Blockholder Short-Term Incentives, Structures, and Governance*

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Abstract
We model blockholder governance as a sequential process, starting from less hostile private intervention, then more confrontational public intervention, and finally exit. When the blockholder is a fund manager facing short-term incentives, stemming from his concerns about investors’ perception of his stock-picking ability, the consequent trading reduction impairs the governance role of exit. The fund manager’s short-term incentives also weaken his incentive to publicly intervene following a failed private intervention; as a result, his threat of escalating the intervention to a more confrontational public stage loses credibility, which causes the firm’s management to be less cooperative in the private intervention stage, lowering the success rate of private intervention.

With multiple blockholders with heterogeneous incentive horizons, the reduction in public intervention by a fund manager with strong short-term incentives strengthens the public intervention incentive by a fund manager with less short-term incentives. The consequent amelioration of the free-rider problem restores the credibility of the threat of public intervention, causing the firm’s management to be more responsive to private intervention and increasing its success rate. Our key message is thus that although blockholder short-term incentives weaken governance in a single blockholder structure, they may enhance governance in a multiple blockholder structure with an appropriate mix of blockholders with various incentive horizons.

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1 Introduction

It has been widely argued that large shareholders (i.e., blockholders) with long-term incentives are instrumental to good corporate governance and long-term corporate investment. The logic is that blockholders with long-term incentives care about a company’s long-term performance and hence have strong incentives to encourage and monitor the management to ensure it delivers long-term value.\(^1\) Therefore, an optimal blockholder structure should consist of only blockholders with long-term incentive horizons. This paper shows that although this argument is true with a single blockholder, it may not hold with multiple blockholders. Specifically, we show how a diverse and asymmetric blockholder structure, consisting of blockholders with both long-term and short-term incentives, can help ameliorate the typical free-rider problem associated with multiple blockholders (e.g., Winton (1993), Noe (2002), and Edmans and Manso (2011)),\(^2\) hence enhance the efficacy of blockholder governance and foster long-term corporate investment. Like in many other economic settings, the free-rider problem arises with multiple symmetric blockholders when each individual blockholder underinvests effort in some value-enhancing intervention (e.g., monitoring the management) that he will gain benefit of regardless of his own effort. In other words, the corporate value enhancement from the intervention resembles a public good, and each individual blockholder believes that effort contributions by others would compensate for the lack of contribution by himself, so his own effort does not make a material difference to the outcome of the intervention. We show that a diverse and asymmetric blockholder structure may make an individual blockholder pivotal in whether the intervention is undertaken, thereby attenuating the free-rider problem.

To fix ideas, we start with a base model adapting the framework of Edmans (2009) with one blockholder. The blockholder sells (a fraction of) his block if he discovers that the firm he invested is of low quality (bad firm), while he does not sell if the firm turns out to be of high quality (good firm). The blockholder’s trading in the financial market is thus informative about firm quality, which encourages a good firm to pursue valuable long-term investment with less fear of producing

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\(^1\)Numerous policies have been proposed to foster investor long-termism, in order to promote long-term corporate investment. The 2009 Aspen Institute proposal, architected by several renowned investors like Warren Buffett and John Bogle, recommends various changes in capital gains tax rules to attract long-term investors in order to overcome corporate myopia; see “Overcoming short-termism: A call for a more responsible approach to investment and business management,” published by The Aspen Institute, September 9, 2009. The EU Commission is consulting on granting “extra voting rights and a bigger slice of dividends” to “loyal” shareholders who are long-term holders of a company’s stock, aiming to promote investor long-termism; see “Brussels aims to reward investor loyalty,” published by Financial Times, January 23, 2013. Bolton and Samama (2012) propose to issue “loyalty-shares” to provide extra reward to shareholders who “have held on to their shares for a contractually specified period of time, the loyalty period.”

\(^2\)McCahery, Sautner, and Starks (2014) show in their survey that the most important impediment to shareholder activism is the free-rider problem.
unfavorable short-term outcomes and hence being misvalued as a bad firm. This governance mechanism via blockholder trading, also examined in Admati and Pfleiderer (2009), albeit in a different setting,\(^3\) is known as exit or the “Wall Street Rule.”

Like Edmans, in this paper we also view the efficacy of blockholder governance as the extent to which the actions taken by the blockholder in a bad firm attenuates the undervaluation of a good firm, thereby overcoming the latter’s (undervaluation-induced) corporate myopia. However, we depart from Edmans in two important ways. First, instead of assuming the blockholder as a principal investor trading on his own account (an assumption also made in Admati and Pfleiderer), we model him as a fund manager acting as an agent of other capital providers. This is motivated by the fact that a significant fraction of U.S. publicly traded companies is collectively held by institutional investors who are often times delegated portfolio managers (e.g., mutual funds, hedge funds, and pension funds).\(^4\) Our starting point is that the fund manager may face short-term incentives and care about short-term fund performance, due to the well documented flow-performance relationship (e.g., Chevalier and Ellison (1997, 1999)). To elaborate, suppose the fund manager, after establishing his block in a firm, discovers that the firm is bad. He can sell a fraction of his block at some interim date. If he sells a large fraction, he will hurt the firm’s short-term stock price, hence depress the fund’s short-term performance relative to other funds, causing a big loss of short-term fund flow. This discourages fund selling, thereby impairing the governance role of exit. To capture the essence of the point in a simple way, the modeling approach we employ assumes that the fund manager cares about how his “stock-picking” ability is perceived by both current and potential capital providers.\(^5\) A low perception, derived from low interim stock price, usually results in redemptions by current investors, and may also discourage capital inflows from potential investors, which can ultimately jeopardize the fund manager’s compensation and even employment. Since selling more a bad firm’s stock sends a clearer signal to the market that he has invested in a bad firm, a reputation-conscious fund manager trades less aggressively. The trading reduction consequently causes the stock price to be less reflective of firm quality, thereby discouraging a good firm from pursuing long-term investment.

\(^3\)Admati and Pfleiderer focus on the disciplinary effect of the threat of blockholder trading in attenuating managerial underperformance.

\(^4\)Over 70% of all outstanding U.S. corporate equities are controlled by institutional money managers, including pension and retirement funds, mutual funds, and hedge funds (see, for example, the Federal Reserve Flow of Funds data), and many of these institutional investors have at least 5% equity ownership in a single firm (which typically defines a blockholder in the empirical literature); see Gopalan (2009).

\(^5\)McCahery, Sautner, and Starks (2014) find that negative inferences made by clients about their stock-picking skills represent an important (or very important) factor affecting institutional money managers’ trading. Section 6 considers alternative modeling assumptions about ability.
Second, while Edmans only considers exit, in our model the fund manager can also undertake corrective actions (referred to as intervention, monitoring, or voice), either publicly or privately, to improve a bad firm’s value. Specifically, we model blockholder governance as a sequential decision process, beginning with private intervention, then public intervention upon a failed private intervention, and finally exit. Private intervention, wherein the fund manager privately engages with a bad firm’s management (e.g., via private letter) by proposing value-enhancing changes, is less hostile and less costly to both the fund and the management.\textsuperscript{6} However, the success of such private engagement depends upon costly and unobservable effort put forth by the management to implement those changes. By contrast, public intervention, wherein the action of intervention is publicly observable (e.g., making a public shareholder proposal), is more confrontational and entails higher costs for both the fund and the management. Thus, blockholder governance in our model follows an escalating sequence from private and less hostile negotiations to public and more confrontational engagement.

We show that career concerns weaken a reputation-conscious fund manager’s incentive to engage in public intervention following a failed private intervention. The reason is that public intervention unambiguously reveals to the market that the fund has invested in a bad firm – we show that the reputational gain to the fund manager from not publicly intervening always outweighs the fund value loss due to the lack of public intervention. As a result, the threat by the reputation-conscious fund manager to escalate his intervention to a more confrontational public stage following a failed private intervention loses credibility, which causes the firm’s management to be less responsive to the changes proposed by the fund manager in the private intervention stage, making private intervention more likely to fail. Furthermore, trading reduction by the reputation-conscious fund manager in the exit stage drives the bad firm’s stock price further above its true value, causing the stock price to be less reflective of the management’s effort, which also weakens the management’s effort incentive in the private intervention stage. Therefore, with a reputation-conscious fund manager the bad firm’s value is less likely to be improved through intervention (public and private); as a result, the consequence to a good firm from being misvalued as a bad firm, following unfavorable short-term performance due to long-term investment, is more dire. The good firm then pursues less long-term investment, and corporate myopia arises.

\textsuperscript{6}The survey by McCahery, Sautner, and Starks (2014) documents widespread behind-the-scenes shareholder intervention through private discussion with management.
In sum, our analysis so far establishes that, with a single blockholder, (reputation-driven) blockholder short-term incentives weaken the effectiveness of blockholder governance and discourage long-term corporate investment.

We then extend the preceding analysis and examine blockholder governance in an asymmetric multiple blockholder structure, wherein the block is split between two funds with two fund managers having heterogeneous degrees of career concerns. Our main result is that a reputation-conscious fund manager’s weakened public intervention incentive strengthens a reputation-unconscious fund manager’s incentive to engage in public intervention. The key idea here is that the lack of public intervention by the reputation-conscious fund manager, due to his career concerns, makes the reputation-unconscious fund manager pivotal in whether the value-enhancing public intervention is undertaken following a failed private intervention – knowing that he cannot free-ride on the reputation-conscious fund manager and hence his effort makes a material difference to the outcome, the reputation-unconscious fund manager has strong incentive to engage in public intervention. Thus, the reputation-conscious fund manager’s career concerns serve as a credible commitment device that strengthens public intervention by the reputation-unconscious fund manager, thereby partially overcoming the free-rider problem typically associated with multiple blockholders. In particular, we show that, compared to a symmetric blockholder structure with two reputation-unconscious fund managers, neither of whom suffers (reputation-driven) short-term incentives, public intervention (following a failed private intervention) may be more intensive in the aforementioned asymmetric blockholder structure.

Furthermore, the amelioration of the free-rider problem in the public intervention stage restores the credibility of the threat that fund intervention will be likely to be escalated to a more confrontational public stage following a failed private intervention. Consequently, the firm’s management, anticipating the high cost that it will bear in the public intervention stage, becomes more responsive to the value-enhancing changes proposed in the private intervention stage, thereby increasing the success rate of private intervention. Therefore, the bad firm’s value may be more likely to be improved in the asymmetric blockholder structure (compared to the aforementioned symmetric structure), which, in turn, reduces a good firm’s undervaluation-induced investment myopia.

Let us summarize. Our key finding in this paper is that while (reputation-driven) blockholder short-term incentives worsen corporate myopia in a single blockholder structure, as is suggested by conventional wisdom, they may enhance blockholder governance (through amelioration of the free-
rider problem) and encourage long-term corporate investment in a multiple blockholder structure with a mixture of blockholders with both long-term and short-term incentives.

At a broad level, the analysis in this paper shows that our understanding of blockholder governance can be further enhanced by considering agency problems at the blockholder level together with firm-level agency problems, given the prevalence of institutional blockholders, many of whom are themselves agents and hence may face short-term incentives that are inconsistent with long-term shareholder value maximization. Our analysis points out that the interaction between the two agency problems should be examined in conjunction with blockholder structures. While blockholder short-term incentives always impair governance and hence induce corporate short-termism in a single blockholder structure, they can help improve the overall quality of governance in a diverse multiple blockholder structure, thereby promoting long-term corporate investment.

Although our reputation-based model of blockholder short-term incentives is rather specific, and there certainly could be other factors that engender blockholder-level agency problems, the core idea of the paper – that is, an individual blockholder’s short-term incentives enable a diverse and asymmetric blockholder structure to be effective in ameliorating the free-rider problem by directly breaking the condition that gives rise to the free-rider problem – has numerous implications (we discuss these implications in detail in Section 6). For example, the idea can be used to shed light on potential governance roles played by passive institutional investors like index funds and exchange-traded funds, whose ownership in U.S. public companies becomes growingly large. These funds normally do not engage in disciplinary trading due to their business models (their trading is largely non-discretionary); they are also typically non-activists, seldom openly engaging with management. Our analysis shows that such passivity may actually have governance value – the presence of such passive funds in a diverse multiple blockholder structure may help strengthen other activists’ incentive to engage in value-enhancing interventions.

The rest is organized as follows. Section 2 reviews the related literature, and delineates the paper’s incremental contributions. Section 3 develops the basic model with a single blockholder, and shows how career concerns weaken blockholder trading. Section 4 introduces intervention (public and private) into the basic model, and shows that career concerns also impair the governance role of intervention with a single blockholder. Section 5 examines the cases with multiple blockholders, and shows how career concerns may ameliorate the free-rider problem and enhance blockholder governance in a multiple blockholder structure. Section 6 discusses model robustness and implications. Section 7 concludes. All proofs are in the Appendix.
2 Related Literature

This paper is related to an extensive literature on blockholder governance. Our review focuses on the theoretical studies that are most relevant to our paper; see Edmans (2014) for a recent comprehensive survey of both the theoretical and empirical literature. The majority studies focus on large blockholders undertaking costly actions to initiate value-enhancing changes. These actions, ranging from correcting managerial wrongdoings to charting corporate policies, are often labeled as blockholder intervention, monitoring or voice. Examples include voting against management, or suggesting a corporate policy change by making a public shareholder proposal or via private communication to management; see McCalhery, Sautner, and Starks (2014) for a survey about specific channels of blockholder voice. Recent advances in blockholder models have started to examine an alternative governance mechanism via the so-called “Wall Street Rule.” In Admati and Pfleiderer (2009) and Edmans (2009), a blockholder who is unsatisfied with the current management can credibly threat to sell his stake. Due to the blockholder’s information advantage over other smaller investors, such trading is informative about the firm’s fundamentals and hence will generate a negative price impact. To the extent that managerial pay is tied to stock prices, the threat of selling by the blockholder may help discipline the management and improve corporate governance. This is commonly referred to as exit. While this literature models blockholders as value-maximizing principals and focuses on how factors like ownership structure and market liquidity affect blockholder voice and exit, we model blockholders as agents facing short-term incentives themselves, and examine how such incentives affect blockholder governance.

Assuming symmetric blockholders, Edmans and Manso (2011) examine their optimal number in a multiple blockholder structure by exploring the following tradeoff – while having more blockholders worsens the typical free-rider problem in intervention, it also causes each blockholder to trade more competitively, thereby impounding more information into prices and strengthening blockholder exit. Our paper extends their analysis by examining asymmetric blockholders. Fixing their number, we show that a greater asymmetry among blockholders, in terms of the degrees

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8 Edmans, Levit, and Reilly (2014) examine exit by investors owning blocks in multiple firms.

9 There is a large empirical literature testing those theories on blockholder voice and exit. For recent studies, see, for example, Carleton, Nelson, and Weisbach (1998), Parrino, Sias, and Starks (2003), Brav, Jiang, Partnoy, and Thomas (2008), Gopalan (2009), Klein and Zur (2009), Becht, Franks, Mayer, and Rossi (2010), Bharath, Jayaraman, and Nagar (2013), Edmans, Fang, and Zur (2013), and Norli, Ostergaard, and Schindele (2015), Gantchev (2013), in a multi-stage representation of blockholder voice, empirically estimates the costs of voice at each stage.
of their (reputation-induced) short-term incentives, can attenuate the free-rider problem and may consequently strengthen blockholder voice.

Most closely related is an interesting recent contribution of Dasgupta and Piacentino (2014), who also model blockholders as fund managers with short-term incentives. Based on the framework of Admati and Pfleiderer (2009), they show that a fund manager who cares about others’ perception of his stock-picking ability is less likely to engage in disciplinary exit, which, in turn, weakens his voice. While they show how a blockholder’s short-term incentives adversely affect corporate governance in a single blockholder structure and their analysis largely focuses on exit, we focus on voice (and we consider both public and private intervention and model them, together with exit, in a sequential decision process) and the central result of our paper reveals the beneficial role of such blockholder short-term incentives in a multiple blockholder structure in ameliorating the free-rider problem associated with multiple blockholders and enhancing blockholder voice.

Goldman and Strobl (2013) model a blockholder as a fund manager who may have to liquidate his shares before the market can correctly evaluate the firm’s investment. This “horizon mismatch” creates an incentive for the blockholder to refrain from disciplinary exit to inflate the short-term stock price, which interacts with the firm manager’s short-term incentives, resulting in excessive, value-destroying investment complexity. Like Dasgupta and Piacentino, their focus is also on the negative effect of blockholder short-term incentives. Also related is Levit (2013), who models private intervention as informal communication between a blockholder and management in a cheap-talk framework. He shows how exit can facilitate such communication by making it more credible from the management’s perspective, thereby improving the effectiveness of private intervention.

