents with wealth level e follow the partial run strategy profile the financial system is not fragile for this type of agent. Financial fragility is thus defined separately by agent type.

4.1 No bailouts implies no relationship

Suppose a strict *no-bailout rule*, which prohibits the policy maker from engaging in any bailouts, is enacted. A strict no-bailouts rule is, in general, not ex post optimal and therefore plagued by credibility issues. Nonetheless, and only for the sake of the discussion in this section, I assume that the policy maker can pre-commit to such a rule. Equilibrium fragility in this case is independent of the distribution of wealth as shown next.

Proposition 1. *Under a strict no-bailouts rule the financial system is either fragile for all agents or fragile for none.*

If bailouts are prohibited and the utility function for the agents is constant relative risk aversion, then the ratio $c_1^e/c_{2\beta}^e$ is independent of wealth levels e , the tax rate τ , and the wealth distribution H . As a result, either the economy is “maximally fragile” in the sense that all types of agents are susceptible to a run, or it is not fragile at all. It is straightforward to show that the former case occurs when we have

$$\frac{R^{(2-\gamma)/\gamma}}{\pi+(1-\pi)R^{(1-\gamma)/\gamma}} < \left(\frac{1-qR}{1-q}\right)^{1/\gamma}. \quad (19)$$

I assume this condition on the parameters holds throughout the remainder of the analysis.¹⁵

¹⁵In the special case where the probability of the run state is zero (i.e. $q = 0$), (19) reduces to condition

Proposition 1 implies that, in the absence of bailouts, changes in the initial distribution of wealth have no effect on financial fragility. In making decisions on redistribution, therefore, the policy maker can ignore financial stability considerations. In the remainder of the paper, I show that this independence breaks down when bailouts occur and derive the implications of wealth inequality for financial fragility.

4.2 Bailouts and the incentive to run

In this section I return to the case where the policy maker is allowed to bail out banks experiencing runs in order to examine the implications for financial fragility. Since the policy maker cannot pre-commit ex ante, the agents and the banks anticipate the ex-post efficient bailout intervention and adjust their behavior accordingly. First, I show that wealthier agents anticipate lower bailouts to their banks and, as a result, have a higher incentive to run. In particular, any equilibrium bailout specification b^* can be characterized as follows.

Proposition 2. *If in equilibrium $b^{e^*} > 0$ and $b^{\tilde{e}^*} > 0$. Then $b^{e^*} > b^{\tilde{e}^*}$ whenever $e < \tilde{e}$.*

In other words, if two banks serving different types of agents both receive bailouts in equilibrium, then the bank serving poorer agents will receive a larger bailout. This result has two implications. First, if two types of agents play the partial run strategy and the type with the lower wealth level is best responding then the type with the higher wealth level is automatically best responding as well. Second, if we can construct equilibrium where agents with wealth level e play the partial run strategy then we can construct a different equilibrium where, in addition to agents with wealth level e , any agent with wealth level above e also plays the partial run strategy.

Next, if any bank e receives a bailout then the losses to its agents, incurred as a result of the run, will be either partially or fully covered. In a given equilibrium, I refer to

(8) in Ennis and Keister (2010).

wealth level e agents as *fully insured* whenever the bailout received by their bank in the event of a run implies that staying invested until the last period incurs no losses, that is, $c_{2\alpha}^e = c_{2\beta}^e$. The next result shows that, in equilibrium, full insurance will be contingent on agents' wealth levels.

Proposition 3. *If, in a given equilibrium, agents with wealth level e are fully insured, then all agents whose wealth lies below e would also be fully insured.*

If the agents in any given bank e are fully insured, then there will be no incentive to run (that is, the partial run strategy is not a best response i.e. $c_1^e < c_{2\beta}^e$) and, as a result, the bailout to this particular bank remains strictly off the equilibrium path of play. Therefore, equilibrium runs (and any equilibrium bailouts) must necessarily be restricted only to partially insured agents and their banks. Proposition 2 shows that agents with wealth level e benefit from full insurance in a given equilibrium only if all agents with lower wealth levels also benefit from full insurance. An implication of this result is that if anyone receives full insurance in equilibrium it must be the agents with relatively low wealth levels.

The properties of the ex-post optimal bailout intervention, derived in Propositions 2 and 3, shape financial fragility by affecting the incentives to run faced by different types of agents. Through this channel, the bailout policy and the distribution of wealth play a central role in the sort of runs that can be sustained in equilibrium as we will see next.

4.3 The cutoff for fragility

This section shows that equilibrium fragility can vary across agents and their banks. As a result, the initial distribution of wealth will have a fragility component that operates through the bailout intervention of the policy maker. In particular, the form of the policy maker's bailout intervention entails that wealthy agents anticipate lower bailouts (if any),

and therefore, will experience larger losses from staying invested in a crisis, which in turn, affects their incentive to run. The following result formalizes this notion.

Proposition 4. *For a given economy, there exists a cutoff e^f such that the financial system is fragile for agents with wealth levels $e > e^f$ and not fragile for agents with wealth levels $e < e^f$.*

The financial system is fragile for all (fragile for none) whenever $e^f < \underline{e}$ ($e^f > \bar{e}$). On the other hand, if the cutoff point is interior $\underline{e} < e^f < \bar{e}$, then the financial system is fragile only for agents located above the $H(e^f)$ percentile of the wealth distribution. The next result shows that the cutoff can be interior in some cases.

Proposition 5. *In some economies, the cutoff point is interior, $\underline{e} < e^f < \bar{e}$.*

Proposition 5 is established by means of the examples in Figures 2a - 2b. The results in Propositions 4 - 5 outline a key feature of the model – equilibrium fragility in this environment tends to be concentrated among the wealthier agents and their banks since, in a crisis, they receive less fiscal support in the form of bailouts. At the same time, all agents with wealth below the cutoff will be *effectively insured* and will not run on the banks in equilibrium. Moreover, since agents with wealth below e^f do not panic in equilibrium, there is no need for the policy maker to step in and to actually bail them out. In other words, we have the following corollary,

Corollary 1. *Equilibrium bailouts will be made only to institutions serving agents with wealth levels above the cutoff e^f .*

That is, equilibrium runs and bailouts will be restricted only to banks that serve agents who are rich enough to remain above the cutoff point. In some cases, all agents with

wealth above e^f will be part of the bailout package. In other cases, there will be an upper bound $e^{NB} < \bar{e}$, also determined in equilibrium, such that only agents with wealth in the interval (e^f, e^{NB}) receive bailout and those with $e > e^{NB}$ receive no bailout .

Before proceeding, note that one can re-interpret the endogenously emerging properties of the banking structure as follows. Agents with wealth below the cutoff e^f belong to the “commercial” part of the financial system where deposit insurance is effective and runs do not occur in equilibrium. Agents with wealth above the cutoff belong to the “shadow banking” sector where both runs, and therefore bailouts, can take place in equilibrium.¹⁶ Moreover, not all agents belonging to the shadow banking sector necessarily anticipate bailouts from the policy maker in a crisis - those whose wealth level is above e^{NB} will be left out of the bailout program altogether. For the analysis to follow, I refer to the interval $[\underline{e}, e^f)$ as the *effective insurance region* and to the interval $[e^f, e^{NB})$ as the *bailouts region*.

4.4 What determines the cutoff for fragility

In this and the next section I study how changes in the initial distribution of wealth shape equilibrium fragility through its effect on the cutoff point e^f . For the analysis that follows, it will be helpful to proceed in steps and to begin by examining the partial effects. First, I fix the initial distribution of wealth and examine how changes in the tax rate affect the cutoff for fragility.

