Lecture Note: Market Microstructure

Albert S. Kyle
University of Maryland

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Overview

Importance of adverse selection in financial market trading.


• Dealer markets and organized exchanges.

• Summary of issues in literature: Grossman and Stiglitz (1980) and related papers.

• One-period model of Kyle (1985).

• Continuous model of Kyle (1985).
Treynor Model of Adverse Selection

Dealer trades against on order which might come from an informed trader or a liquidity (noise) trader.

• Common Prior: Dealer believes value is $V = V_H = 110$ or $V_L = 90$ with equal probabilities.

• Order is from informed trader with probability $\pi = 0.10$, from noise trader with probability $1 - \pi = 0.90$.

• Informed traders observers value of $V_H = 110$ or $V_L = 90$ perfectly. Buys one contract if high, sells one contract if low.

• Noise trader buys of sells one contract randomly with equal probabilities.

What are bid and ask prices at which dealer breaks even?
Solution to Treynor Model

Conditional on buy order arriving, market maker calculates

\[ P_{ASK} = \pi \cdot V_H + (1 - \pi) \cdot E[V] \]
\[ = (0.10) \times (110) + (0.90) \cdot (100) \]
\[ = 101 \]  

(1)

Conditional on sell order arriving, market maker calculates

\[ P_{BID} = \pi \cdot V_L + (1 - \pi) \cdot E[V] \]
\[ = (0.10) \times (90) + (0.90) \cdot (100) \]
\[ = 99 \]  

(2)

Bid-ask spread is 2 dollars as a result of adverse selection. Can turn into dynamic model easily.
Dealer Markets and Organized Exchanges

• Dealer markets: Customers cannot post limit orders. Organized exchanges: respect time and price priority.

• Dealer markets are less anonymous: Dealers know identity of customer. Customer commits to size of order.

• Dealer markets have non-competitive arrangements: opaque inside markets for dealers only, implicit collusion over spreads.

• Literature assumes search is important in dealer markets. PK disagrees. Do customers search for dealers or vice versa?

• Dealer relationships are expensive for customers. PK thinks equilibrium is tournaments.
History

• Organized exchanges set up as cartels with monopolistic fixed commissions, supported by exclusive dealing and extensive self-regulation.

• Regulators banned fixed commissions in 1970s.

• Dealer market allows more rent extraction than centralized exchange. Opaque prices. Costly dealer relationships.

• Regulators opened up dealer markets in 1990s.

• Large tick size and high fees allowed organized exchanges (NYSE) market makers (specialists) to extract rents on organized exchanges.
Markets Today

- Regulators cut tick size to $0.01 in 2001.
- Regulation NMS (National Market System) in U.S. and Mi-FiD in EU broke monopoly franchise of organized exchange.
- Market fragmentation: competing exchanges with low fees.
- Institutional investors less likely to execute large blocks non-anonymously with dealers.
- Traders execute their own orders anonymously in fragmented markets with algorithmic trading.
- High frequency traders with fast algorithms have replaced human market makers.
- Equities, government securities, foreign exchange, commodity futures, corporate bonds changing rapidly.
Some Papers

- Grossman and Stiglitz (1980)
- Diamond and Verrecchia (1981)
- Hellwig (1980)
- Milgrom and Stokey (1982)
- Kyle (1989)
- Glosten and Milgrom (1985)
- Kyle (1985)
- Kyle and Lee (2017): Two papers
- Glosten (1994)
Technical Themes and Issues in the Papers

- Exponential utility and normally distributed random variables ("CARA-normal" assumptions) imply quadratic optimization with linear solutions.
- Rational expectations equilibrium: Traders learn from prices.
- Noisy rational expectations: Prices do not fully reveal the private information of informed traders.
- Need noise trading or overconfidence to generate trading in the presence of adverse selection.
- Competitive rational expectations: Unrealistic idea that traders think they do not affect the price.
- Imperfect competition: Consistent with linearity. Monopoly power over both information and price.
Some Economic Issues

• Relationship to efficient markets hypothesis: Are returns predictable given public information?

• Information content of prices: How much information do price reveal about “fundamental value”?

• Understanding market liquidity: Is liquidity supplied by intermediaries? Or do traders supply liquidity to one another?

• Incentives to acquire costly private information: What is the profitability of trading on costly private information?