Our paper is also rooted in the vast theoretical literature on career concerns starting from the seminal work of Holmstrom (1999/1982). The essence of models in this literature is that an agent’s proclivity to positively influence others’ perception of his ability distorts his action choices.\(^\text{10}\) What is more closely related to our work is a burgeoning literature linking institutional investors’ career concerns to financial market equilibrium and asset price dynamics. Studies in Scharfstein and Stein (1990), Dasgupta and Prat (2006, 2008), Dasgupta, Prat, and Verardo (2011), and Guerrieri and Kondor (2012) show that institutional investors’ career concerns impede information incorporation into asset prices and lead to excessive price volatility, creating inefficiencies in financial market equilibrium.

\(^{10}\)For applications, see, for example, Holmstrom and Ricart i Costa (1986), Boot (1992), Song and Thakor (2006), and Grenadier, Malenko, and Strebulaev (2014).
3 The Basic Model: Exit by a Single Blockholder

Our basic model adapts the framework of Edmans (2009) with one blockholder. An all-equity firm, run by a CEO (she), has one share outstanding, of which $\alpha \in (0, 1)$ units are owned by a blockholder and the rest are collectively held by atomistic shareholders. The blockholder is a delegated portfolio (e.g., mutual fund, hedge fund, or pension fund) manager acting on behalf of many small investors, and henceforth is referred to simply as fund (he). All agents are risk neutral, and the risk-free rate is zero. There are three dates ($t = 0, 1, 2$), and the sequence of events is described below.

- **Events at $t = 0$.** The firm can be good ($G$) or bad ($B$). The firm’s value, which will realize at $t = 2$, is $V \in \{X, 0\}$; $V = X > 0$ ($V = 0$) for a good (bad) firm. The common prior belief at $t = 0$ is that the firm is good with probability (w.p.) $\varphi \in (0, 1)$, and bad w.p. $1 - \varphi$, where $\varphi$ is a random variable with density function $f(\varphi)$. Denote by $E(\varphi) = \bar{\varphi}$ and $\text{Var}(\varphi) = \sigma^2$ the expected value and the variance of $\varphi$, respectively. Nobody knows $\varphi$ a priori.

The firm’s type is then privately revealed to its CEO. A good firm may invest $v \in [0, 1]$ in intangible assets (e.g., brand recognition, patents, or goodwill) at $t = 0$, which increases its value by $gv$ at $t = 2$, with $g > 1$ being a constant. This investment opportunity is unavailable to a bad firm. Moreover, although such opportunity exists for a good firm, only its CEO knows it, while others, including the fund, are unaware of its presence.\(^\text{11}\)

There is also unobservable heterogeneity in fund ability. Specifically, a smarter fund is more likely to invest in a good firm. Without loss of generality we also designate $\varphi$ as fund ability, which nobody knows, including the fund himself when he establishes his block in the firm.

- **Events at $t = 1$.** The firm’s type is now also perfectly revealed to the fund. A public signal $s \in \{s_G, s_B\}$ emits, where $s_G$ is a good signal and $s_B$ is a bad signal; $s$ is the market’s short-term and usually imperfect projection about the firm’s long-term prospect. The common prior belief is $\Pr(s = s_G) = \Pr(s = s_B) = \frac{1}{2}$ for a good firm, while $\Pr(s = s_B) = 1$ for a bad firm. Thus, if $s = s_G$, it is commonly known that the firm is good; if $s = s_B$, the market believes that $\Pr(G) = \frac{\varphi}{2 - \varphi}$ and $\Pr(B) = \frac{2(1 - \varphi)}{2 - \varphi}$.

From a value-maximization perspective, a good firm should invest $v = 1$. However, although investment in intangible assets increases the long-term value, it risks lowering the market’s

\(^\text{11}\)This simplifying assumption follows Edmans (2009) for mathematical brevity in solving for the optimal $v$. We have shown that all the model’s tradeoffs and results sustain if we allow the investment to be anticipated by others. Edmans also shows that his results are unchanged if such investment opportunity is publicly known.
short-term assessment of the good firm. For example, expenditures in intangible investment lower a firm’s short-term earnings, but the market may not be able to fully tell if the weak earnings result from low firm quality or desirable long-term investment. Specifically, if $v \in [0, 1]$ is invested, then for a good firm $\Pr(s = s_G) = \frac{1-v^2}{2}$ and $\Pr(s = s_B) = \frac{1+v^2}{2}$. This reflects the aforementioned tension that more long-term investment (larger $v$) lowers the market’s short-term (imperfect) assessment (lower $\Pr(s = s_G)$).

Since the fund knows the firm’s type, his trading allows the market to draw further inferences about the firm. After $s$ is released, trading occurs in a financial market with a structure similar to Kyle (1985). The fund may sell a fraction of his block, $\beta \in [0, \alpha]$, which will be endogenously determined. We assume $\beta \leq \alpha$ to preclude short selling. There are also liquidity traders who demand $u$, a random variable with density function $r(u)$, where $r(u) = \lambda e^{-\lambda u}$ if $u > 0$, and $r(u) = 0$ if $u \leq 0$, with $\lambda > 1$. A larger $\lambda$ corresponds to a less liquid market; note $\mathbb{E}(u) = \lambda^{-1}$. The market maker, unaware of the value-enhancement opportunity to a good firm, observes total demand $d = u - \beta$, and sets a competitive stock price, $P = \mathbb{E}(V|s, d)$.

- **Events at $t = 2$.** The firm’s type and its value (including the value enhancement, $g\upsilon$, for a good firm) are realized and observed by all. The firm is then liquidated, and agents get paid.

The fund has career concerns in the sense that he cares about others’ perception of his ability. Fund investors (hereafter, investors) update their beliefs about the fund’s ability using Bayes’ rule at $t = 1$. The fund’s utility depends on the investors’ assessment of his ability and fund profit:

$$U_f = \pi_f + \kappa_f \mathbb{E}(\varphi|\mathcal{F}_1),$$

where (i) $\mathbb{E}(\varphi|\mathcal{F}_1)$ is the investors’ ability assessment at $t = 1$, conditional on their information at that date, $\mathcal{F}_1$, including the signal ($s$) and the stock price ($P$); and (ii) $\kappa_f \geq 0$ is the weight that the fund attaches to career concerns relative to fund profit, $\pi_f = \beta P + (\alpha - \beta)V$. Note $\beta P$ is the trading profit from the $\beta$ units sold at $t = 1$, and $(\alpha - \beta)V$ is the $t = 2$ value of unsold units.

The component of the fund’s utility that depends on his perceived ability, $\kappa_f \mathbb{E}(\varphi|\mathcal{F}_1)$, is meant to capture all the consequences that this perception impinges on the fund’s utility calculation. In the delegated portfolio management industry, the investors’ perception of a money manager’s ability affects how the manager’s human capital is valued by the market, hence his compensation and possibly the employment continuation/termination decision. A low perception usually results
in fund redemption, since investors will chase skilled managers and reallocate their capital. To the extent that fund management fees are often tied to the size of assets under management (AUM), it is transparent that a fund has concerns with how his ability is perceived.\footnote{Our exogenous specification of fund career concerns can be endogenized by analyzing commonly used “two-and-twenty” compensation contracts in the money management industry with a linear AUM fee and a convex performance fee (carried interest). Dasgupta and Piacentino (2014) show how such compensation practice generates competition for short-term fund flows among money managers, which hinges on investors’ inferences of the managers’ stock-picking ability from interim stock prices. A low interim stock price hurts a fund’s short-term return relative to others, causing investors to lower their assessment of the fund manager’s ability; the consequent fund outflow reduces the fund’s AUM fee and carried interest. The fund manager thus has an incentive to enhance the investors’ ability assessment by reducing disciplinary trading (which lowers the interim stock price).}

The CEO’s utility places weight $\delta \in (0, 1)$ on the $t = 1$ stock price and $1 - \delta$ on the $t = 2$ firm value $V + gv \times 1_{\{G\}}$ (where $1_{\{G\}} = 1$ ($0$) for a good (bad) firm):\footnote{A good firm’s CEO knows the value-enhancement opportunity, so her utility accounts for the enhancement, $gv$.}

$$U_c = \delta P + (1 - \delta)(V + gv \times 1_{\{G\}}).$$

(2)

### 3.1 Equilibrium in the Financial Market

We begin by characterizing the market maker’s equilibrium pricing function, taking as given fund trading. If $s = s_G$, the market maker knows the firm is good, and therefore sets $P = X$. The fund does not trade in this case ($\beta = 0$) with a fully revealing price. The following analysis considers $s = s_B$, wherein firm type is not fully revealed; the notation “$s_B$” is omitted for brevity. The fund may trade to exploit his information advantage, and the market maker infers the fund’s information from total order $d$. A Nash equilibrium in the market consists of the fund’s trading ($\beta$) and the market maker’s pricing ($P$), such that trading maximizes the fund’s utility in (1) given the pricing, and the pricing uses all the information contained in $d$ and yields zero profit to the market maker.

**Lemma 1.** Upon observing $s_B$ and total order $d$ at $t = 1$, the market maker sets the price:

$$P = \begin{cases} 
0 & \text{if } d \leq 0 \\
\Pr(G|d > 0)X & \text{if } d > 0,
\end{cases}$$

(3)

where

$$\Pr(G|d > 0) = \frac{\varphi}{\varphi + 2(1 - \varphi)e^{-\lambda\beta}} \in (\bar{\varphi}, 1),$$

(4)

which is increasing in $\beta$. The fund sells ($\beta > 0$) if the firm is good, while he does not trade ($\beta = 0$) if the firm is bad.
The fund only sells a bad firm’s stock, whose price is no smaller than the true firm value, thereby generating trading profit. Thus, if \( d = u - \beta \leq 0 \), the market maker knows the fund has sold (since \( u > 0 \)) and the firm must be bad, and therefore sets \( P = 0 \). If \( d > 0 \), the market maker is not sure if the fund has sold, and sets \( P \) based on his posterior belief in (4), which exceeds the true firm value \( (V = 0) \), justifying fund selling. To see why the posterior \( \Pr(G|d > 0) \) is increasing in \( \beta \), note with a larger \( \beta \) the probability of having \( d > 0 \) (upon \( s = s_B \)) is \textit{ceteris paribus} smaller, so the event \( d > 0 \) implies it is more likely that the fund has \textit{not} sold and hence the firm is good.

Although investors do not observe \( d \), they can infer the market maker’s posterior from the signal \( s \) and the price \( P \). If \( s = s_G \), they know the firm is good; if \( s = s_B \), they believe the firm is good w.p. \( \frac{P}{X} \). To ease the exposition, we therefore assume investors update their beliefs as if they also observe \( d \), and use the term “market” to denote both the market maker and investors.

3.2 The Benchmark Case: Exit without Fund Career Concerns

The benchmark case, which is also analyzed in Edmans (2009), considers a reputation-unconscious fund \( (\kappa_f = 0) \). Lemma 1 shows that the fund sells \( \beta \) only if the firm is bad \( (\text{so } s = s_B) \), whose value \( (V = 0) \) may not be fully reflected in its stock price. We analyze a Bayesian Perfect Nash equilibrium, consisting of the fund’s private choice of \( \beta \), and the market’s belief about it \( (\beta^*_n) \), such that the fund’s choice maximizes his utility, and the market’s belief (updated using Bayes’ rule) coincides with the fund’s choice in equilibrium.\(^\text{14}\)

The fund’s problem (condition on the firm being bad) is

\[
\max_{\{\beta\}} U_f = (\alpha - \beta)(0) + \beta e^{-\lambda \beta} \Pr(G|d > 0)X + (1 - e^{-\lambda \beta})(0). \tag{5}
\]

To interpret (5), note (i) the fund makes a trading profit, \( \beta \Pr(G|d > 0)X \), where \( \Pr(G|d > 0) = \frac{\bar{\varphi} e^{-\lambda \beta}}{\bar{\varphi} + 2(1-\bar{\varphi})e^{-\lambda \beta}} \), only if \( d > 0 \) (the bad firm is not fully revealed), which occurs if \( u > \beta \), w.p. \( \int_{\beta}^{\infty} \lambda e^{-\lambda u} du = e^{-\lambda \beta} \); (ii) w.p. \( 1 - e^{-\lambda \beta} \), the bad firm is perfectly revealed \( (d \leq 0) \), and trading yields no gain; and (iii) \( (\alpha - \beta)(0) \) is the \( t = 2 \) value of the fund’s unsold units.

The CEO of a good firm chooses the intangible investment \( \upsilon \) to maximize her utility:

\[
\max_{\{\upsilon\}} U_c = (1 - \delta)(X + g\upsilon) + \delta \left[ \frac{1 - \upsilon^2}{2} X + \frac{1 + \upsilon^2}{2} \Pr(G|d > 0)X \right]. \tag{6}
\]

\(^{14}\)The equilibrium definition does not specify any beliefs corresponding to out-of-equilibrium moves. This is because there are no out-of-equilibrium moves.
The CEO’s utility attaches weight $1-\delta$ on the $t=2$ firm value, $X + g\upsilon$. The second term in (6) captures the CEO’s utility derived from the $t=1$ stock price, with a weighting factor $\delta$. If $s=s_G$ (w.p. $\frac{1-\upsilon^2}{2}$), the market correctly sets $P = X$. If $s=s_B$ (w.p. $\frac{1+\upsilon^2}{2}$), the fund, knowing the firm is good, does not sell; in this case $d > 0$, and $P = \Pr(G|d > 0)X$, where $\Pr(G|d > 0) = \frac{\bar{\phi}}{\bar{\phi} + 2(1-\bar{\phi})e^{-\lambda\beta^*}}$.

**Proposition 1.** A reputation-unconscious fund sells $\beta_s = \min(\lambda^{-1}, \alpha)$ if and only if the firm is bad. A good firm invests $\upsilon_s = \min\left(\frac{1-\delta}{\delta} \frac{\bar{\phi} + \frac{2(1-\bar{\phi})e^{-\lambda\beta^*}}{2}}{\bar{\phi} - \lambda\upsilon_s} \frac{\bar{g}}{X}, 1\right)$, where $\upsilon_s$ and $\beta_s$ are positively correlated.

While selling more a bad firm’s share *ceteris paribus* increases the fund’s trading profit, it also increases the odds of revealing the bad firm. This tradeoff determines the optimal fund trading. Note (i) $\beta_s \leq \alpha$, due to the short-sale constraint; and (ii) in a more liquid market, liquidity traders on average buy more (larger $\mathbb{E}(u) = \lambda^{-1}$), which reduces the odds of revealing the bad firm by fund selling, and hence encourages fund trading (larger $\beta_s$). Proposition 1 is reminiscent of Edmans’s key finding that active blockholder trading can foster, rather than hinder, long-term investment (larger $\upsilon_s$). What impedes long-term investment is the market’s inability to fully tell if a bad signal ($s_B$) arises from bad firm quality or a good firm pursuing long-term investment. A larger $\beta_s$, meaning that the fund would have sold more aggressively if the firm were bad, allows the market to draw sharper inferences when $d > 0$ (note $d > 0$ for sure for a good firm) – with a larger $\beta_s$, the event $d > 0$ implies it is more likely that the fund has not sold and hence the firm is good (Lemma 1). Emitting $s_B$ is thus less dire to the good firm’s CEO, who then chooses more long-term investment.

We subsequently assume $\alpha > \lambda^{-1}$, so $\beta_s = \lambda^{-1}$. This means the market is not sufficiently liquid so the fund does not sell its entire stake in a bad firm.\(^\text{15}\)

### 3.3 Exit with Fund Career Concerns

We now examine a reputation-conscious fund ($\kappa_f > 0$), who may be tempted to distort his trading, relative to the benchmark in Proposition 1, to positively influence others’ perception of his ability. We first characterize the market’s potential assessments of the fund’s ability at $t=1$.

**Lemma 2.** Suppose $\sigma^2 < \bar{\phi}(1-\bar{\phi})$. The market’s posterior belief about the fund’s ability is

$$
\mathbb{E}(\phi|G) = \bar{\phi} + \frac{\alpha^2}{\bar{\phi}} \in (\bar{\phi}, 1),
$$

\(^1\)\^\text{15\textsuperscript{\text{We make this assumption since our focus, unlike Edmans’s, is not on how the block size ($\alpha$) affects exit.}}\)
if the firm is revealed to be good, and

\[ \mathbb{E}(\varphi| B) = \bar{\varphi} - \frac{\sigma^2}{1 - \bar{\varphi}} \in (0, \bar{\varphi}), \]  

(8)

if the firm is revealed to be bad at \( t = 1 \).