Proposition 6. *Suppose that, for a given tax rate τ , we have $\underline{e} < e^f < \bar{e}$. Then, holding H fixed, we have $e^f < \tilde{e}^f$ whenever $\tau < \tilde{\tau}$.*

In other words, increasing the policy maker’s tax revenue will raise the cutoff for fragility, thereby shrinking the fragile region. More tax revenue implies that the policy

¹⁶I show in Section 6 that such an interpretation continues to apply under more general banking arrangements where agents with different wealth levels can belong to the same bank.

maker is willing to provide larger bailouts to banks. Larger bailouts raise the consumption levels, $c_{1\beta}^e$ and $c_{2\beta}^e$, for the remaining agents in banks with a bailout. As a result, an agent located at the old cutoff point will no longer have an incentive to run and the new cutoff point \tilde{e}^f will be higher. Second, I held the tax rate τ fixed and examine how changes in the initial distribution of wealth affect the cutoff point.

Proposition 7. *Suppose that, for a given distribution of wealth H , we have $\underline{e} < e^f < \bar{e}$.*

Holding τ fixed, let \tilde{H} be a mean-preserving spread of H . Then,

(i) If $\tilde{h}(e) > h(e)$ for each $e \in [e^f, e^{NB}]$ then $\tilde{e}^f < e^f$.

(ii) If $\tilde{h}(e) < h(e)$ for each $e \in [e^f, e^{NB}]$ then $\tilde{e}^f > e^f$.

Assume we change the initial distribution of wealth so that mean wealth remains unchanged but the mass of agents at each point in the bailout region is higher as in case (i). Then providing the same level of per-capita bailout for each agent in the bailout region (i.e. between e^f and e^{NB}) requires a bigger economy-wide bailout. This lowered capacity to provide bailouts implies that the bailout payment per agent will be lower. As a result, agents with wealth slightly below the old cutoff will now also have an incentive to run. That is, these agents lose their effective insurance and the new cutoff for fragility \tilde{e}^f will be lower. The opposite result holds in the reverse case as shown in part (ii). Before continuing note the following implication of Proposition 7. Assume we hold tax revenue constant and move from a wealth distribution H to a wealth distribution \tilde{H} such that $\tilde{h}(e) > h(e)$ for each $e \geq e^f$. Then the new cutoff point \tilde{e}^f falls below the old one and the mass of fragile agents in the economy increases.

4.5 Examples

The previous section examined in isolation how changes in the tax rate (Proposition 6) and changes in the mass of agents in the bailout region (Propositions 7) affect the

mass of agents with wealth above the cutoff point. However, as the initial distribution of wealth moves both of these forces will operate jointly on the position of the cutoff point and not always in the same direction. In this section, I present the analysis in the case where both of these forces are simultaneously active. In order to present the results in their simplest form, I focus on the limiting case as the probability of state β goes to zero. The optimal tax rate in this case can be characterized in closed form

$$\tau^* = \frac{1}{1+(1/\delta)[\int e^{-\gamma} dH(e)]^{1/\gamma}}. \quad (20)$$

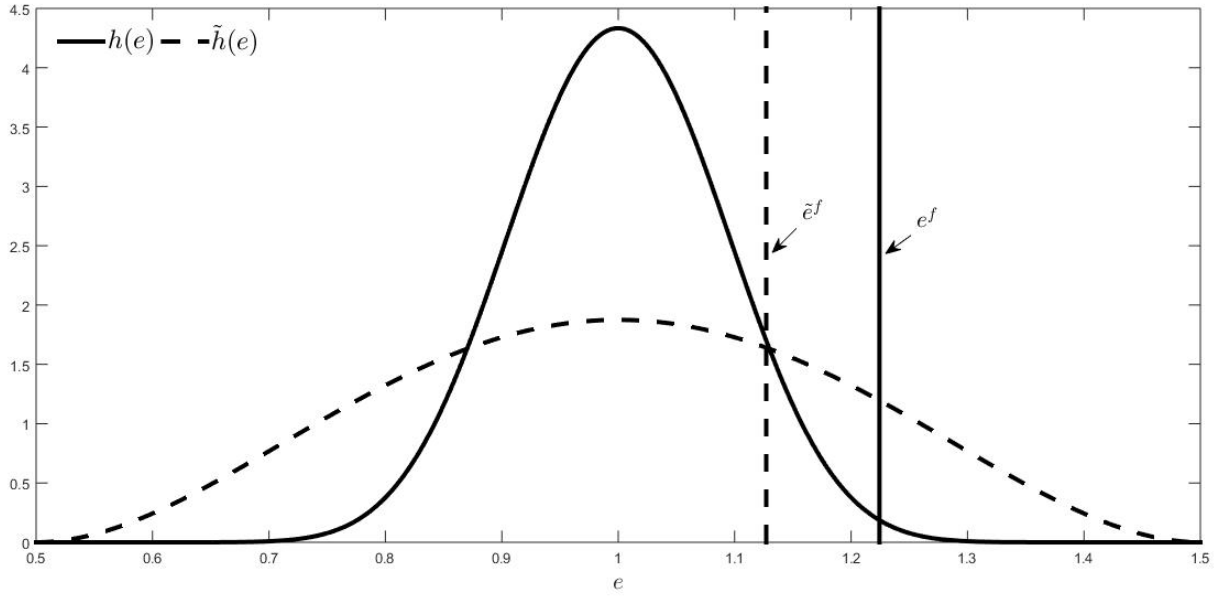
Recall from (2) that the parameter δ governs the relative value of the public good in agents' preferences. If δ is high, the public good is highly valued, and the policy maker sets high tax rate in order to collect more tax revenue and increase the level of the public good consumption. Further, the policy maker responds to a mean preserving of the wealth distribution by choosing a lower tax rate.

The analysis is presented through two numerical examples (in Figures 2a and 2b) which highlight how a more unequal initial wealth distribution can lead to more fragility in some economies and less fragility in others. Figure 2a corresponds to a high value of the parameter δ , and therefore, to an economy where the policy maker sets a high tax rate and is more willing to bail out banks as a result. Figure 2b, on the other hand, corresponds to a low value of δ and therefore to an economy where the policy maker chooses a lower tax rate and will be less willing to bail out banks as a result. In both figures I compare the cutoff for fragility associated to a low level of inequality (represented by a solid line) to the cutoff for fragility associated to a high level of inequality (represented by a dashed line).

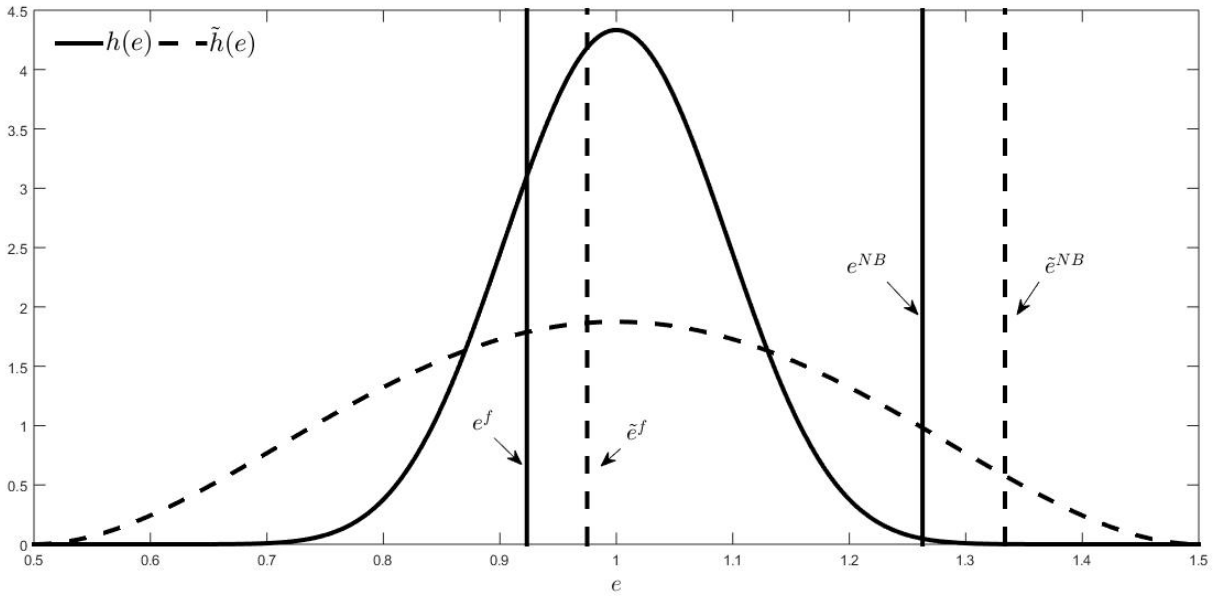
Focus first on Figure 2a, where the fragility cutoff lies significantly to the right of mean wealth. This case represents a situation where most of the agents are effectively insured and the economy remains fragile only for a few relatively wealthy agents. Furthermore, the bailout region in this case covers all agents whose wealth remains above the cutoff for fragility i.e. they all receive some fiscal support but not enough to prevent them from running. Consider a mean preserving spread applied to the original distribution of