• Competition and Monopoly Power: How important is strategic trading in financial markets?
Grossman and Stiglitz (1980)

• Competitive rational expectations equilibrium.
• Identical informed traders with same noisy signal.
• Identical uninformed traders (market makers).
• Exogenous noise traders.
• Exponential utility and normal random variables imply linear solution.
• Price is noisy signal of value (mixed with noise). Imperfect learning from prices.
• Solution is algebraically complicated because of asymmetry between informed traders and uninformed traders.
Diamond and Verrecchia (1982)

- Exponential utility, normally distributed random variables, competitive rational expectations equilibrium.
- Symmetrically informed traders, no uninformed traders or market makers.
- Trade motivated by uncorrelated endowment shocks of same variance.
- Informed traders have different private information of same precision.
- Equilibrium is much less algebraically complicated than Grossman and Stiglitz model.
Smart Money and Noise Trading

Simplest possible model with simplest notation. One Period Model: Informed investors trade with noise traders:

\[
\sigma^2_V = \text{Prior variance of liquidation value} \sim \frac{\text{Dollars}^2}{\text{Shares}^2}
\]

\[ A = \text{Agg. risk aversion (smart investors)} \sim \frac{1}{\text{Dollars}} \tag{3} \]

\[
\sigma^2_U = \text{Variance of noise trading} \sim \text{Shares}^2
\]

\[ \tau = \text{Precision of information} \sim 1 \sim \text{Dimensionless} \]

Dimensional Analysis: Buckingham \( \pi \) Theorem

• Use dimensional parameters \( \sigma^2_V \) and \( A \) for scaling.

• Use dimensionless parameters for solution:

\[ \tau = \text{“Information”}, \quad \theta := A\sigma_V\sigma_U = \text{“Noise”} \tag{4} \]
Setup

Assume $Z_V$, $Z_U$, $Z_I$ are NID(0, 1) and dimensionless.

\[
V = \sigma_V \cdot Z_V = \text{Liquidation Value} \\
U = \sigma_U \cdot Z_U = \text{Noise Trading} \\
I = \tau^{1/2} \cdot Z_V + (1 - \tau)^{1/2} \cdot Z_I = \text{Information}
\]

Timeline:

• Time 0: Informed traders observe $I$.
• Time 1: Trade at price $P$.
• Time 2: Liquidation $V$ realize, returns $V - P$. 
CARA–Normal Framework

With normally distributed $V$, solving

$$\max_x E[- \exp(-A(V - P)x)]$$

is equivalent to solving

$$\max_x E[(V - P)x - \frac{1}{2}A \text{var}[V - P] x^2].$$
Solution

Calculate expectations:

\[ E[Z_V | I] = \tau^{1/2} \cdot I, \quad \text{var}[Z_V | I] = 1 - \tau \]  

(8)

CARA-normal demands and market clearing imply

\[ \text{Demand} = \frac{E[V - P | I, P]}{A \cdot \text{var}[V | I, P]} = U = \text{Supply} \]  

(9)

Solve for \( P \):

\[ \frac{P}{\sigma_V} = \tau^{1/2} \cdot I - A(1 - \tau)\sigma_V^2 \cdot \frac{U}{\sigma_V} \]

\[ = \tau \cdot Z_V + \tau^{1/2}(1 - \tau)^{1/2} \cdot Z_I - (1 - \tau)\theta \cdot Z_U \]  

(10)
Sharpe Ratios

Sharpe ratio and expected squared Sharpe ratio for informed traders:

\[ SR_I = \frac{E[V - P | I]}{(\text{var}[V - P | I])^{1/2}} = \left( \frac{\tau}{1 - \tau} \right)^{1/2} \cdot I, \tag{11} \]

\[ E[(SR_I)^2] = \frac{\tau}{1 - \tau} = \frac{\text{Signal}}{\text{Noise}} \tag{12} \]

Note: In dynamic models, the Sharpe ratio has a time dimension, which is ignored here.
Statistical Identification