Consider a bad firm (so \( s = s_B \)). We again analyze a Bayesian Perfect Nash equilibrium, similarly defined as in Section 3.2. Denote \( \beta_f \) as the market’s equilibrium belief about fund trading. If total share demand is negative (\( d \leq 0 \)), the bad firm is fully revealed, and the market believes

\[ \mathbb{E}(\varphi| d \leq 0) = \mathbb{E}(\varphi| B) = \bar{\varphi} - \frac{\sigma^2}{1 - \bar{\varphi}}. \]  

(9)

If \( d > 0 \), the market cannot unambiguously tell the firm’s type, and its posterior ability inference, given its equilibrium belief about fund trading (\( \beta_f \)), is

\[ \mathbb{E}(\varphi| d > 0) = \mathbb{E}(\varphi| B) + \frac{2(1 - \bar{\varphi})e^{-\lambda \beta_f}}{2 + 2(1 - \bar{\varphi})e^{-\lambda \beta_f}} \left( \bar{\varphi} - \frac{\sigma^2}{1 - \bar{\varphi}} \right). \]  

(10)

With a bad firm the correct ability assessment should be \( \mathbb{E}(\varphi| B) \). However, given the fund’s private choice of \( \beta \), this assessment forms only if \( d \leq 0 \) (w.p. \( 1 - e^{-\lambda \beta} \)); if \( d > 0 \) (w.p. \( e^{-\lambda \beta} \)), the market cannot fully tell the firm’s type, and fund ability is overly assessed in (10). Thus, although the fund’s choice of \( \beta \) cannot influence the market’s equilibrium inferences, \( \mathbb{E}(\varphi| d \leq 0) \) and \( \mathbb{E}(\varphi| d > 0) \),\(^{16}\) it does affect the relative probabilities of the occurrences of the two reputational outcomes.

Conditional on the firm being bad, the fund chooses \( \beta \) to maximize his expected utility:

\[ \max_{\{\beta\}} U_f = \beta e^{-\lambda \beta} \Pr(G| d > 0)X + \kappa_f \left[ (1 - e^{-\lambda \beta})\mathbb{E}(\varphi| d \leq 0) + e^{-\lambda \beta} \mathbb{E}(\varphi| d > 0) \right], \]  

(11)

where \( \Pr(G| d > 0) = \frac{\bar{\varphi}}{\bar{\varphi} + 2(1 - \bar{\varphi})e^{-\lambda \beta_f}} \). Note (i) fund profit, \( \beta e^{-\lambda \beta} \Pr(G| d > 0)X \), is calculated as in (5); and (ii) the second term is the reputation part of fund utility, where the state \( d \leq 0 \) (\( d > 0 \)) occurs w.p. \( 1 - e^{-\lambda \beta} \) (\( e^{-\lambda \beta} \)), and the corresponding ability assessment is \( \mathbb{E}(\varphi| d \leq 0) \) (\( \mathbb{E}(\varphi| d > 0) \)).

\(^{16}\)The two inferences are based on the market’s belief about the fund’s choice of \( \beta \), which is \( \beta_f \).
The problem for a good firm’s CEO is same as in the benchmark case (see (6)), and hence is not repeated here. For our subsequent analysis, it is useful to introduce the following notation:

\[ \Delta \equiv \kappa_f \left[ \mathbb{E}(\varphi|G) - \mathbb{E}(\varphi|B) \right] X = \frac{\kappa_f \sigma^2}{\bar{\varphi}(1 - \bar{\varphi})X}. \] (12)

The numerator of \( \Delta \) equals to the weight that the fund attaches to his career concerns relative to fund profit, \( \kappa_f \), times the wedge between the market’s best and worst possible ability assessments, \( \mathbb{E}(\varphi|G) - \mathbb{E}(\varphi|B) \). Thus, \( \Delta \) captures the degree of fund career concerns, with a larger \( \Delta \) indicating a more reputation-conscious fund with stronger short-term incentives.\(^{17}\)

**Proposition 2.** A reputation-conscious fund sells \( \beta_f = \max(\lambda - 1 - \Delta, 0) < \beta_* \) if and only if the firm is bad. A good firm invests \( \upsilon_f = \min\left(1 - \delta \left( \frac{\bar{\varphi} \kappa_f}{\delta} e^{-\lambda \beta_f} + 2(1 - \bar{\varphi}) X \right) - \lambda \beta_f \right) < \upsilon_* \), which is decreasing in \( \Delta \).

This proposition establishes that a reputation-conscious fund trades less relative to a reputation-unconscious fund (\( \beta_f < \beta_* \)). To see it, suppose the market believes \( \beta_f = \beta_* \) with fund career concerns, so its posterior ability inferences, \( \mathbb{E}(\varphi|d \leq 0) \) and \( \mathbb{E}(\varphi|d > 0) \), are based on \( \beta_* \). Importantly, as noted earlier, the fund’s private choice of \( \beta \) only affects the probabilities of the occurrences of the two reputational outcomes but not the outcomes themselves. Since a lower \( \beta \) increases the likelihood of the occurrence of the state \( d > 0 \) and hence the more favorable outcome, \( \mathbb{E}(\varphi|d > 0) \), the fund has a proclivity to privately lower \( \beta \) to positively influence the market’s ability assessment. However, a lower \( \beta \) also decreases trading profit. This tradeoff between reputational concerns and trading profit results in a choice of \( \beta_f < \beta_* \) by the fund. A fund with sufficiently strong career concerns (\( \Delta \geq \lambda^{-1} \)) fully gives up trading profit by choosing \( \beta = 0 \), so as to maximize the probability of the occurrence of the favorable outcome to one by ensuring \( d > 0 \).

Since \( \beta_f < \beta_* \), the undervaluation of a good firm’s stock upon a bad signal (\( s_B \)) is made worse relative to the benchmark case. Anticipating that, the good firm’s CEO invests less in intangible assets (\( \upsilon_f < \upsilon_* \)) to reduce the odds of emitting \( s_B \). A more reputation-conscious fund (larger \( \Delta \)) trades even less (smaller \( \beta_f \)), further discouraging long-term investment (smaller \( \upsilon_f \)).

In sum, our analysis so far shows that a reputation-conscious fund will internalize the impact of his trading on others’ perceptions of his ability and consequently reduce trading to make these perceptions more favorable. This proclivity to influence ability perceptions through reduced trading

\(^{17}\)In Dasgupta and Piacentino (2014), the cross-sectional variation in fund short-term incentives arises from the variation in fund managers’ contractual incentives (which hinge on fund size and flow, and hence the market’s ability assessment; see discussions in footnote 12) relative to their self-investment in their own funds. They show that while the former gives rise to fund short-term incentives, the latter attenuates such incentives.
impedes information incorporation into prices, and hence weakens blockholder exit and hinders long-term corporate investment.

4 Intervention by a Single Blockholder

We now introduce blockholder intervention into the basic model with one fund, whereby intervention is broadly defined and incorporates any value-enhancing activities the fund may undertake, from monitoring the CEO to prevent managerial wrongdoings to influencing firm strategy.

4.1 Intervention as an Escalation Process

Our model tries to capture several important aspects of shareholder intervention in reality (see Gantchev (2013), and McCahery, Sautner, and Starks (2014)):

- We consider both public intervention and private intervention. Public intervention, wherein the action of intervention is publicly observable, encompasses any open contest with the management, from making shareholder proposals to launching a proxy contest. Besides that, shareholders may also engage in informal and “behind-the-scenes” interactions and negotiations with the management about ways to improve firm value (i.e., private intervention).

- Private intervention is less hostile and less costly to both shareholders and the management, but has a lower success rate compared to public intervention, which is more confrontational and entails higher costs for both shareholders and the management.

- Shareholders usually first try to privately engage with the management about their demand for changes, and take public measures only after private negotiations have failed. That is, the intervention process is, as described by Gantchev (2013), an “escalating sequence from less hostile to more confrontational... from private to more public forms of engagement.”

Specifically, consider a bad firm (so \( s = s_B \)). The sequence of events at \( t = 1 \) of the extended model with intervention, depicted in Figure 1 with three stages, is as follows.

1. **Private intervention stage.** The fund first privately communicates with the CEO about value-enhancing changes. While such communication is costless to the fund, implementing changes requires costly and unobservable CEO effort. There is thus a layer of agency problem here. If the CEO exerts effort \( \sigma \in [0, 1] \), incurring a personal cost \( c(\sigma) \), the bad firm’s value
improves to $X$ (i.e., successful private intervention) w.p. $\sigma$, whereas w.p. $1 - \sigma$ firm value remains 0 (i.e., failed private intervention). We assume $c(\sigma)$ is an increasing and convex function that satisfies the Inada conditions, $c'(0) = 0$ and $c'(1) = \infty$. Upon a successful private intervention, the outcome is publicly observable, and the game ends. A failed private intervention is known only to the fund and the CEO but not the market.

2. **Public intervention stage.** Suppose the private intervention failed. The fund may escalate the engagement into public intervention. While private intervention is costless to the fund, public intervention is costly. The fund’s cost of public intervention is $\tau$, a random variable uniformly distributed on $[0, 1]$. The fund *privately* observes $\tau$, then decides whether to intervene publicly, in which case the intervention is publicly observable and succeeds for sure, improving firm value to $X$. Public confrontation is also costly to the CEO: we assume the CEO’s utility drops to zero upon public intervention by the fund.

3. **Exit stage.** If the fund does not publicly intervene following a failed private intervention, he trades as in the basic model.

---

18 The variable $\tau$ is meant to capture all the costs that the fund bears in engaging in public intervention; examples include costs of monitoring and mounting a contest with the management. The uniform distribution assumption for $\tau$ is an innocuous assumption that is merely made for mathematical simplicity.

19 Nothing changes qualitatively if we assume that public intervention succeeds with some probability less than one.

20 This extreme assumption is meant to capture, in the simplest way for mathematical brevity, the fact that public confrontation with shareholders is very costly to the CEO. Nothing changes as long as the CEO suffers a sufficiently large utility loss upon public intervention.

21 This specification is consistent with the survey finding in McCahery, Sautner, and Starks (2014) that intervention typically precedes exit.
As is standard, our analysis uses backward induction beginning with public intervention, assuming private intervention has failed. We then work backwards to the private intervention stage.

### 4.2 Public Intervention

Suppose private intervention failed (which is known only to the fund and the CEO). We examine fund public intervention, beginning with a reputation-unconscious fund. We analyze a Bayesian Perfect Nash equilibrium, and conjecture the market’s equilibrium beliefs are that the fund publicly intervenes if and only if $\tau \leq \tau^*$, and sells $\beta^{no}_*\phi$ absent public intervention, while abstains from selling with public intervention. Given such beliefs the bad firm’s stock is priced as follows at $t = 1$:

- If total share demand $d \leq 0$, the market knows the fund has sold, hence the firm is bad, private intervention has failed (note the fund will not sell following a successful private intervention), and there is no public intervention; therefore, the market sets $P = 0$.

- If $d > 0$ and public intervention is observed, the market knows the firm is bad but the public intervention increases its value to $X$, and hence sets $P = X$.

- If $d > 0$ without visible intervention, the market cannot tell apart the following possibilities: (i) the firm is good (w.p. $\frac{\phi}{2-\phi}$) that needs no intervention; or (ii) the firm is bad (w.p. $\frac{2(1-\phi)}{2-\phi}$), private intervention has failed but public intervention is not effected (w.p. $\Pr(\tau > \tau^*) = 1 - \tau^*$), so the fund sells $\beta^{no}_*$, but $d = u - \beta^{no}_* > 0$ (w.p. $e^{-\lambda \beta^{no}_*}$). The market thus sets

  $$P = \frac{\phi}{\phi + 2(1-\phi)(1-\tau^*)}e^{-\lambda \beta^{no}_*}X.$$

  (13)

The fund, after observing $\tau$, decides whether to publicly intervene. With public intervention, the market sets $P = X$. The fund is indifferent between selling and holding in this case; we assume holding ($\beta = 0$) due to some unmodeled costs of selling (e.g., short-term capital gains taxes). If the fund does not engage in public intervention, he privately knows the bad firm is still worth $V = 0$, and sells $\beta$ to maximize his expected utility, $U_f = \beta e^{-\lambda \beta}P$, where $P$ is given by (13).\(^{22}\) Clearly, the optimal solution is $\beta_*^{no} = \lambda^{-1}$ as in the basic model. The fund publicly intervenes if and only if $\tau$ is sufficiently small, so that his continuation utility is higher with public intervention than without.

\(^{22}\)Note total share demand $d > 0$ w.p. $e^{-\lambda \beta}$. 
Lemma 3. With a reputation-unconscious fund, there is a unique cutoff, $\tau_*$, defined by

$$
\tau_* = h(\beta_*^{no}, \tau_*) \equiv \left[ \alpha - \frac{\beta_*^{no} e^{-\lambda \beta_*^{no} \bar{\varphi}}}{\bar{\varphi} + 2(1 - \bar{\varphi})(1 - \tau_*) e^{-\lambda \beta_*^{no}}} \right] X, \quad (14)
$$

such that following a failure of private intervention, the fund publicly intervenes if and only if $\tau \leq \tau_*$, and sells $\beta_*^{no} = \lambda^{-1}$ absent public intervention.

Note $h(\beta_*^{no}, \tau_*)$, defined in (14), is the fund’s value enhancement from public intervention instead of trading (i.e., exit), which offsets the fund’s cost of public intervention ($\tau_*$) in equilibrium.

The case with a reputation-conscious fund can be similarly analyzed, except now career concerns also affect fund public intervention and trading. The result is summarized below.

Proposition 3. With a reputation-conscious fund, there is a unique cutoff, $\tau_f$, defined by

$$
\tau_f = h(\beta_f^{no}, \tau_f) - \frac{e^{-\lambda \beta_f^{no} \bar{\varphi}}}{\bar{\varphi} + 2(1 - \bar{\varphi})(1 - \tau_f) e^{-\lambda \beta_f^{no}}} \Delta X, \quad (15)
$$

such that following a failure of private intervention, the fund publicly intervenes if and only if $\tau \leq \tau_f$, and sells $\beta_f^{no} = \max(\lambda^{-1} - \Delta, 0)$ absent public intervention. The likelihood of public intervention by a reputation-conscious fund is lower than that by a reputation-unconscious fund ($\tau_f < \tau_*$), and is further decreasing in the degree of fund career concerns ($\frac{\partial \tau_f}{\partial \Delta} < 0$).

In (15), $h(\beta_f^{no}, \tau_f)$ is the fund’s value enhancement from public intervention compared to trading, similar as $h(\beta_*^{no}, \tau_*)$ in (14). The extra term, $\frac{e^{-\lambda \beta_f^{no} \bar{\varphi}}}{\bar{\varphi} + 2(1 - \bar{\varphi})(1 - \tau_f) e^{-\lambda \beta_f^{no}}} \Delta X$, is the fund’s reputational loss from public intervention instead of trading (i.e., reputational gain from trading instead of public intervention). Note (i) public intervention perfectly reveals the bad firm, causing the market’s ability assessment to drop to its lowest level, $E(\varphi|B)$; and (ii) absent public intervention the fund sells $\beta_f^{no}$ and only reveals the bad firm w.p. $1 - e^{-\lambda \beta_f^{no}}$ (when $d \leq 0$). Reputational gain from abstaining from public intervention arises when $d > 0$, w.p. $e^{-\lambda \beta_f^{no}}$, and the market mistakenly perceives the (bad) firm to be good, w.p. $\frac{\varphi}{\bar{\varphi} + 2(1 - \bar{\varphi})(1 - \tau_f) e^{-\lambda \beta_f^{no}}}$.\footnote{This probability can be derived in the same way as that for (13), with $\tau_*$ and $\beta_*^{no}$ being replaced with $\tau_f$ and $\beta_f^{no}$, respectively.}

When the two events occur, the fund gains reputation, $\kappa_f[E(\varphi|G) - E(\varphi|B)] = \Delta X$. Proposition 3 establishes that a reputation-conscious fund, despite his larger post-trading stake in a bad firm ($\alpha - \beta_f^{no} > \alpha - \beta_*^{no}$), engages in less public intervention than a reputation-unconscious fund ($\tau_f < \tau_*$). This is again rooted in the reputation-conscious fund’s inclination to positively
influence the market’s perception of his ability through distorted action choices – by engaging in less public intervention, hence reducing the odds of revealing the bad firm, the fund’s reputational gain outweighs its value loss due to the reduction in public intervention.\footnote{We show in the Appendix (Proof of Proposition 3) that the utility increase from public intervention instead of trading is always larger for a reputation-conscious fund than for a reputation-unconscious fund. The idea is as follows. Although a reputation-conscious fund sells $\Delta$ less than a reputation-unconscious fund absent public intervention, the reduction in trading profit (due to reduced trading) is exactly offset by the fund’s gain in reputation from trading $\Delta$ less – this is how the reputation-conscious fund’s equilibrium trading is determined. Thus, the reputation-conscious fund’s utility from trading can be calculated as if the fund also sells $\lambda^{-1}$, same as a reputation-unconscious fund. However, the key is that the expected stock price at which trading takes place is higher with a reputation-conscious fund, because his trading reduction lowers the odds of revealing the bad firm. This causes the reputation-conscious fund’s utility increase from public intervention instead of trading to be smaller than that for a reputation-unconscious fund, making public intervention relatively less desirable for the reputation-conscious fund.}

### 4.3 Private Intervention

We now move back to the private intervention stage. We first examine the case with a reputation-unconscious fund. The bad firm’s CEO, being advised by the fund about value-enhancing changes, privately chooses effort $\sigma$ to maximize her utility:

$$\max_{\{\sigma\}} \sigma[\delta X + (1 - \delta)X] + (1 - \sigma)(1 - \tau_*)e^{-\lambda_\beta^\Delta}(\delta P) - c(\sigma),$$

where $P$ is given by (13). To understand (16), note (i) w.p. $\sigma$ firm value is improved, the outcome of the successful private intervention becomes public, the market sets the $t = 1$ stock price as $X$, and the CEO’s corresponding utility is $\delta X + (1 - \delta)X$; (ii) w.p. $1 - \sigma$ private intervention fails, in which case the CEO knows that in the subgame of public intervention which is more confrontational, w.p. $\tau_*$ the fund publicly intervenes, driving the CEO’s utility to zero, and w.p. $1 - \tau_*$ the fund does not engage in public intervention (Lemma 3); and (iii) absent public intervention the fund sells $\beta_{*}^\Delta$, in which case total share demand $d > 0$ w.p. $e^{-\lambda_\beta^\Delta}$, and the corresponding $t = 1$ stock price is given by (13) (the CEO’s utility in this case is $\delta P$), whereas w.p. $1 - e^{-\lambda_\beta^\Delta}$, $d \leq 0$ and the stock price is 0 (the CEO’s utility in this case is 0). The last term is the CEO’s cost of effort.