wealth such that the mass of agents at each point in the old bailout region is higher, as shown on the figure. More inequality leads to lower tax rate (and therefore lower tax revenue), which other things being equal, corresponds to lower cutoff point (Proposition 6). Furthermore, a higher mass of agents at each point above the old cutoff point also pushes down the cutoff for fragility (Proposition 7). Thus, both forces in this case operate in the same direction, generating a lower cutoff for fragility, and leading to financial system with a higher mass of fragile agents. In other words, in this example an increase in inequality leads to an increase in the level of financial fragility

Next, consider Figure 2b where the parameter δ is smaller, and therefore, the policy maker chooses to collect less tax revenue. This represents a case where a relatively smaller fraction (less than half) of the agents are effectively insured. In addition, the bailout region does not cover all fragile agents but only those whose wealth is not too high. Next, apply a mean preserving spread to the original distribution of wealth such that the mass of agents at each point in the old bailout region is lower. The overall effect on the cutoff for fragility would therefore depend on whether the smaller bailout region effect (which pushes the cutoff up) dominates the lower tax revenue effect (which pushes the cutoff down). In Figure 2b the smaller bailout region is the dominant factor and the new cutoff for fragility rises as a result. In other words, in this example an increase in inequality leads to a decrease in the level of financial fragility.



(a) Higher inequality leads to more fragility.



(b) Higher inequality leads to less fragility.

Figure 2: The distribution \tilde{H} (with p.d.f. h) is a mean-preserving spread of the distribution H (with p.d.f. \tilde{h}). The parameter δ is equal to 1 in Figure 2a (high fiscal capacity) and equal to 0.01 in Figure 2b (low fiscal capacity). The other parameter values are the same in both figures and are set to $R = 3$, $\pi = 0.5$, $\gamma = 5$, and $q = 0$. The cutoff for fragility is represented by the vertical solid line when the wealth distribution is H and by the vertical dotted line when the wealth distribution is \tilde{H} . A raise in inequality (as represented by a move from H to \tilde{H}) implies that the mass of fragile agents increases on Figure 2a and decreases on Figure 2b.

4.6 Discussion

A useful way to think of the effect of inequality on the level of fragility is through the notion of *fiscal capacity* (or fiscal space) of the policy maker. High fiscal capacity implies that the policy maker will be willing to provide larger bailouts to banks experiencing runs, which raises the consumption levels for the patient agents in these banks and therefore reduces their incentives to run. Low fiscal capacity operates in the opposite way, by lowering the bailouts payments the policy maker will be willing to make in a crisis.

As the distribution of wealth changes, the fiscal capacity of the policy maker will be simultaneously impacted through two channels. First, the policy maker will change the amount of tax revenue collected. Second, the mass and distribution of agents in the bailout region will change. The effect of each of these two channels in isolation was presented in Propositions 6 and 7. The outcome in the overall economy, where both of these forces are simultaneously at work, depends on parameters and different cases were examined in Figures 2a and 2b. In Figure 2a both of these forces operate in the same direction, namely towards undermining the fiscal capacity of the policy maker. As a result, higher inequality leads to more fragility in this particular economy. In Figure 2b, in contrast, higher inequality strengthens the fiscal capacity of the policy maker since it hollows out the bailout region by placing more agents in the effectively insured part of the wealth distribution. As a result, higher inequality leads to less fragility in this economy. In general, therefore, more inequality can bring either more or less fragility, depending on the overall effect on the policy maker's fiscal capacity.

The results in this section are derived assuming a utilitarian policy maker. It is possible, of course, that policy decisions are made based on some other objective function that gives greater weight to agents who have more political connections or other forms of power. In such a setting, the model I present here would predict that whatever agents have more weight in the objective function will tend to be effectively insured and, hence, have no incentive to run on their banks. Agents with little or no weight in the objective

function, in contrast, will tend to experience fragility. In equilibrium, bank runs and the associated bailouts will tend to be concentrated among agents who are less favored by the policy maker. Viewed this way, the observation that bailouts tend to benefit wealthy individuals can be interpreted as evidence that policy makers may be more utilitarian than is commonly assumed.

The underlying mechanism in the model here does not assume out-size political power or inside connections for the wealthy. Still equilibrium bailouts tend to benefit relatively wealthy agents at the expense of the general public, despite the fact that, by construction, there are no political frictions. In this respect the model I study is different from the existing literature which has proposed a number of political frictions that can generate a link between the distribution of wealth and financial fragility. Rajan (2010), for example, argues that rising inequality unleashes political pressures demanding increased redistribution. In some cases, this redistribution is implemented through means - such as subsidized lending for the poor - which generate distortions in the financial sector and ultimately undermine financial stability. Acemoglu (2011) posits that rising inequality can lead to policies, such as financial deregulation, which tends to disproportionately benefit wealthy individuals, while at the same time, making the financial system more unstable. According to Stiglitz (2009), widening income disparity during the years preceding the 2007-08 financial crisis depressed aggregate demand since it redistributed income from those with high propensity to consume to those with low propensity to consume (i.e. from low to high income individuals). The monetary policy response, pushed on by political factors, triggered a prolonged period of low interest rates in order to stimulate demand. This, in turn, set the stage for the credit expansion followed by the subsequent collapse and ensuing financial crisis.¹⁷

5 Ex-ante redistribution

This section extends the model to allow for *ex-ante* redistribution and examine the

¹⁷Also, Kumhof *et al.* (2015) study a model with non-standard utility function in which rising inequality induces agents in the lower part of the income distribution to take on more debt in order to maintain their relative standards of living resulting in a less stable financial system.

implications for fragility. In particular, I start from a given initial wealth distribution H_0 and allow the policy maker to choose a linear tax and transfer scheme in period 0, which leads to a new distribution H_1 . Let the tax and transfer program of the policy maker in period 0 be characterized by the pair (t, Δ) where t is the marginal tax rate and Δ is a lump-sum transfer. The after-tax wealth level for an agent with wealth level e is hence $(1 - t)e + \Delta$. The amount of tax revenue collected in period 0 is equal to the total tax revenue minus all transfers. That is, $\tau = \int_e^{\bar{e}} (te - \Delta) dH(e)$.