Mean and variance from perspective of “economist”:

$$E[V - P | P] = \frac{-(1 - \tau)^2 \theta^2}{\tau + (1 - \tau)^2 \theta^2} \cdot P \to 0 \text{ as } \theta \to 0,$$  \hspace{1cm} (13)

$$\frac{\text{var}[V - P | P]}{\sigma_V} = \frac{\tau(1 - \tau) + (1 - \tau)^2 \theta^2}{\tau + (1 - \tau)^2 \theta^2} \to 1 - \tau \text{ as } \theta \to 0. \hspace{1cm} (14)$$

$$\frac{\text{var}[P]}{\sigma_V} = \frac{\tau(1 - \tau) + (1 - \tau)^2 \theta^2}{\tau + (1 - \tau)^2 \theta^2} \to \tau \text{ as } \theta \to 0. \hspace{1cm} (15)$$

$$\frac{\text{var}[V - P]}{\sigma_V} = 1 - \tau + (1 - \tau)^2 \theta^2 \to 1 - \tau \text{ as } \theta \to 0. \hspace{1cm} (16)$$

Conclusion: $\tau$ and $\theta$ statistically identified from repeated realizations of $P$ and $V$.

Noise leads to excess volatility and mean reversion.
Homework:

- Verify equations in previous pages.
- Calculate Sharpe ratio and expected squared Sharpe for economist.
Where does research go from here?

“Market Order Model”: Mimics a dealer market by maintaining a distinction between dealers and other traders.


“Limit Order Model”: Equilibrium in demand schedules treats all traders symmetrically with single-price auction which protects time and price priority of orders. Important to consider strategic order submission.

What generates trade?

What prevents trade from collapsing?


• Endogenous hedging incentives large enough to overcome adverse selection. Kyle and Lee (2017)


Insight: Much of intuition from single-period model carries over to dynamic model.
Kyle (1985): Assumptions

• Monopolistic informed trader: Observes private signal. Takes account of impact on price.

• Noise trading: Exogenous normal distributions.

• Market makers: Risk neutral perfect competitors (reduced form). Implies “market efficiency”: Prices follow martingale.

• Note: No risk aversion in model!

• Equilibrium defined by optimal trading strategy for informed trader and pricing rule for market makers.

• One period, multi-period, and continuous-time models.

• “Market-order” model since informed trader conditions on information, not price.
One-Period Model

• Liquidation value $V$ has common prior distribution $V \sim N(P_0, \sigma_V^2)$ (dollars per share).

• Informed trader observes liquidation value $V$. Chooses quantity $X(V)$ to maximize expected profits.

• Noise traders trade exogenous quantity $U \sim N(0, \sigma_U^2)$.

• Market makers set price $P$ as function of “order flow” $Y = X + U$ such that $P(Y) = E[V \mid X + U = Y]$.

• Look for equilibrium with linear price function

$$P(Y) = P_0 + \lambda \cdot Y.$$  (17)
Solution to One-Period Model

• Informed traders choose \( X \) to maximize

\[
\text{Profit} = E \left[ (V - P_0 - \lambda \cdot (X + U)) \cdot X \mid V \right]
\]  
(18)

• First-order condition

\[
V - P_0 - 2 \cdot \lambda \cdot X = 0.
\]  
(19)

• implies

\[
X = \beta \cdot (V - P_0), \quad \text{where} \quad \beta = \frac{1}{2 \cdot \lambda}.
\]  
(20)

• Solution for \( \lambda \) is

\[
\lambda = \frac{\text{cov}[\beta \cdot V + U, V]}{\text{var}[\beta \cdot V + U]} = \frac{\beta \cdot \sigma_V^2}{\beta^2 \cdot \sigma_V^2 + \sigma_U^2}
\]  
(21)

• Model solution is

\[
\beta = \frac{\sigma_U}{\sigma_V}, \quad \lambda = \frac{1}{2} \cdot \frac{\sigma_V}{\sigma_U}.
\]  
(22)
Solution Properties

- Market impact costs arise endogenously in equilibrium. Noise traders’ losses equal informed trader’s profits.
- Market makers offer fixed, break-even supply schedule.
- Model solution can easily be guessed up to constants using dimensional consistency.
- Price is noisy: Only one half of informed trader’s private information is incorporated into prices.
- Equilibrium is unique within class of models offering linear price as function of order flow.
Kyle (1985): Continuous Model Assumptions

- Trading in interval $t \in [0, T]$.
- Liquidation value $V \sim N(P_0, T \cdot \sigma_V)$.
- Noise traders’ exogenous inventory process follows Brownian motion $dU(t) = \sigma_U \cdot dB(t)$.
- Informed trader trades $dX(t)$ to maximize profits, taking into account price impact in present and future.
- Order flow is $dY(t) = dX(t) + dU(t)$.
- Market makers set price so that

$$P(t) = \mathbb{E}[V \mid \text{Past Order Flow} = \{Y(t) : -\infty < s < t\}] . \tag{23}$$
Continuous-time Model Solution