It follows directly from (16) that the CEO’s equilibrium effort, hence the success rate of private intervention with a reputation-unconscious fund, denoted by $\sigma_*$, is given by

$$c'(\sigma_*) = \left[1 - \delta \frac{\varphi(1 - \tau_*)e^{-\lambda_\beta^\Delta}}{\varphi + 2(1 - \varphi)(1 - \tau_*)e^{-\lambda_\beta^\Delta}} \right] X,$$

where $\beta_{*}^\Delta = \lambda^{-1}$ and $\tau_*$ is defined by (14).
We can similarly analyze the case with a reputation-conscious fund, and show that the CEO’s
equilibrium effort, hence the success rate of the reputation-conscious fund’s private intervention,
denoted by $\sigma_f$, is given by
\[
d'(\sigma_f) = \left[ 1 - \frac{\bar{\varphi}(1 - \tau_f)e^{-\lambda\beta_{no}}}{\bar{\varphi} + 2(1 - \bar{\varphi})(1 - \tau_f)e^{-\lambda\beta_{no}}} \right] X, \tag{18}
\]
where $\beta_{no} = \max(\lambda^{-1} - \Delta, 0)$ and $\tau_f$ is defined by (15).\(^{25}\)

**Proposition 4.** The success rate of private intervention is lower with a reputation-conscious fund
than with a reputation-unconscious fund ($\sigma_f < \sigma_*$), and is further decreasing in fund career concerns
($\frac{\partial \sigma_f}{\partial \Delta} < 0$). A CEO whose utility attaches greater weight to the $t = 1$ stock price relative to the $t = 2$
firm value (larger $\delta$) exerts less effort; as a result, the success rate of private intervention decreases
($\frac{\partial \sigma_*}{\partial \delta} < 0$ and $\frac{\partial \sigma_f}{\partial \delta} < 0$), and becomes more sensitive to fund career concerns ($|\frac{\partial \sigma_f}{\partial \Delta}|$ increases).

The CEO’s effort, hence the success rate of private intervention, negatively hinges on her con-
tinuation utility in the subgame of public intervention following a failed private intervention,\(^{26}\)
which depends on the likelihood of public intervention (CEO utility decreases to zero in this case)
and fund trading absent public intervention. We know from Proposition 3 that a more reputation-
conscious fund (larger $\Delta$) (i) is less likely to engage in public intervention; and (ii) trades less absent
public intervention in order to reduce the odds of revealing the bad firm, which, in turn, drives
the bad firm’s stock price further above the true firm value (which is 0 absent public intervention
following a failed private intervention). Consequently, the CEO, whose utility is partially tied to
the inflated stock price, makes a windfall gain. Both (i) and (ii) increase the CEO’s continuation
utility following a failed private intervention, thereby lowering her incentive to exert effort.

Interestingly, in the exit stage the reputation-conscious fund and the CEO have *shared* incentives
to mislead the market in its valuation of the firm – the fund’s incentive is rooted in his inclination to
positively influence the market’s perception of his ability through reduced trading, and the CEO’s
incentive arises from her desire to make windfall gains from inflated stock prices. Furthermore, the
two incentive problems interact with each other – trading reduction by the fund causes the stock
price to be less reflective of the true firm value (hence CEO effort), fueling the CEO’s moral hazard
and lowering her effort incentive in the private intervention stage. This problem worsens when the

\(^{25}\)Inada conditions satisfied by $c(\cdot)$ ensure that $\sigma_* \in (0, 1)$ and $\sigma_f \in (0, 1)$.

\(^{26}\)Note the CEO’s continuation utility following a successful private intervention is $\delta X + (1 - \delta)X = X$. 

20
CEO’s own incentive horizon becomes shorter, i.e., when her utility attaches greater weight to the short-term stock price relative to the long-term firm value (larger $\delta$).

Two layers of complementarity in blockholder governance are identified. First, as discussed above, there is complementarity between fund exit and private intervention – when exit becomes less effective, private intervention is also weakened. This complementarity works through, and is strengthened by, the CEO’s short-term incentives ($\delta$). Second, public intervention and private intervention also exhibit complementarity – fund short-term incentives (induced by reputation concerns) reduce the credibility of the fund’s threat to escalate his intervention to a confrontational public stage following a failed private intervention, which weakens the CEO’s effort incentive, lowering the success rate of fund private intervention.

Proposition 4 highlights the CEO’s incentive horizon as an important driver behind the success/failure of shareholder private intervention. Private intervention is less likely to succeed, hence shareholders are more likely to resort to more confrontational public measures, when the CEO has a shorter incentive horizon (larger $\delta$). Moreover, a shorter CEO incentive horizon further worsens the adverse impact of fund short-term incentives on private intervention ($|\frac{\partial \sigma_f}{\partial \Delta}|$ increases with $\delta$).

Having analyzed blockholder governance in a bad firm, we now examine how that affects a good firm’s long-term investment. Below we show that, same as what is shown in Proposition 2 wherein only exit is considered, in the extended model wherein the fund can govern through private intervention, public intervention and exit, fund career concerns again hinder long-term investment.

**Proposition 5.** When a fund may engage in private intervention, public intervention and exit, a good firm still makes less intangible investment when facing a more reputation-conscious fund.

The fund does not intervene in a good firm, nor does he sell its shares. Thus, upon releasing a bad signal ($s_B$), the only relevant state for the good firm is that total share demand $d > 0$ with no visible intervention. With a less reputation-conscious fund, more visible public intervention can be committed in a bad firm following a failed private intervention (Proposition 3), the event $d > 0$ with no visible intervention thus indicates that the firm is more likely to be good. This makes emitting $s_B$ less dire to the good firm, thereby encouraging its long-term investment.
5 Multiple Blockholders

We now extend the preceding analysis to examine intervention by multiple blockholders. Our main idea can be illustrated in a most parsimonious way with two funds, each owning $\frac{1}{2}$ units of share.\footnote{Our focus with multiple blockholders, unlike Edmans and Manso (2011), is not on their optimal number.}

Again, consider a bad firm (so $s = s_B$). Private intervention is modeled in the same way as that with one fund: value-enhancing changes are proposed to the CEO (by either or both funds), and the success of private intervention depends on CEO effort. If private intervention fails, the public intervention stage ensues. Public intervention is costly to both funds; their costs, $\tau$, are uniformly distributed on $[0, 1]$ and independent of each other. The bad firm’s value, following a failed private intervention, can be improved from 0 to $X$ as long as one fund publicly intervenes.

We examine and compare three blockholder structures:

- A symmetric structure with two reputation-unconscious funds, denoted as UU;
- A symmetric structure with two reputation-conscious funds, denoted as CC; and
- An asymmetric structure with a reputation-unconscious fund and a reputation-conscious fund, denoted as UC.

In each case, we again use backward induction and begin by analyzing public intervention, and then move back to the private intervention stage.

5.1 UU Structure

Suppose private intervention failed. In the ensuing public intervention stage, we analyze a symmetric Bayesian Perfect Nash equilibrium, and conjecture that the market believes that each reputation-unconscious fund (i) publicly intervenes if and only if $\tau \leq \tau_{**}$; and (ii) sells $\beta_{**}$ if neither fund publicly intervenes, while abstains from selling as long as one fund publicly intervenes. Given such beliefs, the market sets the firm’s $t = 1$ stock price as $P = 0$ if total share demand $d \leq 0$, and $P = X$ if $d > 0$ with public intervention being observed. If $d > 0$ without visible intervention, the market weighs the following two possibilities in forming its posterior belief: (i) a good firm that needs no intervention (w.p. $\frac{3}{2-\phi}$); or (ii) the firm is bad (w.p. $\frac{2(1-\phi)}{2-\phi}$), private intervention has
failed but public intervention is not undertaken by either fund (w.p. \((1 - \tau_{*})^2\)), therefore each fund sells \(\beta_{*}\), but \(d = u - 2\beta_{*} > 0\) (w.p. \(e^{-2\lambda\beta_{*}}\)). The market thus sets

\[
P = \frac{\bar{\varphi}}{\bar{\varphi} + 2(1 - \bar{\varphi})(1 - \tau_{*})^2e^{-2\lambda\beta_{*}}X}.
\]

(19)

We assume \(\alpha > 2\lambda^{-1}\) throughout the rest of the analysis. It can be easily verified that, same as the single-blockholder case with a reputation-unconscious fund, with two reputation-unconscious funds each fund sells \(\beta_{*} = \lambda^{-1}\) if neither publicly intervenes.

In the private intervention stage, the problem to the bad firm’s CEO is

\[
\max_{\{\sigma\}} \sigma[\delta X + (1 - \delta)X] + (1 - \sigma)(1 - \tau_{*})^2e^{-2\lambda\beta_{*}}(\delta X) - c(\sigma),
\]

(20)

where \(P\) is given by (19). The CEO’s problem in (20) is similar to that in (16), except that in this case with two funds, the bad firm is not perfectly revealed in the public intervention stage if (i) neither fund publicly intervenes (w.p. \((1 - \tau_{*})^2\)); and (ii) in the subsequent exit stage demand by liquidity traders exceeds aggregate selling by the two funds (w.p. \(e^{-2\lambda\beta_{*}}\)).

**Lemma 4.** In a UU structure, if private intervention fails, there is a unique cutoff, \(\tau_{*}\), defined by

\[
\tau_{*} = (1 - \tau_{*})H(\beta_{*}, \tau_{*}) \equiv (1 - \tau_{*}) \left[ \frac{\alpha}{2} - \frac{\beta_{*}e^{-2\lambda\beta_{*}}\bar{\varphi}}{\bar{\varphi} + 2(1 - \bar{\varphi})(1 - \tau_{*})^2e^{-2\lambda\beta_{*}}} \right] X,
\]

(21)

such that in a symmetric equilibrium each reputation-unconscious fund publicly intervenes if and only if \(\tau \leq \tau_{*}\), and sells \(\beta_{*} = \lambda^{-1}\) if neither publicly intervenes. Private intervention succeeds w.p. \(\sigma_{*}\), given by

\[
c'(\sigma_{*}) = \left[ 1 - \delta \frac{\bar{\varphi}(1 - \tau_{*})^2e^{-2\lambda\beta_{*}}}{\bar{\varphi} + 2(1 - \bar{\varphi})(1 - \tau_{*})^2e^{-2\lambda\beta_{*}}} \right] X.
\]

(22)

\(H(\beta_{*}, \tau_{*})\) differs from \(h(\beta_{*}, \tau_{*})\) in (14) in two aspects. First, each fund now only owns \(\frac{\alpha}{2}\) units of share, which \textit{ceteris paribus} lowers the intervention-induced value enhancement for each fund compared to the single-fund case.

\footnote{H(\beta_{*}, \tau_{*}) differs from h(\beta_{*}, \tau_{*}) in (14) in two aspects. First, each fund now only owns \(\frac{\alpha}{2}\) units of share, which \textit{ceteris paribus} lowers the intervention-induced value enhancement for each fund compared to the single-fund case.}
alizes as long as the other fund publicly intervenes, will incur the cost, $\tau_{ss}$, to publicly intervene by himself only if the other fund does not publicly intervene (which occurs w.p. $1 - \tau_{ss}$ in equilibrium).

### 5.2 CC Structure

The CC structure with two reputation-conscious funds can be similarly analyzed.

**Lemma 5.** In a CC structure, if private intervention fails, there is a unique cutoff, $\tau_{ff}$, defined by

$$\tau_{ff} = \left(1 - \tau_{ff}\right) \left[ H(\beta_{ff}, \tau_{ff}) - \frac{e^{-2\lambda\beta_{ff}\varphi}}{\varphi + 2(1 - \varphi)(1 - \tau_{ff})^2e^{-2\lambda\beta_{ff}}} \Delta X \right],$$  \tag{23}$$

such that in a symmetric equilibrium each reputation-conscious fund publicly intervenes if and only if $\tau \leq \tau_{ff}$, and sells $\beta_{ff} = \max(\lambda^{-1} - \Delta, 0)$ if neither publicly intervenes. Private intervention succeeds w.p. $\sigma_{ff}$, given by

$$c'(\sigma_{ff}) = \left[1 - \delta \frac{\varphi(1 - \tau_{ff})^2e^{-2\lambda\beta_{ff}}}{\varphi + 2(1 - \varphi)(1 - \tau_{ff})^2e^{-2\lambda\beta_{ff}}} \right] X.$$  \tag{24}$$

Equation (23) can be similarly understood as (21). The key difference between the two lies in the additional term, $\frac{e^{-2\lambda\beta_{ff}\varphi}}{\varphi + 2(1 - \varphi)(1 - \tau_{ff})^2e^{-2\lambda\beta_{ff}}} \Delta X$, in (23), which captures each fund’s reputational loss from engaging in public intervention instead of trading.\(^{29}\)

### 5.3 UC Structure

We now examine the UC structure with a reputation-unconscious fund and a reputation-conscious fund. We start by analyzing a Bayesian Perfect Nash equilibrium in the public intervention stage. Suppose the market’s equilibrium beliefs are that the reputation-unconscious (reputation-conscious) fund (i) publicly intervenes if and only if $\tau \leq \tau_{sf}$ ($\tau \leq \tau_{fs}$); and (ii) sells $\beta_{sf}$ ($\beta_{fs}$) if neither fund publicly intervenes, while refrains from selling as long as one fund publicly intervenes.\(^{30}\)

As in other cases, the market sets the firm’s $t = 1$ stock price as $P = 0$ if total share demand $d \leq 0$, and $P = X$ if $d > 0$ with visible intervention. If $d > 0$ with no visible intervention, the market weighs the following two possibilities in forming its posterior belief: (i) a good firm requiring no intervention (w.p. $\frac{\varphi}{2 - \varphi}$); or (ii) the firm is bad (w.p. $\frac{2(1 - \varphi)}{2 - \varphi}$), private intervention has failed but

\(^{29}\)The intuition for this extra term is similar as that for the extra term in (15) compared to (14).

\(^{30}\)Note the difference in subscripts in different notations: “$f$” means that the fund in consideration is reputation-unconscious while the other fund is reputation-conscious; “$s$” represents the opposite case.
public intervention is not undertaken by either fund (w.p. \((1 - \tau_f)(1 - \tau_{f*})\)), so both funds sell, but \(d = u - (\beta_{sf} + \beta_{f*}) > 0\) (w.p. \(e^{-\lambda(\beta_{sf} + \beta_{f*})}\)). Therefore, the market sets

\[
P = \frac{\bar{\psi}}{\bar{\psi} + 2(1 - \bar{\psi})(1 - \tau_{sf})(1 - \tau_{f*})e^{-\lambda(\beta_{sf} + \beta_{f*})}X. \tag{25}
\]

In the private intervention stage, the problem to the bad firm’s CEO is

\[
\max_{\sigma} \sigma[\delta X + (1 - \delta)X] + (1 - \sigma)(1 - \tau_{sf})(1 - \tau_{f*})e^{-\lambda(\beta_{sf} + \beta_{f*})}(\delta P) - c(\sigma), \tag{26}
\]

where \(P\) is given by (25). The problem in (26) is similar as that in (20), except that in this case with an asymmetric structure, following a failure of private intervention the bad firm is not perfectly revealed w.p. \((1 - \tau_{sf})(1 - \tau_{f*})e^{-\lambda(\beta_{sf} + \beta_{f*})}\). Solving (26) yields the CEO’s equilibrium effort, hence the success rate of private intervention with the asymmetric structure (denoted by \(\sigma_{asy}\)):

\[
c'(\sigma_{asy}) = \left[1 - \delta \frac{\bar{\psi}(1 - \tau_{sf})(1 - \tau_{f*})e^{-\lambda(\beta_{sf} + \beta_{f*})}}{\bar{\psi} + 2(1 - \bar{\psi})(1 - \tau_{sf})(1 - \tau_{f*})e^{-\lambda(\beta_{sf} + \beta_{f*})}}\right] X. \tag{27}
\]

It is straightforward to verify that, same as the single-fund cases (Lemma 3 and Proposition 3), in a UC structure absent public intervention by either fund, the reputation-unconscious (reputation-conscious) fund sells \(\beta_{sf} = \lambda^{-1} (\beta_{f*} = \max(\lambda^{-1} - \Delta, 0))\). The complexity of the analysis for this case with an asymmetric structure arises from the possibility that the two funds may publicly intervene with different probabilities – it is possible \(\tau_{sf} \neq \tau_{f*}\) – and they need to be jointly determined.