The policy maker has the option of collecting τ in tax revenue without any redistribution by selecting $t = \tau$ and $\Delta = 0$. At the same time, the policy maker can raise the same amount in tax revenue while fully redistributing all wealth in the process by selecting $t = 1$ and $\Delta = 1 - \tau$. Henceforth, the *level of redistribution* is measured by $\rho = \frac{\Delta}{1 - \tau}$, where $\rho = 0$ corresponds to no redistribution (i.e. as in the baseline case) and $\rho = 1$ corresponds to full redistribution.

There are a number of well-known efficiency losses associated with redistribution that are examined extensively in the literature on optimal taxation.¹⁸ It is beyond the scope of the paper to explicitly model the determination of these costs. Instead, I augment the model with a loss function $L(t, \Delta)$ expressed in utility terms and subtracted from total welfare in period 0. The function L is assumed to be (weakly) increasing in both arguments.¹⁹

Given a loss function L , the “traditional” approach will be to balance the welfare benefits of redistribution against the costs embedded in L . As long as the exogenous cost of redistribution prevents the policy maker from choosing to make H_1 a degenerate distribution in which all agents have equal wealth level, I can apply the baseline model by simply defining H_1 to be the initial distribution of wealth. More interestingly, however, the (after-tax and transfer) distribution of wealth in period 0 has a fragility component that operates entirely separately from any efficiency costs. Specifically, for each tax and transfer pair (t, Δ) there is an associated cutoff point $e^f(t, \Delta)$ such that an agent is fragile

¹⁸See Diamond and Saez (2011) for an overview of the literature on optimal taxation.

¹⁹As will become clear, the precise way one chooses to model the cost of taxation and redistribution will not matter for the remainder of the analysis in this section.

if and only if his net wealth level remains above that cutoff. As a result, when choosing the optimal level of redistribution, the policy maker must take into account this fragility component as well.

In order to isolate the impact of this fragility component on the optimal choice of ex-ante redistribution, from now on I assume that ex-ante redistribution carries no efficiency costs i.e. $L(t, \Delta) = 0$ for all t and Δ . This allows me to focus on the novel factors operating for and against redistribution in this environment without the confounding impact of efficiency gains or losses. We obtain the following result.

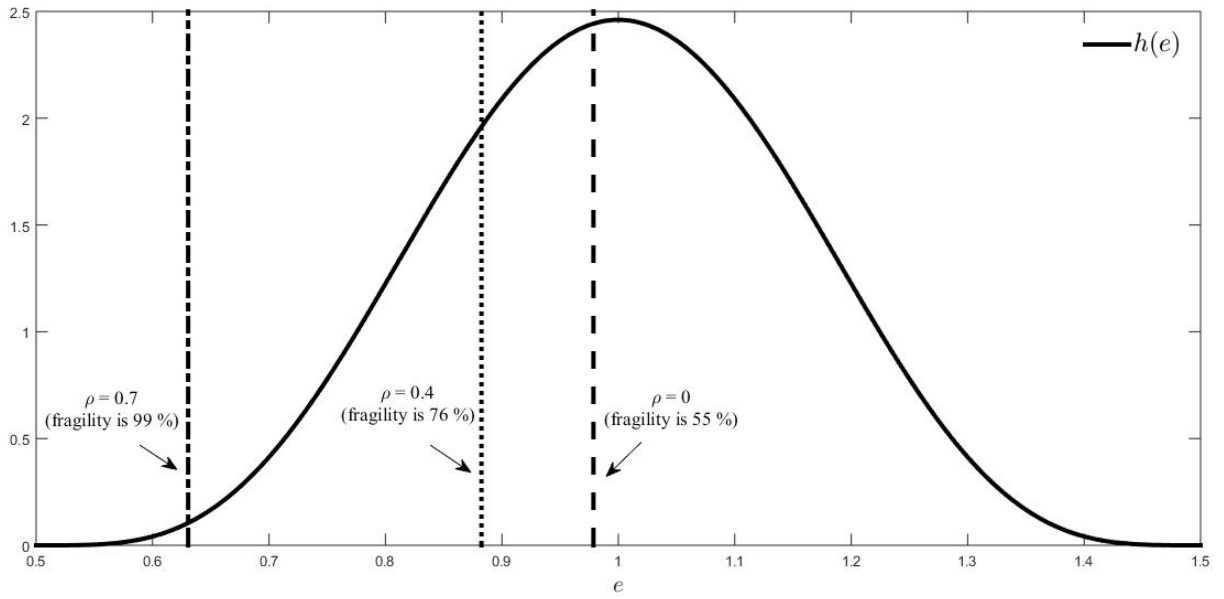
Proposition 8. *The optimal level of ex-ante redistribution in some economies remains partial even if period-0 redistribution carries no efficiency losses.*

Proposition 8 is established in Figures 3a and 3b. First, refer to Figure 3a, which shows the initial distribution of wealth together with the fragility cutoff associated with three different levels of redistribution ρ . In this example, more ex ante redistribution moves the cutoff to the left and therefore pulls more agents into the fragile region. Thus, for the level of redistribution, ρ , respectively equal to 0, 0.4 and 0.7 and 1 the corresponding cutoff point implies that the fraction of fragile agents in the economy is 0.55, 0.76 and 0.99 and 1 respectively. Figure 3b shows the level of redistribution on the horizontal axis and total welfare on the vertical axis and is constructed under the same parameters as in Figure 3a. The gain from redistribution is non-monotone, meaning that one must search globally for the optimum. In this case total welfare reaches a maximum at an *interior* level of redistribution approximately equal to 0.27. For lower values of ρ the utilitarian benefits of a more equal wealth distribution exceed the fragility costs in terms of a larger mass of agents whose wealth falls above the cutoff for fragility. For higher values of ρ , however, the utilitarian gains are not sufficient to offset the higher fragility and total welfare starts to decrease.²⁰

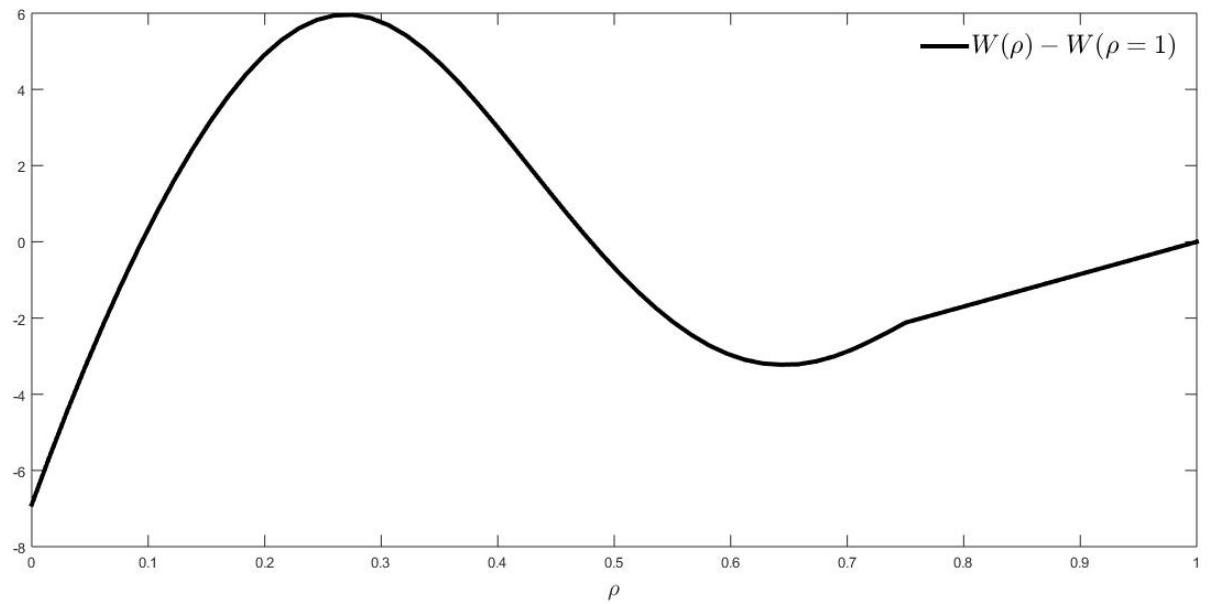
To summarize, when redistribution affects fragility, a utilitarian policy maker will not