• Supply schedule is a fixed linear schedule offering “instantaneous liquidity”:

\[ P(t) = P_0 + \lambda \cdot Y(t), \quad \lambda = \frac{\sigma_V}{\sigma_U}. \]  
(24)

• Informed trader moves price linearly toward \( V \):

\[ \frac{dX(t)}{dt} = \beta(t) \cdot (V - P(t))dt, \quad \beta(t) = \frac{1}{T - t} \cdot \frac{\sigma_U}{\sigma_V}. \]  
(25)

• Error variance of market makers is

\[ \text{var}[V \mid \text{Past Order Flow}] = (T - t) \cdot \sigma_V^2. \]  
(26)
Properties of Continuous Solution

• Derivative $1/(T - t)$ replaces $1/2$ in one-period model.

• Informed trader trades like perfectly discriminating monopolist, moving gradually along supply schedule of market makers.

• Informed trader does not intentionally camouflage his trading, but price is noisy estimate of value because informed trade is hidden in noise trading.

• Returns volatility is constant $\sigma^2_V$ (dollars per share per square root of time)

• Noise traders trade too aggressively. Could reduce transactions costs by one half by smoothing out trading over an arbitrarily short period of time!

- Number of traders within each group $M$
- Number of groups of traders $N$
- Fundamental volatility $\sigma_V$
- Residual uncertainty $\sigma_Y$
- Risk aversion $A$
- Private information $\tau_I$
- Endowment shock $\sigma_S$
- Exogenous noise trading $\Sigma_Z$
Measuring Information and Competition

• Equilibrium in demand schedules
• Traders learn from prices
• Informational efficiency $\phi$

$$\tau^* := \frac{\sigma_V^2}{\text{var} \{ v \mid i_l, s_l, p \}} = 1 + \tau_I + (N - 1) \tau_I \varphi. \quad (27)$$

• Traders trade strategically
• Competition $\chi$

$$\chi := \frac{x_i^*}{x_i^{PT}} = \frac{A\sigma_V^2 \left( \frac{1}{\tau^*} + \sigma_Y^2 \right)}{2\lambda + A\sigma_V^2 \left( \frac{1}{\tau^*} + \sigma_Y^2 \right)}. \quad (28)$$
Equilibrium

• Existence condition:

\[ \varphi < \varphi_{soc} := \frac{MN - 2}{MN - 2 + N}. \] (29)

• Informational efficiency:

\[
\frac{1}{\varphi} - 1 = \frac{(A\sigma_V \sigma_S)^2}{\tau_I} \left(1 + \sigma_Y^2 \tau^*\right)^2
+ \frac{(A\sigma_V \Sigma_Z)^2}{\tau_I M^2 (N - 1) \left(\frac{MN - 1}{MN - 2}\right)^2 \left(\frac{\varphi_{soc}}{\varphi_{soc} - \varphi}\right)^2 \left(1 + \sigma_Y^2 \tau^*\right)^2}.
\] (30)

• Competition and Information:

\[
\chi = \frac{\varphi_{soc} - \varphi}{\varphi_{soc} - \varphi + \frac{2(1+(N-1)\varphi)}{(MN-2+N)}}.
\] (31)
Take-aways

• Information is in the price. Competition is in the quantity.
• \( \varphi \) and the price are independent of \( M \) when \( \Sigma_Z^2 = 0 \).
• There is an inverse relationship between \( \varphi \) and \( \chi \).
• Perfect competition requires \( M \to \infty \) or \( \varphi \to 0 \)
• Grossman-Stiglitz paradox goes away because prices are never fully revealing in a perfectly competitive market.
• The puzzle is there may be no trade even when there are ex-ante gains from trade: No-trade theorem.
• If no trade, prices are not fully revealing unless \( M \to \infty \)
• Vanishing noise (\( \Sigma_Z^2 \to 0 \)) limit is well-defined.
Conclusion

Are there better mechanisms than single-price auctions for generating both better risk sharing and more informative prices simultaneously?

Can such a better mechanism be found in the standard CARA–normal setting?

QUESTIONS?