To start with, we first prove that, interestingly, in a UC structure wherein the reputation-conscious fund’s career concerns are not too strong (\(\Delta \in (0, \lambda^{-1}]\)), both the reputation-unconscious fund and the reputation-conscious fund publicly intervene with the same, but lower, probability relative to that under the UU structure (wherein each fund publicly intervenes w.p. \(\tau_{**}\); see (21)).

**Lemma 6.** When \(\Delta \in (0, \lambda^{-1}]\), comparing the UC and UU structures we have \(\tau_{sf} = \tau_{f*} < \tau_{**}\).

The intuition is as follows. When \(\Delta \in (0, \lambda^{-1}]\), in the exit stage the reputation-conscious fund sells \(\Delta\) less compared to the reputation-unconscious fund,\(^{31}\) but the consequent reduction in the reputation-conscious fund’s trading profit, relative to that earned by the reputation-unconscious fund, is exactly offset by his gain in reputation from trading \(\Delta\) less, instead of selling as much as

\(^{31}\)Note \(\beta_{sf} - \beta_{f*} = \lambda^{-1} - (\lambda^{-1} - \Delta) = \Delta\) in this case.
the reputation-unconscious fund. Thus, the reputation-conscious fund’s utility from trading can be calculated as if he also sells $\lambda^{-1}$, same as the reputation-unconscious fund. Since the expected stock price at which trading takes place is the same for the two funds, both derive the same utility from trading, hence will publicly intervene with the same probability ($\tau_{fs} = \tau_{sf}$). Furthermore, compared to the UU structure, in the UC structure the reduction in total fund trading absent public intervention lowers the odds that the bad firm is revealed through trading, which increases each fund’s utility from trading, weakening each’s public intervention incentive ($\tau_{fs} = \tau_{sf} < \tau_{ss}$).

Next, we examine the case with $\Delta > \lambda^{-1}$: a UC structure in which the reputation-conscious fund’s career concerns are sufficiently strong so that he does not trade ($\beta_{fs} = 0$) in the exit stage.

**Proposition 6.** Suppose $\Delta > \lambda^{-1}$. In a UC structure the reputation-conscious fund is less likely to engage in public intervention than the reputation-unconscious fund ($\tau_{fs} < \tau_{sf}$). When the reputation-conscious fund’s career concerns become stronger ($\Delta$ increases), he publicly intervenes less ($\tau_{fs}$ decreases), while the reputation-unconscious fund publicly intervenes more ($\tau_{sf}$ increases).

This is one main result of the paper. To understand it, note that, different from the case in which $\Delta \in (0, \lambda^{-1}]$ (see Lemma 6), any increase in $\Delta$ beyond $\lambda^{-1}$ (so $\beta_{fs} = 0$) does not further reduce total fund trading in the exit stage in a UC structure – total fund trading stays constant at $\beta_{sf} + \beta_{fs} = \lambda^{-1} + 0 = \lambda^{-1}$ in this case. Thus, different from the case wherein $\Delta \in (0, \lambda^{-1}]$, with $\Delta > \lambda^{-1}$ the odds that the bad firm may be revealed to the market through fund trading cannot be lowered further via trading reduction by the reputation-conscious fund, simply because his trading cannot be reduced further. Consequently, as the reputation-conscious fund’s career concerns become stronger, he has to resort to more reduction in public intervention ($\tau_{fs}$ decreases) to reduce the odds of revealing the bad firm. Anticipating this, the reputation-unconscious fund intervenes more ($\tau_{sf}$ increases). Interestingly, thus, the reputation-conscious fund’s weakened incentive to engage in public intervention, due to his strong career concerns, serves as a **credible commitment device** that strengthens the reputation-unconscious fund’s public intervention incentive. That is, fund career concerns may help overcome the free-rider problem typically associated with multiple blockholders.

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32 Total trading by the two funds is also lowered by $\Delta$, which reduces the odds of revealing the bad firm, and consequently improves the market’s assessment of the reputation-conscious fund’s ability, compared to the (counterfactual) case wherein the reputation-conscious fund were also to sell $\lambda^{-1}$.

33 The expected stock price is $e^{-\lambda(\beta_{sf} + \beta_{fs})} P$, depending on total trading by the two funds but not individual trading, where $P$ is given by (25) and $e^{-\lambda(\beta_{sf} + \beta_{fs})}$ is the probability with which total share demand $d > 0$.

34 The idea here is different from that for the comparison between a reputation-conscious fund and a reputation-unconscious fund in the single-blockholder cases (see footnote 24). There, the reputation-conscious fund and the reputation-unconscious fund are examined in two separate cases, and the expected stock prices at which trading takes place (hence trading profits) are different, depending on each fund’s individual trading in each case.
5.4 Comparing the Three Blockholder Structures

Having examined the three blockholder structures separately, we now compare their relative corporate governance efficacy. Below we first summarize, for each structure, the findings on (i) the success rate of private intervention; (ii) the probability that at least one fund publicly intervenes following a failed private intervention; and (iii) aggregate fund trading in the exit stage.

<table>
<thead>
<tr>
<th>Blockholder structure</th>
<th>(i) Private intervention</th>
<th>(ii) Public intervention</th>
<th>(iii) Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>UU</td>
<td>$\sigma_{**}$</td>
<td>$1 - (1 - \tau_{**})^2$</td>
<td>$2\lambda^{-1}$</td>
</tr>
<tr>
<td>CC</td>
<td>$\sigma_{ff}$</td>
<td>$1 - (1 - \tau_{ff})^2$</td>
<td>$2\max(\lambda^{-1} - \Delta, 0)$</td>
</tr>
<tr>
<td>UC</td>
<td>$\sigma_{asy}$</td>
<td>$1 - (1 - \tau_{f})(1 - \tau_{f^*})$</td>
<td>$\lambda^{-1} + \max(\lambda^{-1} - \Delta, 0)$</td>
</tr>
</tbody>
</table>

We first compare the two symmetric blockholder structures (UU and CC).

**Lemma 7.** The success rate of private intervention, the probability that at least one fund publicly intervenes following a failed private intervention, and aggregate fund trading in the exit stage are all higher in the UU structure compared to the CC structure.

This lemma shows that the blockholder structure with two reputation-conscious funds (CC) is always dominated by the one with two reputation-unconscious funds (UU) in terms of governance efficacy. The intuition is that fund career concerns in the CC structure weaken exit and public intervention, both of which also lower the CEO’s effort incentive in the private intervention stage.

The next comparison is between the UU structure and the UC structure. We first compare the UU structure with a UC structure in which the reputation-conscious fund’s career concerns are not too strong (i.e., the UC structure considered in Lemma 6 with $\Delta \in (0, \lambda^{-1}]$). We note that both fund exit and public intervention are stronger in the UU structure: (i) following a failed private intervention, the probability that at least one fund publicly intervenes in the UU structure, $1 - (1 - \tau_{**})^2$, is higher than that in the UC structure, $1 - (1 - \tau_{sf})(1 - \tau_{f^*})$ (following from Lemma 6); and (ii) aggregate fund trading in the exit stage is $2\beta_{**} = 2\lambda^{-1}$ in the UU structure, also larger than that in the UC structure, $\beta_{sf} + \beta_{f^*} = 2\lambda^{-1} - \Delta$. Both (i) and (ii) reduce the CEO’s effort incentive in the private intervention stage – comparing (27) with (22), it follows immediately that $\sigma_{asy} < \sigma_{**}$. The intuition, similar as that for Proposition 4, is as follows. Compared to the UU structure, in the UC structure the fact (i) reduces the credibility of the funds’ threat to take more confrontational public measures following a failed private intervention, and the fact (ii) drives the firm’s stock price further above its true value in the exit stage. Both weaken the CEO’s effort incentive in the private intervention stage, lowering the success rate of private intervention.
The discussions are summarized in the following lemma:

**Lemma 8.** A UC structure in which the reputation-conscious fund’s career concerns are not sufficiently strong \((\Delta \in (0, \lambda^{-1}])\) is strictly dominated by the UU structure – the success rate of private intervention, the probability that at least one fund publicly intervenes following a failure of private intervention, and aggregate fund trading in the exit stage are all higher in the former structure.

Finally, we compare the UU structure with a UC structure in which the reputation-conscious fund’s career concerns are sufficiently strong (i.e., the UC structure considered in Proposition 6 with \(\Delta > \lambda^{-1}\)).

**Proposition 7.** Suppose the reputation-conscious fund’s career concerns are sufficiently strong (\(\Delta\) sufficiently bigger than \(\lambda^{-1}\)). When a fund’s block size \((\frac{\alpha}{2})\) is sufficiently large and/or financial market liquidity \((\lambda^{-1})\) is sufficiently low, the UC structure dominates the UU structure in terms of governance efficacy, manifested by (i) a higher probability in the UC structure that at least one fund publicly intervenes following a failed private intervention, i.e., \(1 - (1 - \tau_{sf})(1 - \tau_{fs}) > 1 - (1 - \tau_{**})^2\); and (ii) a higher success rate of private intervention in the UC structure, i.e., \(\sigma_{asy} > \sigma_{**}\). Consequently, a good firm makes more intangible investment in the UC structure.

This is the other main result, with the following intuition. First, consider public intervention. In the UC structure, as \(\Delta\) increases further beyond \(\lambda^{-1}\), although the reputation-conscious fund intervenes less (\(\tau_{fs}\) decreases), the reputation-unconscious fund intervenes more (\(\tau_{sf}\) increases), due to the positive externality that the former’s (reputation-induced) reduction in public intervention exerts on the latter’s intervention incentive (Proposition 6). When the fund’s block size \((\frac{\alpha}{2})\) is sufficiently large, such externality will be sufficiently strong, so that the negative effect of the reputation-conscious fund’s career concerns in weakening his own public intervention incentive is outweighed by its positive effect in strengthening the reputation-unconscious fund’s incentive. Consequently, following a failed private intervention, public intervention (by at least one fund) is more likely in the UC structure, i.e., \(1 - (1 - \tau_{sf})(1 - \tau_{fs}) > 1 - (1 - \tau_{**})^2\).

The efficacy of blockholder governance also depends upon the success rate of private intervention and fund trading in the exit stage. Consider trading. In the UU structure, trading is stronger – in the exit stage each reputation-unconscious fund sells \(\lambda^{-1}\) and total trading is \(2\lambda^{-1}\), while in the UC structure (with \(\Delta > \lambda^{-1}\)) total trading is only \(\lambda^{-1}\), due to the reputation-conscious fund’s trading reduction. However, as discussed above, public intervention may be stronger in the UC structure, due to the amelioration of the free-rider problem in the public intervention stage. Thus,
it is possible that in the UC structure the strengthened fund public intervention outweighs the weakened fund exit, causing the UC structure to dominate the UU structure in terms of the overall governance efficacy following a failed private intervention. This dominance occurs when (i) a fund’s block size ($\alpha^2$) is sufficiently large; and/or (ii) market liquidity ($\lambda^{-1}$) is sufficiently low, which effectively diminishes the comparative disadvantage of the UC structure relative to the UU structure in terms of fund trading.

When blockholder governance becomes stronger following a failed private intervention, the bad firm CEO’s effort incentive in the private intervention stage also becomes stronger, thereby increasing the success rate of fund private intervention. Therefore, under the same two conditions above (i.e., $\alpha^2$ is sufficiently large and/or $\lambda^{-1}$ is sufficiently small), private intervention is also more likely to succeed in the UC structure.

Finally, when blockholder governance becomes more effective, undervaluation upon releasing an interim bad signal ($s_B$) becomes less of a concern for a good firm, which then chooses more long-term investment in the UC structure.

6 Model Robustness and Implications

6.1 Modeling Choices and Robustness

In our main model, a fund develops his reputation for selecting good firms \textit{ex ante}, i.e., stock-picking skill. While this modeling choice is consistent with the survey finding in McCahery, Sautner, and Starks (2014) (see footnote 5), and can capture the well documented fund flow-performance relationship, there could be multiple dimensions of ability. Therefore, it is natural to ask, at least from a theoretical perspective, how our analysis would be affected if a fund’s ability is modeled as his ability to gather private information about firm type \textit{ex post} (i.e., distinguish between good and bad firms) and act upon that information by coming out with right strategies to enhance a bad firm’s value. That is, the fund builds his reputation for being able to identify a bad firm \textit{ex post} and turn it around.\footnote{Some activists may get their reputation from \textit{deliberately} picking bad firms \textit{ex ante} and turning them around. While we do not model this possibility (for example, imagine that high uncertainty may cloud turnaround prospects, so the majority of investors will first try to select good firms and only deal with turnaround issues if they find out later that they have picked a bad firm, rather than “picking a fight” intentionally), we discuss later that it is unlikely to change our main results qualitatively; see footnote 37.} There are two layers of ability in this alternative modeling assumption,
and we consider each in turn. In what follows, we describe the main results from the analysis of this alternative model of fund ability and explain the intuition.\textsuperscript{36}

First, suppose, same as in the main model, the fund can unambiguously identify a bad firm ex post at $t = 1$, and his reputation rests in his ability to turn around the bad firm by proposing \textit{right} value-enhancing changes to its CEO. Note in our main model, we assume that in the private intervention stage changes proposed by the fund are always right, and the success of private intervention only depends upon the CEO’s effort to implement those changes. Here, the success also depends on whether the proposed changes are right (i.e., relevant and effective to the specific firm).

The thought process goes as follows. Suppose the fund privately proposes some strategic changes to the bad firm’s CEO, who then exerts (hidden) effort to implement those changes, but firm value fails to improve in this private intervention stage (otherwise, the game ends); the failure is known only to the fund and the CEO as in the main model. The fund, being aware that the failure could be due to his strategies being wrong or shirking by the CEO, needs to decide whether to escalate his intervention to a public stage and forcibly implement those changes by himself or simply exit. With public intervention, the bad firm is revealed to the market and the potential reputation cost to the fund arises if his public intervention fails (in which case the market knows unambiguously that the failure must be due to the fund’s wrong strategies). In the case of exit, selling more increases the odds of revealing to the market that the fund lacks the ability to turn around a bad firm – the market knows that the fund sells only when private intervention has failed and he is sufficiently pessimistic about the effectiveness of his turnaround strategies. Therefore, fund career concerns in this alternative model also cause him to reduce public intervention and trading following a failed private intervention, thereby jeopardizing the success of private intervention.\textsuperscript{37} The results in our main model thus sustain qualitatively.

Second, suppose, different from the main model, conditional on the firm releasing a bad public signal ($s_B$) at $t = 1$, the fund only receives an informative but imprecise (private) signal about firm type, and develops his reputation for being able to identify firm type ex post (i.e., high signal precision). We show that when the market’s prior belief about firm type is sufficiently high ($\bar{\phi}$ sufficiently big), the results in our main model remain qualitatively unaltered.

The intuition is as follows. With this alternative model of ability, the fund tends to distort his action choices to conform with the market’s prior belief about firm type to manipulate the

\textsuperscript{36}The formal analysis, omitted here for the sake of the paper’s length, is available upon request.

\textsuperscript{37}We do not expect the results to change qualitatively if we consider activists who deliberately select bad firms (see footnote 35), since they also care about the market’s assessment of their turnaround ability.
market’s perception that he has received a signal with high precision. If the prior is that the firm is likely to be good (\(\bar{\varphi}\) sufficiently big), then trading and public intervention following a failed private intervention, which are against the prior, will adversely affect the market’s inferences about fund ability – the market will place a higher weight that the fund has received a wrong signal when trading and public intervention are detected. Knowing this, the fund will sell less and engage in less public intervention to protect his reputation. Therefore, the results in the main model remain.