²⁰Note that for ρ above about 0.65, the financial system becomes fragile for almost all of the agents and therefore the utilitarian effects starts to dominate again since fragility cannot increase further.

necessarily choose to fully redistribute wealth even when the standard costs of redistribution are completely absent. The reason is that financial fragility in this model is shaped by the distribution of wealth in the initial period and any redistribution undertaken in period 0 would potentially have a fragility component in addition to the standard efficiency costs. In some cases, this fragility component might increase the benefit of redistribution, while in others the effect will be the opposite. In particular, if a fully equal distribution of wealth also entails that fragility is not present (as in Figure 2a), then the policy maker wants to engage in more redistribution not only because it brings utilitarian benefits, but also because it brings financial stability gains. In contrast, if a fully equal wealth distribution also entails maximum fragility (as in Figures 2b and 3a) then more progressive taxation must be traded-off against the financial fragility costs it entails. In such cases, the policy maker may choose to stop short of fully equalizing wealth levels even when all of the standard efficiency costs to redistribution are absent (as in Figure 3b).



(a) Ex-ante redistribution and fragility.



(b) Optimal level of redistribution.

Figure 3: The parameter values are as follows $R = 3$, $\pi = 0.5$, $\gamma = 5$, $\delta = 0.01$, and $q = 0.05$. In addition, there is no efficiency cost of period-0 redistribution, i.e. $L(t, \Delta) = 0$ for each combination of t and Δ . Figure 3a shows that for level of redistribution, ρ , respectively equal to 0, 0.4 and 0.7 and 1 the corresponding cutoff point implies that the fraction of fragile agents in the economy is 0.55, 0.76 and 0.99 and 1 respectively. Figure 3b shows that total welfare in period 0 is maximized at an interior level of redistribution approximately equal to 0.27.

6 Banking arrangements and fragility

For my purposes a *banking arrangement* represents a specific grouping of agents into banks. In the baseline version of the model, each bank services only one agent type. Although this assumption simplified the analysis considerably, it raises two further questions. How robust is the model to alternative banking arrangements, and what could be the counterpart in reality to the financial institutions assumed in the model? Accordingly in this section, I show what sort of banking arrangements in this environment are equivalent to type-specific banks and then link these arrangements to real world financial practices.

Suppose we were to consider arrangements in which agents of more than one type are grouped together in a single bank. The operation of the bank then has distributional consequences and some assumption must be made about how different types of agents are treated. One natural benchmark is the *proportional principle*. Under this principle, all agents who withdraw in the same period and in the same state from a given bank k must be given the same rate of return, that is, $\frac{c_1^{e_1,k}}{e_1} = \frac{c_1^{e_2,k}}{e_2}$, $\frac{c_2^{e_1,k}}{e_1} = \frac{c_2^{e_2,k}}{e_2}$, and so forth. Note that the allocation in any type-specific bank automatically satisfies the proportional principle. At the same time, if more than one agent types are grouped into the same bank then the proportional principle is imposed as an additional constraint on the payments within the bank. One justification for imposing such simple rules for the operation of the bank is that more complicated rules may be plagued by credibility issues (recall that banks have limited commitment in this setting). In addition, the proportional principle implies that banks do not redistribute on their own among their agents.

Next, I assume that the bailout intervention of the policy maker is directly making payouts to agents in banks experiencing a run – in addition to any payments these agents receive from their banks. In particular, an agent with wealth level e withdrawing after his bank infers that the state is β consumes $c_{t\beta}^{e,k} + d_t^{e,k}$ where $d_t^{e,k}$ is the additional payout provided by the policy maker in the period of the withdrawal. The bailout given to any bank is then obtained as the sum of all extra payouts given by the policy maker to agents

in the bank. This is equivalent to assuming that the policy maker chooses not only the bailout given to a particular bank, but also how to distribute this bailout across the remaining agents within the bank (as was the case in Cyprus, Iceland, and elsewhere).²¹

Suppose we derive the equilibrium under the assumption from the baseline model in which agents with different wealth levels are grouped into different banks. We could then ask what other grouping(s) of agents into banks will generate the same equilibrium allocation? The following result provides an answer.

Proposition 9. *Suppose banks follow the proportional principle. Starting from any equilibrium of the economy with type-specific banks, suppose that (i) all agents who follow the no-run strategy in that equilibrium are grouped into one or more banks, and (ii) all agents who follow the partial-run strategy in that equilibrium are separately grouped into one or more banks. Then there exists an equilibrium under the new grouping that delivers the same allocation as the original equilibrium with type-specific banks.*

In other words, the equilibrium allocations obtained under type-specific banks will be robust to different groupings of agents into banks as long as agents who follow the partial run strategy are not grouped together with agents who follow the no-run strategy. So, suppose that in equilibrium an agent follows the partial run strategy if and only if his wealth level is greater than the cutoff for fragility e^f . According to Proposition 9 there always exists an equilibrium which replicates the equilibrium under type-specific banks as long as agents with wealth below e^f are grouped into banks separately from those with wealth above e^f .

There are, of course, a variety of different ways for the agents both above and below the cutoff to be grouped into banks. One way is to form type-specific banking arrangements as in the baseline version of the model. Another way is to form only two types of banks

²¹In order to ensure consistency with the baseline model, I impose the same limits on the amount of ex-post redistribution that can be undertaken through bailouts. Namely, taxation after the initial period is not allowed, a bailout can be made only to banks experiencing a run and super-insurance is strictly prohibited. As in the baseline model, these restrictions are necessary to ensure that the bailout program is not used as a means of redistribution independently of any banking crises.

- one “commercial bank” for agents below the cutoff and one “shadow bank” for agents above the cutoff. Both of these approaches yield the same outcome, but working with type-specific banks, as in the baseline case, turns out to be much simpler.²² Finally, notice that all banking arrangements that are equivalent to type-specific banks have one feature in common. Namely they are consistent with a type of “Volker rule” in the sense that risky deposits (i.e. those that will be withdrawn in a run) are not mixed in the same institution with safe deposits.

7 Conclusion

I analyzed a model in which the initial wealth distribution is linked to the allocation of bailouts during crises and shapes the sort of runs that can arise in equilibrium. This approach is motivated, in part, by some of the characteristics of the recent financial crises. In particular, the fiscal support to the distressed part of the financial system was based mainly on the policy maker’s willingness to bail out different type of agents and their institutions rather than by existing contractual obligations.

The model predicts that wealthier agents tend to experience fragility under a utilitarian policy maker because they anticipate the bailout intervention to favor them less compared to agents that are not as wealthy. In particular, for given distribution of wealth there is an endogenously determined cutoff point. Agents whose wealth level lies above the cutoff will be in the fragile region of the wealth distribution. That is, these agents will have incentive to panic and, in equilibrium, the institutions in which they invest are endogenously more likely to experience a run and receive a bailout.

I used the model to characterize how changes in the distribution of wealth affect financial fragility. More inequality leads to more fragility if it pulls more agents into the fragile region of the wealth distribution and to less fragility if the effect is the opposite. Moreover, the fragile region in the distribution of wealth has implications for the optimal

²²The analysis of a decentralized banking formation game, including how agents endogenously sort themselves into banks, is interesting but beyond the scope of this paper.

level of redistribution ex ante. In particular, redistribution ex ante will be more attractive if it pulls more agents away from the fragile region of the distribution of wealth and less attractive if the opposite effect occurs. The recognition of this fact implies that the optimal level of ex-ante redistribution can remain partial even absent the standard efficiency costs associated to progressive taxation. Finally, I showed how the banking arrangements underlying the model can be linked to real world financial organizations.

Appendix. Proofs of Selected Propositions

Proposition 1. *Under a strict no-bailouts rule the financial system is either fragile for all agents or fragile for none.*

Proof. The discussion in the text established that any type e of agent would best respond with the partial run strategy in (13) if and only if the condition in (14) holds. That is, the ratio $c_1^e/c_{2\beta}^e$ must be greater than one. Under a no-bailouts rule, this ratio does not depend on e and is greater than one only if the condition on the parameters in (19) holds. In order to see that, start by dividing both sides in (15) by $u'(c_{2\beta}^e)$ to obtain,

$$\frac{u'(c_1^e)}{u'(c_{2\beta}^e)} = R(1 - q)\frac{u'(c_{2\alpha}^e)}{u'(c_{2\beta}^e)} + Rq. \quad (21)$$

From (7), (8) and the fact that the function u exhibits constant relative risk aversion, the solution for $c_{2\beta}^e$ in terms of c_1^e satisfies the relationship

$$c_{2\beta}^e = R^{1/\gamma} \frac{(1-\tau) - \pi c_2^e}{(1-\pi)(\pi + (1-\pi)R^{1/(1-\gamma)})}.$$

where $\gamma > 1$ is the coefficient of relative risk aversion and λ_β depends only on parameters

$$\lambda_\beta = \frac{1}{(1-\pi)(\pi + (1-\pi)R^{1/(1-\gamma)})}.$$

Inserting the expression for $c_{2\beta}^e$ into (21) and using (4), it follows that the right hand side in (21) is independent of e . Moreover, from (21) it follows that $c_1^e/c_{2\beta}^e$ is also not a function of e . Thus, the financial system in the no-bailouts economy is either fragile for all agents (i.e. what was referred to as *maximally fragile* in the text) or fragile for none.

The main text also stated that the condition in (19) implies that the no-bailouts economy is maximally fragile. Indeed, set $b^e = 0$ and use (2), (4), (7), (8) and (15) to solve explicitly for c_1^e . That is,

$$c_1^e = \frac{(1-\tau)e}{\pi + ((1-q)\lambda_\alpha^{-\gamma} + q\lambda_\beta^{-\gamma})^{1/\gamma}},$$

where $\lambda_\alpha = ((1 - \pi)R^{1/(1-\gamma)})^{-1}$. The partial run strategy profile in (13) is a best response for any type e agents if the condition in (14) holds. That is,

$$\frac{c_1^e}{c_{2\beta}^e} = \lambda_\beta((1 - q)\lambda_\alpha^{-\gamma} + q\lambda_\beta^{-\gamma})^{-1/\gamma} > 1.$$

The restriction on parameters in (19) implies that the right hand side of the above is greater than one, yielding the desired result. □

Proposition 2. *If in equilibrium $b^{e*} > 0$ and $b^{\tilde{e}*} > 0$. Then $b^{e*} > b^{\tilde{e}*}$ whenever $e < \tilde{e}$.*

Proof. Suppose that type- e agents follow the partial run strategy in (13), then the bailout to bank e must be strictly below the full-insurance level of \bar{b}^e . Indeed, assume the bailout to bank e provides full insurance to its investors, i.e. $b^e = \bar{b}^e$ then we must have $c_{2\alpha}^e = c_{2\beta}^e$. Combining this last equality with (15) yields

$$u'(c_1^e) = Ru'(c_{2\beta}^e),$$

and therefore $c_1^e < c_{2\beta}^e$ which violates the best response condition in (14). In other words, if in equilibrium type- e agents follow the partial run strategy then $b^e < \bar{b}^e$. Next, assume that type- e agents and type- \tilde{e} agents follow the partial run strategy and their banks receive a bailout of $b^e > 0$ and $b^{\tilde{e}} > 0$ respectively. Then we must have

$$0 < b^e < \bar{b}^e \quad \text{and} \quad 0 < b^{\tilde{e}} < \bar{b}^{\tilde{e}}. \quad (22)$$

For fixed b^e , I write $c_1^e(b^e)$, $c_{2\alpha}^e(b^e)$, $c_{1\beta}^e(b^e)$, and $c_{2\beta}^e(b^e)$, to highlight that the payment plan in any bank e is a function of b^e . From (7), (4), (8) and (15), we have

$$\frac{\partial c_1^e(b^e)}{\partial b^e} > 0, \quad \frac{\partial c_{2\alpha}^e(b^e)}{\partial b^e} < 0, \quad \frac{\partial c_{1\beta}^e(b^e)}{\partial b^e} > 0 \quad \text{and} \quad \frac{\partial c_{2\beta}^e(b^e)}{\partial b^e} > 0, \quad (23)$$

as well as

$$\frac{\partial c_1^e(b^e)}{\partial e} > 0, \quad \frac{\partial c_{2\alpha}^e(b^e)}{\partial e} > 0, \quad \frac{\partial c_{1\beta}^e(b^e)}{\partial e} > 0 \quad \text{and} \quad \frac{\partial c_{2\beta}^e(b^e)}{\partial e} > 0. \quad (24)$$

Moreover, if $b^e > 0$ and $b^{\tilde{e}} > 0$, then (17) and (22) imply

$$c_{2\beta}^e(b^e) = c_{2\beta}^{\tilde{e}}(b^{\tilde{e}}).$$

Combining the above equality with (23) and (24) yields $b^e > b^{\tilde{e}}$ whenever $e < \tilde{e}$. The main text also stated that if two types of agents play the partial run strategy and the type with the lower wealth level best responds then the type with the higher wealth level best responds as well. This result follows immediately from (17) and (23). □

Proposition 3. *If, in a given equilibrium, agents with wealth level e are fully insured, then all agents whose wealth lies below e would also be fully insured.*

Proof. The definition of full insurance in Section 3.3 implies that if the level of the per-agent bailout payment in bank e corresponds to full insurance, $b^e = \bar{b}^e$, then the payment to an agent with wealth level e in period 2 does not depend on the sunspot state $c_{2\alpha}^e = c_{2\beta}^e$ (the reverse also holds: if $c_{2\alpha}^e = c_{2\beta}^e$ then $b^e = \bar{b}^e$). Using (2) and (15) we can express \bar{b}^e as follows

$$\bar{b}^e = \bar{b}(1 - \tau)e,$$

where \bar{b} depends only on parameters, and therefore, \bar{b}^e is an increasing function of e . Combined with Proposition 2 this yields the desired result. Namely, in any equilibrium if type- e agents receive full insurance then any type with lower wealth level will be fully insured as well. □

Proposition 4. *For a given economy, there exists a cutoff e^f such that the financial system is fragile for agents with wealth levels $e \geq e^f$ and not fragile for agents with wealth*

levels $e < e^f$.

Proof. Consider a cutoff strategy profile y_ζ such that any agent with wealth level e plays the partial run strategy in (13) if and only if his wealth level is greater than or equal to some cutoff point ζ , which belongs to the domain of the wealth distribution, i.e. $\zeta \in [\underline{e}, \bar{e}]$. Denote with $Y(\tau)$ the set of all cutoff strategy profiles consistent with equilibrium for given tax rate τ and let $e^f(\tau)$ denote the smallest of all cutoff points belonging to the set $Y(\tau)$. That is, $y_{e^f(\tau)} \in Y(\tau)$ and $e^f(\tau) \leq \zeta$ whenever $y_\zeta \in Y(\tau)$.