If the firm is a priori very likely to be bad (\(\bar{\varphi}\) sufficiently small), then fund trading and intervention are consistent with the market’s prior, and career concerns may induce the fund to sell and intervene even when he receives a positive private signal about firm type at \(t = 1\) (so his private signal indicates a good firm that does not warrant intervention or exit). In this case, the alternative ability assumption changes our results on exit and intervention in the main model.\(^{38}\) However, what remains unaffected is that fund career concerns again adversely affects a good firm’s long-term investment, since excessive selling and intervention lowers the market’s ability to draw correct inferences about firm type, which again worsens the misvaluation-induced corporate myopia.

6.2 Implications

6.2.1 The Corporate Governance Value of Passivity

Index funds seldom perform active monitoring, nor are they likely to engage in disciplinary trading. Index funds’ such passivity in both voice and exit may make one wonder about their corporate governance value; this question becomes increasingly important due to the recent proliferation of index funds whose ownership in U.S. publicly traded companies has been growingly significant.\(^{39}\) Although index funds’ passivity is largely driven by their business strategies, whereas in our model the passivity of a reputation-conscious fund (in public intervention and discretionary trading) arises from its short-term incentives, the model’s core idea can still be applied to shed light on the potential corporate governance value of index funds – in firms owned by both index funds and non-index institutional blockholders, the former’s (credible) passivity may strengthen the latter’s activity in performing active monitoring and thus ameliorate the free-rider problem. There is some anecdotal evidence that seems to be consistent with this explanation. According to a report by Reuters, with increasingly many index and exchange-traded funds who are “passive investors which own millions

\(^{38}\)The result that exit could be a sign of high (rather than low) ability in this case echoes the finding in Dow and Gorton (1997) that fund managers may trade excessively because they want to signal that they have information.

\(^{39}\)According to Morningstar, as of year end of 2012 about 24% of U.S. institutional assets are owned by index funds.
of shares of U.S. companies but rarely say much about how they should be run,” activists complain that they “often cannot find another investor to compare notes with” and hence have to conduct deep research on companies by themselves.  

A recent study by Mullins (2014) offers some empirical support to our explanation. He finds that firms experiencing exogenous increase in ownership by index funds due to reconstitution of Russell equity indices, despite their total blockholdings staying unchanged after the reconstitution, display stronger shareholder voting at annual meetings, manifested by the increasing (decreasing) failure rate of management (shareholder) proposals. Mullins interprets his findings as suggesting that passive index funds may be complementary to other non-index institutional blockholders, augmenting the latter’s incentive and ability to engage in active monitoring, which is consistent with our model’s core idea.

At a broad level, our analysis sheds light on the prevalence of diverse blockholder structures, in which blockholders with various incentive horizons (various $\Delta$ in our model) co-exist. Think about mutual funds and hedge funds. To the extents that a fund manager’s perceived ability affects fund flow and a mutual fund manager’s compensation is relatively more tied to the size of assets under management (and hence fund flow) than a hedge fund manager, our analysis suggests that mutual funds tend to have longer stock holding horizons and are less likely to engage with management (both publicly and privately) than hedge funds. This is consistent with casual observations that hedge funds are usually more active in both exit and intervention (regardless public confrontation or private negotiation). However, our analysis reveals some potential corporate governance value arising from the co-existence of mutual funds and hedge funds in a firm’s blockholder structure, wherein the inactivity of the mutual funds can strengthen the hedge funds’ incentive to undertake corrective actions. That is, a heterogeneous blockholder structure with a mutual fund and a hedge fund can be more valuable (in terms of corporate governance) than a homogeneous structure with two hedge funds.

### 6.2.2 Private Intervention

The survey by McCahery, Sautner, and Starks (2014) documents widespread behind-the-scenes shareholder engagement with management. Given the importance of both private and public en-

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41Hedge fund managers typically self-invest more in their own funds than mutual fund managers, which causes them to care relatively more about long-term fund value (and hence have lower relatively lower $\Delta$ in our model); see Dasgupta and Piacentino (2014) for a formal analysis distinguishing between hedge funds and mutual funds along this line.
gagement, it is empirically interesting to examine the factors that drive the shareholders’ choice between the two.\textsuperscript{42} Our analysis suggests that besides the usual cost considerations, the CEO’s incentive horizon (e.g., the weight that the CEO’s compensation attaches to short-term stock price relative to long-term firm value, with a lower weight corresponding to a longer incentive horizon; for such a measure, see Gopalan, Milbourn, Song, and Thakor (2014)) is also an important driving factor. Specifically, Proposition 4 shows that shareholders are \textit{ceteris paribus} more likely to privately engage with a CEO with longer incentive horizon, and such private intervention is also more likely to be successful.

6.2.3 The Dark Side of Investor “Long-Termism”

Long investment horizon by large institutional investors has been widely lauded as a key instrument to encourage companies to pursue long-term goals, while short investment horizon has been blamed as a significant contributing factor to corporate myopia. This view, largely mixing investor long-termism with stock holding horizon, seems to be ubiquitous and influences much of the policy debate (see footnote 1).

In academia a large body of empirical research employs fund turnover measures to gauge whether an institutional investor is long-term or short-term oriented, with investors with low fund turnover (and hence long stock holding horizon) being classified as long-term investors; see Gaspar, Massa, and Matos (2005) and Bushee (1998). This literature is usually grounded on the following two arguments. First, the fact that an investor holds a company’s stock for a long period, rather than selling it upon short-term market fluctuations, is because he cares about the company’s long-term performance. In turn, this long-term view taken by the investor encourages the company to focus on long-term value creation rather than increasing short-term metrics. Second, since the investor has a prolonged (and usually large) stake in the company, he has strong incentive to monitor the management to ensure it delivers long-term value.\textsuperscript{43}

Our analysis shows that such conventional wisdom that long stock holding horizon signifies an investor with a long-term view should be taken with caution.\textsuperscript{44} For an investor who is the sole or

\textsuperscript{42}McCahery, Sautner, and Starks (2014) construct a voice index summing the different types of corrective (public and private) actions taken by an institutional investor to measure the voice intensity and examine its determinants. Their measure of voice intensity does not differentiate between private and public engagement.

\textsuperscript{43}Empirical studies include, among others, Chen, Harford, and Li (2007), Derrien, Kecskés, and Thesmar (2013), and Cella, Ellul, and Giannetti (2013).

\textsuperscript{44}Edmans (2009), through a different mechanism in his model, also suggests a dark side of investor long-termism – he shows that policies “to reduce liquidity and thus create unconditionally long-run shareholders who never sell” may backfire and discourage long-term corporate investment.
dominant blockholder in a firm, his long stock holding horizon should not be viewed synonymously as investor long-termism, if what drives the long holding horizon is the investor’s short-term (reputational) concerns as modeled in this paper. Such seemingly investor long-termism may impede information incorporation into prices and exacerbate corporate myopia. Such an investor may not necessarily be a dedicated monitor either, despite his endured and potentially large stake. Thus, our analysis suggests that empirical research on investor horizon and policy prescriptions aiming to promote investor long-termism should adopt a finer approach by heeding investors’ motives in stock holding as well as blockholder structure.

7 Conclusion

We model blockholder governance as a sequential process, beginning with less hostile private intervention, then more confrontational and costly public intervention, and finally exit. The blockholder is a fund manager who may face short-term incentives and care about the market’s assessment of his stock-picking ability. Such career concerns cause the fund manager to reduce disciplinary trading and undertake less value-enhancing public interventions following a failed private intervention, in order to positively influence others’ perception of his ability. As a result, the threat by the fund manager to escalate his intervention to a more confrontational public stage loses credibility, jeopardizing the success of private intervention. Consequently, the effectiveness of blockholder governance is weakened in a single blockholder structure.

The main result of the paper is that, interestingly, the very cause of the ineffectiveness of blockholder governance in a single blockholder structure can help improve the quality of governance in an asymmetric multiple blockholder structure by ameliorating the free-rider problem typically associated with multiple blockholders and enhancing blockholder voice. Specifically, in an asymmetric blockholder structure with a reputation-unconscious fund manager and a reputation-conscious fund manager, the weakened public intervention incentive by the latter credibly makes the former pivotal in whether the value-enhancing public intervention is undertaken following a failed private intervention, thereby strengthening the former’s incentive to engage in public intervention. Consequently, the credibility of the threat that intervention is likely to be escalated to a more confrontational public stage (following a failed private intervention) is restored, which enhances the success probability of private intervention, hence the efficacy of blockholder governance.
The paper’s central message is thus that a diverse blockholder structure with heterogeneous blockholders in terms of the degrees of their short-term incentives may help improve the overall quality of blockholder governance, and consequently promote long-term corporate investment. That is, quite contrary to conventional wisdom, agency problems at the blockholder level do not necessarily exacerbate, but instead may ameliorate agency problems at the firm level.

Future research could go in various directions. One direction is to explore the multiple blockholder structure further. While we only model two blockholders in order to deliver our key message in a most direct and parsimonious way, it would be interesting to simultaneously endogenize both the optimal number of blockholders and the degree of asymmetry among them. The other direction is to endogenize the CEO’s incentive horizon which we have taken as given in the current paper, as in most existing models of blockholder governance. However, it is likely that CEO incentive and blockholder governance are jointly determined. It would be fruitful to examine how the incentive horizon of blockholders and the CEO’s pay horizon (i.e., the mix of short-term and long-term pay, as in Gopalan, Milbourn, Song, and Thakor (2014)) interact with each other in influencing long-term corporate investment. Current corporate governance discussions have centered around executive compensation, with the focus there being how to structure the compensation to curb myopic managerial behaviors. If executive compensation and block holdings are indeed jointly determined, perhaps an equally important issue is how to design blockholder structure to complement executive compensation to promote long-term corporate investment.
Appendix

Proof of Lemma 1. Consider $s = s_B$. If the firm is good, the fund knows there is no gain from selling, so he chooses $\beta = 0$. If the firm is bad, the fund knows the stock price ($P$) will be no smaller than the firm’s true value ($V = 0$), so he sells ($\beta > 0$). Thus, if $d = u - \beta \leq 0$, the market maker knows the fund must have sold (since $u > 0$) and hence the firm is bad, so $P = 0$. If $d > 0$, the market maker cannot tell if the fund has sold. The market maker’s posterior belief in this case is

$$\Pr(G|d > 0) = \frac{\Pr(d > 0|G)\Pr(G)}{\Pr(d > 0|G)\Pr(G) + \Pr(d > 0|B)\Pr(B)} = \frac{\varphi(2 - \varphi)^{-1}}{\varphi(2 - \varphi)^{-1} + 2(1 - \varphi)(2 - \varphi)^{-1}e^{-\lambda \beta}} = \frac{\varphi}{\varphi + 2(1 - \varphi)e^{-\lambda \beta}}, \quad (A1)$$

which is increasing in $\beta$. In deriving the posterior, we have used the facts: (i) with a good firm, $\beta = 0$, so $d = u > 0$ for sure; and (ii) with a bad firm, $\beta > 0$, so $d = u - \beta > 0$ occurs w.p. $\int_0^\infty e^{-\lambda u} du = e^{-\lambda \beta}$. □

Proof of Proposition 1. The first-order condition for (5) is

$$(1 - \lambda \beta)e^{-\lambda \beta} \frac{\varphi}{\varphi + 2(1 - \varphi)e^{-\lambda \beta}} X = 0, \quad (A2)$$

so the fund chooses $\beta = \min(\lambda^{-1}, \alpha)$, given that $\beta \leq \alpha$, which must coincide with $\beta^*$ in equilibrium. The second-order condition is satisfied, $[-\lambda - \lambda(1 - \lambda \beta)]e^{-\lambda \beta} \frac{\varphi}{\varphi + 2(1 - \varphi)e^{-\lambda \beta}} X < 0$.

To analyze the CEO’s problem in (6), denote

$$\Omega(v, \beta^*) \equiv \frac{\partial U_c}{\partial v} = (1 - \delta)g - \delta v \frac{2(1 - \varphi)e^{-\lambda \beta^*}}{\varphi + 2(1 - \varphi)e^{-\lambda \beta^*}} X.$$

It is clear $U_c$ is a concave function of $v$, since $\frac{\partial \Omega(v, \beta^*)}{\partial v} < 0$. Note

$$\Omega(0, \beta^*) = (1 - \delta)g > 0,$$

$$\Omega(1, \beta^*) = (1 - \delta)g - \delta \frac{2(1 - \varphi)e^{-\lambda \beta^*}}{\varphi + 2(1 - \varphi)e^{-\lambda \beta^*}} X.$$

If $\delta \leq \delta_* \equiv g \left[ \frac{2(1 - \varphi)e^{-\lambda \beta^*}}{\varphi + 2(1 - \varphi)e^{-\lambda \beta^*}} X \right]^{-1} \in (0, 1)$, then $\Omega(1, \beta^*) \geq 0$; this shows $\Omega(v, \beta^*) > 0 \forall v \in [0, 1)$ and hence $v^* = 1$. If $\delta > \delta_*$, then $\Omega(1, \beta^*) < 0$, and hence $v^*$ is given by the first-order condition, $\Omega(v^*, \beta^*) = 0$, where $v^* \in (0, 1)$ is exactly ensured by $\delta > \delta_*$. In this case, it is clear $v^*$ and $\beta^*$ are positively correlated. □

Proof of Lemma 2. If the firm is revealed to be good, then

$$f(\varphi|G) = \frac{\varphi f(\varphi)}{\int_0^1 \varphi f(\varphi) d\varphi} = \frac{\varphi f(\varphi)}{\varphi}.$$
Substituting this into \( E(\varphi|G) = \int_0^1 \varphi f(\varphi|G) d\varphi \) yields (7). If the firm is revealed to be bad, then
\[
f(\varphi|B) = \frac{(1 - \varphi)f(\varphi)}{\int_0^1 (1 - \varphi)f(\varphi)d\varphi} = \frac{(1 - \varphi)f(\varphi)}{1 - \varphi}.
\]
Substituting this into \( E(\varphi|B) = \int_0^1 \varphi f(\varphi|B) d\varphi \) yields (8).

**Proof of Proposition 2.** To analyze the fund’s problem in (11), denote
\[
\Lambda(\beta, \beta_f) \equiv \frac{\partial U_f}{\partial \beta} = e^{-\lambda \beta} \frac{\bar{\varphi}}{\varphi + 2(1 - \varphi)e^{-\lambda \beta_f}} \left[ (1 - \lambda \beta)X - \frac{\kappa_f \lambda \sigma^2}{\bar{\varphi}(1 - \varphi)} \right].
\]
It is clear \( \Lambda(\lambda^{-1}, \beta_f) < 0 \), so in any equilibrium we must have \( \beta_f < \lambda^{-1} \).

The CEO’s optimization problem is similar to that in the benchmark case, stated in Equation (6), except here \( \Pr(G|d > 0) = \frac{\bar{\varphi}}{\bar{\varphi} + 2(1 - \bar{\varphi})e^{-\lambda \sigma^2}} \). Denote
\[
\Omega(\upsilon, \beta_f) \equiv \frac{\partial U_c}{\partial \upsilon} = (1 - \delta)g - \delta \upsilon \frac{2(1 - \bar{\varphi})e^{-\lambda \beta_f}}{\bar{\varphi} + 2(1 - \bar{\varphi})e^{-\lambda \beta_f}}X.
\]
Note \( U_c \) is a concave function of \( \upsilon \), since \( \frac{\partial U_c(\upsilon, \beta_f)}{\partial \upsilon} < 0 \).

If \( \lambda \Delta \geq 1 \), then \( \Lambda(\beta, \beta_f) < 0 \) \( \forall \beta \in [0, \lambda^{-1}] \). In this case, \( U_f \) is monotonically decreasing in \( \beta \) over \( \beta \in [0, \lambda^{-1}] \), and hence \( \beta_f = 0 \) in the unique equilibrium. For the CEO’s problem, \( \Omega(\upsilon, 0) = (1 - \delta)g - 2\delta \upsilon(1 - \bar{\varphi})(2 - \bar{\varphi})^{-1}X \), we have \( \Omega(1, 0) \geq 0 \); this shows \( \Omega(\upsilon, 0) > 0 \) \( \forall \upsilon \in [0, 1] \), and hence \( \upsilon_f = 1 \) in equilibrium. If \( \delta \geq \delta_f \), then \( \Omega(1, 0) < 0 \), and hence \( \upsilon_f \) is given by the first-order condition \( \Omega(\upsilon_f, 0) = 0 \), which yields \( \upsilon_f = \frac{1 - \delta}{\delta} \frac{2 - \bar{\varphi}}{2(1 - \bar{\varphi})} X \); note \( \upsilon_f \in (0, 1) \) is exactly ensured by \( \delta \geq \delta_f \). It is clear that \( \upsilon_f < \upsilon_* \), where \( \upsilon_* \) is given by Proposition 1.

If \( \lambda \Delta \in (0, 1) \), then \( \Lambda(0, \beta_f) > 0 \). Note
\[
\frac{\partial \Lambda(\beta, \beta_f)}{\partial \beta} \propto \left[ \frac{\kappa_f \lambda \sigma^2}{\bar{\varphi}(1 - \varphi)} - X \right] - (1 - \lambda \beta)X < 0,
\]
so now \( U_f \) is a concave function of \( \beta \), and \( \beta_f \) is given by the first-order condition, \( \Lambda(\beta_f, \beta_f) = 0 \), which yields
\[
(1 - \lambda \beta_f)X = \frac{\kappa_f \lambda \sigma^2}{\bar{\varphi}(1 - \varphi)} \Rightarrow \beta_f = \frac{1}{\lambda} - \frac{\kappa_f \sigma^2}{\bar{\varphi}(1 - \varphi)}X = \lambda^{-1} - \Delta.
\]
We then turn to the CEO’s optimization problem. If \( \delta \leq \delta_f \equiv \frac{g + 2(1 - \bar{\varphi})e^{-\lambda \beta_f}}{\bar{\varphi} + 2(1 - \bar{\varphi})e^{-\lambda \beta_f}}X \), then \( \Omega(1, \beta_f) \geq 0 \); this shows \( \Omega(\upsilon, \beta_f) > 0 \) \( \forall \upsilon \in [0, 1] \) and hence \( \upsilon_f = 1 \). If \( \delta > \delta_f \), then \( \Omega(1, \beta_f) < 0 \) and \( \upsilon_f \) is given by the first-order condition, \( \Omega(\upsilon_f, \beta_f) = 0 \), which yields \( \upsilon_f = \frac{1 - \delta}{\delta} \frac{2 - \bar{\varphi}}{2(1 - \bar{\varphi})} \frac{g}{X} \), where \( \upsilon_f \in (0, 1) \) is exactly ensured by \( \delta > \delta_f \). Also, since \( \beta_f < \beta_* \), we have \( \upsilon_f < \upsilon_* \). Note \( \upsilon_f \) and \( \beta_f \) are positively related, so the result that \( \upsilon_f \) is decreasing in \( \Delta \) follows from the fact that \( \beta_f \) is decreasing in \( \Delta \). \( \square \)
Proof of Lemma 3. If the fund publicly intervenes, he will not sell, and his continuation utility is

\[ U_{f}^{\text{pub}} = \alpha X - \tau. \]

If the fund does not publicly intervene, he will sell \( \beta_{*}^{\text{no}} = \lambda^{-1} \), and his continuation utility is

\[ U_{f}^{\text{no}} = \beta_{*}^{\text{no}} e^{\lambda \beta_{*}^{\text{no}}} \frac{\bar{\phi}}{\bar{\phi} + 2(1 - \bar{\phi})(1 - \tau_{*})e^{\lambda \beta_{*}^{\text{no}}} X}. \]

Public intervention occurs if and only if \( U_{f}^{\text{pub}} \geq U_{f}^{\text{no}} \), which yields (14). The problem in (14) is a fixed-point problem. Since \( h(\beta_{*}^{\text{no}}, \tau) \) is a continuous and decreasing function of \( \tau \), the condition \( h(\beta_{*}^{\text{no}}, 1) \in (0, 1) \), i.e.,

\[ (\alpha - \lambda^{-1}e^{-1})X \in (0, 1), \tag{A4} \]

ensures the existence and uniqueness of an interior solution, \( \tau_{*} \in (0, 1) \), by the Fixed-Point Theorem. Note \( (\alpha - \lambda^{-1}e^{-1})X > 0 \) is guaranteed by \( \alpha > \lambda^{-1} \), so we only need to assume \( (\alpha - \lambda^{-1}e^{-1})X < 1 \). If \( (\alpha - \lambda^{-1}e^{-1})X \geq 1 \), we will have \( h(\beta_{*}^{\text{no}}, \tau) \geq \tau \) for \( \forall \tau \in [0, 1] \), so \( \tau_{*} = 1 \), i.e., the fund always publicly intervenes as the fund’s value enhancement from public intervention instead of trading is too high compared to the cost of public intervention. To knock out this uninteresting boundary case, we assume \( (\alpha - \lambda^{-1}e^{-1})X < 1 \) throughout. \( \square \)

Proof of Proposition 3. Suppose the market’s equilibrium beliefs are that the fund publicly intervenes if and only if \( \tau \leq \tau_{f} \), and sells \( \beta_{*}^{\text{no}} \) absent public intervention, while abstains from selling with public intervention. With public intervention, the fund improves the bad firm’s value to \( X \) (and hence chooses \( \beta = 0 \)), but also reveals its type. His continuation utility is

\[ U_{f}^{\text{pub}} = \alpha X + \kappa_{f} E(\varphi|B) - \tau. \]

If the fund does not publicly intervene, he will sell \( \beta \) to maximize his expected continuation utility:

\[ U_{f}^{\text{no}} = \beta e^{-\lambda \beta} \Pr(G|d > 0)X + \kappa_{f}[(1 - e^{-\lambda \beta})E(\varphi|d \leq 0) + e^{-\lambda \beta}E(\varphi|d > 0)], \]

where

- \( \Pr(G|d > 0) = \frac{\bar{\phi}}{\bar{\phi} + 2(1 - \bar{\phi})(1 - \tau_{*})e^{-\lambda \beta} X} \);
- \( E(\varphi|d \leq 0) = E(\varphi|B) \), since \( d \leq 0 \) perfectly reveals the bad firm’s type; and
- \( E(\varphi|d > 0) = \Pr(G|d > 0)E(\varphi|G) + \Pr(B|d > 0)E(\varphi|B) \).

It is straightforward to show that the solution is same as that in the basic model, i.e., \( \beta_{*}^{\text{no}} = \max(\lambda^{-1} - \Delta, 0) \).
The fund publicly intervenes if and only if $U_{f^{pub}} \geq U_{f^{no}}$, which yields

$$
\tau_f = [\alpha - \beta_f^{\alpha} e^{-\lambda \beta_f^{\alpha}} \Pr(G|d > 0)]X - e^{-\lambda \beta_f^{\alpha}} \Pr(G|d > 0) \kappa_f [\mathbf{E}(\varphi|\mathbf{G}) - \mathbf{E}(\varphi|\mathbf{B})]
$$

$$
= \left[\alpha - \frac{\beta_f^{\alpha} e^{-\lambda \beta_f^{\alpha}} \varphi}{\varphi + 2(1 - \varphi)(1 - \tau_f) e^{-\lambda \beta_f^{\alpha}}}\right] X - \frac{e^{-\lambda \beta_f^{\alpha}} \varphi}{\varphi + 2(1 - \varphi)(1 - \tau_f) e^{-\lambda \beta_f^{\alpha}}} \Delta X
$$

$$
= h(\beta_f^{\alpha}, \tau_f) - \frac{e^{-\lambda \beta_f^{\alpha}} \varphi}{\varphi + 2(1 - \varphi)(1 - \tau_f) e^{-\lambda \beta_f^{\alpha}}} \Delta X
$$

$$
\equiv \hat{h}(\beta_f^{\alpha}, \tau_f).
$$

(A5)

It is clear $\frac{\partial \hat{h}(\beta_f^{\alpha}, \tau_f)}{\partial \tau_f} < 0$. We consider several cases, depending on the value of $\Delta$. First, suppose $\Delta \geq \lambda^{-1}$, so $\beta_f^{\alpha} = 0$. In this case, $\hat{h}(0, 0) = [\alpha - \varphi(2 - \varphi)^{-1}]X$. There are two subcases:

1. If $\Delta \geq \alpha(2 - \varphi)\varphi^{-1}$, we have $\hat{h}(0, 0) \leq 0$, and hence $\tau_f = 0$, always smaller than $\tau^*$.

2. If $\Delta \in [\lambda^{-1}, \alpha(2 - \varphi)\varphi^{-1})$, we have $\hat{h}(0, 0) > 0$. In this case $\hat{h}(0, 1) = (\alpha - \Delta)X \leq (\alpha - \lambda^{-1})X < (\alpha - \lambda^{-1}e^{-1})X < 1$. This ensures a unique interior solution, $\tau_f \in (0, 1)$. To show $\tau_f < \tau^*$ in this case, it is sufficient to show $\hat{h}(0, \varphi) < \hat{h}(\lambda^{-1}, \varphi)$ for all $\varphi \in [0, 1]$. To see this, note

$$
\hat{h}(0, \varphi) - \hat{h}(\lambda^{-1}, \varphi) = \left[\frac{\varphi}{\varphi e + 2(1 - \varphi)(1 - \tau_f) - \Delta \frac{\varphi}{\varphi + 2(1 - \varphi)(1 - \tau_f)}\right] X,
$$

which is clearly negative, since $\Delta \geq \lambda^{-1}$.

Second, suppose $\Delta \in (0, \lambda^{-1})$, so $\beta_f^{\alpha} = \lambda^{-1} - \Delta > 0$. In this case, the existence and uniqueness of $\tau_f$ can be shown by observing (i) $\frac{\partial \hat{h}(\beta_f^{\alpha}, \tau_f)}{\partial \tau_f} < 0$; and (ii) $\hat{h}(\beta_f^{\alpha}, 1) \in (0, 1)$, which requires

$$(\alpha - \beta_f^{\alpha} e^{-\lambda \beta_f^{\alpha}})X - e^{-\lambda \beta_f^{\alpha}} \Delta X = (\alpha - \lambda^{-1}e^{-1} + \lambda \Delta)X \in (0, 1).$$

Note (i) $(\alpha - \lambda^{-1}e^{-1} + \lambda \Delta)X < (\alpha - \lambda^{-1}e^{-1})X < 1$, where the second inequality follows from (A4); and (ii) $(\alpha - \lambda^{-1}e^{-1} + \lambda \Delta)X > (\alpha - \lambda^{-1})X > 0$. 

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To prove \( \tau_f < \tau_* \) in this case with \( \Delta \in (0, \lambda^{-1}) \), it is sufficient to show \( \hat{h}(\beta_f^{\text{no}}, \tau) < h(\beta_*^{\text{no}}, \tau) \) for \( \forall \tau \in [0, 1] \). Substituting \( \beta_f^{\text{no}} = \lambda^{-1} - \Delta \) into \( \hat{h}(\beta_f^{\text{no}}, \tau) \) yields

\[
\hat{h}(\beta_f^{\text{no}}, \tau) = \left[ \alpha - (\lambda^{-1} - \Delta)e^{-1+\lambda \Delta} \frac{\varphi}{\phi + 2(1-\varphi)(1-\tau)e^{-1+\lambda \Delta}} \right] X
- e^{-1+\lambda \Delta} \frac{\varphi + 2(1-\varphi)(1-\tau)e^{-1+\lambda \Delta} \Delta X}{\phi + 2(1-\varphi)(1-\tau)e^{-1+\lambda \Delta}}
= \left[ \alpha - \lambda^{-1} e^{-\lambda \Delta} \frac{\varphi}{\phi + 2(1-\varphi)(1-\tau)e^{-1+\lambda \Delta}} \right] X
= \left[ \alpha - \lambda^{-1} e^{-\lambda \Delta} \phi \frac{\varphi}{\phi e^{-\lambda \Delta} + 2(1-\varphi)(1-\tau)e^{-1}} \right] X
< \left[ \alpha - \lambda^{-1} e^{-1} \frac{\varphi}{\phi + 2(1-\varphi)(1-\tau)e^{-1}} \right] X
= h(\beta_*^{\text{no}}, \tau).
\]

Finally, the result \( \frac{\partial \sigma_f}{\partial \Delta} < 0 \) directly follows from the fact \( \frac{\partial \hat{h}(\beta_f^{\text{no}}, \tau)}{\partial \Delta} < 0 \).

**Proof of Proposition 4.** The result \( \sigma_f < \sigma_* \) follows directly by comparing (17) and (18), noting that \( (1-\tau_f)e^{-\lambda \beta_f^{\text{no}}} > (1-\tau_*)e^{-\lambda \beta_*^{\text{no}}} \). The right-hand side of (18) is decreasing in \( \Delta \) (because it is increasing in \( \beta_f^{\text{no}} \) and \( \tau_f \), both of which are decreasing in \( \Delta \)), which leads to \( \frac{\partial \sigma_f}{\partial \Delta} < 0 \). The results \( \frac{\partial \sigma_*}{\partial \delta} < 0 \) and \( \frac{\partial \sigma_f}{\partial \delta} < 0 \) follow by noting that the right-hand sides of (17) and (18) are decreasing in \( \delta \). Finally,

\[
|\frac{\partial \sigma_f}{\partial \Delta}| = \delta \frac{\partial}{\partial \Delta} \frac{\varphi^{(1-\tau_f)e^{-\lambda \beta_f^{\text{no}}}}}{\phi + 2(1-\varphi)(1-\tau_f)e^{-\lambda \beta_f^{\text{no}}}} \frac{\partial}{\partial \Delta} c''(\sigma_f),
\]

which is increasing in \( \delta \) (note both the numerator and denominator are positive).

**Proof of Proposition 5.** The investment by a good firm is

\[
v_* = \min \left( \frac{1 - \delta \varphi + 2(1-\varphi)(1-\tau_*)e^{-1} g X}{2(1-\varphi)(1-\tau_*)e^{-1}}, 1 \right), \tag{A6}
\]

with a reputation-unconscious fund (which can be derived in the same way as in the Proof of Proposition 1 wherein only exit is modeled), and

\[
v_f = \min \left( \frac{1 - \delta \varphi + 2(1-\varphi)(1-\tau_f)e^{-\lambda \beta_f^{\text{no}}} g X}{2(1-\varphi)(1-\tau_f)e^{-\lambda \beta_f^{\text{no}}}}, 1 \right), \tag{A7}
\]

with a reputation-conscious fund (which can be derived in the same way as in the Proof of Proposition 2 wherein only exit is modeled). It is clear \( v_f < v_* \), since \( \tau_f < \tau_* \) and \( \beta_f^{\text{no}} < \beta_*^{\text{no}} = \lambda^{-1} \).
**Proof of Lemma 4.** If a reputation-unconscious fund publicly intervenes by incurring the cost \( \tau \), his block is worth \( \frac{\alpha X}{2} \), and his continuation utility is

\[
U^\text{pub}_f = \frac{\alpha X}{2} - \tau.
\]

If the fund does not publicly intervene, he knows (i) w.p. \( \tau^{**} \) the other fund publicly intervenes, in which case neither fund sells and his own block is worth \( \frac{\alpha X}{2} \); and (ii) w.p. \( 1 - \tau^{**} \) the other fund does not publicly intervene either, in which case each fund sells \( \beta^{**} = \lambda^{-1} \), with a stock price being given by (19) if \( d > 0 \) (w.p. \( e^{-2\lambda\beta^{**}} = e^{-2} \)), and 0 if \( d \leq 0 \) (w.p. 1 - \( e^{-2} \)). His continuation utility in this case is

\[
U^{\tau^{**}}_f = \frac{\alpha X}{2} + (1 - \tau^{**}) \left( \frac{\lambda^{-1}e^{-2\varphi}}{\varphi + 2(1 - \varphi)(1 - \tau^{**})e^{-2\lambda\beta^{**}}} \right) X.
\]

Public intervention occurs if and only if \( U^\text{pub}_f \geq U^{\tau^{**}}_f \), which yields (21). Equation (22) follows directly by solving (20).