I show that the financial system is not fragile for any agent whose wealth lies below the cutoff point $e^f(\tau)$. Indeed, assume $\tilde{e} \in [\underline{e}, \bar{e}]$ is the lowest wealth level for which one can construct an equilibrium such that agents with wealth $e = \tilde{e}$ play the partial run strategy. It must then be the case that $e^f(\tau) \geq \tilde{e}$. If one can construct an equilibrium in which only a fraction of the agents with wealth level greater than or equal to \tilde{e} follow the partial run strategy, then clearly one can also construct an equilibrium where all of the agents with wealth level greater than or equal to \tilde{e} follow the partial run strategy (since augmenting the set of agents who run on the bank decreases the per-agent bailout payment in each bank). Therefore, $y_{\tilde{e}}$ is a cutoff strategy profile $y_{\tilde{e}} \in Y(\tau)$, and hence, $e^f(\tau) \geq \tilde{e}$. In other words, $e^f(\tau) \geq \tilde{e}$ and, at the same time, $e^f(\tau) \leq \tilde{e}$ which yields $e^f(\tau) = \tilde{e}$.

The following procedure can be used to establish if the financial system is fragile for any given type of agents. Fix a tax rate τ , derive the lowest cutoff point associated to a cutoff strategy profile and call this point $e^f(\tau)$. The financial system for this particular tax rate is then fragile for all agents whose wealth level is greater than or equal to $e^f(\tau)$. For each tax rate τ the *maximally fragile equilibrium* is the one where all agents with wealth greater than or equal to $e^f(\tau)$ play the partial run strategy in (13). Denote with $W_{MF}(\tau)$ the sum of all agents expected utilities in period 0 assuming that the maximally fragile equilibrium obtains and let $\tau^* = \operatorname{argmax}_{\tau \in [0,1]} W_{MF}(\tau)$ denote the optimal choice of tax rate within the maximally fragile equilibria. Finally, the financial system in the overall economy will not be fragile for any agent whose wealth level is below $e^f \equiv e^f(\tau^*)$.

□

Proposition 5. *In some economies, the cutoff point is interior, $\underline{e} < e^f < \bar{e}$.*

Proof. Consider equilibrium where each agent follows the partial run strategy if and only if his wealth level is greater than or equal to $e^f \equiv e^f(\tau^*)$. Based on the partial run strategy in (13) any bank e can experience a run only when the sunspot state is β . Assume that the probability of state β is equal to zero, that is $q = 0$. In this case, the optimal tax rate τ^* can be obtained in closed form as shown in (20). Moreover, when $q = 0$ we can solve explicitly for the profile of early payments in all banks $c_1^* = \{c_1^{e*}\}_{\underline{e}}^{\bar{e}}$ and the bailout specification $b^* = \{b^{e*}\}_{\underline{e}}^{\bar{e}}$. That is, the allocation for each agent type with wealth level below e^f is given in (16). Moreover for each agent type with wealth level greater than or equal to the cutoff point e^f we have

$$c_1^{e*} = \frac{(1-\tau^*)e}{\pi+(1-\pi)R^{(1-\gamma)/\gamma}}, \quad (25)$$

as well as,

$$b^{e*} = (1 - \tau^*)(a_1 e^f - a_2 e) \quad (26)$$

where both a_1 and a_2 depend only on parameters. From (26) the upper limit on the bailout region is therefore equal to $e^{NB} \equiv \min \left\{ \frac{a_1}{a_2} e^f, \bar{e} \right\}$ and the economy-wide bailout is

$$\begin{aligned} b^* &= (1 - \tau^*) \int_{e^f}^{e^{NB}} (a_1 e^f - a_2 e) dH(e) \\ &= (1 - \tau^*) \left[a_1 (e^f H(e^{NB}) - H(e^f)) - a_2 \int_{e^f}^{e^{NB}} e dH(e) \right], \end{aligned} \quad (27)$$

If the cutoff point e^f belongs to the interior of the wealth distribution (that is $\underline{e} < e^f < \bar{e}$) the incentive to run in (14) must hold with equality. That is, $c_1^{e*} = c_{2\beta}^{e*}$ for $e = e^f$. To see that, suppose this were not true and $c_1^{e*} > c_{2\beta}^{e*}$ when $e = e^f$. Then there exist another cutoff strategy profile y_ζ for some $\zeta \in (\underline{e}, e^f)$ which is also consistent with equilibrium.

But this leads to a contradiction since by construction e^f is the smallest point associated to a cutoff strategy profile. Next, applying (17) and using $c_1^{e^f} = c_{2\beta}^{e^f}$ yields,

$$Ru'(c_1^{e^*}) = Ru'(c_{2\beta}^{e^*}) = v(\tau^* - b^*) \quad \text{for } e = e^f. \quad (28)$$

Plugging the expression for b^* in (27) into the equation in (28) it follows that any interior cutoff point e^f also satisfies the following equation

$$\left(\frac{(1-\tau^*)e^f}{\pi+(1-\pi)R^{(1-\gamma)/\gamma}} \right)^{-\gamma} = \frac{\delta}{R} \left(\tau^* - (1-\tau^*) \left[a_1 (e^f H(e^{NB}) - H(e^f)) - a_2 \int_{e^f}^{e^{NB}} e dH(e) \right] \right)^{-\gamma}. \quad (29)$$

The existence of an interior solution to the above equation for various wealth distributions is established numerically on Figures 2a though 3a. Note that if the equation in (29) has more than one root then the cutoff point for fragility e^f is equal to the smallest root in the domain of the wealth distribution.

□

Proposition 6. *Suppose that for given τ we have $\underline{e} < e^f < \bar{e}$. Then, holding H fixed, we have $e^f < \tilde{e}^f$ whenever $\tau < \tilde{\tau}$.*

Proof. Fix a tax rate τ and assume the point cutoff point for fragility for this particular tax rate $e^f \equiv e^f(\tau)$ belongs to an interior point of the wealth distribution. Consider an equilibrium where type- e agents follow the partial run strategy in (13) only if their wealth level is greater than or equal to the cutoff point. In the general case where the probability of state β is positive i.e. $q > 0$ the early payments $c_1^{e^*}$ in each bank for which $e \geq e^f$ can be written as

$$c_1^{e^*} = g(e/e^f)(1-\tau)e, \quad (30)$$

whereas the bailout payment for each $e \geq e^f$ can be expressed as

$$b^{e^*} = f(e/e^f)(1-\tau)e. \quad (31)$$

The function $f(x)$ is strictly decreasing in x . The upper limit on the bailout region is obtained from $e^{NB} = \min \{\hat{e}^{NB}, \bar{e}\}$ where $\hat{e}^{NB} = \bar{x}e^f$ and $f(\bar{x}) = 0$. The economy-wide bailout package is

$$b^* = \int_{e^f}^{e^{NB}} f(e/e^f)(1-\tau)e dH(e). \quad (32)$$

Plugging (32) into the expression for (28) then implies that the interior cutoff point e^f must satisfy the following equation

$$(g(1)e^f)^{-\gamma} = \frac{\delta}{R} \left(\frac{\tau}{1-\tau} - \int_{e^f}^{e^{NB}} f(e/e^f) dH(e) \right)^{-\gamma}. \quad (33)$$

The above yields the desired result. Namely, holding the distribution of wealth fixed H fixed, the above implies that if the tax rate increases $\tilde{\tau} > \tau$ then the cutoff point also raises $\tilde{e}^f > e^f$. Note that, in the special case $q = 0$, we have

$$g(e/e^f) = \frac{1}{\pi + (1-\pi)R^{(1-\gamma)/\gamma}} \quad \text{and} \quad f(e/e^f) = a_1(e/e^f)^{-1} - a_2,$$

and thus the equation in (33) reduces to the equation in (29). □

Proposition 7. *Suppose that, for a given distribution of wealth H , we have $\underline{e} < e^f < \bar{e}$.*