**\( \Box \)**

**Proof of Lemma 5.** If a reputation-conscious fund publicly intervenes, he improves the value of his block to \( \frac{\alpha X}{2} \), but also reveals the bad firm’s type. His continuation utility is

\[
U^\text{pub}_f = \frac{\alpha X}{2} + \kappa_f \mathbb{E}(\varphi|B) - \tau.
\]

If the fund does not publicly intervene, he knows (i) w.p. \( \tau_{ff} \) the other fund publicly intervenes, in which case neither fund sells and his own block is worth \( \frac{\alpha X}{2} \), but the bad firm’s type is fully revealed; and (ii) w.p. \( 1 - \tau_{ff} \) the other fund does not publicly intervene either, in which case each fund sells \( \beta_{ff} = \max(\lambda^{-1} - \Delta, 0) \) (this can be easily verified). His continuation utility in this case is

\[
U^{\tau_{ff}}_f = \tau_{ff} \left[ \frac{\alpha X}{2} + \kappa_f \mathbb{E}(\varphi|B) \right] + (1 - \tau_{ff}) \left( \frac{\beta_{ff}e^{-2\lambda\beta_{ff}\varphi}}{\varphi + 2(1 - \varphi)(1 - \tau_{ff})e^{-2\lambda\beta_{ff}}} \right) X
\]

\[
+ \left( 1 - \tau_{ff} \right) \kappa_f \left[ (1 - e^{-2\lambda\beta_{ff}}) \mathbb{E}(\varphi|B) + e^{-2\lambda\beta_{ff}} \mathbb{E}(\varphi|d > 0) \right].
\]

To understand the last term in the above expression, note when neither fund publicly intervenes:

- the bad firm’s type is fully revealed when \( d \leq 0 \), which occurs w.p. \( 1 - e^{-2\lambda\beta_{ff}} \); and
- when \( d > 0 \), which occurs w.p. \( e^{-2\lambda\beta_{ff}} \), the market’s posterior belief is

\[
\mathbb{E}(\varphi|d > 0) = \Pr(G|d > 0)\mathbb{E}(\varphi|G) + \Pr(B|d > 0)\mathbb{E}(\varphi|B)
= \frac{\varphi}{\varphi + 2(1 - \varphi)(1 - \tau_{ff})e^{-2\lambda\beta_{ff}}} \mathbb{E}(\varphi|G) + \frac{2(1 - \varphi)(1 - \tau_{ff})e^{-2\lambda\beta_{ff}}}{\varphi + 2(1 - \varphi)(1 - \tau_{ff})e^{-2\lambda\beta_{ff}}} \mathbb{E}(\varphi|B).
\]

Public intervention occurs if and only if \( U^\text{pub}_f \geq U^{\tau_{ff}}_f \), which yields (23). Equation (24) can be derived in the same way as (22).
Proof of Lemma 6. For the reputation-unconscious fund, if he publicly intervenes, his continuation utility is

\[ U_{f}^{\text{pub}} = \frac{\alpha X}{2} - \tau. \]

If he does not publicly intervene, he knows (i) w.p. \( \tau_{f*} \) the other fund publicly intervenes, in which case neither fund sells and his own block is worth \( \frac{\alpha X}{2} \); and (ii) w.p. \( 1 - \tau_{f*} \) the other fund does not intervene either, in which case he sells \( \lambda^{-1} \) and the other fund sells \( \max(\lambda^{-1} - \Delta, 0) \). His continuation utility is

\[ U_{f}^{\text{no}} = \tau_{f*} \frac{\alpha X}{2} + (1 - \tau_{f*}) \cdot \frac{\lambda^{-1} e^{-1 - \max(1 - \Delta, 0)} \varphi}{\varphi + 2(1 - \varphi)(1 - \tau_{f*})(1 - \tau_{f*})e^{-1 - \max(1 - \Delta, 0)}} X. \]

The reputation-unconscious fund publicly intervenes if and only if \( U_{f}^{\text{pub}} \geq U_{f}^{\text{no}} \), which yields

\[ \tau_{sf} = (1 - \tau_{f*}) \left[ \frac{\alpha}{2} - \frac{\lambda^{-1} e^{-1 - \max(1 - \Delta, 0)} \varphi}{\varphi + 2(1 - \varphi)(1 - \tau_{f*})(1 - \tau_{f*})e^{-1 - \max(1 - \Delta, 0)}} \right] X. \quad (A8) \]

For the reputation-conscious fund, public intervention reveals the bad firm’s type, and his corresponding continuation utility is

\[ U_{f}^{\text{pub}} = \frac{\alpha X}{2} + \kappa_{f} \mathbb{E}(\varphi|B) - \tau. \]

If he does not publicly intervene, he knows (i) w.p. \( \tau_{sf} \) the other fund intervenes, in which case neither fund sells, but the bad firm’s type is fully revealed; and (ii) w.p. \( 1 - \tau_{sf} \) the other fund does not intervene either, in which case he sells \( \max(\lambda^{-1} - \Delta, 0) \) and the other fund sells \( \lambda^{-1} \). His continuation utility is

\[ U_{f}^{\text{no}} = \tau_{sf} \left[ \frac{\alpha X}{2} + \kappa_{f} \mathbb{E}(\varphi|B) \right] + (1 - \tau_{sf}) \cdot \frac{\max(\lambda^{-1} - \Delta, 0) e^{-1 - \max(1 - \Delta, 0)} \varphi}{\varphi + 2(1 - \varphi)(1 - \tau_{sf})(1 - \tau_{f*})e^{-1 - \max(1 - \Delta, 0)}} X \]

\[ + (1 - \tau_{sf}) \kappa_{f} \left\{ [1 - e^{-1 - \max(1 - \Delta, 0)}] \mathbb{E}(\varphi|B) + e^{-1 - \max(1 - \Delta, 0)} \mathbb{E}(\varphi|d > 0) \right\}. \]

To understand the last term in the above expression, note when neither fund publicly intervenes: (i) the bad firm’s type is fully revealed when \( d \leq 0 \), w.p. \( 1 - e^{-\lambda(\beta_{f} + \beta_{f*})} = 1 - e^{-\max(1 - \Delta, 0)} \); and (ii) when \( d > 0 \), w.p. \( e^{-1 - \max(1 - \Delta, 0)} \), the market’s posterior belief about the reputation-conscious fund’s ability is

\[ \mathbb{E}(\varphi|d > 0) = \text{Pr}(G|d > 0) \mathbb{E}(\varphi|G) + \text{Pr}(B|d > 0) \mathbb{E}(\varphi|B) \]

\[ = \frac{\varphi}{\varphi + 2(1 - \varphi)(1 - \tau_{sf})(1 - \tau_{sf})e^{-1 - \max(1 - \Delta, 0)}} \mathbb{E}(\varphi|G) \]

\[ + \frac{2(1 - \varphi)(1 - \tau_{sf})(1 - \tau_{sf})e^{-1 - \max(1 - \Delta, 0)}}{\varphi + 2(1 - \varphi)(1 - \tau_{sf})(1 - \tau_{sf})e^{-1 - \max(1 - \Delta, 0)}} \mathbb{E}(\varphi|B). \]
where the first inequality is ensured by \( \alpha > \tau \) which yields

\[
\tau_{fs} = (1 - \tau_{sf}) \left[ \frac{\alpha}{2} - \frac{\lambda^{-1} e^{-1} \phi}{\phi + 2(1 - \phi)(1 - \tau_{sf})(1 - \tau_{fs}) e^{-\lambda \Delta}} \right] X
\]

\[
- (1 - \tau_{sf}) \frac{\lambda^{-2} e^{2+\lambda \Delta \phi}}{\phi + 2(1 - \phi)(1 - \tau_{sf})(1 - \tau_{fs}) e^{2+\lambda \Delta}} X.
\]

When \( \Delta \in (0, \lambda^{-1}] \), (A8) and (A9) can be simplified as, respectively,

\[
\tau_{sf} = (1 - \tau_{sf}) \left[ \frac{\alpha}{2} - \frac{\lambda^{-1} e^{-2+\lambda \Delta \phi}}{\phi + 2(1 - \phi)(1 - \tau_{sf})(1 - \tau_{fs}) e^{2+\lambda \Delta}} \right] X,
\]

(A10)

\[
\tau_{fs} = (1 - \tau_{sf}) \left[ \frac{\alpha}{2} - \frac{\lambda^{-1} e^{-2+\lambda \Delta \phi}}{\phi + 2(1 - \phi)(1 - \tau_{sf})(1 - \tau_{fs}) e^{2+\lambda \Delta}} \right] X.
\]

(A11)

From (A10) and (A11), we have \( \frac{\tau_{sf}}{1 - \tau_{sf}} = \frac{\tau_{fs}}{1 - \tau_{sf}} \Rightarrow (\tau_{sf} - \tau_{fs})(1 - \tau_{sf} - \tau_{fs}) = 0 \). Note \( \tau_{sf} + \tau_{fs} < 1 \): observe

\[
\left[ \frac{\alpha}{2} - \frac{\lambda^{-1} e^{-2+\lambda \Delta \phi}}{\phi + 2(1 - \phi)(1 - \tau_{sf})(1 - \tau_{fs}) e^{2+\lambda \Delta}} \right] X < (\alpha - \lambda^{-1} e^{-1}) X < 1,
\]

where the first inequality is ensured by \( \alpha > 2\lambda^{-1} \), and the second inequality follows from (A4). Denote \( \tau_{sf} = \tau_{fs} \equiv \hat{\tau} \). We need to show that \( \hat{\tau} \), given by

\[
\hat{\tau} = (1 - \hat{\tau}) \left[ \frac{\alpha}{2} - \frac{\lambda^{-1} e^{-2+\lambda \Delta \phi}}{\phi + 2(1 - \phi)(1 - \hat{\tau})^2 e^{2+\lambda \Delta}} \right] X
\]

(A12)

is smaller than \( \tau_{ss} \), given by (21). Suppose \( \hat{\tau} \geq \tau_{ss} \), so \( (1 - \hat{\tau})^2 \leq (1 - \tau_{ss})^2 \). Then, the right-hand side of (A12) is smaller than the right-hand side of (21), which leads to \( \hat{\tau} < \tau_{ss} \), a contradiction.

\( \Box \)

\textbf{Proof of Proposition 6.} In this case with \( \Delta > \lambda^{-1} \), (A8) and (A9) can be written as, respectively,

\[
\tau_{sf} = (1 - \tau_{sf}) \left[ \frac{\alpha}{2} - \frac{\lambda^{-1} e^{-1} \phi}{\phi + 2(1 - \phi)(1 - \tau_{sf})(1 - \tau_{fs}) e^{-\lambda \Delta}} \right] X,
\]

(A13)

\[
\tau_{fs} = (1 - \tau_{sf}) \left[ \frac{\alpha}{2} - \frac{\Delta e^{-1} \phi}{\phi + 2(1 - \phi)(1 - \tau_{sf})(1 - \tau_{fs}) e^{-\lambda \Delta}} \right] X.
\]

(A14)

Note

\[
\frac{\tau_{sf}}{\tau_{fs}} = \frac{1 - \tau_{sf}}{1 - \tau_{sf}} \frac{\alpha}{2} - \frac{\lambda^{-1} e^{-1} \phi}{\phi + 2(1 - \phi)(1 - \tau_{sf})(1 - \tau_{fs}) e^{-\lambda \Delta}} \frac{\Delta e^{-1} \phi}{\phi + 2(1 - \phi)(1 - \tau_{sf})(1 - \tau_{fs}) e^{-\lambda \Delta}} > \frac{1 - \tau_{fs}}{1 - \tau_{sf}},
\]

which yields \( (\tau_{sf} - \tau_{fs})(1 - \tau_{sf} - \tau_{fs}) > 0 \). Note \( \tau_{sf} + \tau_{fs} < 1 \), so \( \tau_{fs} < \tau_{sf} \). When \( \Delta \) increases, holding \( \tau_{sf} \) fixed, the right-hand side of (A14) falls. The solution to the fixed-point problem in (A14), \( \tau_{fs} \), falls as

\[\text{[Following similar arguments as in the Proof of Lemma 6, we can show \( \left[ \frac{\alpha}{2} - \frac{\lambda^{-1} e^{-1} \phi}{\phi + 2(1 - \phi)(1 - \tau_{sf})(1 - \tau_{fs}) e^{-\lambda \Delta}} \right] X < (\alpha - \lambda^{-1} e^{-1}) X < 1 \), which is what is needed to show \( \tau_{sf} + \tau_{fs} < 1 \).} \]
well. The decrease in $\tau_f$ causes the right-hand side of (A13) to increase, and therefore the solution to the fixed-point problem in (A13), $\tau_{sf}$, also increases.

**Proof of Lemma 7.** The proof for $\tau_{ff} < \tau_{ss}$ is similar to that for $\tau_f < \tau_s$ with one fund (see the Proof of Proposition 3). Note $\frac{\tau_{ff}}{1-\tau_{ff}} = H(\beta_{ff}, \tau_{ff}) = \frac{e^{2\lambda\beta_{ff}\varphi}}{\varphi + 2(1-\varphi)(1-\tau_{ff})e^{-2\lambda\Delta}} \Delta X \equiv \tilde{H}(\beta_{ff}, \tau_{ff})$ and $\frac{\tau_{ss}}{1-\tau_{ss}} = H(\beta_{ss}, \tau_{ss})$, so it is sufficient show $\tilde{H}(\beta_{ff}, \tau) < H(\beta_{ss}, \tau)$ for $\forall \tau \in [0,1]$. Consider the case in which $\Delta \in (0, \lambda^{-1})$. Substituting $\beta_{ff} = \lambda^{-1} - \Delta$ into $\tilde{H}(\beta_{ff}, \tau)$ yields

$$
\tilde{H}(\beta_{ff}, \tau) = \left[ \frac{\alpha}{2} - (\lambda^{-1} - \Delta)e^{-2+2\lambda\Delta} \frac{\varphi}{\varphi + 2(1-\varphi)(1-\tau)^2e^{-2+2\lambda\Delta}} \right] X
$$

$$
= \left[ \frac{\alpha}{2} - \lambda^{-1}e^{-2+2\lambda\Delta} \frac{\varphi}{\varphi + 2(1-\varphi)(1-\tau)^2e^{-2+2\lambda\Delta}} \right] X
$$

$$
= \left[ \frac{\alpha}{2} - \lambda^{-1}e^{-2} \frac{\varphi e^{-2\lambda\Delta} + 2(1-\varphi)(1-\tau)^2e^{-2}}{\varphi + 2(1-\varphi)(1-\tau)^2e^{-2}} \right] X
$$

$$
= \left[ \frac{\alpha}{2} - \frac{\lambda^{-1}e^{-2}\varphi}{\varphi + 2(1-\varphi)(1-\tau)^2e^{-2}} \right] X
$$

$$
= H(\beta_{ss}, \tau).
$$

The proof for the case with $\Delta \geq \lambda^{-1}$ (so $\beta_{ff} = 0$) is similar as the corresponding proof for $\tau_f < \tau_s$ with $\Delta \geq \lambda^{-1}$, and hence is omitted here (see the Proof of Proposition 3 with one fund). The result $\sigma_{ff} < \sigma_{ss}$ follows directly by comparing (22) and (24), noting that $(1 - \tau_{ff})^2e^{-2\lambda\beta_{ff}} > (1 - \tau_{ss})^2e^{-2\lambda\beta_{ss}}$.

**Proof of Lemma 8.** We only need to show $\sigma_{ss} > \sigma_{asy}$, which directly follows by comparing (22) and (27), noting that $(1 - \tau_{sf})(1 - \tau_{fs})e^{-\lambda(\beta_{sf} + \beta_{fs})} > (1 - \tau_{ss})^2e^{-2\lambda\beta_{ss}}$ when $\Delta \in (0, \lambda^{-1})$.

**Proof of Proposition 7.** It is sufficient to examine the case wherein $\Delta$ is sufficiently big such that $\tau_{fs} = 0$. In this case,

$$
\tau_{sf} = \left[ \frac{\alpha}{2} - \frac{\lambda^{-1}e^{-1}\varphi}{\varphi + 2(1-\varphi)(1-\tau_{sf})e^{-1}} \right] X. \quad (A15)
$$

Comparing (A15) with (21), wherein $\tau_{ss}$ is determined, note when $\frac{\alpha}{2}$ is sufficiently large and/or $\lambda^{-1}$ is sufficiently small, $\tau_{sf} \approx \frac{\alpha}{1-\tau_{ss}} > \tau_{ss}$. Comparing (22) and (27), we know that establishing the possibility of $\sigma_{asy} > \sigma_{ss}$ is equivalent to establishing the possibility of $(1 - \tau_{sf})(1 - \tau_{fs})e^{-\lambda(\beta_{sf} + \beta_{fs})} (1 - \tau_{ss})^2e^{-2\lambda\beta_{ss}}$, i.e., $1 - \tau_{sf} < (1 - \tau_{ss})^2e^{-1}$. Since $\tau_{sf} \approx \frac{\alpha}{1-\tau_{ss}}$, it is thus sufficient to show $1 - \frac{\alpha}{1-\tau_{ss}} < (1 - \tau_{ss})^2e^{-1} \Leftrightarrow J(\tau_{ss}) \equiv \tau_{ss}^3 - 3\tau_{ss}^2 + (3 - 2e)\tau_{ss} + (e - 1) < 0$. Note $J(0) = e^{-1} > 0$, $J(1) = -e < 0$, and $J'(\tau_{ss}) < 0$. Thus, there exists a cutoff value of $\tau_{ss}$, such that if $\tau_{ss}$ is bigger than that cutoff, $J(\tau_{ss}) < 0$. The result follows by noting that $\tau_{ss}$ is sufficiently big when $\frac{\alpha}{2}$ is sufficiently big and/or $\lambda^{-1}$ is sufficiently small. Finally, note $(1 - \tau_{sf})(1 - \tau_{fs})e^{-\lambda(\beta_{sf} + \beta_{fs})} < (1 - \tau_{ss})^2e^{-2\lambda\beta_{ss}} \Rightarrow 1 - (1 - \tau_{sf})(1 - \tau_{fs}) > 1 - (1 - \tau_{ss})^2$. 

\[\square\]
References