Holding τ fixed, let \tilde{H} be a mean-preserving spread of H . Then,

- (i) *If $\tilde{h}(e) > h(e)$ for each $e \in [e^f, e^{NB}]$ then $\tilde{e}^f < e^f$.*
- (ii) *If $\tilde{h}(e) < h(e)$ for each $e \in [e^f, e^{NB}]$ then $\tilde{e}^f > e^f$.*

Proof. Start from a given distribution of wealth H with an associated cutoff for fragility e^f and a bailout region $[e^f, e^{NB}]$. Then, consider a new distribution of wealth \tilde{H} with a cutoff for fragility \tilde{e}^f and a bailout region $[\tilde{e}^f, \tilde{e}^{NB}]$. Suppose $\tilde{h}(e) > h(e)$ for each $e \in [e^f, e^{NB}]$, then the following relation must hold

$$b = (1-\tau) \int_{e^f}^{e^{NB}} f(e/e^f) e h(e) de < (1-\tau) \int_{e^f}^{e^{NB}} f(e/e^f) e \tilde{h}(e) de = \tilde{b}.$$

Applying (33) it follows that the cutoff for fragility in the economy corresponding to \tilde{H} is lower than the cutoff for fragility in the economy corresponding to H , that is $\tilde{e}^f < e^f$. Further, in some cases, we can determine whether the cutoff point will raise or fall as a result of a change in the initial distribution of wealth under more general shifts. In particular, suppose we move to a new distribution of wealth \tilde{H} such that tax revenue remains the same but the total fraction of wealth belonging to agents in the bailout region corresponding to H increases. That is,

$$\int_{e^f}^{e^{NB}} eh(e)de < \int_{e^f}^{e^{NB}} e\tilde{h}(e)de,$$

Note that $\tilde{h}(e) > h(e)$ for $e \in [e^f, e^{NB}]$ is sufficient but not necessary for the above relation to hold. Then as the probability of state β converges to zero (i.e. as q goes to zero) the equation in (29) combined with the above relation implies $\tilde{e}^f < e^f$.

□

Proposition 9. *Suppose all banks follow the proportional principle. Starting from any equilibrium of the economy with type-specific banks, suppose that (i) all agents who follow the no-run strategy in that equilibrium are grouped into one or more banks, and (ii) all agents who follow the partial-run strategy in that equilibrium are separately grouped into one or more banks. Then there exists an equilibrium under the new grouping that delivers exactly the same allocation as the original equilibrium with type-specific banks.*

Proof. To begin, fix a complete strategy profile $y = \{y^e\}_e^e$ for all agents and denote with $c^e \equiv \{c_1^e, c_{2\alpha}^e, \tilde{c}_{1\beta}^e, \tilde{c}_{2\beta}^e\}$ the allocation for type- e agents which obtains under a type-specific banking arrangement (i.e. when all types are grouped separately into banks). Suppose we form a new banking arrangement by grouping agents into banks in the manner stated in the proposition. I show that there always exists an equilibrium under this alternative banking arrangement which exactly coincides to the equilibrium obtained under type-specific banks.

The proof is by construction. Keep the same strategy profile y , and denote with $c^{e,k} \equiv$

$\{c_1^{e,k}, c_{2\alpha}^{e,k}, \tilde{c}_{1\beta}^{e,k}, \tilde{c}_{2\beta}^{e,k}\}$ the resulting allocation for type- e agents when they are grouped into bank k . The objective of any bank k is to maximize the sum of all its agents expected utilities subject to the proportional principle and taking as given the strategy profile of the investors, the bailout specification of the policy maker, and the payments made by all other banks. In equilibrium the allocation in any bank k must satisfy the following resource constraint in state α

$$\int_{\underline{e}}^{\bar{e}} \left[\pi c_1^{e,k} + (1 - \pi) \frac{c_{2\alpha}^{e,k}}{R} \right] dH^k(e) = (1 - \tau) \int_{\underline{e}}^{\bar{e}} e dH^k(e),$$

as well as the following resource constraint in state β

$$\int_{\underline{e}}^{\bar{e}} \left[\pi c_1^{e,k} + \pi_{\beta}^{e,k} \tilde{c}_{1\beta}^{e,k} + (1 - \pi - \pi_{\beta}^{e,k}) \frac{\tilde{c}_{2\beta}^{e,k}}{R} \right] dH^k(e) = (1 - \tau) \int_{\underline{e}}^{\bar{e}} e dH^k(e) + b^k.$$

where b^k is the total per-agent bailout to bank k . These two resource constraints simply state that the total outflow of resources in period 1 and 2 from any bank k in each state must come either internally (from endowments obtained as deposits in period 0 and placed into the constant return to scale technology) or externally by receiving additional resources from the policy maker in the form of a bailout transfer. It will be useful to define $b^{e,k}$ as per-agent bailout for type- e agents when they are grouped into bank k . That is,

$$b^{e,k} \equiv \pi_{\beta}^{e,k} (\tilde{c}_{1\beta}^{e,k} - c_{1\beta}^{e,k}) + (1 - \pi - \pi_{\beta}^{e,k}) \frac{\tilde{c}_{2\beta}^{e,k} - c_{2\beta}^{e,k}}{R}.$$

The total bailout to any bank k is then obtained by summing over all $b^{e,k}$ in bank k to obtain $b^k = \int_{\underline{e}}^{\bar{e}} b^{e,k} dH^k(e)$. Since by assumption all agents grouped into the same bank follow the same withdrawal strategy we have $\pi_{\beta}^{e,k} = \pi_{\beta}^k$ for each wealth level e . In addition, the proportional principle requires that the allocation for any two agents types, say e and \tilde{e} , grouped into the same bank k must satisfy $\frac{c^{e,k}}{e} = \frac{c^{\tilde{e},k}}{\tilde{e}}$. The first order condition for the optimal choice of payment within any bank k therefore reduce to the

following expressions for each agent type grouped into bank k

$$u'(\tilde{c}_{1\beta}^{e,k}) = Ru'(\tilde{c}_{2\beta}^{e,k}),$$

as well as,

$$u'(c_1^{e,k}) = (1 - q)Ru'(c_{2\alpha}^{e,k}) + qRu'(\tilde{c}_{2\beta}^{e,k}).$$

Moreover, if any bank k is bailed out then from (17) the consumption levels for each agent type in that bank must satisfy $\tilde{c}_{1\beta}^{e,k} = \tilde{c}_{1\beta}$ and $\tilde{c}_{2\beta}^{e,k} = \tilde{c}_{2\beta}$, where $\tilde{c}_{1\beta}$ and $\tilde{c}_{2\beta}$ are set to ensure that the marginal utilities in any bank with a bailout is equal to the marginal utility from the public good. That is,

$$u'(\tilde{c}_{1\beta}^{e,k}) = Ru'(\tilde{c}_{2\beta}^{e,k}) = v\left(\tau - \int_0^1 b^k \sigma_k dk\right).$$

where σ_k is the mass of agents grouped into bank k . From the above equilibrium conditions it follows that the allocation for any agent type e in any bank k coincides with the allocation for this type under type-specific banking, that is, $c^{e,k} = c^e$ and $b^e = b^{e,k}$. This can be verified by plugging the allocation and the bailout payment under type-specific banks in the above equilibrium conditions. In other words, we have constructed an equilibrium allocation under this alternative banking arrangement which exactly coincides with the equilibrium under type-specific banks. The reverse statement holds as well. That is, we can fix a complete strategy profile for all agents and then form a banking arrangement such that all agents grouped into the same bank follow the same withdrawal strategy and each bank follows the proportional principle. Then, keeping the same strategy profile for the agents, an identical equilibrium will result under a type-specific banking arrangement.

□

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